Contents

1 Setting 1
- 1.1 Default code 1
- 1.2 SIMD 2

2 Math 2
- 2.1 Extended Euclidean Algorithm 2
- 2.2 Linear Sieve 2
- 2.3 Primality Test 3
- 2.4 Integer Factorization (Pollard's rho) 3
- 2.5 Chinese Remainder Theorem 3
- 2.6 Query of nCr mod M in $O(Q+M)$ (extended Lucas Theorem) 3
- 2.7 Kirchoff's Theorem 4
- 2.8 FFT(Fast Fourier Transform) 4
- 2.9 NTT(Number Theoretic Transform) 4
- 2.10 FWHT(Fast Walsh-Hadamard Transform) and Convolution 4
- 2.11 Matrix Operations 5
- 2.12 Gaussian Elimination 5
- 2.13 Simplex Algorithm 5
- 2.14 Discrete Mathematics 6
- 2.15 DLAS Heuristic 6
- 2.16 Special Nim Game 7
- 2.17 Lifting The Exponent 7

3 Data Structure 7
- 3.1 Order statistic tree(Policy Based Data Structure) 7
- 3.2 Hash Table 7
- 3.3 Rope 7
- 3.4 Fenwick Tree 7
- 3.5 2D Fenwick Tree 7
- 3.6 Segment Tree with Lazy Propagation 8
- 3.7 Persistent Segment Tree 8
- 3.8 Persistent Segment Tree with Lazy Propagation 8
- 3.9 Splay Tree 9
- 3.10 Bitset to Set 9
- 3.11 Li-Chao Tree 10
- 3.12 Wavelet Tree 10

4 DP 11
- 4.1 Convex Hull Optimization 11
- 4.2 Divide & Conquer Optimization 11
- 4.3 Knuth Optimization 11
- 4.4 Bitset Optimization 12
- 4.5 Kitamasa & Berlekamp-Massey 12
- 4.6 SOS(Subset of Sum) DP 12

5 Graph 13
- 5.1 SCC 13
- 5.2 2-SAT 13
- 5.3 BCC, Cut vertex, Bridge 13
- 5.4 Block-cut Tree 13
- 5.5 Shortest Path Faster Algorithm 13
- 5.6 Centroid Decomposition 14
- 5.7 Lowest Common Ancestor 14
- 5.8 Heavy-Light Decomposition 14
- 5.9 Hall's Theorem 14
- 5.10 Stable Marriage 15
- 5.11 Bipartite Matching (Kuhn) 15
- 5.12 Maximum Flow (Dinic) 15
- 5.13 Maximum Flow with Edge Demands 15
- 5.14 Min-cost Maximum Flow 16
- 5.15 General Min-cut (Stoer-Wagner) 17
- 5.16 Hungarian Algorithm 17
- 5.17 General Unweighted Maximum Matching(Tutte) 18
- 5.18 General Weighted Maximum Matching(Blossom) 18
- 5.19 Offline Dynamic Connectivity 19

6 Geometry 20
- 6.1 Basic Operations 20
- 6.2 Convex Hull & Rotating Calipers 21
- 6.3 Half Plane Intersection 21
- 6.4 Minimum Permimeter Triangle 21
- 6.5 Minimum Enclosing Circle 21
- 6.6 Point in Polygon Test 22
- 6.7 Polygon Cut 22
- 6.8 Number of Point in Triangle 22
- 6.9 Voronoi Diagram 23
- 6.10 KD-Tree 24
- 6.11 Pick's theorem 24

7 String 24
- 7.1 KMP 24
- 7.2 Z Algorithm 25
- 7.3 Aho-Corasick 25
- 7.4 Suffix Array with LCP 25
- 7.5 Manacher's Algorithm 25
- 7.6 EERTREE 25

1 Setting

1.1 Default code

```cpp
#pragma GCC optimize ("O3,unroll-loops")
#pragma GCC target ("avx,avx2,fma")
#define debug(...) __dbg(#__VA_ARGS__, __VA_ARGS__)
template<typename T>
ostream& operator<<(ostream& out, vector<T> v) {
  string _;out << '('; for (T x : v) out << _ << x, _ = " ";out << ')';
  return out;
}
void __dbg(string s, auto... x) {
  string _;cout<<'('<<s<<") : ";(..., (cout << _ << x, _ = ", "));cout<<'\n';
```

```
}
auto gen_tree = [](int n) {
  auto prufer_decode = [](const vector<int>& v) {
    const int n = v.size() + 2; int p = 1, leaf = 1;
    vector deg(n + 1, 1);
    for (int i : v) deg[i]++; while (deg[p] != 1) p++, leaf++;
    vector res(0, pair(0, 0));
    for (int i : v) {
      res.push_back({ leaf, i });
      if (--deg[i] == 1 && i < p) leaf = i;
      else { do p++; while (deg[p] != 1); leaf = p; }
    }
    res.push_back({ leaf, n });
    return res;
  };
  vector v(n - 2, 0);
  for (int& i : v) i = gen_rand(1, n);
  return prufer_decode(v);
};
auto vectors(const int n, auto&& val) {
  return vector(n, val);
}
auto vectors(const int n, auto&&... args) {
  return vector(n, vectors(args...));
}
struct query { // mo's algorithm
  int l, r, i;
  bool operator< (const query& x) {
  if ((l ^ x.l) >> 9) return l < x.l;
    return l >> 9 & 1 ^ r < x.r;
  }
};
template<typename T>
struct custom_hash {
  static uint64_t splitmix64(uint64_t x) {
    x += 0x9e3779b97f4a7c15;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
  size_t operator() (T x) const {
    static const uint64_t seed = chrono::steady_clock::now().time_since_epoch().count();
      return splitmix64(x + seed);
  }
};
```

1.2 SIMD

```
#include <immintrin.h>
alignas(32) int A[8]{ 1, 2, 3, 1, 2, 3, 1, 2 }, B[8]{ 1, 2, 3, 4, 5, 6, 7, 8 };
alignas(32) int C[8]; // alignas(bit size of <type>) <type> var[256/(bit size)]
// Must compute "index is multiply of 256bit"(ex> short->16k, int->8k, ...)
__m256i a = _mm256_load_si256((__m256i*)A);
__m256i b = _mm256_load_si256((__m256i*)B);
__m256i c = _mm256_add_epi32(a, b);
void _mm256_store_si256((__m256i*)C, c);
// d : double precision(64-bit), s : single precision(32-bit), i : integer
__m256i _mm256_abs_epi32 (__m256i a)
__m256i _mm256_set1_epi32 (int a) // set all elements to a
__m256i _mm256_and_si256 (__m256i a, __m256i b)
__m256i _mm256_setzero_si256 (void)
__m256d _mm256_add_pd(__m256d a, __m256d b) // double precision(64-bit)
__m256d _mm256_sub_pd(__m256 a, __m256 b) // double precision(64-bit)
__m256d _mm256_addsub_pd(__m256d m1, __m256d m2) // a0-b0,a1+b1,...
__m256d _mm256_andnot_pd (__m256d a, __m256d b) // (~a)&b
__m256i _mm256_avg_epu16 (__m256i a, __m256i b) // unsigned, (a+b+1)>>1
__m256d _mm256_ceil_pd (__m256d a)
__m256i _mm256_cmpeq_epi64 (__m256i a, __m256i b)
__m256i _mm256_cmpgt_epi16 (__m256i a, __m256i b)
__m256d _mm256_div_pd (__m256d a, __m256d b)
__m256i _mm256_max_epi32 (__m256i a, __m256i b)
__m256i _mm256_mul_epi32 (__m256i a, __m256i b)
__m256 _mm256_rcp_ps (__m256 a) // 1/a
__m256 _mm256_rsqrt_ps (__m256 a) // 1/sqrt(a)
__m256i _mm256_set1_epi64x (long long a)
__m256i _mm256_sign_epi16 (__m256i a, __m256i b) // a*(sign(b))
__m256i _mm256_sll_epi32 (__m256i a, __m128i count) // a << count
__m256d _mm256_sqrt_pd (__m256d a)
__m256i _mm256_sra_epi16 (__m256i a, __m128i count)
__m256i _mm256_xor_si256 (__m256i a, __m256i b)
void _mm256_zeroall (void)
void _mm256_zeroupper (void)
// Example codes for sum and min over [l,r) in arr[]
alignas(32) int arr[100000],tmp[8];
// sum
int ans=0;
while(l&7 && l<r)ans+=arr[l++];
while(r&7 && l<r)ans+=arr[--r];
__m256i sum=_mm256_setzero_si256();
while(l<r){
  __m256i a=_mm256_load_si256((__m256i*)(arr+l));
  sum=_mm256_add_epi32(sum,a);
  l+=8;
}
_mm256_store_si256((__m256i*)tmp,sum);
for(j=8;j;)ans+=tmp[--j];
// min
int ans=1e9;
while(l&7&&l<r)ans=min(ans,arr[l++]);
while(r&7&&l<r)ans=min(ans,arr[--r]);
__m256i minv=_mm256_set1_epi32(1e9);
while(l<r){
  __m256i v=_mm256_load_si256((__m256i*)(arr+l));
  minv=_mm256_min_epi32(minv,v);
  l+=8;
}
_mm256_store_si256((__m256i*)tmp, minv);
ans=min(ans,*min_element(tmp,tmp+8));
```

2 Math

2.1 Extended Euclidean Algorithm

```
// Extended Euclidean Algorithm, O(lgn)
// ax+by=g, return (g,x,y)
tuple<ll, ll, ll> extended_gcd(ll a, ll b){
  if (a == 0) {b, 0, 1};
  auto [g, x, y] = extended_gcd(b % a, a);
  return {g, y - (b / a) * x, x};
}
// find x in [0,m) s.t. ax === gcd(a, m) (mod m)
ll modinverse(ll a, ll m) {
    return (get<1>(extended_gcd(a, m))%m+m)%m;
}
```

2.2 Linear Sieve

```
struct sieve {
  const ll MAXN = 101010;
  vector<ll> sp, e, phi, mu, tau, sigma, primes;
  // sp : smallest prime factor, e : exponent, phi : euler phi, mu : mobius
  // tau : num of divisors, sigma : sum of divisors
```

```
  sieve(ll sz):sp(sz+1),e(sz+1),phi(sz+1),mu(sz+1),tau(sz+1),sigma(sz+1) {
    phi[1] = mu[1] = tau[1] = sigma[1] = 1;
    for (ll i = 2; i <= sz; i++) {
      if (!sp[i]) {
        primes.push_back(i), e[i] = 1, phi[i] = i - 1, mu[i] = -1, tau[i] = 2, sigma[i] = i + 1;
      }
      for (auto j : primes) {
        if (i * j > sz) break;
        sp[i * j] = j;
        if (i % j == 0) {
          e[i * j] = e[i] + 1, phi[i * j] = phi[i] * j, mu[i * j] = 0,
                tau[i * j] = tau[i] / e[i * j] * (e[i * j] + 1),
                sigma[i * j] = sigma[i] * (j - 1) / (powm(j, e[i * j]) - 1) *
                               (powm(j, e[i * j] + 1) - 1) / (j - 1);
          break;
        }
        e[i * j] = 1, phi[i * j] = phi[i] * phi[j], mu[i * j] = mu[i] * mu[j],
              tau[i * j] = tau[i] * tau[j], sigma[i * j] = sigma[i] * sigma[j];
      }
    }
  }
  sieve() : sieve(MAXN) {}
};
```

2.3 Primality Test

```
// test whether n is prime based on miller-rabin test
// O(logn*logn)
bool is_prime(ll n) {
  if (n < 2 || n % 2 == 0 || n % 3 == 0) return n == 2 || n == 3;
  ll k = __builtin_ctzll(n - 1), d = n - 1 >> k;
  for (ll a : { 2, 325, 9375, 28178, 450775, 9780504, 1795265022 }) {
    ll p = modpow(a % n, d, n), i = k;
    while (p != 1 && p != n - 1 && a % n && i--) p = modmul(p, p, n);
    if (p != n - 1 && i != k) return 0;
  }
  return 1;
}
```

2.4 Integer Factorization (Pollard's rho)

```
ll pollard(ll n) {
  auto f = [n](ll x) { return modadd(modmul(x, x, n), 3, n); };
  ll x = 0, y = 0, t = 30, p = 2, i = 1, q;
  while (t++ % 40 || gcd(p, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if (q = modmul(p, abs(x - y), n)) p = q;
    x = f(x), y = f(f(y));
  }
  return gcd(p, n);
}
// integer factorization
// O(n^0.25 * logn)
vector<ll> factor(ll n) {
  if (n == 1) return {};
  if (is_prime(n)) return { n };
  ll x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), r.begin(), r.end());
  sort(l.begin(), l.end());
  return l;
}
```

2.5 Chinese Remainder Theorem

```
// x = r_i mod m_i
// (y, m) 'x = y mod m' 'm = lcm(m_i)', if not exists return (0, 0)
auto crt = [](auto r, auto m) {
  const int n = r.size(); i64 r0 = 0, m0 = 1;
  for (int i = 0; i < n; i++) {
    i64 r1 = r[i], m1 = m[i];
    if (m0 < m1) swap(r0, r1), swap(m0, m1);
    if (m0 % m1 == 0 && r0 % m1 != r1) return pair(0LL, 0LL);
    if (m0 % m1 == 0) continue;
    i64 g = gcd(m0, m1);
    if ((r1 - r0) % g) return pair(0LL, 0LL);
    i64 u0 = m0 / g, u1 = m1 / g;
    i64 x = (r1 - r0) / g % u1 * modinv(u0, u1) % u1;
    r0 += x * m0, m0 *= u1; if (r0 < 0) r0 += m0;
  }
  return pair(r0, m0);
};
```

2.6 Query of nCr mod M in $O(Q + M)$ (extended Lucas Theorem)

```
auto sol_p_e = [](int q, const auto& qs, const int p, const int e, const int mod) {
  // qs[i] = {n, r}, nCr mod p^e in O(p^e)
  vector dp(mod, 1);
  for (int i = 0; i < mod; i++) {
    if (i) dp[i] = dp[i - 1];
    if (i % p == 0) continue;
    dp[i] = mul(dp[i], i);
  }
  auto f = [&](i64 n) {
    i64 res = 0;
    while (n /= p) res += n;
    return res;
  };
  auto g = [&](i64 n) {
    auto rec = [&](const auto& self, i64 n) -> int {
      if (n == 0) return 1;
      int q = n / mod, r = n % mod;
      int ret = mul(self(self, n / p), dp[r]);
      if (q & 1) ret = mul(ret, dp[mod - 1]);
      return ret;
    };
    return rec(rec, n);
  };
  auto bino = [&](i64 n, i64 r) {
    if (n < r) return 0;
    if (r == 0 || r == n) return 1;
    i64 a = f(n) - f(r) - f(n - r);
    if (a >= e) return 0;
    int b = mul(g(n), modinv(mul(g(r), g(n - r)), mod));
    return mul(pow(p, a), b);
  };
  vector res(q, 0);
  for (int i = 0; i < q; i++) {
    auto [n, r] = qs[i];
    res[i] = bino(n, r);
  }
  return res;
};
auto sol = [](int q, const auto& qs, const int mod) {
  vector fac = factor(mod);
  vector r(q, vector(fac.size(), 0));
  vector m(fac.size(), 1);
  for (int i = 0; i < fac.size(); i++) {
    auto [p, e] = fac[i];
    for (int j = 0; j < e; j++) m[i] *= p;
    auto res = sol_p_e(q, qs, p, e, m[i]);
    for (int j = 0; j < q; j++) r[j][i] = res[j];
```

```
  }
  vector res(q, 0);
  for (int i = 0; i < q; i++) res[i] = crt(r[i], m).first;
  return res;
};
```

2.7 Kirchoff's Theorem

무향 그래프의 Laplacian matrix L : (정점의 차수 대각 행렬) - (인접행렬)이다. L에서 행과 열을 하나씩 제거한 것을 L'라 하자. 어느 행/열이든 관계 없다. 그래프의 스패닝 트리의 개수는 $det(L')$

2.8 FFT(Fast Fourier Transform)

```
void fft(int sign, int n, double *real, double *imag) {
  double theta = sign * 2 * pi / n;
  for (int m = n; m >= 2; m >>= 1, theta *= 2) {
    double wr = 1, wi = 0, c = cos(theta), s = sin(theta);
    for (int i = 0, mh = m >> 1; i < mh; ++i) {
      for (int j = i; j < n; j += m) {
        int k = j + mh;
        double xr = real[j] - real[k], xi = imag[j] - imag[k];
        real[j] += real[k], imag[j] += imag[k];
        real[k] = wr * xr - wi * xi, imag[k] = wr * xi + wi * xr;
      }
      double _wr = wr * c - wi * s, _wi = wr * s + wi * c;
      wr = _wr, wi = _wi;
    }
  }
  for (int i = 1, j = 0; i < n; ++i) {
    for (int k = n >> 1; k > (j ^= k); k >>= 1){}
    if (j < i) swap(real[i], real[j]), swap(imag[i], imag[j]);
  }
}
// Compute Poly(a)*Poly(b), write to r; Indexed from 0
// O(n*logn)
int mult(int *a, int n, int *b, int m, int *r) {
  const int maxn = 100; int fn = 1;
  static double ra[maxn], rb[maxn], ia[maxn], ib[maxn];
  while (fn < n + m) fn <<= 1;  // n + m: interested length
  for (int i = 0; i < n; ++i) ra[i] = a[i], ia[i] = 0;
  for (int i = n; i < fn; ++i) ra[i] = ia[i] = 0;
  for (int i = 0; i < m; ++i) rb[i] = b[i], ib[i] = 0;
  for (int i = m; i < fn; ++i) rb[i] = ib[i] = 0;
  fft(1, fn, ra, ia);fft(1, fn, rb, ib);
  for (int i = 0; i < fn; ++i) {
    double real = ra[i] * rb[i] - ia[i] * ib[i];
    double imag = ra[i] * ib[i] + rb[i] * ia[i];
    ra[i] = real, ia[i] = imag;
  }
  fft(-1, fn, ra, ia);
  for (int i = 0; i < fn; ++i) r[i] = (int)floor(ra[i] / fn + 0.5);
  return fn;
}
```

2.9 NTT(Number Theoretic Transform)

```
void ntt(poly& f, bool inv = 0) {
  int n = f.size(), j = 0;
  vector<ll> root(n >> 1);
  for (int i = 1; i < n; i++) {
    int bit = (n >> 1);
    while (j >= bit) {
      j -= bit;
      bit >>= 1;
    }
    j += bit;
    if (i < j) swap(f[i], f[j]);
  }
  ll ang = pw(w, (mod - 1) / n);
  if (inv) ang = pw(ang, mod - 2);
  root[0] = 1;
  for (int i = 1; i < (n >> 1); i++) root[i] = root[i - 1] * ang % mod;
  for (int i = 2; i <= n; i <<= 1) {
    int step = n / i;
    for (int j = 0; j < n; j += i) {
      for (int k = 0; k < (i >> 1); k++) {
        ll u = f[j | k], v = f[j | k | i >> 1] * root[step * k] % mod;
        f[j | k] = (u + v) % mod; f[j | k | i >> 1] = (u - v) % mod;
        if (f[j | k | i >> 1] < 0) f[j | k | i >> 1] += mod;
      }
    }
  }
  ll t = pw(n, mod - 2);
  if (inv) for (int i = 0; i < n; i++) f[i] = f[i] * t % mod;
}

vector<ll> multiply(poly& _a, poly& _b) {
  vector<ll> a(all(_a)), b(all(_b));
  int n = 2;
  while (n < a.size() + b.size()) n <<= 1;
  a.resize(n);b.resize(n);ntt(a);ntt(b);
  for (int i = 0; i < n; i++) a[i] = a[i] * b[i] % mod;
  ntt(a, 1);
  return a;
}
```

$998\,244\,353 = 119 \times 2^{23} + 1$. Primitive root: 3.
$985\,661\,441 = 235 \times 2^{22} + 1$. Primitive root: 3.
$1\,012\,924\,417 = 483 \times 2^{21} + 1$. Primitive root: 5.

2.10 FWHT(Fast Walsh-Hadamard Transform) and Convolution

```
// (fwht_or(a))_i = sum of a_j for all j s.t. i | j = j
// (fwht_and(a))_i = sum of a_j for all j s.t. i & j = i
// x @ y = popcount(x & y) mod 2
// (fwht_xor(a))_i = (sum of a_j for all j s.t. i @ j = 0)
//                   - (sum of a_j for all j s.t. i @ j = 1)
// inv = 0 for fwht, 1 for ifwht(inverse fwht)
// {convolution(a,b)}_i = sum of a_j * b_k for all j,k s.t. j op k = i
//   = ifwht(fwht(a) * fwht(b))
vector<ll> fwht_or(vector<ll> &x, bool inv) {
  vector<ll> a = x; ll n = a.size();
  int dir = inv ? -1 : 1;
  for(int s = 2, h = 1; s <= n; s <<= 1, h <<= 1) {
    for(int l = 0; l < n; l += s) {
      for(int i = 0; i < h; i++)a[l + h + i] += dir * a[l + i];
    }
  }
  return a;
}
vector<ll> fwht_and(vector<ll> &x, bool inv) {
  vector<ll> a = x; ll n = a.size();
  int dir = inv ? -1 : 1;
  for(int s = 2, h = 1; s <= n; s <<= 1, h <<= 1) {
    for(int l = 0; l < n; l += s) {
      for(int i = 0; i < h; i++)a[l + h] += dir * a[l + h + i];
    }
  }
  return a;
}
vector<ll> fwht_xor(vector<ll> &x, bool inv) {
  vector<ll> a = x;
  ll n = a.size();
```

```
  for(int s = 2, h = 1; s <= n; s <<= 1, h <<= 1) {
    for(int l = 0; l < n; l += s) {
      for(int i = 0; i < h; i++) {
        int t = a[l + h + i];a[l + h + i] = a[l + i] - t;a[l + i] += t;
        if(inv) a[l + h + i] /= 2, a[l + i] /= 2;
      }
    }
  }
  return a;
}
```

2.11 Matrix Operations

```
inline bool is_zero(ld a) { return abs(a) < eps; }
// returns {det(A), A^-1, rank(A), tr(A)}
// A becomes invalid after call this O(n^3)
tuple<ld,vector<vector<ld>>,ll,ll> inv_det_rnk(auto A) {
  ld n=A.size(); ld det = 1; vector out(n, vector<ld>(n)); ld tr=0;
  for (int i = 0; i < n; i++) {
    out[i][i] = 1; tr+=A[i][i];
  }
  for (int i = 0; i < n; i++) {
    if (is_zero(A[i][i])) {
      ld maxv = 0; int maxid = -1;
      for (int j = i + 1; j < n; j++) {
        auto cur = abs(A[j][i]);
        if (maxv < cur) {
          maxv = cur;
          maxid = j;
        }
      }
      if (maxid == -1 || is_zero(A[maxid][i])) return {0, out, i, tr};
      for (int k = 0; k < n; k++) {
        A[i][k] += A[maxid][k]; out[i][k] += out[maxid][k];
      }
    }
    det *= A[i][i];
    ld coeff = 1.0 / A[i][i];
    for (int j = 0; j < n; j++) A[i][j] *= coeff,out[i][j] *= coeff;
    for (int j = 0; j < n; j++) if (j != i) {
      ld mp = A[j][i];
      for (int k = 0; k < n; k++) A[j][k] -= A[i][k] * mp;
      for (int k = 0; k < n; k++) out[j][k] -= out[i][k] * mp;
    }
  }
  return {det, out, n, tr};
}
```

2.12 Gaussian Elimination

```
const double EPS = 1e-10;
typedef vector<vector<double>> VVD;

// Gauss-Jordan elimination with full pivoting.
// solving systems of linear equations (AX=B)
// INPUT:    a[][] = an n*n matrix
//           b[][] = an n*m matrix
// OUTPUT:   X      = an n*m matrix (stored in b[][])
//           A^{-1} = an n*n matrix (stored in a[][])
// O(n^3)
bool gauss_jordan(VVD& a, VVD& b) {
    const int n = a.size();
    const int m = b[0].size();
    vector<int> irow(n), icol(n), ipiv(n);
    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k; }
        if (fabs(a[pj][pk]) < EPS) return false; // matrix is singular
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        irow[i] = pj;
        icol[i] = pk;
        double c = 1.0 / a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];
            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
        }
    }
    for (int p = n - 1; p >= 0; p--) if (irow[p] != icol[p]) {
        for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return true;
}
```

2.13 Simplex Algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//   maximize   c^T x s.t.    Ax <= b; x >= 0
// A -- m x n mat, b -- m-dimensional vec, c -- n-dimensional vec
// return {value of the optimal solution, solution vector}
struct LPSolver {
  ll m, n;
  vector<ll> B, N;
  vector<vector<ld>> D;
  LPSolver(const vector<vector<ld>>& A, const vector<ld>& b, const vector<ld>& c):
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, vector<ld>(n + 2)) {
    for (ll i = 0; i < m; i++) for (ll j = 0; j < n; j++) D[i][j] = A[i][j];
    for (ll i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1] = b[i]; }
    for (ll j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }
  void pivot(ll r, ll s) {
    ld inv = 1.0 / D[r][s];
    for (ll i = 0; i < m + 2; i++) if (i != r)
      for (ll j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (ll j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (ll i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv; swap(B[r], N[s]);
  }
  bool simplex(ll phase) {
    ll x = phase == 1 ? m + 1 : m;
    while (true) {
      ll s = -1;
      for (ll j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      ll r = -1;
      for (ll i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] < B[r]) r = i;
```

```
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }
  pair<ld,vector<ld>> solve() {
    ll r = 0; vector<ld> x(n);
    for (ll i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      pivot(r, n);
      if (!simplex(1) || D[m + 1][n + 1] < -EPS) return -numeric_limits<ld>::infinity();
      for (ll i = 0; i < m; i++) if (B[i] == -1) {
        ll s = -1;
        for (ll j = 0; j <= n; j++)if(s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] <
            N[s])s=j;
        pivot(i, s);
      }
    }
    if (!simplex(2)) return numeric_limits<ld>::infinity();
    for (ll i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};
```

2.14 Discrete Mathematics

```
/* Solve x for x^P = A mod Q
 * O((lgQ)^2 + Q^0.25 (lgQ)^3)
 * (P, Q-1) = 1 -> P^-1 mod (Q-1) exists
 * x has solution iff A^((Q-1) / P) = 1 mod Q
 * PP | (Q-1) -> P < sqrt(Q), solve lgQ rounds of discrete log
 * else -> find a s.t. s | (Pa - 1) -> ans = A^a */
const int X = 1e5;
ll base, ae[X], aXe[X], iaXe[X];
unordered_map<ll, ll> ht;
#define FOR(i, c) for (int i = 0; i < (c); ++i)
#define REP(i, l, r) for (int i = (l); i <= (r); ++i)
// discrete log : O(sqrt(Q))
void build(ll a) { // ord(a) = P < sqrt(Q)
  base = a; ht.clear();
  ae[0] = 1; ae[1] = a; aXe[0] = 1; aXe[1] = pw(a, X, Q);
  iaXe[0] = 1; iaXe[1] = pw(aXe[1], Q-2, Q);
  REP(i, 2, X-1) {
    ae[i]=mul(ae[i-1],ae[1],Q);aXe[i]=mul(aXe[i-1],aXe[1],Q);iaXe[i]=mul(iaXe[i-1],iaXe[1],Q);
  }
  FOR(i, X) ht[ae[i]] = i;
}
ll dis_log(ll x) {
  FOR(i, X) {
    ll iaXi = iaXe[i]; ll rst = mul(x, iaXi, Q);
    if (ht.count(rst)) return i*X + ht[rst];
  }
}
ll main2() { // solve x^P = A mod Q
  ll g; ll t = 0, s = Q-1;
  while (s % P == 0) ++t,s /= P;
  if (A == 0) return 0;
  if (t == 0) {
    // a^{P^-1 mod phi(Q)}
    auto [x, y, _] = extended_gcd(P, Q-1);
    if (x < 0) {
      x = (x % (Q-1) + Q-1) % (Q-1);
    }
    ll ans = pw(A, x, Q);
    if (pw(ans, P, Q) != A) while(1);
    return ans;
  }
  // A is not P-residue
  if (pw(A, (Q-1) / P, Q) != 1) return -1;
  for (g = 2; g < Q; ++g) if (pw(g, (Q-1) / P, Q) != 1) break;
  ll alpha = 0; ll y, _; gcd(P, s, alpha, y, _);
  if (alpha < 0) alpha = (alpha % (Q-1) + Q-1) % (Q-1);
  if (t == 1) return pw(A, alpha, Q);
  ll a = pw(g, (Q-1) / P, Q);
  build(a);
  ll b = pw(A, add(mul(P%(Q-1), alpha, Q-1), Q-2, Q-1), Q);
  ll c = pw(g, s, Q); ll h = 1; ll e = (Q-1) / s / P; // r^{t-1}
  REP(i, 1, t-1) {
    e /= P; ll d = pw(b, e, Q); ll j = 0;
    if (d != 1) {
      j = -dis_log(d);
      if (j < 0) j = (j % (Q-1) + Q-1) % (Q-1);
    }
    b = mul(b, pw(c, mul(P%(Q-1), j, Q-1), Q), Q);
    h = mul(h, pw(c, j, Q), Q); c = pw(c, P, Q);
  }
  return mul(pw(A, alpha, Q), h, Q);
}
// only for sqrt
void calcH(int &t, int &h, const int p) {
  int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(int a, int p, int &x, int &y) {
  if(p == 2) { x = y = 1; return true; }
  int p2 = p / 2, tmp = pw(a, p2, p);
  if (tmp == p - 1) return false;
  if ((p + 1) % 4 == 0) {
    x=pw(a,(p+1)/4,p); y=p-x; return true;
  } else {
    int t, h, b, pb; calcH(t, h, p);
    if (t >= 2) {
      do {b = rand() % (p - 2) + 2;
      } while (pw(b, p / 2, p) != p - 1);
      pb = pw(b, h, p);
    } int s = pw(a, h / 2, p);
    for (int step = 2; step <= t; step++) {
      int ss = (((ll)(s * s) % p) * a) % p;
      for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
      if (ss + 1 == p) s = (s * pb) % p;
      pb = ((ll)pb * pb) % p;
    } x = ((ll)s * a) % p; y = p - x;
  } return true;
}
```

2.15 DLAS Heuristic

```
auto dlas = [](const auto& state, int iter) {
  vector s(3, state);
  vector buc(5, s[0].score());
  auto cur_score = buc[0], min_score = cur_score;
  int cur_pos = 0, min_pos = 0, k = 0;
  for (int i = 0; i < iter; i++) {
    auto prv_score = cur_score;
    int nxt_pos = cur_pos + 1 < 3 ? cur_pos + 1 : 0;
    if (nxt_pos == min_pos) nxt_pos = nxt_pos + 1 < 3 ? nxt_pos + 1 : 0;
    auto& cur_state = s[cur_pos];
    auto& nxt_state = s[nxt_pos];
    nxt_state = cur_state;
    nxt_state.mutate();
    auto nxt_score = nxt_state.score();
    if (min_score > nxt_score) {
```

```
    i = 0;
    min_pos = nxt_pos;
    min_score = nxt_score;
  }
  if (nxt_score == cur_score || nxt_score < ranges::max(buc)) {
    cur_pos = nxt_pos;
    cur_score = nxt_score;
  }
  auto& fit = buc[k];
  if (cur_score > fit || cur_score < min(fit, prv_score)) {
    fit = cur_score;
  }
  k = k + 1 < 5 ? k + 1 : 0;
 }
 return pair(s[min_pos], min_score);
};
```

2.16 Special Nim Game

Subtraction Game : 한 번에 k개까지의 돌만 가져갈 수 있는 경우, 각 더미의 돌의 개수를 $k+1$로 나눈 나머지를 XOR 합하여 판단한다.
Index-k Nim : 한 번에 최대 k개의 더미를 골라 각각의 더미에서 아무렇게나 돌을 제거할 수 있을 때, 각 binary digit에 대하여 합을 $k+1$로 나눈 나머지를 계산한다. 만약 이 나머지가 모든 digit에 대하여 0이라면 두번째, 하나라도 0이 아니라면 첫번째 플레이어가 승리.

2.17 Lifting The Exponent

For any integers x, y a positive integer n, and a prime number p such that $p \nmid x$ and $p \nmid y$, the following statements hold:

- When p is odd:
  - If $p \mid x-y$, then $\nu_p(x^n - y^n) = \nu_p(x-y) + \nu_p(n)$.
  - If n is odd and $p \mid x+y$, then $\nu_p(x^n + y^n) = \nu_p(x+y) + \nu_p(n)$.
- When $p = 2$:
  - If $2 \mid x-y$ and n is even, then $\nu_2(x^n - y^n) = \nu_2(x-y) + \nu_2(x+y) + \nu_2(n) - 1$.
  - If $2 \mid x-y$ and n is odd, then $\nu_2(x^n - y^n) = \nu_2(x-y)$.
  - Corollary:
    - If $4 \mid x-y$, then $\nu_2(x+y) = 1$ and thus $\nu_2(x^n - y^n) = \nu_2(x-y) + \nu_2(n)$.
- For all p:
  - If $\gcd(n,p) = 1$ and $p \mid x-y$, then $\nu_p(x^n - y^n) = \nu_p(x-y)$.
  - If $\gcd(n,p) = 1$, $p \mid x+y$ and n odd, then $\nu_p(x^n + y^n) = \nu_p(x+y)$.

3 Data Structure

3.1 Order statistic tree(Policy Based Data Structure)

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
// order_of_key (k) : Number of items strictly smaller than k
// find_by_order(k) : -Kth element in a set (counting from zero)
// O(lgn)
using ordered_set =
    tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update>;
using ordered_multi_set = tree<int, null_type, less_equal<int>, rb_tree_tag,
                               tree_order_statistics_node_update>;
void m_erase(ordered_multi_set &OS, int val) {
  int index = OS.order_of_key(val);
  ordered_multi_set::iterator it = OS.find_by_order(index);
  if (*it == val) OS.erase(it);
}
```

3.2 Hash Table

```
// gp_hash_table, cc_hash_table, hash for pair
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch().count();
struct chash {
  int operator()(int x) const { return x ^ RANDOM; }
};
gp_hash_table<int, int, chash> table;
struct pair_hash {
  template <class T1, class T2>
  size_t operator () (const pair<T1,T2> &p) const {
    auto h1 = hash<T1>{}(p.first);
    auto h2 = hash<T2>{}(p.second);
    return h1 ^ h2;
  }
};
gp_hash_table<int, int, chash> table;
unordered_set<pll, pair_hash> st;
```

3.3 Rope

```
#include<ext/rope>
using namespace __gnu_cxx;
crope arr; string str; // or rope<T> arr; vector<T> str;
arr.insert(i, str); // Insert at position i with O(log n)
arr.erase(i, n);// Delete n characters from position i with O(log n)
arr.replace(i, n, str); // Replace n characters from position i with str with O(log n)
crope sub = arr.substr(i, n); // Get substring of length n starting from position i with O(log n)
```

3.4 Fenwick Tree

```
struct Fenwick {
  const ll MAXN = 100000;
  vector<ll> tree;
  Fenwick(ll sz) : tree(sz + 1) {}
  Fenwick() : Fenwick(MAXN) {}
  ll query(ll p) {  // sum from index 1 to p, inclusive
    ll ret = 0;
    for (; p > 0; p -= p & -p) ret += tree[p];
    return ret;
  }
  void add(ll p, ll val) {
    for (; p <= TSIZE; p += p & -p) tree[p] += val;
  }
};
```

3.5 2D Fenwick Tree

```
// Call with size of the grid
// Example: fenwick_tree_2d<int> Tree(n+1,m+1) for n x m grid indexed from 1
template <class T>struct fenwick_tree_2d {
  vector<vector<T>> x;
  fenwick_tree_2d(int n, int m) : x(n, vector<T>(m)) { }
  void add(int k1, int k2, int a) { // x[k] += a
    for (; k1 < x.size(); k1 |= k1 + 1)
      for (int k = k2; k < x[k1].size(); k |= k + 1) x[k1][k] += a;
  }
  T sum(int k1, int k2) { // return x[0] + ... + x[k]
    T s = 0;
    for (; k1 >= 0; k1 = (k1 & (k1 + 1)) - 1)
      for (int k = k2; k >= 0; k = (k & (k + 1)) - 1) s += x[k1][k];
    return s;
  }
};
```

3.6 Segment Tree with Lazy Propagation

```
struct Segment_Lazy {
#ifdef ONLINE_JUDGE
  const int TSIZE = 1 << 20;  // always 2^k form && n <= TSIZE
#else
  const int TSIZE = 1 << 3;  // always 2^k form && n <= TSIZE
#endif
  vector<ll> segtree, prop, dat;
  Segment_Lazy() {
    segtree.resize(TSIZE * 2);
    prop.resize(TSIZE * 2);
    dat.resize(1);
  }
  Segment_Lazy(int n){
    segtree.resize(2<<(32-__builtin_clz(n)));
    prop.resize(2<<(32-__builtin_clz(n)));
    dat.resize(n);
  }
  void seg_init(int nod, int l, int r) {
    if (l == r) {
      segtree[nod] = dat[l];
    } else {
      int m = (l + r) >> 1;
      seg_init(nod << 1, l, m);
      seg_init(nod << 1 | 1, m + 1, r);
      segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
    }
  }
  void seg_relax(int nod, int l, int r) {
    if (prop[nod] == 0) return;
    if (l < r) {
      int m = (l + r) >> 1;
      segtree[nod << 1] += (m - l + 1) * prop[nod];
      prop[nod << 1] += prop[nod];
      segtree[nod << 1 | 1] += (r - m) * prop[nod];
      prop[nod << 1 | 1] += prop[nod];
    }
    prop[nod] = 0;
  }
  ll seg_query(int nod, int l, int r, int s, int e) {
    if (r < s || e < l) return 0;
    if (s <= l && r <= e) return segtree[nod];
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    return seg_query(nod << 1, l, m, s, e) +
           seg_query(nod << 1 | 1, m + 1, r, s, e);
  }
  void seg_update(int nod, int l, int r, int s, int e, int val) {
    if (r < s || e < l) return;
    if (s <= l && r <= e) {
      segtree[nod] += (r - l + 1) * val;
      prop[nod] += val;
      return;
    }
    seg_relax(nod, l, r);
    int m = (l + r) >> 1;
    seg_update(nod << 1, l, m, s, e, val);
    seg_update(nod << 1 | 1, m + 1, r, s, e, val);
    segtree[nod] = segtree[nod << 1] + segtree[nod << 1 | 1];
  }
  // usage:
  // seg_update(1, 0, n - 1, qs, qe, val);
  // seg_query(1, 0, n - 1, qs, qe);
};
```

3.7 Persistent Segment Tree

```
struct PST{
  struct Node{ll l,r,v;};
  vector<Node>nodes;vector<ll>root;
  PST(ll n=0,ll qmax=0){
    root.reserve(qmax+5);
    nodes.reserve(4*(n+1)+20*qmax+5);
    root.push_back(init(0,n));
  }
  ll init(ll s,ll e){
    ll cur=nodes.size();
    nodes.push_back({-1,-1,0});
    if(s<e){
        ll m=s+e>>1;
        nodes[cur].l=init(s,m),nodes[cur].r=init(m+1,e);
    }
    return cur;
  }
  ll add(ll s,ll e,ll pre,ll pos,ll val){
    ll cur=nodes.size();
    nodes.push_back(nodes[pre]);
    if(s==e){
        nodes[cur].v+=val;
        return cur;
    }
    ll m=s+e>>1;
    if(pos<=m)nodes[cur].l=add(s,m,nodes[pre].l,pos,val);
    else nodes[cur].r=add(m+1,e,nodes[pre].r,pos,val);
    nodes[cur].v=nodes[nodes[cur].l].v+nodes[nodes[cur].r].v;
    return cur;
  }
  ll query(ll s,ll e,ll u,ll v,ll l,ll r){
    if(r<s||e<l)return 0;
    if(l<=s&&e<=r)return nodes[v].v-nodes[u].v;
    ll m=s+e>>1;
    return query(s,m,nodes[u].l,nodes[v].l,l,r)
         +query(m+1,e,nodes[u].r,nodes[v].r,l,r);
  }
  // pst.init(0,n);
  // pst.add(0,n,prev_root,pos,val);
  // pst.query(0,n,root_u,root_v,l,r);
};
```

3.8 Persistent Segment Tree with Lazy Propagation

```
struct PST_Lazy{
    struct Node{ll l,r,v,prop;};
    vector<Node>nodes;vector<ll>root;vector<ll>arr;
    PST_Lazy(ll n=0,ll qmax=0){
        root.reserve(qmax+5);
        nodes.reserve(4*(n+1)+20*qmax+5);
        arr.resize(n+1);
        root.push_back(init(0,n));
    }
    ll init(ll s,ll e){
        ll cur=nodes.size();
        nodes.push_back({-1,-1,0,0});
        if(s<e){
            ll m=s+e>>1;
            nodes[cur].l=init(s,m),nodes[cur].r=init(m+1,e);
        }
        return cur;
    }
    ll add(ll s,ll e,ll pre,ll pos_l,ll pos_r,ll val){
        ll cur=nodes.size();
```

```
        nodes.push_back(nodes[pre]);
        if(pos_r<s||e<pos_l)return cur;
        if(pos_l<=s&&e<=pos_r){
            nodes[cur].v+=val*(e-s+1);
            nodes[cur].prop+=val;
            return cur;
        }
        ll m=s+e>>1;
        nodes[cur].l=add(s,m,nodes[pre].l,pos_l,pos_r,val);
        nodes[cur].r=add(m+1,e,nodes[pre].r,pos_l,pos_r,val);
        nodes[cur].v=nodes[nodes[cur].l].v+nodes[nodes[cur].r].v+nodes[cur].prop*(e-s+1);
        return cur;
    }
    ll query(ll s,ll e,ll u,ll v,ll l,ll r){
        if(r<s||e<l)return 0;
        if(l<=s&&e<=r)return nodes[v].v-nodes[u].v;
        ll m=s+e>>1;
        ll left=query(s,m,nodes[u].l,nodes[v].l,l,r);
        ll right=query(m+1,e,nodes[u].r,nodes[v].r,l,r);
        ll overlap=(min(e,r)-max(s,l)+1)*(nodes[v].prop-nodes[u].prop);
        return left+right+overlap;
    }
};
```

3.9 Splay Tree

```
struct Splay{
  struct node {
    node* l, * r, * p;
    int cnt, min, max, val;
    long long sum;
    bool inv;
    node(int _val):
      cnt(1), sum(_val), min(_val), max(_val), val(_val), inv(false),
      l(nullptr), r(nullptr), p(nullptr) {}
  };
  node* root;
  void update(node* x) {
    x->cnt = 1;
    x->sum = x->min = x->max = x->val;
    if (x->l) {
      x->cnt += x->l->cnt;x->sum += x->l->sum;
      x->min = min(x->min, x->l->min); x->max = max(x->max, x->l->max);
    }
    if (x->r) {
      x->cnt += x->r->cnt; x->sum += x->r->sum;
      x->min = min(x->min, x->r->min); x->max = max(x->max, x->r->max);
    }
  }
  void rotate(node* x) {
    node* p = x->p;
    node* b = nullptr;
    if (x == p->l) {
      p->l = b = x->r;
      x->r = p;
    }
    else {
      p->r = b = x->l;
      x->l = p;
    }
    x->p = p->p;
    p->p = x;
    if (b) b->p = p;
    x->p ? (p == x->p->l ? x->p->l : x->p->r) = x : (root = x);
    update(p);
    update(x);
  }
  // make x into root
  void splay(node* x) {
    while (x->p) {
      node* p = x->p;
      node* g = p->p;
      if (g) rotate((x == p->l) == (p == g->l) ? p : x);
      rotate(x);
    }
  }
  void relax_lazy(node* x) {
    if (!x->inv) return;
    swap(x->l, x->r);
    x->inv = false;
    if (x->l) x->l->inv = !x->l->inv;
    if (x->r) x->r->inv = !x->r->inv;
  }
  // find kth node in splay tree
  void find_kth(int k) {
    node* x = root;
    relax_lazy(x);
    while (true) {
      while (x->l && x->l->cnt > k) {
        x = x->l;
        relax_lazy(x);
      }
      if (x->l) k -= x->l->cnt;
      if (!k--) break;
      x = x->r;
      relax_lazy(x);
    }
    splay(x);
  }
  // collect [l, r] nodes into one subtree and return its root
  node* interval(int l, int r) {
    find_kth(l - 1);
    node* x = root;
    root = x->r;
    root->p = nullptr;
    find_kth(r - l + 1);
    x->r = root;
    root->p = x;
    root = x;
    return root->r->l;
  }
  void traverse(node* x) {
    relax_lazy(x);
    if (x->l) {
      traverse(x->l);
    }
    // do something
    if (x->r) {
      traverse(x->r);
    }
  }
  void uptree(node* x) {
    if (x->p) {
      uptree(x->p);
    }
    relax_lazy(x);
  }
};
```

3.10 Bitset to Set

```
typedef unsigned long long ull;
```

```cpp
const int sz = 100001 / 64 + 1;
struct bset {
  ull x[sz];
  bset(){
    memset(x, 0, sizeof x);
  }
  bset operator|(const bset &o) const {
    bset a;
    for (int i = 0; i < sz; i++)a.x[i] = x[i] | o.x[i];
    return a;
  }
  bset &operator|=(const bset &o) {
    for (int i = 0; i < sz; i++)x[i] |= o.x[i];
    return *this;
  }
  inline void add(int val){
    x[val >> 6] |= (1ull << (val & 63));
  }
  inline void del(int val){
    x[val >> 6] &= ~(1ull << (val & 63));
  }
  int kth(int k){
    int i, cnt = 0;
    for (i = 0; i < sz; i++){
      int c = __builtin_popcountll(x[i]);
      if (cnt + c >= k){
        ull y = x[i];
        int z = 0;
        for (int j = 0; j < 64; j++){
          z += ((x[i] & (1ull << j)) != 0);
          if (cnt + z == k)return i * 64 + j;
        }
      }
      cnt += c;
    }
    return -1;
  }
  int lower(int z){
    int i = (z >> 6), j = (z & 63);
    if (x[i]){
      for (int k = j - 1; k >= 0; k--)if (x[i] & (1ull << k))return (i << 6) | k;
    }
    while (i > 0)
    if (x[--i])
    for (j = 63;; j--)
    if (x[i] & (1ull << j))return (i << 6) | j;
    return -1;
  }
  int upper(int z){
    int i = (z >> 6), j = (z & 63);
    if (x[i]){
      for (int k = j + 1; k <= 63; k++)if (x[i] & (1ull << k))return (i << 6) | k;
    }
    while (i < sz - 1)if (x[++i])for (j = 0;; j++)if (x[i] & (1ull << j))return (i << 6) | j;
    return -1;
  }
};
```

3.11 Li-Chao Tree

```cpp
struct Line {
  ll a, b;
  ll get(ll x) { return a * x + b; }
};
struct Node {
  int l, r;  // child
  ll s, e;   // range
  Line line;
};
struct Li_Chao {
  vector<Node> tree;
  void init(ll s, ll e) { tree.push_back({-1, -1, s, e, {0, -INF}}); }
  void update(int node, Line v) {
    ll s = tree[node].s, e = tree[node].e, m;
    m = (s + e) >> 1;
    Line low = tree[node].line, high = v;
    if (low.get(s) > high.get(s)) swap(low, high);
    if (low.get(e) <= high.get(e)) {
      tree[node].line = high;
      return;
    }
    if (low.get(m) < high.get(m)) {
      tree[node].line = high;
      if (tree[node].r == -1) {
        tree[node].r = tree.size();
        tree.push_back({-1, -1, m + 1, e, {0, -INF}});
      }
      update(tree[node].r, low);
    } else {
      tree[node].line = low;
      if (tree[node].l == -1) {
        tree[node].l = tree.size();
        tree.push_back({-1, -1, s, m, {0, -INF}});
      }
      update(tree[node].l, high);
    }
  }
  ll query(int node, ll x) {
    if (node == -1) return -INF;
    ll s = tree[node].s, e = tree[node].e, m;
    m = (s + e) >> 1;
    if (x <= m)
      return max(tree[node].line.get(x), query(tree[node].l, x));
    else
      return max(tree[node].line.get(x), query(tree[node].r, x));
  }
  // usage : seg.init(-2e8, 2e8); seg.update(0, {-c[i], c[i] * a[i - 1]});
  // seg.query(0, a[n - 1]);
};
```

3.12 Wavelet Tree

```cpp
struct bit_array { // 0-indexed
  using u64 = unsigned long long;
  explicit bit_array(int sz) : n(sz + 64 >> 6), data(n), psum(n) {}
  void set(int i) { data[i >> 6] |= u64(1) << (i & 63); }
  int rank(int i, bool x) const {
    auto res = rank(i);
    return x ? res : i - res;
  }
  int rank(int l, int r, bool x) const {
    auto res = rank(r) - rank(l);
    return x ? res : r - l - res;
  }
  bool operator[](int i) const {
    return data[i >> 6] >> (i & 63) & 1;
  }
  void init() {
    for (int i = 1; i < n; i++)
      psum[i] = psum[i - 1] + __builtin_popcountll(data[i - 1]);
  }
private:
```

```cpp
  int n;
  vector<u64> data;
  vector<int> psum;
  int rank(int i) const {
    return psum[i >> 6] + __builtin_popcountll(data[i >> 6] & (u64(1) << (i & 63)) - 1);
  }
};
// 전처리 O(nlgn) 각쿼리별  O(lgn)
template<typename T, enable_if_t<is_integral_v<T>, int> = 0>
struct wavelet_matrix { // 0-indexed
  explicit wavelet_matrix(vector<T> v) :
    n(v.size()),
    lg(__lg(*max_element(v.begin(), v.end())) + 1),
    data(lg, bit_array(n)),
    zero(lg, 0) {
    for (int i = lg - 1; i >= 0; i--) {
      for (int j = 0; j < n; j++) if (v[j] >> i & 1) data[i].set(j);
      data[i].init();
      auto it = stable_partition(v.begin(), v.end(), [&](T x) { return ~x >> i & 1; });
      zero[i] = it - v.begin();
    }
  }
  int rank(int l, int r, T x) const { // count i s.t. (l <= i < r) && (v[i] == x)
    if (x >> lg) return 0;
    for (int i = lg - 1; i >= 0; i--) {
      bool f = x >> i & 1;
      adjust(i, l, r, f);
    }
    return r - l;
  }
  int count(int l, int r, T x) const { // count i s.t. (l <= i < r) && (v[i] < x)
    if (x >> lg) return r - l + 1;
    int res = 0;
    for (int i = lg - 1; i >= 0; i--) {
      bool f = x >> i & 1;
      if (f) res += data[i].rank(l, r, 0);
      adjust(i, l, r, f);
    }
    return res;
  }
  T quantile(int l, int r, int k) const { // kth (0-indexed) smallest number in v[l, r)
    T res = 0;
    for (int i = lg - 1; i >= 0; i--) {
      int c = data[i].rank(l, r, 0);
      bool f = c <= k;
      if (f) res |= T(1) << i, k -= c;
      adjust(i, l, r, f);
    }
    return res;
  }
private:
  int n, lg;
  vector<bit_array> data;
  vector<int> zero;
  void adjust(int i, int& l, int& r, bool f) const {
    if (!f) {
      l = data[i].rank(l, 0);
      r = data[i].rank(r, 0);
    }
    else {
      l = zero[i] + data[i].rank(l, 1);
      r = zero[i] + data[i].rank(r, 1);
    }
  }
};
```

4 DP

4.1 Convex Hull Optimization

$O(n^2) \to O(n \log n)$
DP 점화식 꼴
$D[i] = \max_{j<i}(D[j] + b[j] * a[i])$ $(b[k] \leq b[k+1])$
$D[i] = \min_{j<i}(D[j] + b[j] * a[i])$ $(b[k] \geq b[k+1])$
특수조건) $a[i] \leq a[i+1]$ 도 만족하는 경우, 마지막 쿼리의 위치를 저장해두면 이분검색이 필요없어지기 때문에 amortized $O(n)$ 에 해결할 수 있음

```cpp
struct CHTLinear {
  struct Line {
    ll a, b;
    ll y(ll x) const { return a * x + b; }
  };
  vector<Line> stk; int qpt;
  CHTLinear() : qpt(0) { }
  // when you need maximum :  (previous l).a < (now l).a
  // when you need minimum :  (previous l).a > (now l).a
  void pushLine(const Line& l) {
    while (stk.size() > 1) {
      Line& l0 = stk[stk.size() - 1]; Line& l1 = stk[stk.size() - 2];
      if ((l0.b - l.b) * (l0.a - l1.a) > (l1.b - l0.b) * (l.a - l0.a)) break;
      stk.pop_back();
    }
    stk.push_back(l);
  }
  // (previous x) <= (current x)
  // it calculates max/min at x
  ll query(ll x) {
    while (qpt + 1 < stk.size()) {
      Line& l0 = stk[qpt]; Line& l1 = stk[qpt + 1];
      if (l1.a - l0.a > 0 && (l0.b - l1.b) > x * (l1.a - l0.a)) break;
      if (l1.a - l0.a < 0 && (l0.b - l1.b) < x * (l1.a - l0.a)) break;
      ++qpt;
    }
    return stk[qpt].y(x);
  }
};
```

4.2 Divide & Conquer Optimization

$O(kn^2) \to O(kn \log n)$
조건 1) DP 점화식 꼴
$D[t][i] = \min_{j<i}(D[t-1][j] + C[j][i])$
조건 2) $A[t][i]$는 $D[t][i]$의 답이 되는 최소의 j라 할 때, 아래의 부등식을 만족해야 함
$A[t][i] \leq A[t][i+1]$
조건 2-1) 비용C가 다음의 사각부등식을 만족하는 경우도 조건 2)를 만족하게 됨
$C[a][c] + C[b][d] \leq C[a][d] + C[b][c]$ $(a \leq b \leq c \leq d)$

```cpp
//To get D[t][s...e] and range of j is [l, r]
void f(int t, int s, int e, int l, int r){
  if(s > e) return;
  int m = s + e >> 1; int opt = l;
  for(int i=l; i<=r; i++){
    if(D[t-1][opt] + C[opt][m] > D[t-1][i] + C[i][m]) opt = i;
  }
  D[t][m] = D[t-1][opt] + C[opt][m];
  f(t, s, m-1, l, opt); f(t, m+1, e, opt, r);
}
```

4.3 Knuth Optimization

$O(n^3) \to O(n^2)$

조건 1) DP 점화식 꼴
$D[i][j] = \min_{i<k<j}(D[i][k] + D[k][j]) + C[i][j]$
조건 2) 사각 부등식
$C[a][c] + C[b][d] \le C[a][d] + C[b][c] \ \ (a \le b \le c \le d)$
조건 3) 단조성
$C[b][c] \le C[a][d] \ \ (a \le b \le c \le d)$
결론) 조건 2, 3을 만족한다면 $A[i][j]$를 $D[i][j]$의 답이 되는 최소의 k라 할 때, 아래의 부등식을 만족하게 됨
$A[i][j-1] \le A[i][j] \le A[i+1][j]$
3중 루프를 돌릴 때 위 조건을 이용하면 최종적으로 시간복잡도가 $O(n^2)$ 이 됨

```
for (i = 1; i <= n; i++) {
  cin >> a[i];
  s[i] = s[i - 1] + a[i]; dp[i - 1][i] = 0; assist[i - 1][i] = i;
}
for (i = 2; i <= n; i++) {
  for (j = 0; j <= n - i; j++) {
    dp[j][i + j] = 1e9 + 7;
    for (k = assist[j][i + j - 1]; k <= assist[j + 1][i + j]; k++) {
      if (dp[j][i + j] > dp[j][k] + dp[k][i + j] + s[i + j] - s[j]) {
        dp[j][i + j] = dp[j][k] + dp[k][i + j] + s[i + j] - s[j];
        assist[j][i + j] = k;
      }
    }
  }
}
```

4.4 Bitset Optimization

```
#define private public
#include <bitset>
#undef private
#include <x86intrin.h>
template <size_t _Nw>
void _M_do_sub(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
  for (int i = 0, c = 0; i < _Nw; i++)
    c = _subborrow_u64(c, A._M_w[i], B._M_w[i], (unsigned long long *)&A._M_w[i]);
}
template <>
void _M_do_sub(_Base_bitset<1> &A, const _Base_bitset<1> &B) {
  A._M_w -= B._M_w;
}
template <size_t _Nb>
bitset<_Nb> &operator-=(bitset<_Nb> &A, const bitset<_Nb> &B) {
  _M_do_sub(A, B);
  return A;
}
template <size_t _Nb>
inline bitset<_Nb> operator-(const bitset<_Nb> &A, const bitset<_Nb> &B) {
  bitset<_Nb> C(A);
  return C -= B;
}
template <size_t _Nw>
void _M_do_add(_Base_bitset<_Nw> &A, const _Base_bitset<_Nw> &B) {
  for (int i = 0, c = 0; i < _Nw; i++)
    c = _addcarry_u64(c, A._M_w[i], B._M_w[i], (unsigned long long *)&A._M_w[i]);
}
template <>
void _M_do_add(_Base_bitset<1> &A, const _Base_bitset<1> &B) {
  A._M_w += B._M_w;
}
template <size_t _Nb>
bitset<_Nb> &operator+=(bitset<_Nb> &A, const bitset<_Nb> &B) {
  _M_do_add(A, B);
  return A;
}
template <size_t _Nb>
inline bitset<_Nb> operator+(const bitset<_Nb> &A, const bitset<_Nb> &B) {
  bitset<_Nb> C(A);
  return C += B;
}
```

4.5 Kitamasa & Berlekamp-Massey

```
// linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$
// Time: O(n^2 \log k)
ll get_nth(Poly S, Poly tr, ll k) {  // get kth term of recurrence
  int n = sz(tr);
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i, 0, n + 1) rep(j, 0, n + 1) res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i=2*n;i>n;--i)rep(j,0,n)res[i-1-j]=(res[i-1-j]+res[i]*tr[j])%mod;
    res.resize(n + 1);
    return res;
  };
  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  ll res = 0;
  rep(i, 0, n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
// Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
// Time: O(N^2)
vector<ll> berlekampMassey(vector<ll> s) {
  ll n = s.size(), L = 0, m = 0, d, coef, b = 1;
  vector<ll> C(n), B(n), T; C[0] = B[0] = 1;
  for (ll i = 0; i < n; i++) {
    ++m, d = s[i] % mod;
    for (ll j = 1; j <= L; j++) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C, coef = d * modpow(b, mod - 2) % mod;
    for (j = m; j < n; j++) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L, B = T, b = d, m = 0;
  }
  C.resize(L + 1), C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
ll guess_nth_term(vector<ll> x, lint n) {
  if (n < x.size()) return x[n];
  vector<ll> v = berlekamp_massey(x);
  if (v.empty()) return 0;
  return get_nth(v, x, n);
}
```

4.6 SOS(Subset of Sum) DP

```
//iterative version O(N*2^N) with TC, MC
for(int mask = 0; mask < (1<<N); ++mask){
  dp[mask][-1] = A[mask];  //handle base case separately (leaf states)
  for(int i = 0;i < N; ++i){
    if(mask & (1<<i)) dp[mask][i] = dp[mask][i-1] + dp[mask^(1<<i)][i-1];
    else dp[mask][i] = dp[mask][i-1];
  }
```

```
  F[mask] = dp[mask][N-1];
}
// toggling, O(N*2^N) with TC, O(2^N) with MC
for(int i = 0; i<(1<<N); ++i) F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
  if(mask & (1<<i)) F[mask] += F[mask^(1<<i)];
}
```

5 Graph

5.1 SCC

```
// find SCCs in given directed graph
// O(V+E)
// the order of scc_idx constitutes a reverse topological sort
auto get_scc = [](const auto& adj) { // 1-indexed
  const int n = adj.size() - 1; int dfs_cnt = 0, scc_cnt = 0;
  vector scc(n + 1, 0), dfn(n + 1, 0), s(0, 0);
  auto dfs = [&](const auto& self, int cur) -> int {
    int ret = dfn[cur] = ++dfs_cnt; s.push_back(cur);
    for (int nxt : adj[cur]) {
      if (!dfn[nxt]) ret = min(ret, self(self, nxt));
      else if (!scc[nxt]) ret = min(ret, dfn[nxt]);
    }
    if (ret == dfn[cur]) {
      scc_cnt++;
      while (s.size()) {
        int x = s.back(); s.pop_back(); scc[x] = scc_cnt;
        if (x == cur) break;
      }
    }
    return ret;
  };
  for (int i = 1; i <= n; i++) if (!dfn[i]) dfs(dfs, i);
  return pair(scc_cnt, scc);
};
```

5.2 2-SAT

boolean variable b_i 마다 b_i를 나타내는 정점, $\neg b_i$를 나타내는 정점 2개를 만듦. 각 clause $b_i \vee b_j$ 마다 $\neg b_i \to b_j$, $\neg b_j \to b_i$ 이렇게 edge를 이어줌. 그렇게 만든 그래프에서 SCC를 다 구함. 어떤 SCC 안에 b_i 와 $\neg b_i$가 같이 포함되어있다면 해가 존재하지 않음. 아니라면 해가 존재함. 해가 존재할 때 구체적인 해를 구하는 방법. 위에서 SCC를 구하면서 SCC DAG를 만들어준다. 거기서 위상정렬을 한 후, 앞에서부터 SCC를 하나씩 봐준다. 현재 보고있는 SCC에 b_i가 속해있는데 얘가 $\neg b_i$보다 먼저 등장했다면 b_i = false, 반대의 경우라면 b_i = true, 이미 값이 assign되었다면 pass.

5.3 BCC, Cut vertex, Bridge

```
const int MAXN = 100;
vector<pair<int, int>> graph[MAXN];  // { next vertex id, edge id }
int up[MAXN], visit[MAXN], vtime;
vector<int> stk;

int is_cut[MAXN];             // v is cut vertex if  is_cut[v] > 0
vector<int> bridge;           // list of edge ids
vector<int> bcc_edges[MAXN];  // list of edge ids in a bcc
int bcc_cnt;

void dfs(int nod, int par_edge) {
    up[nod] = visit[nod] = ++vtime;
    int child = 0;
    for (const auto& e : graph[nod]) {
        int next = e.first, eid = e.second;
        if (eid == par_edge) continue;
        if (visit[next] == 0) {
            stk.push_back(eid);
            ++child;
            dfs(next, eid);
            if (up[next] == visit[next]) bridge.push_back(eid);
            if (up[next] >= visit[nod]) {
                ++bcc_cnt;
                do {
                    auto lasteid = stk.back();
                    stk.pop_back();
                    bcc_edges[bcc_cnt].push_back(lasteid);
                    if (lasteid == eid) break;
                } while (!stk.empty());
                is_cut[nod]++;
            }
            up[nod] = min(up[nod], up[next]);
        }
        else if (visit[next] < visit[nod]) {
            stk.push_back(eid);
            up[nod] = min(up[nod], visit[next]);
        }
    }
    if (par_edge == -1 && is_cut[nod] == 1)
        is_cut[nod] = 0;
}

// find BCCs & cut vertexs & bridges in undirected graph
// O(V+E)
void get_bcc() {
    vtime = 0;
    memset(visit, 0, sizeof(visit));
    memset(is_cut, 0, sizeof(is_cut));
    bridge.clear();
    for (int i = 0; i < n; ++i) bcc_edges[i].clear();
    bcc_cnt = 0;
    for (int i = 0; i < n; ++i) {
        if (visit[i] == 0)
            dfs(i, -1);
    }
}
```

5.4 Block-cut Tree

각 BCC 및 cut vertex가 block-cut tree의 vertex가 되며, BCC와 그 BCC에 속한 cut vertex 사이에 edge를 이어주면 된다.

5.5 Shortest Path Faster Algorithm

```
// shortest path faster algorithm
// average for random graph : O(E) , worst : O(VE)
const int MAXN = 20001;
const int INF = 100000000;
int n, m;
vector<pair<int, int>> graph[MAXN];
bool inqueue[MAXN];
int dist[MAXN];

void spfa(int st) {
    for (int i = 0; i < n; ++i) {
        dist[i] = INF;
    }
    dist[st] = 0;

    queue<int> q;
    q.push(st);
    inqueue[st] = true;
    while (!q.empty()) {
        int u = q.front();
        q.pop();
```

```
        inqueue[u] = false;
        for (auto& e : graph[u]) {
            if (dist[u] + e.second < dist[e.first]) {
                dist[e.first] = dist[u] + e.second;
                if (!inqueue[e.first]) {
                    q.push(e.first);
                    inqueue[e.first] = true;
                }
            }
        }
    }
}
```

5.6 Centroid Decomposition

```
// O(n lg n) for centroid decomposition
auto cent_decom = [](const auto& adj) {
  const int n = adj.size() - 1;
  vector sz(n + 1, 1), dep(n + 1, 0), par(n + 1, 0);
  auto dfs = [&](const auto& self, int cur, int prv) -> void {
    for (auto [nxt, cost] : adj[cur]) {
      if (nxt == prv) continue;
      self(self, nxt, cur);
      sz[cur] += sz[nxt];
    }
  };
  auto adjust = [&](int cur) {
    while (1) {
      int f = 0;
      for (auto [nxt, cost] : adj[cur]) {
        if (dep[nxt] || sz[cur] >= 2 * sz[nxt]) continue;
        sz[cur] -= sz[nxt], sz[nxt] += sz[cur];
        cur = nxt, f = 1;
        break;
      }
      if (!f) return cur;
    }
  };
  auto rec = [&](const auto& self, int cur, int prv) -> void {
    cur = adjust(cur);
    par[cur] = prv;
    dep[cur] = dep[prv] + 1;
    for (auto [nxt, cost] : adj[cur]) {
      if (dep[nxt]) continue;
      self(self, nxt, cur);
    }
  };
  dfs(dfs, 1, 0);
  rec(rec, 1, 0);
  return pair(dep, par);
};
```

5.7 Lowest Common Ancestor

```
const int MAXN = 100;
const int MAXLN = 9;
vector<int> tree[MAXN];
int depth[MAXN];
int par[MAXLN][MAXN];

void dfs(int nod, int parent) {
    for (int next : tree[nod]) {
        if (next == parent) continue;
        depth[next] = depth[nod] + 1;
        par[0][next] = nod;
        dfs(next, nod);
    }
}

void prepare_lca() {
    const int root = 0;
    dfs(root, -1);
    par[0][root] = root;
    for (int i = 1; i < MAXLN; ++i)
        for (int j = 0; j < n; ++j)
            par[i][j] = par[i - 1][par[i - 1][j]];
}

// find lowest common ancestor in tree between u & v
// assumption : must call 'prepare_lca' once before call this
// O(logV)
int lca(int u, int v) {
    if (depth[u] < depth[v]) swap(u, v);
    if (depth[u] > depth[v]) {
        for (int i = MAXLN - 1; i >= 0; --i)
            if (depth[u] - (1 << i) >= depth[v])
                u = par[i][u];
    }
    if (u == v) return u;
    for (int i = MAXLN - 1; i >= 0; --i) {
        if (par[i][u] != par[i][v]) {
            u = par[i][u];
            v = par[i][v];
        }
    }
    return par[0][u];
}
```

5.8 Heavy-Light Decomposition

```
// heavy-light decomposition in O(n)
auto get_hld = [](auto adj) {
  const int n = adj.size() - 1;
  int ord = 0;
  vector sz(n + 1, 1), dep(n + 1, 0), par(n + 1, 0);
  vector in(n + 1, 0), out(n + 1, 0), top(n + 1, 0);
  auto dfs1 = [&](const auto& self, int cur, int prv) -> void {
    if (prv) adj[cur].erase(ranges::find(adj[cur], prv));
    for (int& nxt : adj[cur]) {
      dep[nxt] = dep[cur] + 1;
      par[nxt] = cur;
      self(self, nxt, cur);
      sz[cur] += sz[nxt];
      if (sz[adj[cur][0]] < sz[nxt]) swap(adj[cur][0], nxt);
    }
  };
  auto dfs2 = [&](const auto& self, int cur) -> void {
    in[cur] = ++ord;
    for (int nxt : adj[cur]) {
      top[nxt] = adj[cur][0] == nxt ? top[cur] : nxt;
      self(self, nxt);
    }
    out[cur] = ord;
  };
  dfs1(dfs1, 1, 0);
  dfs2(dfs2, top[1] = 1);
  return tuple(sz, dep, par, in, out, top);
};
```

5.9 Hall's Theorem

- Let $G = (L \cup R, E)$ be a bipartite graph. For $S \subseteq L$, let $N(S) \subseteq R$ be the set of vertices adjacent to some vertex in S. Then, $\exists M$ matching in G that covers all vertex of $L \Leftrightarrow \forall S \subseteq L, |S| \leq |N(S)|$

- Hall's Theorem is equivalent to the following statement: Let $S = \{S_1, S_2, \ldots, S_n\}$ be a set of sets. Then, we can choose $x_i \in S_i$ for all i such that $x_i \neq x_j$ for all $i \neq j$ iff. $\forall T \subseteq \{1, 2, \ldots, n\}, |\bigcup_{i \in T} S_i| \geq |T|$.

5.10 Stable Marriage

```cpp
// man : 1~n, woman : n+1~2n, O(n^2) stable marriage
struct StableMarriage{
  int n; vector<vector<int>> g;
  StableMarriage(int n) : n(n), g(2*n+1) { for(int i=1; i<=n+n; i++) g[i].reserve(n); }
  void addEdge(int u, int v){  g[u].push_back(v); } // insert in decreasing order of preference.
  vector<int> run(){
    queue<int> q; vector<int> match(2*n+1), ptr(2*n+1);
    for(int i=1; i<=n; i++) q.push(i);
    while(q.size()){
      int i = q.front(); q.pop();
      for(int &p=ptr[i]; p<g[i].size(); p++){
        int j = g[i][p];
        if(!match[j]){ match[i] = j; match[j] = i; break; }
        int m = match[j], u = -1, v = -1;
        for(int k=0; k<g[j].size(); k++){
          if(g[j][k] == i) u = k; if(g[j][k] == m) v = k;
        }
        if(u < v){
          match[m] = 0; q.push(m); match[i] = j; match[j] = i; break;
    } /*if u < v*/ } /*for-p*/ } /*while*/
    return match; } /*vector<int> run*/
};
```

5.11 Bipartite Matching (Kuhn)

```cpp
auto bipartite_matching = [](const auto& adj) { // O(VE)
  const int n = adj.size() - 1;
  vector par(n + 1, 0), c(n + 1, 0);
  auto dfs = [&](const auto& self, int cur) -> bool {
    if (c[cur]++) return 0;
    for (int nxt : adj[cur])
      if (!par[nxt] || self(self, par[nxt]))
        return par[nxt] = cur, 1;
    return 0;
  };
  int res = 0;
  for (int i = 1; i <= n; i++) {
    fill(c.begin(), c.end(), 0);
    if (dfs(dfs, i)) res++;
  }
  return res;
};
```

5.12 Maximum Flow (Dinic)

```cpp
// usage:
// MaxFlowDinic::init(n);
// MaxFlowDinic::add_edge(0, 1, 100, 100); // for bidirectional edge
// MaxFlowDinic::add_edge(1, 2, 100); // directional edge
// result = MaxFlowDinic::solve(0, 2); // source -> sink
// graph[i][edgeIndex].res -> residual
//
// in order to find out the minimum cut, use `l'.
// if l[i] == 0, i is unrechable.
//
// O(V*V*E)
// with unit capacities, O(min(V^(2/3), E^(1/2)) * E)
struct MaxFlowDinic {
    typedef int flow_t;
    struct Edge {
        int next;
        size_t inv; /* inverse edge index */
        flow_t res; /* residual */
    };
    int n;
    vector<vector<Edge>> graph;
    vector<int> q, l, start;

    void init(int _n) {
        n = _n;
        graph.resize(n);
        for (int i = 0; i < n; i++) graph[i].clear();
    }
    void add_edge(int s, int e, flow_t cap, flow_t caprev = 0) {
        Edge forward{ e, graph[e].size(), cap };
        Edge reverse{ s, graph[s].size(), caprev };
        graph[s].push_back(forward);
        graph[e].push_back(reverse);
    }
    bool assign_level(int source, int sink) {
        int t = 0;
        memset(&l[0], 0, sizeof(l[0]) * l.size());
        l[source] = 1;
        q[t++] = source;
        for (int h = 0; h < t && !l[sink]; h++) {
            int cur = q[h];
            for (const auto& e : graph[cur]) {
                if (l[e.next] || e.res == 0) continue;
                l[e.next] = l[cur] + 1;
                q[t++] = e.next;
            }
        }
        return l[sink] != 0;
    }
    flow_t block_flow(int cur, int sink, flow_t current) {
        if (cur == sink) return current;
        for (int& i = start[cur]; i < graph[cur].size(); i++) {
            auto& e = graph[cur][i];
            if (e.res == 0 || l[e.next] != l[cur] + 1) continue;
            if (flow_t res = block_flow(e.next, sink, min(e.res, current))) {
                e.res -= res;
                graph[e.next][e.inv].res += res;
                return res;
            }
        }
        return 0;
    }
    flow_t solve(int source, int sink) {
        q.resize(n);
        l.resize(n);
        start.resize(n);
        flow_t ans = 0;
        while (assign_level(source, sink)) {
            memset(&start[0], 0, sizeof(start[0]) * n);
            while (flow_t flow = block_flow(source, sink, numeric_limits<flow_t>::max()))
                ans += flow;
        }
        return ans;
    }
};
```

5.13 Maximum Flow with Edge Demands

그래프 $G = (V, E)$ 가 있고 source s와 sink t가 있다. 각 간선마다 $d(e) \leq f(e) \leq c(e)$ 를 만족하도록 flow $f(e)$를 흘려야 한다. 이 때의 maximum flow를 구하는 문제다.
먼저 모든 demand를 합한 값 D를 아래와 같이 정의한다.

$$D = \sum_{(u \to v) \in E} d(u \to v)$$

이제 G 에 몇개의 정점과 간선을 추가하여 새로운 그래프 $G' = (V', E')$ 을 만들 것이다. 먼저 새로운 source s' 과 새로운 sink t' 을 추가한다. 그리고 s' 에서 V 의 모든 점마다 간선을 이어주고, V 의 모든 점에서 t' 로 간선을 이어준다.
새로운 capacity function c' 을 아래와 같이 정의한다.

1. V 의 점 v에 대해 $c'(s' \to v) = \sum_{u \in V} d(u \to v)$, $c'(v \to t') = \sum_{w \in V} d(v \to w)$
2. E 의 간선 $u \to v$에 대해 $c'(u \to v) = c(u \to v) - d(u \to v)$
3. $c'(t \to s) = \infty$

이렇게 만든 새로운 그래프 G' 에서 maximum flow를 구했을 때 그 값이 D 라면 원래 문제의 해가 존재하고, 그 값이 D가 아니라면 원래 문제의 해는 존재하지 않는다.
위에서 maximum flow를 구하고 난 상태의 residual graph 에서 s' 과 t' 을 떼버리고 s 에서 t 사이의 augument path 를 계속 찾으면 원래 문제의 해를 구할 수 있다.

```
struct MaxFlowEdgeDemands
{
    MaxFlowDinic mf;
    using flow_t = MaxFlowDinic::flow_t;

    vector<flow_t> ind, outd;
    flow_t D; int n;

    void init(int _n) {
        n = _n; D = 0; mf.init(n + 2);
        ind.clear(); outd.clear();
        ind.resize(n, 0); outd.resize(n, 0);
    }

    void add_edge(int s, int e, flow_t cap, flow_t demands = 0) {
        mf.add_edge(s, e, cap - demands);
        D += demands; ind[e] += demands; outd[s] += demands;
    }

    // returns { false, 0 } if infeasible
    // { true, maxflow } if feasible
    pair<bool, flow_t> solve(int source, int sink) {
        mf.add_edge(sink, source, numeric_limits<flow_t>::max());

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.add_edge(n, i, ind[i]);
            if (outd[i]) mf.add_edge(i, n + 1, outd[i]);
        }

        if (mf.solve(n, n + 1) != D) return{ false, 0 };

        for (int i = 0; i < n; i++) {
            if (ind[i]) mf.graph[i].pop_back();
            if (outd[i]) mf.graph[i].pop_back();
        }

        return{ true, mf.solve(source, sink) };
    }
};
```

5.14 Min-cost Maximum Flow

```
// precondition: there is no negative cycle.
// usage:
// MinCostFlow mcf(n);
// for(each edges) mcf.addEdge(from, to, cost, capacity);
// mcf.solve(source, sink); // min cost max flow
// mcf.solve(source, sink, 0); // min cost flow
// mcf.solve(source, sink, goal_flow); // min cost flow with total_flow >= goal_flow if possible
struct MinCostFlow {
    typedef int cap_t;
    typedef int cost_t;

    bool iszerocap(cap_t cap) { return cap == 0; }

    struct edge {
        int target;
        cost_t cost;
        cap_t residual_capacity;
        cap_t orig_capacity;
        size_t revid;
    };

    int n;
    vector<vector<edge>> graph;

    MinCostFlow(int n) : graph(n), n(n) {}

    void addEdge(int s, int e, cost_t cost, cap_t cap) {
        if (s == e) return;
        edge forward{ e, cost, cap, cap, graph[e].size() };
        edge backward{ s, -cost, 0, 0, graph[s].size() };
        graph[s].emplace_back(forward);
        graph[e].emplace_back(backward);
    }

    pair<cost_t, cap_t> augmentShortest(int s, int e, cap_t flow_limit) {
        auto infinite_cost = numeric_limits<cost_t>::max();
        auto infinite_flow = numeric_limits<cap_t>::max();
        vector<pair<cost_t, cap_t>> dist(n, make_pair(infinite_cost, 0));
        vector<int> from(n, -1), v(n);

        dist[s] = pair<cost_t, cap_t>(0, infinite_flow);
        queue<int> q;
        v[s] = 1; q.push(s);
        while(!q.empty()) {
            int cur = q.front();
            v[cur] = 0; q.pop();
            for (const auto& e : graph[cur]) {
                if (iszerocap(e.residual_capacity)) continue;
                auto next = e.target;
                auto ncost = dist[cur].first + e.cost;
                auto nflow = min(dist[cur].second, e.residual_capacity);
                if (dist[next].first > ncost) {
                    dist[next] = make_pair(ncost, nflow);
                    from[next] = e.revid;
                    if (v[next]) continue;
                    v[next] = 1; q.push(next);
                }
            }
        }

        auto p = e;
        auto pathcost = dist[p].first;
        auto flow = dist[p].second;
        if (iszerocap(flow)|| (flow_limit <= 0 && pathcost >= 0)) return pair<cost_t, cap_t>(0, 0)
            ;
        if (flow_limit > 0) flow = min(flow, flow_limit);

        while (from[p] != -1) {
            auto nedge = from[p];
            auto np = graph[p][nedge].target;
```

```
            auto fedge = graph[p][nedge].revid;
            graph[p][nedge].residual_capacity += flow;
            graph[np][fedge].residual_capacity -= flow;
            p = np;
        }
        return make_pair(pathcost * flow, flow);
    }

    pair<cost_t,cap_t> solve(int s, int e, cap_t flow_minimum = numeric_limits<cap_t>::max()) {
        cost_t total_cost = 0;
        cap_t total_flow = 0;
        for(;;) {
            auto res = augmentShortest(s, e, flow_minimum - total_flow);
            if (res.second <= 0) break;
            total_cost += res.first;
            total_flow += res.second;
        }
        return make_pair(total_cost, total_flow);
    }
};
```

5.15 General Min-cut (Stoer-Wagner)

```
// implementation of Stoer-Wagner algorithm
// O(V^3)
//usage
// MinCut mc;
// mc.init(n);
// for (each edge) mc.addEdge(a,b,weight);
// mincut = mc.solve();
// mc.cut = {0,1}^n describing which side the vertex belongs to.
struct MinCutMatrix
{
    typedef int cap_t;
    int n;
    vector<vector<cap_t>> graph;

    void init(int _n) {
        n = _n;
        graph = vector<vector<cap_t>>(n, vector<cap_t>(n, 0));
    }
    void addEdge(int a, int b, cap_t w) {
        if (a == b) return;
        graph[a][b] += w;
        graph[b][a] += w;
    }

    pair<cap_t, pair<int, int>> stMinCut(vector<int> &active) {
        vector<cap_t> key(n);
        vector<int> v(n);
        int s = -1, t = -1;
        for (int i = 0; i < active.size(); i++) {
            cap_t maxv = -1;
            int cur = -1;
            for (auto j : active) {
                if (v[j] == 0 && maxv < key[j]) {
                    maxv = key[j];
                    cur = j;
                }
            }
            t = s; s = cur;
            v[cur] = 1;
            for (auto j : active) key[j] += graph[cur][j];
        }
        return make_pair(key[s], make_pair(s, t));
    }

    vector<int> cut;

    cap_t solve() {
        cap_t res = numeric_limits<cap_t>::max();
        vector<vector<int>> grps;
        vector<int> active;
        cut.resize(n);
        for (int i = 0; i < n; i++) grps.emplace_back(1, i);
        for (int i = 0; i < n; i++) active.push_back(i);
        while (active.size() >= 2) {
            auto stcut = stMinCut(active);
            if (stcut.first < res) {
                res = stcut.first;
                fill(cut.begin(), cut.end(), 0);
                for (auto v : grps[stcut.second.first]) cut[v] = 1;
            }

            int s = stcut.second.first, t = stcut.second.second;
            if (grps[s].size() < grps[t].size()) swap(s, t);

            active.erase(find(active.begin(), active.end(), t));
            grps[s].insert(grps[s].end(), grps[t].begin(), grps[t].end());
            for (int i = 0; i < n; i++) { graph[i][s] += graph[i][t]; graph[i][t] = 0; }
            for (int i = 0; i < n; i++) { graph[s][i] += graph[t][i]; graph[t][i] = 0; }
            graph[s][s] = 0;
        }
        return res;
    }
};
```

5.16 Hungarian Algorithm

```
int n, m;
int mat[MAX_N + 1][MAX_M + 1];

// hungarian method : bipartite min-weighted matching
// O(n^3) or O(m*n^2)
// http://e-maxx.ru/algo/assignment_hungary
// mat[1][1] ~ mat[n][m]
// matched[i] : matched column of row i
int hungarian(vector<int>& matched) {
    vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1), minv(m + 1);
    vector<char> used(m + 1);
    for (int i = 1; i <= n; ++i) {
        p[0] = i;
        int j0 = 0;
        fill(minv.begin(), minv.end(), INF);
        fill(used.begin(), used.end(), false);
        do {
            used[j0] = true;
            int i0 = p[j0], delta = INF, j1;
            for (int j = 1; j <= m; ++j) {
                if (!used[j]) {
                    int cur = mat[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
            }
            for (int j = 0; j <= m; ++j) {
                if (used[j])
                    u[p[j]] += delta, v[j] -= delta;
                else
                    minv[j] -= delta;
            }
            j0 = j1;
```

```
        } while (p[j0] != 0);
        do {
            int j1 = way[j0];
            p[j0] = p[j1];
            j0 = j1;
        } while (j0);
    }
    for (int j = 1; j <= m; ++j) matched[p[j]] = j;
    return -v[0];
}
```

5.17 General Unweighted Maximum Matching(Tutte)

그래프 $G = (V, E)$에 대해 랜덤한 소수 p를 골라 다음과 같은 $|V| \times |V|$ 행렬 T를 만들자. 이 때 $r_{i,j}$는 $[1, p-1]$ 사이의 랜덤한 정수이다. 최대 매칭의 크기는 높은 확률로 $rank(T)/2$이다.

$$T_{i,j} = \begin{cases} r_{i,j} & \text{if } (i,j) \in E \wedge i < j \\ r_{j,i} & \text{if } (i,j) \in E \text{ and } i > j \\ 0 & \text{otherwise} \end{cases}$$

5.18 General Weighted Maximum Matching(Blossom)

```
// O(N^3) (but fast in practice)
static const int INF = INT_MAX;
static const int N = 514;
struct edge{
  int u,v,w; edge(){}
  edge(int ui,int vi,int wi)
    :u(ui),v(vi),w(wi){}
};
int n,n_x;
edge g[N*2][N*2];
int lab[N*2];
int match[N*2],slack[N*2],st[N*2],pa[N*2];
int flo_from[N*2][N+1],S[N*2],vis[N*2];
vector<int> flo[N*2];
queue<int> q;
int e_delta(const edge &e){
  return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
}
void update_slack(int u,int x){
  if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x]))slack[x]=u;
}
void set_slack(int x){
  slack[x]=0;
  for(int u=1;u<=n;++u)
    if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
      update_slack(u,x);
}
void q_push(int x){
  if(x<=n)q.push(x);
  else for(size_t i=0;i<flo[x].size();i++)
    q_push(flo[x][i]);
}
void set_st(int x,int b){
  st[x]=b;
  if(x>n)for(size_t i=0;i<flo[x].size();++i)
    set_st(flo[x][i],b);
}
int get_pr(int b,int xr){
  int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
  if(pr%2==1){
    reverse(flo[b].begin()+1,flo[b].end());
    return (int)flo[b].size()-pr;
  }else return pr;
}
void set_match(int u,int v){
  match[u]=g[u][v].v;
  if(u<=n) return;
  edge e=g[u][v];
  int xr=flo_from[u][e.u],pr=get_pr(u,xr);
  for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i^1]);
  set_match(xr,v);
  rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end());
}
void augment(int u,int v){
  for(;;){
    int xnv=st[match[u]];
    set_match(u,v);
    if(!xnv)return;
    set_match(xnv,st[pa[xnv]]);
    u=st[pa[xnv]],v=xnv;
  }
}
int get_lca(int u,int v){
  static int t=0;
  for(++t;u||v;swap(u,v)){
    if(u==0)continue;
    if(vis[u]==t)return u;
    vis[u]=t;
    u=st[match[u]];
    if(u)u=st[pa[u]];
  }
  return 0;
}
void add_blossom(int u,int lca,int v){
  int b=n+1;
  while(b<=n_x&&st[b])++b;
  if(b>n_x)++n_x;
  lab[b]=0,S[b]=0;
  match[b]=match[lca];
  flo[b].clear();
  flo[b].push_back(lca);
  for(int x=u,y;x!=lca;x=st[pa[y]])
    flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q_push(y);
  reverse(flo[b].begin()+1,flo[b].end());
  for(int x=v,y;x!=lca;x=st[pa[y]])
    flo[b].push_back(x),flo[b].push_back(y=st[match[x]]),q_push(y);
  set_st(b,b);
  for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
  for(int x=1;x<=n;++x)flo_from[b][x]=0;
  for(size_t i=0;i<flo[b].size();++i){
    int xs=flo[b][i];
    for(int x=1;x<=n_x;++x)
      if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b][x]))
        g[b][x]=g[xs][x],g[x][b]=g[x][xs];
    for(int x=1;x<=n;++x)
      if(flo_from[xs][x])flo_from[b][x]=xs;
  }
  set_slack(b);
}
void expand_blossom(int b){
  for(size_t i=0;i<flo[b].size();++i)
    set_st(flo[b][i],flo[b][i]);
  int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
  for(int i=0;i<pr;i+=2){
    int xs=flo[b][i],xns=flo[b][i+1];
    pa[xs]=g[xns][xs].u;
    S[xs]=1,S[xns]=0;
    slack[xs]=0,set_slack(xns);
    q_push(xns);
  }
```

```
  S[xr]=1,pa[xr]=pa[b];
  for(size_t i=pr+1;i<flo[b].size();++i){
    int xs=flo[b][i];
    S[xs]=-1,set_slack(xs);
  }
  st[b]=0;
}
bool on_found_edge(const edge &e){
  int u=st[e.u],v=st[e.v];
  if(S[v]==-1){
    pa[v]=e.u,S[v]=1;
    int nu=st[match[v]];
    slack[v]=slack[nu]=0;
    S[nu]=0,q_push(nu);
  }else if(S[v]==0){
    int lca=get_lca(u,v);
    if(!lca)return augment(u,v),augment(v,u),true;
    else add_blossom(u,lca,v);
  }
  return false;
}
bool matching(){
  memset(S+1,-1,sizeof(int)*n_x);
  memset(slack+1,0,sizeof(int)*n_x);
  q=queue<int>();
  for(int x=1;x<=n_x;++x)
    if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
  if(q.empty())return false;
  for(;;){
    while(q.size()){
      int u=q.front();q.pop();
      if(S[st[u]]==1)continue;
      for(int v=1;v<=n;++v)
        if(g[u][v].w>0&&st[u]!=st[v]){
          if(e_delta(g[u][v])==0){
            if(on_found_edge(g[u][v]))return true;
          }else update_slack(u,st[v]);
        }
    }
    int d=INF;
    for(int b=n+1;b<=n_x;++b)
      if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
    for(int x=1;x<=n_x;++x)
      if(st[x]==x&&slack[x]){
        if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
        else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2);
      }
    for(int u=1;u<=n;++u){
      if(S[st[u]]==0){
        if(lab[u]<=d)return 0;
        lab[u]-=d;
      }else if(S[st[u]]==1)lab[u]+=d;
    }
    for(int b=n+1;b<=n_x;++b)
      if(st[b]==b){
        if(S[st[b]]==0)lab[b]+=d*2;
        else if(S[st[b]]==1)lab[b]-=d*2;
      }
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
      if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]][x])==0)
        if(on_found_edge(g[slack[x]][x]))return true;
    for(int b=n+1;b<=n_x;++b)
      if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(b);
  }
  return false;
}
pair<long long,int> solve(){
  memset(match+1,0,sizeof(int)*n);
  n_x=n;
  int n_matches=0;
  long long tot_weight=0;
  for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
  int w_max=0;
  for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v){
      flo_from[u][v]=(u==v?u:0);
      w_max=max(w_max,g[u][v].w);
    }
  for(int u=1;u<=n;++u)lab[u]=w_max;
  while(matching())++n_matches;
  for(int u=1;u<=n;++u)
    if(match[u]&&match[u]<u)
      tot_weight+=g[u][match[u]].w;
  return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
  g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
  n = _n;
  for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v)
      g[u][v]=edge(u,v,0);
}
```

5.19 Offline Dynamic Connectivity

```
struct OFDC{
  // offline dynamic connectivity in O(q lg^2 q)
  ll n,q; vector<ll>par, sz; vector<pll>query; stack<ll>st;
  vector<vector<pll>>tree; map<pll,ll>at;
  void update(ll node, ll tl, ll tr, ll l, ll r, pll v){
    if(r<tl||tr<l)return;
    if(l<=tl&&tr<=r) { tree[node].push_back(v); return; }
    ll tm=tl+tr>>1;
    update(node<<1,tl,tm,l,r,v); update(node<<1|1,tm+1,tr,l,r,v);
  }
  ll _find(ll x){ return x==par[x]?x:_find(par[x]); }
  bool _same(pll a){return _find(a.first)==_find(a.second);}
  bool _union(ll x, ll y){
    x=_find(x),y=_find(y); if(x==y)return false;
    if(sz[x]<sz[y])swap(x,y); par[y]=x; sz[x]+=sz[y]; st.push(y);
    return true;
  }
  void _delete(){
    if(st.empty())return;
    ll x=st.top(); st.pop(); sz[par[x]]-=sz[x]; par[x]=x;
  }
  void dfs(ll node, ll tl, ll tr){
    ll cnt=0;
    for(auto [x,y]:tree[node]) if(_union(x,y)) cnt++;
    if(tl==tr){if(query[tl].first)cout<<(_same(query[tl])?1:0)<<'\n';}
    else{ll tm=tl+tr>>1;dfs(node<<1,tl,tm), dfs(node<<1|1,tm+1,tr);}
    while(cnt--)_delete();
  }
  void run(ll _n, ll _q, vector<tlll>_query){
    // 1 : add, 2 : del, 3 : query
    n=_n, q=_q; query.resize(q); tree.resize(q<<2|1),par.resize(n+1);
    sz.assign(n+1,1),iota(par.begin(),par.end(),0);
    for(ll i=0;i<q;i++){
      auto [op,a,b]=_query[i]; if(a>b)swap(a,b);
```

```
      if(op==1) at[{a,b}]=i;
      else if(op==2) update(1,0,q-1,at[{a,b}],i,{a,b}), at.erase({a,b});
      else query[i]={a,b};
    }
    for(auto [x,y]:at) update(1,0,q-1,y,q-1,x);
    dfs(1,0,q-1);
  }
};
```

6 Geometry

6.1 Basic Operations

```
const ld eps = 1e-12;
inline ll diff(ld lhs, ld rhs) {
  if (lhs - eps < rhs && rhs < lhs + eps) return 0;
  return (lhs < rhs) ? -1 : 1;
}
inline bool is_between(ld check, ld a, ld b) {
  return (a < b) ? (a - eps < check && check < b + eps)
                 : (b - eps < check && check < a + eps);
}
struct Point {
  ld x, y;
  bool operator==(const Point& rhs) const {
    return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
  }
  Point operator+(const Point& rhs) const { return Point{x + rhs.x, y + rhs.y}; }
  Point operator-(const Point& rhs) const { return Point{x - rhs.x, y - rhs.y}; }
  Point operator*(ld t) const { return Point{x * t, y * t}; }
  int pos() const {
    if (y < 0) return -1;
    if (y == 0 && 0 <= x) return 0;
    return 1;
  }
  bool operator<(Point r) const { // sort by angle, ccw order from half line ≤x0,y=0
      if (pos() != r.pos()) return pos() < r.pos();
      return 0 < (x * r.y - y * r.x);
  }
  Point rotate(ld theta) const {// rotate ccw by theta
    return Point{x * cos(theta) - y * sin(theta), x * sin(theta) + y * cos(theta)};
  }
};
struct Circle {
  Point center;
  ld r;
};
struct Line {
  Point pos, dir;
};
inline ld inner(const Point& a, const Point& b) { return a.x * b.x + a.y * b.y; }
inline ld outer(const Point& a, const Point& b) { return a.x * b.y - a.y * b.x; }
inline ll ccw_line(const Line& line, const Point& point) {
  return diff(outer(line.dir, point - line.pos), 0);
}
inline ll ccw(const Point& a, const Point& b, const Point& c) {
  return diff(outer(b - a, c - a), 0);
}
inline ld dist(const Point& a, const Point& b) { return sqrt(inner(a - b, a - b)); }
inline ld dist2(const Point& a, const Point& b) { return inner(a - b, a - b); }
inline ld dist(const Line& line, const Point& point, bool segment = false) {
  ld c1 = inner(point - line.pos, line.dir);
  if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);
  ld c2 = inner(line.dir, line.dir);
  if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);
  return dist(line.pos + line.dir * (c1 / c2), point);
}
bool get_cross(const Line& a, const Line& b, Point& ret) {
  ld mdet = outer(b.dir, a.dir);
  if (diff(mdet, 0) == 0) return false;
  ld t2 = outer(a.dir, b.pos - a.pos) / mdet;
  ret = b.pos + b.dir * t2;
  return true;
}
bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
  ld mdet = outer(b.dir, a.dir);
  if (diff(mdet, 0) == 0) return false;
  ld t1 = -outer(b.pos - a.pos, b.dir) / mdet;
  ld t2 = outer(a.dir, b.pos - a.pos) / mdet;
  if (!is_between(t1, 0, 1) || !is_between(t2, 0, 1)) return false;
  ret = b.pos + b.dir * t2;
  return true;
}
Point inner_center(const Point& a, const Point& b, const Point& c) {
  ld wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
  ld w = wa + wb + wc;
  return Point{(wa * a.x + wb * b.x + wc * c.x) / w,
               (wa * a.y + wb * b.y + wc * c.y) / w};
}
Point outer_center(const Point& a, const Point& b, const Point& c) {
  Point d1 = b - a, d2 = c - a;
  ld area = outer(d1, d2);
  ld dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y + d1.y * d2.y * (d1.y - d2.y);
  ld dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x + d1.x * d2.x * (d1.x - d2.x);
  return Point{a.x + dx / area / 2.0, a.y - dy / area / 2.0};
}
vector<Point> circle_line(const Circle& circle, const Line& line) {
  vector<Point> result;
  ld a = 2 * inner(line.dir, line.dir);
  ld b = 2 * (line.dir.x * (line.pos.x - circle.center.x) +
              line.dir.y * (line.pos.y - circle.center.y));
  ld c = inner(line.pos - circle.center, line.pos - circle.center) - circle.r * circle.r;
  ld det = b * b - 2 * a * c;
  ll pred = diff(det, 0);
  if (pred == 0)
    result.push_back(line.pos + line.dir * (-b / a));
  else if (pred > 0) {
    det = sqrt(det);
    result.push_back(line.pos + line.dir * ((-b + det) / a));
    result.push_back(line.pos + line.dir * ((-b - det) / a));
  }
  return result;
}
vector<Point> circle_circle(const Circle& a, const Circle& b) {
  vector<Point> result;
  ll pred = diff(dist(a.center, b.center), a.r + b.r);
  if (pred > 0) return result;
  if (pred == 0) {
    result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
    return result;
  }
  ld aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
  ld bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
  ld tmp = (bb - aa) / 2.0;
  Point cdiff = b.center - a.center;
  if (diff(cdiff.x, 0) == 0) {
    if (diff(cdiff.y, 0) == 0) return result;
    return circle_line(a, Line{Point{0, tmp / cdiff.y}, Point{1, 0}});
  }
  return circle_line(a, Line{Point{tmp / cdiff.x, 0}, Point{-cdiff.y, cdiff.x}});
}
Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
  Point ba = b - a, cb = c - b;
```

```
  Line p{(a + b) * 0.5, Point{ba.y, -ba.x}};
  Line q{(b + c) * 0.5, Point{cb.y, -cb.x}};
  Circle circle;
  if (!get_cross(p, q, circle.center))
    circle.r = -1;
  else
    circle.r = dist(circle.center, a);
  return circle;
}
Circle circle_from_2pts_rad(const Point& a, const Point& b, ld r) {
  ld det = r * r / dist2(a, b) - 0.25;
  Circle circle;
  if (det < 0)
    circle.r = -1;
  else {
    ld h = sqrt(det);
    // center is to the left of a->b
    circle.center = (a + b) * 0.5 + Point{a.y - b.y, b.x - a.x} * h;
    circle.r = r;
  }
  return circle;
}
Circle circle_from_2pts(const Point& a, const Point& b) {
  Circle circle;
  circle.center = (a + b) * 0.5;
  circle.r = dist(a, b) / 2;
  return circle;
}
```

6.2 Convex Hull & Rotating Calipers

```
// get all antipodal pairs with O(n)
// calculate convex hull with O(nlgn)
void antipodal_pairs(vector<Point>& pt, vector<Point>& convex_hull) {
  sort(pt.begin(), pt.end(), [](const Point& a, const Point& b) {
    return (a.x == b.x) ? a.y < b.y : a.x < b.x;
  });
  vector<Point> up, lo;
  for (const auto& p : pt) {
    while (up.size() >= 2 && ccw(*++up.rbegin(), *up.rbegin(), p) >= 0) up.pop_back();
    while (lo.size() >= 2 && ccw(*++lo.rbegin(), *lo.rbegin(), p) <= 0) lo.pop_back();
    up.push_back(p);
    lo.push_back(p);
  }
  for (int i = 0, j = (int)lo.size() - 1; i + 1 < up.size() || j > 0;) {
    get_pair(up[i], lo[j]);  // DO WHAT YOU WANT
    if (i + 1 == up.size()) --j;
    else if (j == 0) ++i;
    else if ((up[i + 1].y - up[i].y) * (lo[j].x - lo[j - 1].x) >
             (up[i + 1].x - up[i].x) * (lo[j].y - lo[j - 1].y))++i;
    else--j;
  }
  upper.insert(upper.end(), ++lower.rbegin(), --lower.rend());
  swap(upper, convex_hull);
}
```

6.3 Half Plane Intersection

```
typedef pair<ld, ld> pi;
bool z(ld x) { return fabs(x) < eps; }
struct line {
  ld a, b, c;
  bool operator<(const line &l) const {
    bool flag1 = pi(a, b) > pi(0, 0); bool flag2 = pi(l.a, l.b) > pi(0, 0);
    if (flag1 != flag2) return flag1 > flag2;
    ld t = ccw(pi(0, 0), pi(a, b), pi(l.a, l.b));
    return z(t) ? c * hypot(l.a, l.b) < l.c * hypot(a, b) : t > 0;
  }
  pi slope() { return pi(a, b); }
};
pi cross(line a, line b) {
  ld det = a.a * b.b - b.a * a.b;
  return pi((a.c * b.b - a.b * b.c) / det, (a.a * b.c - a.c * b.a) / det);
}
bool bad(line a, line b, line c) {
  if (ccw(pi(0, 0), a.slope(), b.slope()) <= 0) return false;
  pi crs = cross(a, b);
  return crs.first * c.a + crs.second * c.b >= c.c;
}
bool solve(vector<line> v, vector<pi> &solution) {  // ax + by <= c;
  sort(v.begin(), v.end());
  deque<line> dq;
  for (auto &i : v) {
    if (!dq.empty() && z(ccw(pi(0, 0), dq.back().slope(), i.slope()))) continue;
    while (dq.size() >= 2 && bad(dq[dq.size() - 2], dq.back(), i)) dq.pop_back();
    while (dq.size() >= 2 && bad(i, dq[0], dq[1])) dq.pop_front();
    dq.push_back(i);
  }
  while (dq.size() > 2 && bad(dq[dq.size() - 2], dq.back(), dq[0])) dq.pop_back();
  while (dq.size() > 2 && bad(dq.back(), dq[0], dq[1])) dq.pop_front();
  vector<pi> tmp;
  for (int i = 0; i < dq.size(); i++) {
    line cur = dq[i], nxt = dq[(i + 1) % dq.size()];
    if (ccw(pi(0, 0), cur.slope(), nxt.slope()) <= eps) return false;
    tmp.push_back(cross(cur, nxt));
  }
  solution = tmp;
  return true;
}
```

6.4 Minimum Permimeter Triangle

```
bool cmp_x(pt a, pt b) {return a.x < b.x;}
bool cmp_y(pt a, pt b) {return a.y < b.y;}
double dist(pt a, pt b) {return hypot(abs(a.x - b.x), abs(a.y - b.y));}
double perimeter(pt a, pt b, pt c) {return dist(a, b) + dist(b, c) + dist(c, a);}
double dac3(int l, int r) {
  // get the smallest triangle perimeter in pts[l, r]
  if (r - l <= 1) return INF;
  if (r - l == 2) return perimeter(pts[l], pts[l + 1], pts[l + 2]);
  int mid = (l + r) / 2;
  double d1 = dac3(l, mid), d2 = dac3(mid + 1, r);
  double ans = min(d1, d2);
  vector<pt> strip;
  for (int i = l; i <= r; i++) {
    if (abs(pts[i].x - pts[mid].x) < ans) strip.push_back(pts[i]);
  }
  sort(strip.begin(), strip.end(), cmp_y);
  for (int i = 0; i < strip.size(); i++) {
    for (int j = i + 1; j < strip.size() && (strip[j].y - strip[i].y) < ans; j++) {
      for (int k = j + 1; k < strip.size() && (strip[k].y - strip[j].y) < ans; k++) {
        ans = min(ans, perimeter(strip[i], strip[j], strip[k]));
      }
    }
  }
  return ans;
}
double closest_triple(vector<pt> &pts) {
  sort(pts.begin(), pts.end(), cmp_x);return dac3(0, pts.size() - 1);
}
```

6.5 Minimum Enclosing Circle

```
Circle minimumEnclosingCost(vector<Point> v){
  // O(n^3) but if random_shuffle is used, it is amortized O(n)
  random_shuffle(v.begin(), v.end());
  Point p = {0, 0};
  ld r = 0; int n = v.size();
  for(int i=0; i<n; i++) if(dist(p, v[i]) > r){
    p = v[i], r = 0;
    for(int j=0; j<i; j++) if(dist(p, v[j]) > r){
      auto tmp=circle_from_2pts(v[i], v[j]); p = tmp.center, r = tmp.r;
      for(int k=0; k<j; k++) if(dist(p, v[k]) > r){
        auto tmp=circle_from_3pts(v[i], v[j], v[k]); p = tmp.center, r = tmp.r;
      }
    }
  }
  return {p, r};
}
```

6.6 Point in Polygon Test

```
inline ld is_left(Point p0, Point p1, Point p2) {
  return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
}
// point in polygon test
bool is_in_polygon(Point p, vector<Point>& poly) {
  int wn = 0;
  for (int i = 0; i < poly.size(); ++i) {
    int ni = (i + 1 == poly.size()) ? 0 : i + 1;
    if (poly[i].y <= p.y) {
      if (poly[ni].y > p.y) {
        if (is_left(poly[i], poly[ni], p) > 0) {
          ++wn;
        }
      }
    } else {
      if (poly[ni].y <= p.y) {
        if (is_left(poly[i], poly[ni], p) < 0) {
          --wn;
        }
      }
    }
  }
  return wn != 0;
}
```

6.7 Polygon Cut

```
// left side of a->b
vector<Point> cut_polygon(const vector<Point>& polygon, Line line) {
    if (!polygon.size()) return polygon;
    typedef vector<Point>::const_iterator piter;
    piter la, lan, fi, fip, i, j;
    la = lan = fi = fip = polygon.end(); i = polygon.end() - 1;
    bool lastin = diff(ccw_line(line, polygon[polygon.size() - 1]), 0) > 0;
    for (j = polygon.begin() ; j != polygon.end() ; j++) {
        bool thisin = diff(ccw_line(line, *j), 0) > 0;
        if (lastin && !thisin) la = i, lan = j;
        if (!lastin && thisin) fi = j, fip = i;
        i = j;
        lastin = thisin;
    }
    if (fi == polygon.end()) {
        if (!lastin) return vector<Point>();
        return polygon;
    }
    vector<Point> result;
    for (i = fi ; i != lan ; i++) {
        if (i == polygon.end()) {
            i = polygon.begin();
            if (i == lan) break;
        }
        result.push_back(*i);
    }
    Point lc, fc;
    get_cross(Line{ *la, *lan - *la }, line, lc);
    get_cross(Line{ *fip, *fi - *fip }, line, fc);
    result.push_back(lc);
    if (diff(dist2(lc, fc), 0) != 0) result.push_back(fc);
    return result;
}
```

6.8 Number of Point in Triangle

```
// N arr , M brr points, O(NMLg(NM)+Q) solution
// query : 3 points a,b,c : arr index
// find brr points in triangle arr_abc(line excluded)
template<class Int = long long, class Int2 = long long>
struct VecI2 {
  Int x, y;
  VecI2() : x(0), y(0) {}
  VecI2(Int _x, Int _y) : x(_x), y(_y) {}
  VecI2 operator+(VecI2 r) const { return VecI2(x+r.x, y+r.y); }
  VecI2 operator-(VecI2 r) const { return VecI2(x-r.x, y-r.y); }
  VecI2 operator-() const { return VecI2(-x, -y); }
  Int2 operator*(VecI2 r) const { return Int2(x) * Int2(r.x) + Int2(y) * Int2(r.y); }
  Int2 operator^(VecI2 r) const { return Int2(x) * Int2(r.y) - Int2(y) * Int2(r.x); }
  static bool compareYX(VecI2 a, VecI2 b){ return a.y < b.y || (!(b.y < a.y) && a.x < b.x); }
  static bool compareXY(VecI2 a, VecI2 b){ return a.x < b.x || (!(b.x < a.x) && a.y < b.y); }
};
using namespace std;
using Vec = VecI2<ll>;

void func(vector<Vec>& A, vector<Vec>& B){
  auto pointL = vector<int>(N); // bx < Ax
  auto pointM = vector<int>(N); // bx = Ax
  rep(i,N) rep(j,M) if(A[i].y == B[j].y){
    if(B[j].x < A[i].x) pointL[i]++;
    if(B[j].x == A[i].x) pointM[i]++;
  }
  auto edgeL = vector<vector<int>>(N, vector<int>(N)); // bx < lerp(Ax, Bx)
  auto edgeM = vector<vector<int>>(N, vector<int>(N)); // bx = lerp(Ax, Bx)
  rep(a,N){
    struct PointId { int i; int c; Vec v; };
    vector<PointId> points;
    rep(b,N) if(A[a].y < A[b].y) points.push_back({ b, 0, A[b] - A[a] });
    rep(b,M) if(A[a].y < B[b].y) points.push_back({ b, 1, B[b] - A[a] });
    rep(b,N) if(A[a].y < A[b].y) points.push_back({ b, 2, A[b] - A[a] });
    sort(points.begin(), points.end(), [&](const PointId& l, const PointId& r){
      ll det = l.v ^ r.v;
      if(det != 0) return det < 0;
      return l.c < r.c;
    });
    int qN = points.size();
    vector<int> queryOrd(qN); rep(i,qN) queryOrd[i] = i;
    sort(queryOrd.begin(), queryOrd.end(), [&](int l, int r){
      return pll{points[l].v.y, points[l].c%2} < pll{points[r].v.y, points[r].c%2};
    });
    vector<int> BIT(qN);
    for(int qi=0; qi<qN; qi++){
      int q = queryOrd[qi];
      if(points[q].c == 0){
        int buf = 0, p = q+1;
        while(p > 0){ buf += BIT[p-1]; p -= p & -p; }
```

```
      edgeL[a][points[q].i] = buf;
    } else if(points[q].c == 1) {
      int p = q+1;
      while(p <= qN){ BIT[p-1]++; p += p & -p; }
    } else {
      int buf = 0, p = q+1;
      while(p > 0){ buf += BIT[p-1]; p -= p & -p; }
      edgeM[a][points[q].i] = buf;
    }
  }
  rep(b,N) edgeM[a][b] -= edgeL[a][b];
}

int Q; cin >> Q;
rep(qi, Q){
  int a,b,c; cin >> a >> b >> c;
  if(Vec::compareYX(A[b], A[a])) swap(a, b);if(Vec::compareYX(A[c], A[b])) swap(b, c);
  if(Vec::compareYX(A[b], A[a])) swap(a, b);
  auto det = (A[a] - A[c]) ^ (A[b] - A[c]); int ans = 0;
  if(det != 0){
    if(A[a].y == A[b].y){ // A[a].x < A[b].x
      ans = edgeL[b][c] - (edgeL[a][c] + edgeM[a][c]);
    } else if(A[b].y == A[c].y){ // A[b].x < A[c].x
      ans = edgeL[a][c] - (edgeL[a][b] + edgeM[a][b]);
    } else if(det < 0){
      ans += edgeL[a][c]-edgeL[b][c]-edgeM[b][c]-edgeL[a][b]-edgeM[a][b]-pointL[b]-pointM[b];
    } else ans += edgeL[a][b]+edgeL[b][c]+pointL[b]-edgeL[a][c]-edgeM[a][c];
  }
  cout << ans << '\n';
}
}
```

6.9 Voronoi Diagram

```
typedef pair<ld, ld> pdd;
const ld EPS = 1e-12;
ll dcmp(ld x){ return x < -EPS? -1 : x > EPS ? 1 : 0; }
ld operator / (pdd a, pdd b){ return a.first * b.second - a.second * b.first; }
pdd operator * (ld b, pdd a){ return pdd(b * a.first, b * a.second); }
pdd operator + (pdd a,pdd b){ return pdd(a.first + b.first, a.second + b.second); }
pdd operator - (pdd a,pdd b){ return pdd(a.first - b.first, a.second - b.second); }
ld sq(ld x){ return x*x; }
ld size(pdd p){ return hypot(p.first, p.second); }
ld sz2(pdd p){ return sq(p.first) + sq(p.second); }
pdd r90(pdd p){ return pdd(-p.second, p.first); }
pdd inter(pdd a, pdd b, pdd u, pdd v){ return u+(((a-u)/b)/(v/b))*v; }
pdd get_circumcenter(pdd p0, pdd p1, pdd p2){
  return inter(0.5*(p0+p1), r90(p0-p1), 0.5*(p1+p2), r90(p1-p2)); }
ld pb_int(pdd left, pdd right, ld sweepline){
  if(dcmp(left.second-right.second) == 0) return (left.first + right.first) / 2.0;
  ll sign = left.second < right.second ? -1 : 1;
  pdd v = inter(left, right-left, pdd(0, sweepline), pdd(1, 0));
  ld d1 = sz2(0.5 * (left+right) - v), d2 = sz2(0.5 * (left-right));
  return v.first + sign * sqrt(max(0.0, d1 - d2)); }
class Beachline{
  public:
    struct node{
      node(){}
      node(pdd point, ll idx):point(point), idx(idx), end(0),
        link{0, 0}, par(0), prv(0), nxt(0) {}
      pdd point; ll idx; ll end;
      node *link[2], *par, *prv, *nxt;
    };
    node *root;
    ld sweepline;
    Beachline() : sweepline(-1e20), root(NULL){ }
    inline ll dir(node *x){ return x->par->link[0] != x; }
    void rotate(node *n){
      node *p = n->par; ll d = dir(n); p->link[d] = n->link[!d];
      if(n->link[!d]) n->link[!d]->par = p; n->par = p->par;
      if(p->par) p->par->link[dir(p)] = n; n->link[!d] = p; p->par = n;
    } void splay(node *x, node *f = NULL){
      while(x->par != f){
        if(x->par->par == f);
        else if(dir(x) == dir(x->par)) rotate(x->par);
        else rotate(x);
        rotate(x);
      }
      if(f == NULL) root = x;
    } void insert(node *n, node *p, ll d){
      splay(p); node* c = p->link[d];
      n->link[d] = c; if(c) c->par = n; p->link[d] = n; n->par = p;
      node *prv = !d?p->prv:p, *nxt = !d?p:p->nxt;
      n->prv = prv; if(prv) prv->nxt = n; n->nxt = nxt; if(nxt) nxt->prv = n;
    } void erase(node* n){
      node *prv = n->prv, *nxt = n->nxt;
      if(!prv && !nxt){ if(n == root) root = NULL; return; }
      n->prv = NULL;   if(prv) prv->nxt = nxt;
      n->nxt = NULL;   if(nxt) nxt->prv = prv;
      splay(n);
      if(!nxt){
        root->par = NULL; n->link[0] = NULL;
        root = prv;
      }
      else{
        splay(nxt, n);     node* c = n->link[0];
        nxt->link[0] = c; c->par = nxt; n->link[0] = NULL;
        n->link[1] = NULL; nxt->par = NULL; root = nxt;
      }
    } bool get_event(node* cur, ld &next_sweep){
      if(!cur->prv || !cur->nxt) return false;
      pdd u = r90(cur->point - cur->prv->point);
      pdd v = r90(cur->nxt->point - cur->point);
      if(dcmp(u/v) != 1) return false;
      pdd p = get_circumcenter(cur->point, cur->prv->point, cur->nxt->point);
      next_sweep = p.second + size(p - cur->point); return true;
    } node* find_bl(ld x){
      node* cur = root;
      while(cur){
        ld left = cur->prv ? pb_int(cur->prv->point, cur->point, sweepline) : -1e30;
        ld right = cur->nxt ? pb_int(cur->point, cur->nxt->point, sweepline) : 1e30;
        if(left <= x && x <= right){ splay(cur); return cur; }
        cur = cur->link[x > right];
      }
    }
};
using BNode = Beachline::node; static BNode* arr; static ll sz;
static BNode* new_node(pdd point, ll idx){
  arr[sz] = BNode(point, idx); return arr + (sz++); }
struct event{
  event(ld sweep, ll idx):type(0), sweep(sweep), idx(idx){}
  event(ld sweep, BNode* cur):type(1), sweep(sweep), prv(cur->prv->idx), cur(cur), nxt(cur->nxt->
    idx){}
  ll type, idx, prv, nxt;
  BNode* cur;
  ld sweep;
  bool operator>(const event &l)const{ return sweep > l.sweep; }
};
void Voronoi(vector<pdd> &input, vector<pdd> &vertex, vector<pll> &edge, vector<pll> &area){
  Beachline bl = Beachline();
  priority_queue<event, vector<event>, greater<event>> events;
  auto add_edge = [&](ll u, ll v, ll a, ll b, BNode* c1, BNode* c2){
```

```
    if(c1) c1->end = edge.size()*2;
    if(c2) c2->end = edge.size()*2 + 1;
    edge.emplace_back(u, v);
    area.emplace_back(a, b);
  };
  auto write_edge = [&](ll idx, ll v){ idx%2 == 0 ? edge[idx/2].first = v : edge[idx/2].second = v
    ; };
  auto add_event = [&](BNode* cur){ ld nxt; if(bl.get_event(cur, nxt)) events.emplace(nxt, cur);
    };
  ll n = input.size(), cnt = 0;
  arr = new BNode[n*4]; sz = 0;
  sort(input.begin(), input.end(), [](const pdd &l, const pdd &r){
    return l.second != r.second ? l.second < r.second : l.first < r.first; });
  BNode* tmp = bl.root = new_node(input[0], 0), *t2;
  for(ll i = 1; i < n; i++){
    if(dcmp(input[i].second - input[0].second) == 0){
      add_edge(-1, -1, i-1, i, 0, tmp);
      bl.insert(t2 = new_node(input[i], i), tmp, 1);
      tmp = t2;
    }
    else events.emplace(input[i].second, i);
  }
  while(events.size()){
    event q = events.top(); events.pop();
    BNode *prv, *cur, *nxt, *site;
    ll v = vertex.size(), idx = q.idx;
    bl.sweepline = q.sweep;
    if(q.type == 0){
      pdd point = input[idx];
      cur = bl.find_bl(point.first);
      bl.insert(site = new_node(point, idx), cur, 0);
      bl.insert(prv = new_node(cur->point, cur->idx), site, 0);
      add_edge(-1, -1, cur->idx, idx, site, prv);
      add_event(prv); add_event(cur);
    }
    else{
      cur = q.cur, prv = cur->prv, nxt = cur->nxt;
      if(!prv || !nxt || prv->idx != q.prv || nxt->idx != q.nxt) continue;
      vertex.push_back(get_circumcenter(prv->point, nxt->point, cur->point));
      write_edge(prv->end, v); write_edge(cur->end, v);
      add_edge(v, -1, prv->idx, nxt->idx, 0, prv);
      bl.erase(cur);
      add_event(prv); add_event(nxt);
    }
  }
  delete arr;
}
```

6.10 KD-Tree

```
// k-d tree : find closest point from arbitrary point
// Time Complexity : average O(log N), worst O(N)
struct KDNode{
    pll v; bool dir;
    ll sx, ex, sy, ey;
    KDNode(){ sx = sy = inf; ex = ey = -inf; }
};
const auto xcmp = [](pll a, pll b){ return tie(a.x, a.y) < tie(b.x, b.y); };
const auto ycmp = [](pll a, pll b){ return tie(a.y, a.x) < tie(b.y, b.x); };
struct KDTree{
    // Segment Tree Size
    static const int S = 1 << 18;
    KDNode nd[S]; int chk[S];
    vector<pll> v;
    KDTree(){ init(); }
    void init(){ memset(chk, 0, sizeof chk); }
    void _build(int node, int s, int e){
        chk[node] = 1;nd[node].dir = !nd[node/2].dir;
        nd[node].sx = min_element(v.begin()+s, v.begin()+e+1, xcmp)->x;
        nd[node].ex = max_element(v.begin()+s, v.begin()+e+1, xcmp)->x;
        nd[node].sy = min_element(v.begin()+s, v.begin()+e+1, ycmp)->y;
        nd[node].ey = max_element(v.begin()+s, v.begin()+e+1, ycmp)->y;
        if(nd[node].dir) sort(v.begin()+s, v.begin()+e+1, ycmp);
        else sort(v.begin()+s, v.begin()+e+1, xcmp);
        int m = s + e >> 1; nd[node].v = v[m];
        if(s <= m-1) _build(node << 1, s, m-1);
        if(m+1 <= e) _build(node << 1 | 1, m+1, e);
    }
    void build(const vector<pll> &_v){
        v = _v; sort(all(v));_build(1, 0, v.size()-1);
    }
    ll query(pll t, int node = 1){
        ll tmp, ret = inf;
        if(t != nd[node].v) ret = min(ret, dst(t, nd[node].v));
        bool x_chk = (!nd[node].dir && xcmp(t, nd[node].v));
        bool y_chk = (nd[node].dir && ycmp(t, nd[node].v));
        if(x_chk || y_chk){
            if(chk[node << 1]) ret = min(ret, query(t, node << 1));
            if(chk[node << 1 | 1]){
                if(nd[node].dir) tmp = nd[node << 1 | 1].sy - t.y;
                else tmp = nd[node << 1 | 1].sx - t.x;
                if(tmp*tmp < ret) ret = min(ret, query(t, node << 1 | 1));
            }
        }
        else{
            if(chk[node << 1 | 1]) ret = min(ret, query(t, node << 1 | 1));
            if(chk[node << 1]){
                if(nd[node].dir) tmp = nd[node << 1].ey - t.y;
                else tmp = nd[node << 1].ex - t.x;
                if(tmp*tmp < ret) ret = min(ret, query(t, node << 1));
            }
        }
        return ret;
    }
};
```

6.11 Pick's theorem

격자점으로 구성된 simple polygon에 대해 i는 polygon 내부의 격자수, b는 polygon 선분 위 격자수, A는 polygon 넓이라고 할 때 $A = i + \frac{b}{2} - 1$.

7 String

7.1 KMP

```
void calculate_pi(vector<int>& pi, const string& str) {
  pi[0] = -1;
  for (int i = 1, j = -1; i < str.size(); i++) {
    while (j >= 0 && str[i] != str[j + 1]) j = pi[j];
    if (str[i] == str[j + 1]) pi[i] = ++j;
    else pi[i] = -1;
  }
}
// returns all positions matched
// O(|text|+|pattern|)
vector<int> kmp(const string& text, const string& pattern) {
  vector<int> pi(pattern.size()), ans;
  if (pattern.size() == 0) return ans;
  calculate_pi(pi, pattern);
  for (int i = 0, j = -1; i < text.size(); i++) {
    while (j >= 0 && text[i] != pattern[j + 1]) j = pi[j];
    if (text[i] == pattern[j + 1]) {
      j++;
```

```
      if (j + 1 == pattern.size()) ans.push_back(i - j), j = pi[j];
    }
  }
  return ans;
}
```

7.2 Z Algorithm

```
// Z[i] : maximum common prefix length of &s[0] and &s[i] with O(|s|)
auto get_z = [](const string& s) {
  const int n = s.size(); vector z(n, 0); z[0] = n;
  for (int i = 1, l = -1, r = -1; i < n; i++) {
  if (i <= r) z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
    if (r < i + z[i] - 1) l = i, r = i + z[i] - 1;
  }
  return z;
};
```

7.3 Aho-Corasick

```
struct aho_corasick_with_trie {
  const ll MAXN = 100005, MAXC = 26;
  ll trie[MAXN][MAXC], fail[MAXN], term[MAXN], piv = 0;
  void init(vector<string> &v) {
    memset(trie, 0, sizeof(trie));memset(fail, 0, sizeof(fail));
    memset(term, 0, sizeof(term));piv = 0;
    for (auto &i : v) {
      ll p = 0;
      for (auto &j : i) {
        if (!trie[p][j]) trie[p][j] = ++piv;
        p = trie[p][j];
      }
      term[p] = 1;
    }
    queue<ll> que;
    for (ll i = 0; i < MAXC; i++) if (trie[0][i]) que.push(trie[0][i]);
    while (!que.empty()) {
      ll x = que.front(); que.pop();
      for (ll i = 0; i < MAXC; i++) if (trie[x][i]) {
        ll p = fail[x];
        while (p && !trie[p][i]) p = fail[p];
        p = trie[p][i];
        fail[trie[x][i]] = p;
        if (term[p]) term[trie[x][i]] = 1;
        que.push(trie[x][i]);
      }
    }
  }
  bool query(string &s) {
    ll p = 0;
    for (auto &i : s) {
      while (p && !trie[p][i]) p = fail[p];
      p = trie[p][i]; if (term[p]) return 1;
    }
    return 0;
  }
};
```

7.4 Suffix Array with LCP

```
// calculates suffix array with O(n*logn)
auto get_sa(const string& s) {
  const int n = s.size(), m = max(256, n) + 1;
  vector<int> sa(n), r(n << 1), nr(n << 1), cnt(m), idx(n);
  for (int i = 0; i < n; i++) sa[i] = i, r[i] = s[i];
  for (int d = 1; d < n; d <<= 1) {
    auto cmp = [&](int a, int b) { return r[a] < r[b] || r[a] == r[b] && r[a + d] < r[b + d];};
    for (int i = 0; i < m; ++i) cnt[i] = 0;
    for (int i = 0; i < n; ++i) cnt[r[i + d]]++;
    for (int i = 1; i < m; ++i) cnt[i] += cnt[i - 1];
    for (int i = n - 1; ~i; --i) idx[--cnt[r[i + d]]] = i;
    for (int i = 0; i < m; ++i) cnt[i] = 0;
    for (int i = 0; i < n; ++i) cnt[r[i]]++;
    for (int i = 1; i < m; ++i) cnt[i] += cnt[i - 1];
    for (int i = n - 1; ~i; --i) sa[--cnt[r[idx[i]]]] = idx[i];
    nr[sa[0]] = 1;
    for (int i = 1; i < n; ++i) nr[sa[i]] = nr[sa[i - 1]] + cmp(sa[i - 1], sa[i]);
    for (int i = 0; i < n; ++i) r[i] = nr[i];
    if (r[sa[n - 1]] == n) break;
  }
  return sa;
}
// calculates lcp array. it needs suffix array & original sequence with O(n)
auto get_lcp(const string& s, const auto& sa) {
  const int n = s.size(); vector lcp(n - 1, 0), isa(n, 0);
  for (int i = 0; i < n; i++) isa[sa[i]] = i;
  for (int i = 0, k = 0; i < n; i++) if (isa[i]) {
    for (int j = sa[isa[i] - 1]; s[i + k] == s[j + k]; k++);
    lcp[isa[i] - 1] = k ? k-- : 0;
  }
  return lcp;
}
```

7.5 Manacher's Algorithm

```
// find longest palindromic span for each element in str with O(|str|)
auto manacher = [](const string& s) {
  const int n = s.size(); vector d(n, 0);
  for (int i = 0, l = -1, r = -1; i < n; i++) {
    if (i < r) d[i] = min(r - i, d[l + r - i]);
    while (d[i] < min(i + 1, n - i) && s[i - d[i]] == s[i + d[i]]) d[i]++;
    if (i + d[i] > r) l = i - d[i], r = i + d[i];
  }
  return d;
};
```

7.6 EERTREE

```
template<class S = string , class T = typename S::value_type>
struct eertree {
  struct node { int len, link;map<T, int> child; };
  S s; vector<node> data; int max_suf;
  eertree() : max_suf(1) {
    data.push_back({ -1, 0 }); data.push_back({ 0, 0 });
  }
  void add(T c) {
    s.push_back(c); int i = max_suf;
    while (data[i].len + 2 > s.size() || s[s.size() - data[i].len - 2] != c) i = data[i].link;
    if (data[i].child.count(c) == 0) {
      if (i == 0) data[i].child[c] = data.size(), data.push_back({ data[i].len + 2, 1 });
      else {
        int j = data[i].link; while (s[s.size() - data[j].len - 2] != c) j = data[j].link;
        data[i].child[c] = data.size(); data.push_back({ data[i].len + 2, data[j].child[c] });
      }
    }
    i = data[i].child[c];
    max_suf = i;
  }
};
```