lec12

August 12, 2016

0.0.1 Searching via sorting

Consider the phone book for Addis Ababa. Suppose that it has 1 million names in it. But still, we can find a number easily because it is **alphabetically sorted**.

What would happen if the names were listed in the phone book in random order?

This is true in general - we can find items much faster in arrays that are **sorted**:

(In Python an array and a list are basically the same thing, in other programming languages they can be different.)

```
In [2]: def search(L,item):
            """Search in an unorted array"""
            for i in range(len(L)):
                sys.stdout.write('*')
                if L[i]==item:
                    return i
            return -1

In [3]: L = range(200)

In [4]: search(L,100)

**********************************************************************************************************************************************************************

Out[4]: 100
```

Can we do it faster using the fact that `L` is **sorted**?

Turns out the answer is **yes**

0.1 Binary Search

Input: Sorted array L of length n, item $item$

Output: Index i such that $L[i] == item$ or -1 if no such i exists.

Operation: Check if $L[n/2] > item$.

If YES, then check if $L[n/4] > item$, if NO then check if $L[3n/4] > item$.

If first check was YES and second YES, check if $L[n/8] > item$.

If first check was YES and second NO, check if $L[3n/8] > item$.

If first check was NO and second NO, check if $L[7n/8] > item$.

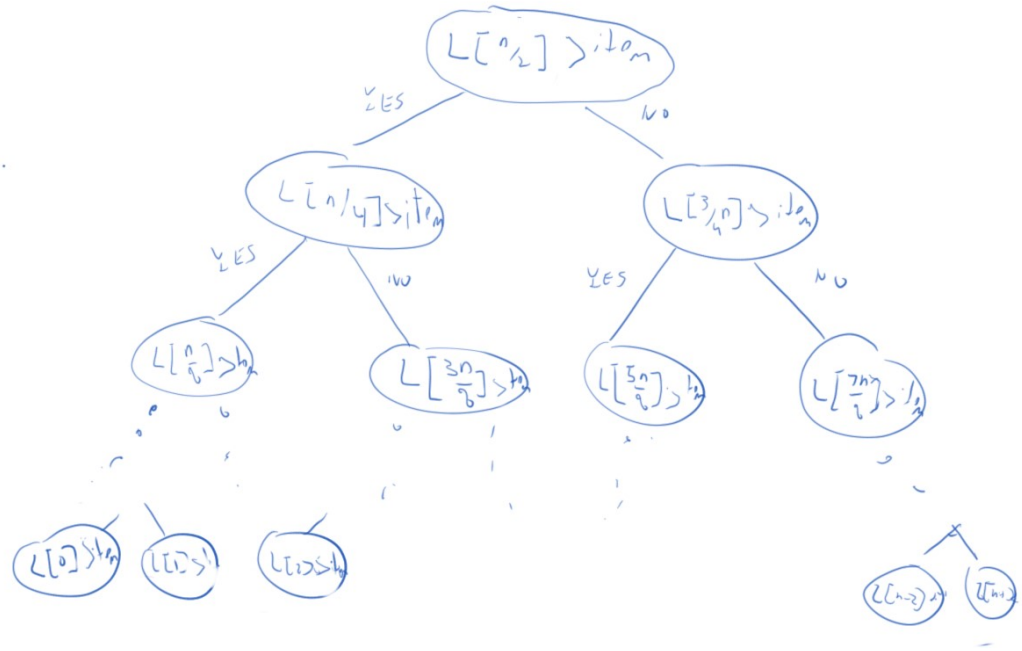
If first check was NO and second YES, check if $L[5n/8] > item$.

....

continue in this way

```
In [5]: #overview of binary search
```

```
Out[5]:
```



0.2 Binary Search

(a bit more formal operation)

Input: Sorted array L of length n, item $item$
Output: Index i such that $L[i] == item$ or -1 if no such i exists.
Operation:
Check if $L[n/2] < item$:

- if YES, continue search in $L[0 : n/2]$
- if NO, continue search in $L[n/2 : n]$

```
In [6]: def bin_search(L,item):
            sys.stdout.write('*')
            n = len(L)
            if not n:
                return -1
            m = int(n/2)
            if L[m]==item:
                return m
            if L[m]>item:
                return bin_search(L[:m],item)
```

```
        res = bin_search(L[m+1:n],item)
        if res==-1:
            return -1
        return m+1+res
```

```
In [7]: search(L,100)
```

```
**********************************************************************************************
```

```
Out[7]: 100
```

```
In [8]: bin_search(L,100)
```

```
********
```

```
Out[8]: 100
```

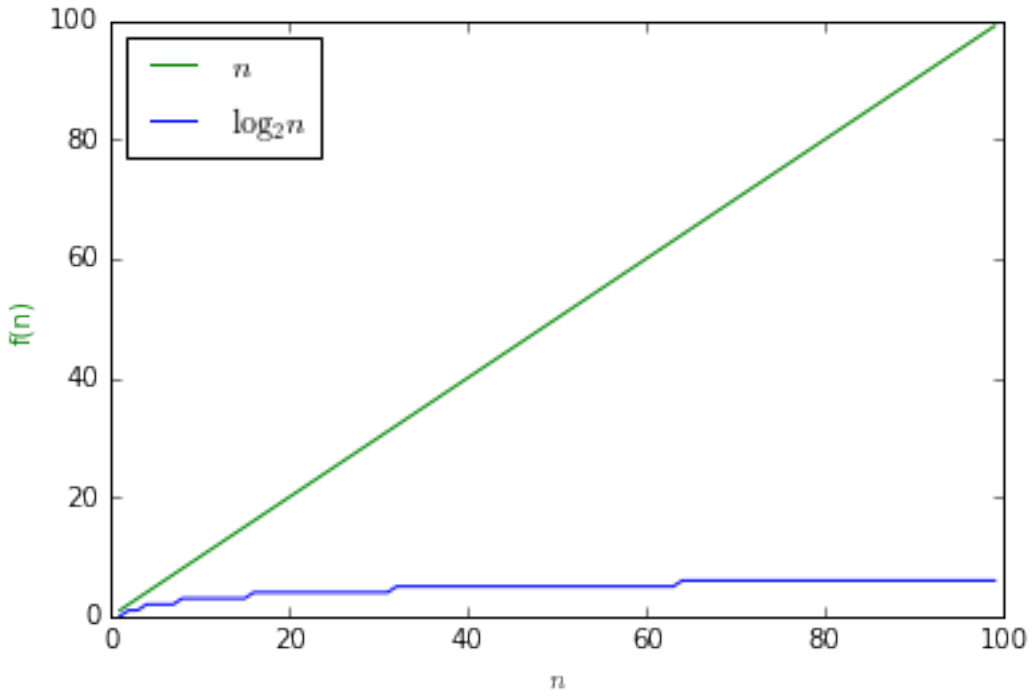
If you run a binary search on a string of length n, then in one step we reduce the problem to a string of length $n/2$, in another step to a string of length $n/4$, and so on.

So the number of steps is the number of items in the sequence $n, n/2, n/4, n/8, \ldots, 1$.

In other words, the number of steps binary search takes is the number t such that $n/2^t \leq 1$, which means $t = \lceil log_2 n \rceil \leq \log_2 n + 1$.

$\log_2 n$ is much much smaller than n.

```
In [26]: # compare n with log_2 n
```



In particular, if Facebook wants to search for a user in the data base of 10^9 users, then if they keep the list sorted, they can do it in 30 steps instead of $1,000,000,000$.

For example, Facebook can have a list of all the emails of their users, sorted by their name. Now, given any string `name`, in 30 steps they can find the email corresponding to this user.

Let's be more specific and, since we don't have the list of Facebook users, consider the list of students in this course.

Suppose we have this list of teams in this course:

```
In [16]: groups = ['1:asmare habtemu, biniam kidane, lewi mekonnen', '2:gatlyak chr
```

We can make it into a list of pairs of the form *(team number, student)*

```
In [17]: pairs = []
         for s in groups:
             i = s.index(":")
             team = int(s[0:i])
             for name in s[i+1:].split(', '):
                 pairs.append([team,name])
         print pairs
```

```
[[1, 'asmare habtemu'], [1, 'biniam kidane'], [1, 'lewi mekonnen'], [2, 'gatlyak ch
```

Now we can sort these pairs based on student name:

```
In [18]: def name(pair):
             return pair[1]

         pairs = sorted(pairs,key=name)
         print pairs
```

```
[[7, 'abdurezak temam'], [28, 'abinet mulugeta'], [5, 'abreham tuna'], [24, 'adem m
```

```
In [19]: teams, names = zip(*pairs)
         print names
```

```
('abdurezak temam', 'abinet mulugeta', 'abreham tuna', 'adem mohammed', 'adonay ger
```

And define binary search to use this key too:

```
In [20]: def bin_search_name(L,s_name):
             n = len(L)
             if not n:
                 return -1
             m = int(n/2)
             if name(L[m])==s_name:
```

```
            return m
        if name(L[m])>s_name:
            return bin_search_name(L[:m],s_name)
        res = bin_search_name(L[m+1:n],s_name)
        if res==-1:
            return -1
        return m+res+1
```

```
In [21]: idx = bin_search_name(pairs,'Abinet Mulugeta')
         print "Group number", pairs[idx][0]
```

```
Group number 29
```

```
In [22]: idx = bin_search_name(pairs,'Yonatan Wesenyelah')
         print "Group number", pairs[idx][0]
```

```
Group number 29
```

1 Dictionaries

This kind of tasks - storing information that you want to access using some *key*, is so common that python has a special data structure for it called a **dictionary**

```
In [132]: groups_dict = { 'adem mohammed': 24, 'samuel testage': 2, 'Kalkidan Mulu
```

```
In [1]: groups_dict['samuel testage']
```

```
2
```

We can add to `groups_dict` *all* the pairs as follows:

```
In [156]: for pair in pairs:
              groups_dict[pair[1]] = int(pair[0])
          print groups_dict
```

```
{'betelhem walelign': 26, 'Eman Hassen': 4, 'Aman Musa-Umare': 22, 'Haymanot Gidena
```

1.0.1 Are dictionaries implemented using sorted arrays?

Problem: What if you add a new element - do you need to re-sort the whole array?

Two main solutions: *Binary search trees* and *hash tables*.

Python uses hash tables and this is what we explain next.

2 Hash tables*

Idea: Design a function `h(s)` such that for every string `s` , `h(s)` is a number between 0 and n. Then, we can have a list `L` of length n such that `L[h(s)]` will be the group of student `s`

So, to get the group of the student with name `name`, all we need to do is to compute `h(name)` and then we can get this student in only one step.

Problems:

- How do you find such a function?
- What do you do if you have two different students of with names `name1` and `name2` such that `h(name1)==h(name2)`?
- How small can we make n? Note that it costs us in computer *memory* to make n too big.

Let's start with the first problem: we want to find a function h that takes every string s to a somewhat "random number" between 1 and n. (in our case $n = 81$)

One simple function is the following: treat each letter as a number from 1 to 26 and add all the letters in the name modulo n .

(note that this is a simple function that works well sometimes but not always, and in particular will always map "boaz" and "azbo" to the same number; there are better "hash functions" that are used in Python and other systems.)

```
In [9]: def letter_to_number(c):
            if c==' ':
                return 27
            if c=='-':
                return 28
            return 1+ord(c)-ord('a')

        letter_to_number('c')
```

```
Out[9]: 3
```

```
In [10]: def h(s,n):
             res = 0
             for c in s.lower():
                 res += letter_to_number(c)
             return res % n
```

```
In [14]: h("boaz barak",83)
```

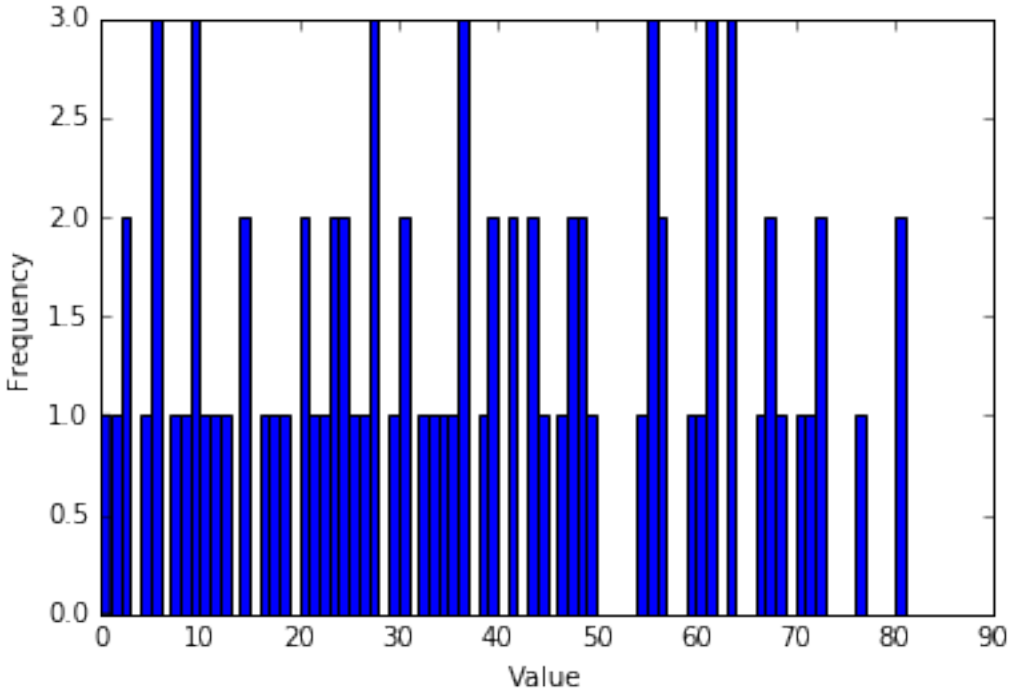
```
Out[14]: 21
```

We don't have the list of all Facebook users, so let's test how well it works for the students in this class:

```
In [23]: len(pairs)
```

```
Out[23]: 83
```

```
In [24]: integer_hist([h(pair[1],83) for pair in pairs])
```



We see that the function is "almost" good, in that most places only have one student matched to it, but several places have two students and a few have three students.

So now we can have a list `groups_list` of length 83, where for every `i`, `groups_list[i]` will contains the list of all pairs corresponding to the students with name `s` such that `h(s)==i`.

For every `i`, the list `groups_list[i]` will have at most three pairs.

So, if we want to get the group of a student with name `s` we need to do it in at most four steps:

- We let `L=groups_list[h(s)]`
- Then we scan this short list `L` to find the pair of the form `[ t , s]`

2.1 Summary of data structures

Often the right data structure can make all the difference:

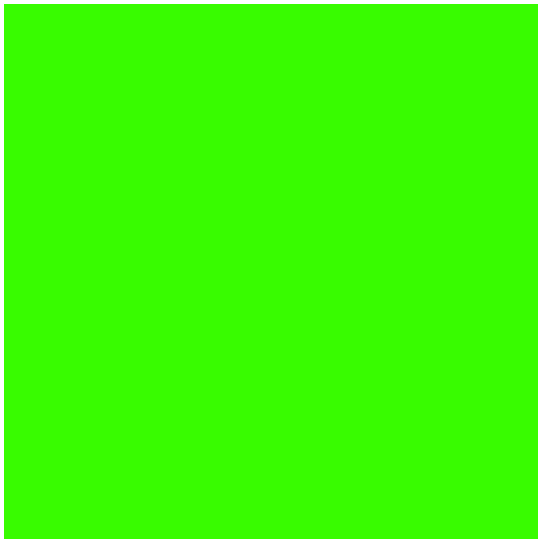
Data structure	Get(key).....	Insert(key)..	Other properties
Unsorted list	n	< 10 (*)	Supports any objects
Sorted array	$\log n$	n	Supports range queries
Search trees	$\log n$	$\log n$	Support range queries
Hash table	< 10	< 10	Supports non-comparable keys

Note: Data structures is a *huge* topic and if you study more computer science you will hear about more concepts such as stacks, queues, heaps, and many more.

3 Graphics on a computer

The image you see is composed of about 1,000,000 little dots known as *pixels* (1024×768).
Each pixel can be set to a different color, and that produces the image.
All colors are obtained by mixing red, green and blue

```
In [30]: demo_RGB()
```



Python allows us to take an array of color values for the pixels and plot it on the screen. To make things easier, we implemented some helper functions to do it:

- `color(red,green,blue)`: takes three numbers and simply returns a list of these three numbers, but it can also be called with named parameters and has defeault values.
- `empty_screen(height,width)`: returns an `width` × `height` array `s` (namely a list of `width` lists, each of them is of length `height`). For every `x` between 0 and `width` and `y` between 0 and `height`, `s[x][y] = color(255,255,255)`.
- `plot_array(s)`: plots the array `s` on the screen where `s[0][0]` corresponds to the bottom left corner and `s[width][length]` corresponds to the top right corner.

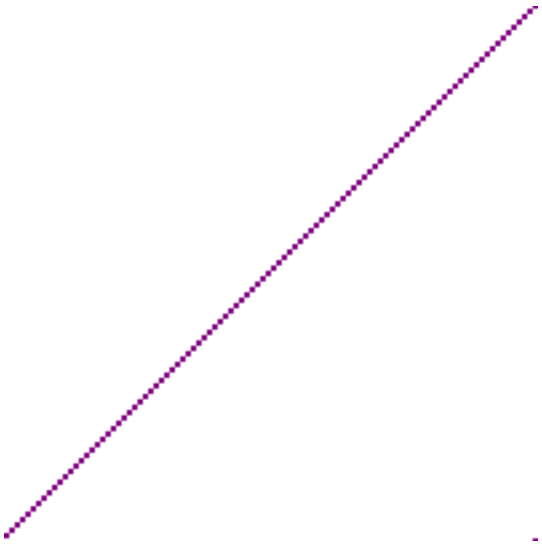
We will now demonstrate how to use these functions:

```
In [82]: color(20,30,40)

Out[82]: (20, 30, 40)

In [83]: color(blue=255)

Out[83]: (0, 0, 255)

In [84]: s = empty_screen(100,100)
```

In [85]:
```
plot_array(s)
```

In [86]:
```
s[50][50] = color(red=255,blue=0,green=0)
plot_array(s)
```


In [87]:
```
import time
for i in range(100):
    s[i][i]=color(red=128,blue=128)
    plot_array(s)
    # time.sleep(0.01)
```



```
In [88]: import math
         def sine(angle):
             return math.sin((angle/360.0)*2*math.pi)
         def cosine(angle):
             return math.cos((angle/360.0)*2*math.pi)

In [125]: def cannon(angle,speed,time, gravity=9.8):
              x = speed*time*cosine(angle)
              y = speed*time*sine(angle) -(gravity/2.0)*(time**2)
              return round(x,3),round(y,3)

In [110]: cannon(45,10,100,gravity=0)

Out[110]: (707.107, 707.107)

In [111]: cannon(45,10,200,gravity=0)

Out[111]: (1414.214, 1414.214)

In [112]: cannon(30,10,100,gravity=0)

Out[112]: (866.025, 500.0)

In [113]: cannon(70,10,100,gravity=0)

Out[113]: (342.02, 939.693)

In [123]: cannon(45,100,10)

Out[123]: (707.107, 217.107)
```

```
In [126]: cannon(45,100,20)

Out[126]: (1414.214, -545.786)

In [120]: cannon(30,100,10)

Out[120]: (866.025, 10.0)

In [121]: cannon(60,100,10)

Out[121]: (500.0, 376.025)

In [90]: def draw_cannon(angle,speed):
             s = empty_screen(100,100)
             x =0
             y =0
             t=0.0
             while x<100 and y>=0 and y<100:
                 s[x][y] = color(red=255)
                 (x,y) = cannon(angle,speed,t)
                 x= int(x)
                 y = int(y)
                 t += 1.0/speed
                 plot_array(s)


In [92]: draw_cannon(45,30)
```

4 Quiz tomorrow

- One hour on the computer.
- Be here before **9:15am**: half the people will take the quiz, half will do lab work, and then switch.
- Some questions would be easier and some harder - *don't feel bad if you can't solve them all!* (or even most)
- quiz is mostly for us, so we know what concepts you know and what concepts you don't.
- some questions would be very similar to your labworks.
- if you didn't complete the labworks: talk to your friends that did complete it, and make sure you understand the solutions.

5 Labwork

Choose one of two projects to complete: (if you finish one, you can also try the other)

In these projects you can use all the functions that you have built in previous lab works.

5.0.1 Project 1: Linear equations solver

Write a function `equation_solver` that asks a user for the number n of equations and variables, and then for n equations of the form `10y-0.5x+25z + 50 = 0` and prints a solution of the form `x=10.0, y=0.5, z=2.0`

The function can use the function `solve` we've seen before, any helper functions you already made or new ones. In particular, write the helper functions

- `parse_equation(s)` that takes a string representing an equation in n variables and returns a list of $n+1$ numbers that represents the coefficients for all variables (in alphabetical order) and the constant coefficient.

- `get_variables(user_equations)` that takes a list of strings corresponding to equations and returns a sorted list of all variables appearing in those equations. Each variable should only appear once so if the equations are in n variables then the length of the list that is returned should be n.

The function `equation_solver` itself needs to be of the form below.

bonus: handle equations with missing variables, equations where the right hand side is not just `= 0`, equations that have infinitely many or no solutions (for the latter one you might need to modify `solve`)

```
In [14]: ### HELPER FUNCTIONS:

         def make_first_coeff_nonzero_general(eqs):
             for i in range(len(eqs)):
                 if eqs[i][0]:
                     eqs[0], eqs[i] = eqs[i], eqs[0]
                     return
             sys.exit("oops:  all first coefficients are zero")
             return
```

```
        def multiply_equation(eq,num):
            """Multiply all coefficients of equation eq by number num.
               Return result"""
            res = []
            for x in eq:
                res += [x*num]
            return res

        def add_equations(eq1,eq2):
            """Add eq1 and eq2. Return result"""
            res = []
            for i in range(len(eq1)):
                res.append(eq1[i]+eq2[i])
            return res

        def solve(eqs):
            n = len(eqs)
            if n==0:
                return []
            make_first_coeff_nonzero_general(eqs)  # make 1st coef of 1st equation
            eqs[0] = multiply_equation(eqs[0],1/eqs[0][0])
            # make 1st coef of 1st  equation equal 1

            for i in range(1,n-1):
                eqs[i] = add_equations(eqs[i],multiply_equation(eqs[0],-eqs[i][0]))
            # make 1st coef of 2nd .. n-th equation equal zero

            rest_equations = []
            for i in range(1,n):
                rest_equations.append(eqs[i][1:n+1])

            solutions = solve(rest_equations)
            # solve remainder of  equations for remainder of  variables

            x =  - eqs[0][n]
            for i in range(1,n):
                x -= eqs[0][i]*solutions[i-1]
            # solve 1st variable using solution for 2nd and 3rd variable

            return [x] + solutions

In [129]: def get_variables(user_equations):
              """
              Gets list of strings including some equations and returns a sorted li
              appear in these equations.
              You can assume all variables are a single lower-case letter.
              """
```

```
    # YOUR CODE HERE
    # needs to return a list
```

```
In [ ]: def parse_equation(s,variables):
            """
            Gets string input representing an equation in the variables in the sort
            and outputs a list `eq` of length `len(variables)+1` that contains the
            each variable (in alphabetical order) and the coefficient for the const
            You can assume that each variable is a single lower-case letter
            """

            # YOUR CODE HERE
            # needs to return a list of length len(variables)+1
```

```
In [ ]: def equation_solver():
            n = int(raw_input("Enter the number of equations / variables"))

            user_equations = []
            for i in range(n):
                s = raw_input("Enter equation number "+str(i)+":")
                user_equations.append(s)

            variables = get_variables(user_equations) # you need to write get_varia
            eqs  = []
            for s in user_equations:
                eqs.append(parse_equation(s,variables)) # you need to write parse_e

            solutions = solve(eqs)

            for i in len(solutions):
                print variables[i]+ " = "+str(solutions[i])
```

5.0.2 Project 2: Cannon game

Prepare a game for two players that will work as follows:

Each player chooses a *position*, *angle* and *speed* for their cannon.

Then the two cannons shoot and each player scores if they hit the other player's cannon.

bonus: handle the case that the cannonball goes higher than the size of the screen, showing when it goes back down.

Feel free to add other features to this game to make it more interesting.

```
In [1]: # run this code to get graphics to work

In [2]: %pylab

Using matplotlib backend: Qt4Agg
Populating the interactive namespace from numpy and matplotlib
```

```
In [3]: %matplotlib inline

In [12]: import sys
         import matplotlib
         import numpy as np
         import matplotlib.pyplot as plt
         from PIL import Image
         import math
         import IPython.display as dsp

         def empty_screen(w=100,h=100):
             res = [ [color(255,255,255) for i in range(h)] for j in range(w)]
             return res

         def color(red=0,green=0,blue=0):
             return (red,green,blue)

         def array_to_image(array, scale=2):
             w = len(array)
             h = len(array[0])
             data = np.zeros((h*scale, w*scale, 3), dtype=np.uint8)
             for i in range(w):
                 for j in range(h):
                     for k in range(scale):
                         for l in range(scale):
                             data[(h-1)*scale-(j*scale + k),i*scale+l] = array[i][j
             img = Image.fromarray(data, 'RGB')
             img.save('my.png')
             return dsp.Image(filename='my.png')


         def plot_array(array, scale=2):
             dsp.clear_output(True)
             dsp.display_png(array_to_image(array,scale))
             # sys.stdout.flush()

         def sine(angle):
             return math.sin((angle/360.0)*2*math.pi)
         def cosine(angle):
             return math.cos((angle/360.0)*2*math.pi)

In [2]: def cannon_game(width,height):
            angle1 = int(raw_input("Player 1: enter your angle"))
            speed1 = int(raw_input("Player 1: enter your speed"))
            location1 = int(raw_input("Player 1: enter your location"))

            angle2 = 180-int(raw_input("Player 2: enter your angle"))
            speed2 = int(raw_input("Player 2: enter your speed"))
```

```
            location2 = width - int(raw_input("Player 2: enter your location"))

            draw_two_cannons(angle1,speed1,location1,angle2,speed2,location2,width,
            # function you need to write to draw the path of the two cannons

            if will_it_hit(angle1,speed1,location2-location1):
                print "Player 1 hit player 2"
            if will_it_hit(180-angle2,speed2,location2-location1):
                print "Player 2 hit player 1"
```

In [ ]:
```
def will_it_hit(angle,speed,distance):
    """
    Return True if a cannon ball fired at this speed and angle will land ba
    at distance d that is at most one meter away from distance.
    """

    ### YOUR CODE HERE
```

In [ ]:
```
def draw_two_cannons(angle1,speed1,location1,angle2,speed2,location2,width,
    """
    Draw in two colors the trajectory of two cannonballs shot from these lo
    Use a screen of the given width and height
    """

    ### YOUR CODE HERE
    ### You can learn from the draw_cannon function below.
```

In [5]:
```
def draw_cannon(angle,speed):
    s = empty_screen(100,100)
    x =0
    y =0
    t=0.0
    while x<100 and y>=0 and y<100:
        s[x][y] = color(red=255)
        (x,y) = cannon(angle,speed,t)
        x= int(x)
        y = int(y)
        t += 1.0/speed
        plot_array(s)
```

In [10]:
```
def cannon(angle,speed,time, gravity=9.8):
    x = speed*time*cosine(angle)
    y = speed*time*sine(angle) -(gravity/2.0)*(time**2)
    return round(x,3),round(y,3)
```

In [13]:
```
draw_cannon(45,20)
```

In [ ]: