DERIVATION OF GRAVITATIONAL FORMULA

Using Kepler's law

$$p^2 = k r^3$$

and p is the period,

$$p = \frac{2\pi r}{v}$$ (circumference by the velocity of planet)

and thus substituting it in earlier Kepler's equation

$$p = \frac{2\pi r}{v} \Rightarrow p^2 = k r^3$$

$$\frac{4\pi^2 r^2}{v^2} = k r^3$$

Now multiplying both side by m (say m was absorbed in proportionality constant k)

$$\Rightarrow \frac{mv^2}{r} = \frac{4\pi^2 m}{kr^2}$$

This we know as centripetal force so,

$$F = \frac{4\pi^2 m}{kr^2}$$

Therefore, this must be the gravitational force keeping m in orbit around M.

Then Newton from his 3rd law says that magnitude of force on and by should be equal. So, to establish the symmetricity

$$F = \frac{4\pi^2 M}{k' r^2}(m)$$

and take $\frac{4\pi^2}{k''} = G$

We are taking constant as

$$K = \frac{K''}{M} \quad \text{and} \quad K' = \frac{K''}{m}$$

and G = Law of Universal Gravitation

$$\boxed{F = G\frac{Mm}{r^2}}$$

Force of gravitation formula.

$$\boxed{G = 6.673 \times 10^{-11}\ N\,m^2\,kg^{-2}}$$