Sorting

In the lab works you've seen how to sort any list. One way to write this sorting function is as follows:

In [2]:
```
def find_min_index(L):
    current_index = 0
    current_min = L[0]
    for j in range(1,len(L)):
        if current_min > L[j]:
            current_min = L[j]
            current_index = j
    return current_index
```

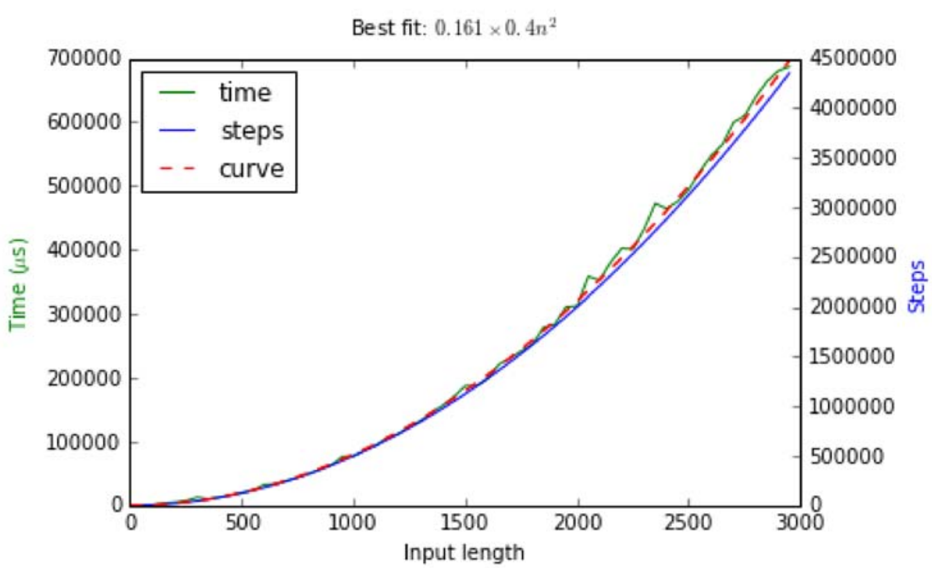
In [3]:
```
def sort(L):
    if len(L)<=1:
        return L # a one-element list is always sorted
    min_idx = find_min_index(L)

    L[0], L[min_idx] = L[min_idx], L[0]
    # switch minimum element to first location

    return [L[0]] + sort(L[1:len(L)])
```

In [4]:
```
sort([3,1,4,1,5,9,2])
```

Out[4]:
```
[1, 1, 2, 3, 4, 5, 9]
```

In [24]:
```
# running time of our sort() function on inputs from size 0 to 3000
```

```
............................................................plot_steps: True
0.161 micro-seconds per step
(array([2], dtype=int64),)
Curve (steps): $0.4n^2$
```



What is the shape of this curve? Have you seen it before?

It turns out that for sorting a list of n elements, our algorithm will take about $10^{-7}n^2$ seconds.

Facebook has $1,000,000,000 = 10^9$users. If they wanted to sort the list of their users using this algorithm it will take them about $10^{-7}(10^9)^2 = 10^{-7}10^{18} = 10^{11}$seconds.

In [7]:
```
print (10**11 / (60*60*24*365)), " years!"
```

```
3170  years!
```

The problem is that when n becomes big, n^2 becomes much bigger.

If we had an algorithm that runs in $10^{-7}n$ seconds, then we could sort Facebook's users in 0.01 seconds. Even an algorithm that runs in time $10^{-2}n$ would take less than twenty minutes to do it.

We see that the effect of n vs. n^2 is much more important than the effect of the constant.

Where does the n^2 come from?

In [9]:
```
def find_min_index(L):
    current_index = 0
    current_min = L[0]
    for j in range(1,len(L)):
        sys.stdout.write('*')
        if current_min > L[j]:
            current_min = L[j]
            current_index = j
    return current_index
```

In [10]:
```
def sort(L):
    if len(L)<=1:
        return L # a one-element list is always sorted
    min_idx = find_min_index(L)
    print ""
    L[0], L[min_idx] = L[min_idx], L[0]
    # switch minimum element to first location

    return [L[0]] + sort(L[1:len(L)])
```

In [11]:
```
sort([10,9,8,7,6,5,4,3,2,1])
```

```
*********
********
*******
******
*****
****
***
**
*
```

Out[11]:
```
[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```

The number of steps we take to sort a list of length n is about
$n + (n-1) + (n-2) + \cdots + 2 + 1 = \frac{n(n+1)}{2} = 0.5n^2 + 0.5n$

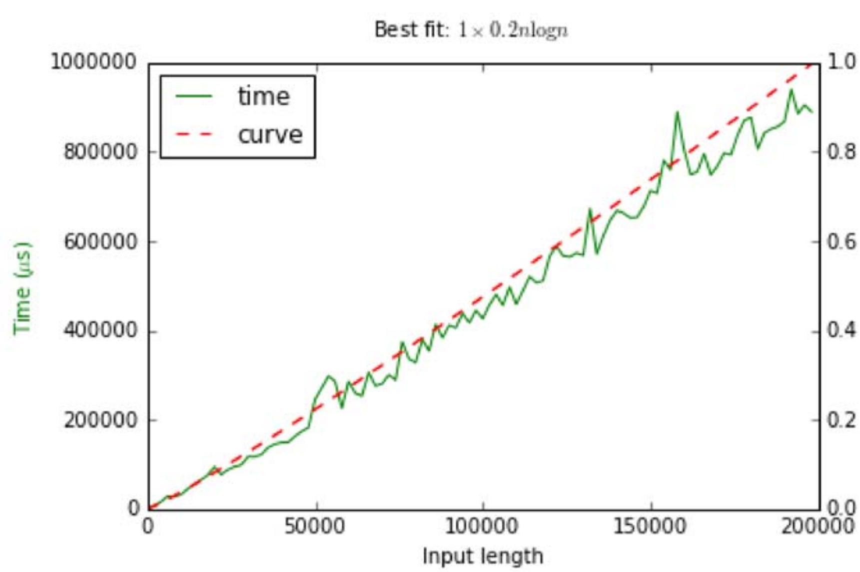
Can we do better?

Turns out that the answer is **yes**

There is a different sorting algorithm for which the running time looks like:

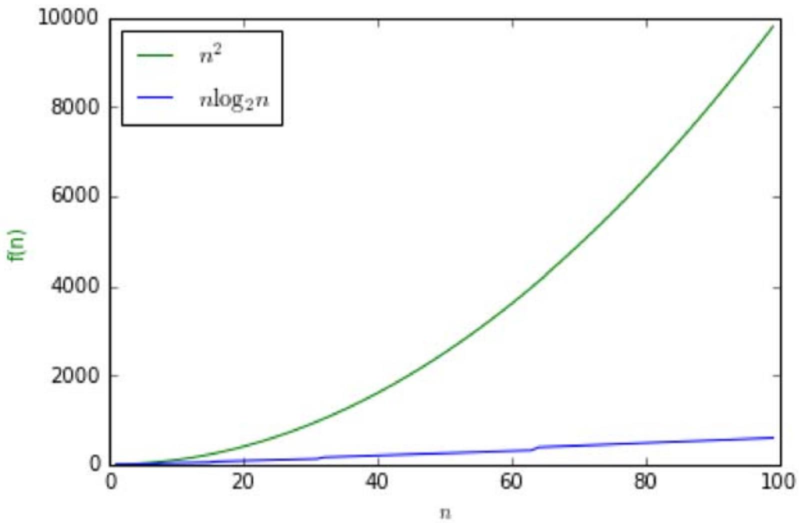
In [21]:
```
# running time of alternative sorting algorithm for inputs from size 0 to 200,
000
```

```
...............................................................................
plot_steps: False
1.000 micro-seconds per step
(array([3, 4], dtype=int64),)
Curve (steps): $0.2n\log n$
```



That is, sorting n elements takes about $5 \cdot 10^{-7} n \log_2 n$ seconds. $\log_2 n$ is much smaller than n ($\log_2 (10,000,000) < 30$) and so this is much better.

```
In [30]: # compare n*Log(n) vs n^2
```



In particular this algorithm will take about $5 \cdot 10^{-7} \cdot 10^9 \cdot 30 = 150 \cdot 10^2 = 1500$ seconds or 25 minutes to sort the Facebook user list

This is 10^9 times faster than what it would take in the slower sorting algorithm!!

Even if Facebook has a computer that is a million times faster than my laptop, it would still take them more time than for me to sort this list.

The cleverness of the algorithm is more important than the speed of the machine!

A smarter sorting algorithm

Our sorting algorithm had the following general operation on a list of size n:

1. Find the minimum element and put it at the beginning.
2. Sort the last $n - 1$ elements.

As we saw the number of steps looked something like this:

We will show the **merge sort** algorithm that does the following on a list of size n:

1. Sort the first $n/2$ elements to get a list $L1$ and the last $n/2$ elements to get a list $L2$.
2. Merge the two lists together to one sorted list.

It turns out that the number of steps it takes looks like the following:

In [31]:
```
# illustration of number of steps sorting 20 numbers in selection sort vs merge sort
```

```
******************************          ******************************
*****************************           ***************
****************************            *******
***************************             ***
**************************              *
*************************               *
************************                ***
***********************                 *
**********************                  *
*********************                   *******
********************                    ***
*******************                     *
******************                      *
*****************                       ***
****************                        *
***************                         *
**************                          ***************
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*******                                 *
******                                  *******
*****                                   ***
****                                    *
***                                     *
**                                      ***
*                                       *
                                        *
```
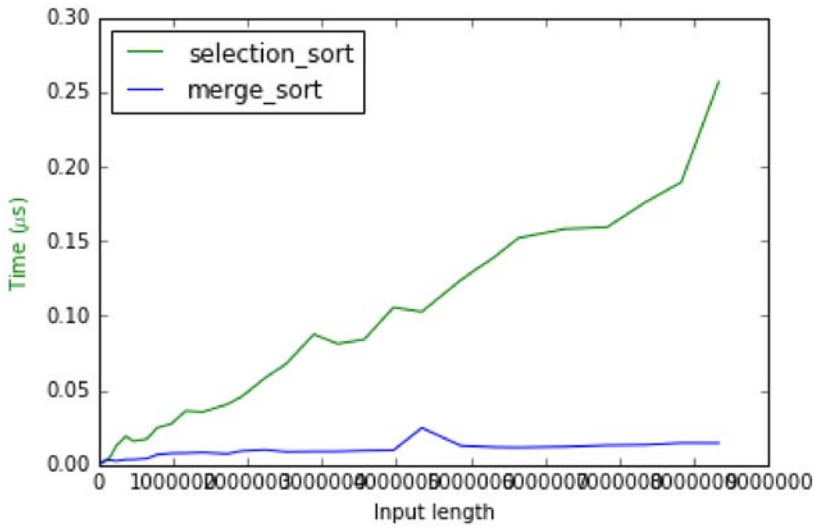

In [16]:
```
# comparison of running time of selection sort and merge sort
```

```
..............................plot_steps: False
1.000 micro-seconds per step
..............................plot_steps: True
0.611 micro-seconds per step
```



[Sorting demo by Justin Johnson (http://cs.stanford.edu/people/jcjohns/sorting.js/)](http://cs.stanford.edu/people/jcjohns/sorting.js/)

Sorting with different keys

Recall the bonus homework exercise of sorting an array by **last name**:

In [12]:
```
names = ['abinet mulugeta', 'urgie  huseien', '...', 'mentesenot tefera']
```

In [13]:
```
'zelalem ades' < 'betelehem eshetu'
```

Out[13]: False

Our approach:

We define a function `last_name` such that:

In [15]:
```
last_name('zelalem ades')
```

Out[15]: 'ades'

In [16]:
```
last_name('betelehem eshetu')
```

Out[16]: 'eshetu'

In [17]:
```
last_name('zelalem ades') < last_name('betelehem eshetu')
```

Out[17]: True

So we only have to change our comparisons from using `s > t` to using `last_name(s) > last_name(t)`

In particular it is enough to modify `find_min_index` as follows.

From:

In [18]:
```
def find_min_index(L):
    current_index = 0
    current_min = L[0]
    for j in range(1,len(L)):
        if current_min > L[j]:
            current_min = L[j]
            current_index = j
    return current_index
```

To:

In [19]:
```
def find_min_index(L):
    current_index = 0
    current_min = L[0]
    for j in range(1,len(L)):
        if last_name(current_min) > last_name(L[j]):
            current_min = L[j]
            current_index = j
    return current_index
```

Recall that the code of sort was the following:

In [20]:
```
def sort(L):
    if len(L)<=1:
        return L # a one-element list is always sorted
    min_idx = find_min_index(L)
    print ""
    L[0], L[min_idx] = L[min_idx], L[0]
    # switch minimum element to first location

    return [L[0]] + sort(L[1:len(L)])
```

There is no need to change it!

In [21]:
```
sort(names)
```

Out[21]:
```
['shambel abate',
 'shafi abdi',
 ...
 'yonatan wosenyeleh',
 'hailegbrel wudneh',
 'wondimu yohanes']
```

Now all that is left is to write the function `last_name`, which will be an exercise for you.

Sorting in python

Python provides a built-in function ```sorted''

In [22]:
```
sorted(names)
```

Out[22]:
```
['abdurezak temam',
 'abinet mulugeta',
 'abrham tuna',
...
 'yosef tadiwos',
 'zakira tebarek',
 'zelalem ades']
```

The function can even sort in reverse:

In [23]:
```
sorted(names, reverse=True)
```

Out[23]:
```
['zelalem ades',
 'zakira tebarek',
 'yosef tadiwos',
...
 'abrham tuna',
 'abinet mulugeta',
 'abdurezak temam']
```

and take a key:

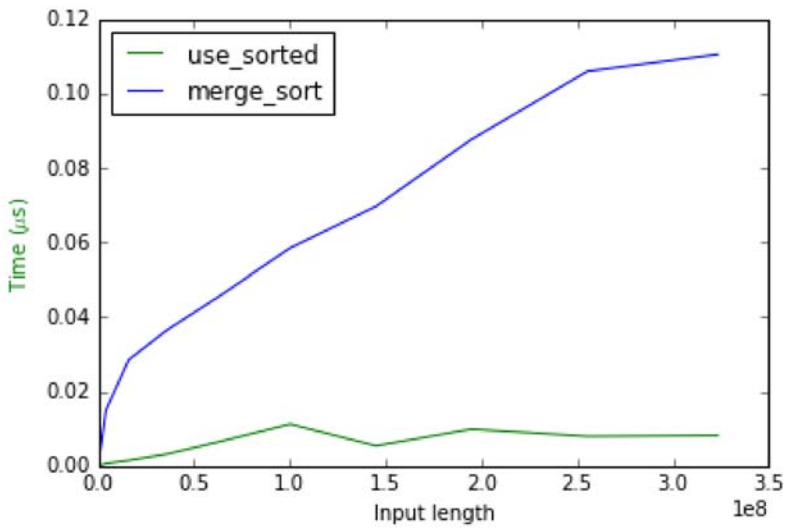
```
In [24]: sorted(names,key=last_name)
```

```
Out[24]: ['shambel abate',
 'shafi abdi',
 'habene abdi',
...
 'yonatan wosenyeleh',
 'hailegbrel wudneh',
 'wondimu yohanes']
```

It is also quite fast:

```
In [47]: # merge sort vs Python built-in sorted algorithm
```

```
..........plot_steps: False
1.000 micro-seconds per step
..........plot_steps: True
0.506 micro-seconds per step
```



Why teach sorting?

We saw that Python gives you sorting "for free"

and it's even faster than the best algorithm we could write.

So why force you to learn and code sorting algorithms?

Answer 1: Computer Science and programming is more than just Python.

Answer 2: I am not trying to teach you to sort numbers. I am trying to teach you how to think.

Lab work

Exercise 1

Write a function `sort4` that sorts a list of 4 elements. The function should make two calls to `sort2`

In [25]:
```
def sort4(L):
    L_first_sorted = sort2(L[0:2])
    L_last_sorted = sort2(L[2:4])
    #
    # do something to return a sorted list
    #
```

In [27]:
```
sort4([10,2,5,7])
```

Out[27]:
```
[2, 5, 7, 10]
```

Exercise 2

Write a function `merge_lists` that takes two sorted lists L1 and L2 and returns a sorted list that of length `len(L1)+len(L2)` that contains all their elements.

Exercise 3

Write a function `sort32` that sorts a list of 32 elements. The function should make two recursive calls to a provided function `sort16`

In [51]:
```
def sort32(L):
    L_first_sorted = sort16(L[0:4])
    L_last_sorted = sort16(L[4:8])
    #
    # do something to return a sorted list
    #
```

In [29]:
```
sort16 = sorted
```

In [31]:
```
sort32([32,31,...,2,1])
```

Out[31]:
```
[1,
 2,
 3,
..
 32]
```

Exercise 4

Write a function `merge_sort` that will sort a list of *any* size. The function should make two recursive calls to itself

In [5]:
```
def merge_sort(L):
    #
    # do something here
    #
    L_first_sorted = merge_sort(L[0:int(len(L)/2)])
    L_last_sorted = merge_sort(L[int(len(L)/2):len(L)])
    #
    # do something to return a sorted list
```

In [33]:
```
merge_sort([78, 39, 50, 43, 3, 30, 34, 75, 33, 7, 30, 71, 76, 44, 27, 4, 68, 2
1, 51, 78, 11, 53, 71, 60, 64, 9, 28, 63, 55, 34, 44, 52, 28, 52, 43, 44, 4, 4
1, 40, 17])
```

Out[33]:
```
[3,
 4,
 4,
...
 76,
 78,
 78]
```

Exercise 5

Use a stopwatch to compare selection sort and merge sort on random lists of 800 numbers. Run each one of them 10 times and record the average time.

In [12]:
```
def gen_random_list(n):
    return [random.randint(0,2*n) for i in range(n)]
```

Exercise 6

Write a function `last_name` that on input a string s, will find the first space character `' '` in s, and will return the rest of s. You don't have to worry about strings that don't contain spaces or contain more than one space.

In [34]:
```
last_name('boaz barak')
```
Out[34]:
```
'barak'
```