RADIATING ENERGY TRANSPORT

$$\# \text{ of interaction} = n\sigma dx$$

- σ → effective cross section for absorption and scattering
- dx → photon covers path distance 'dx'
- n → number density

Now,

for # of interactions = 1 and $dx = l$

$$\therefore \boxed{l = \frac{1}{n\sigma}}$$ → This distance 'l' is called "mean free path"

and there will be variety of absorbers and scatterers, each with its own density n_i and cross section σ_i

$$l = \frac{1}{\sum n_i \sigma_i} \equiv \frac{1}{\rho K} \qquad \left\{ \begin{array}{l} \rho \to \text{mass density} \\ K \to \text{opacity} \end{array} \right\}$$

Thomson Scattering of photons on free electrons

The Thomson cross section is,

$$\sigma_T = \underbrace{\frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2}_{\text{independent of } T \text{ and } h\nu.} = 6.7 \times 10^{-25} \text{ cm}^2$$

and in interior of sun, we can assume all hydrogen are ionised, so,

$$n_e \approx \rho / m_H \quad (\text{as there is 1 electron per atom of mass } m_H)$$

So, Mean free path for electron scattering will be,

$$l_{es} = \frac{1}{n_e \sigma_T} \approx \frac{m_H}{\rho \sigma_T} \approx \frac{1.7 \times 10^{-24}\, g}{1.4\, g\, cm^{-3} \times 6.7 \times 10^{-25}\, cm^2} \approx 2\, cm$$

ρ → solar mean density

Now, this was calculated mean free path but in reality, density of Sun is higher in places where electron scattering is dominant.

And owed to this reason, the mean free path of typical photon is less,

$$\therefore\ l \approx 1\, mm.$$

Thus, photons can travel only a tiny distance inside the Sun before being scattered or absorbed and re-emitted in a new direction. Since the new direction is random, the emergence of photons from the Sun is necessarily a random walk process.

D → change in position of a photon after N steps

l_i → each step described by l_i having length l.

$$D = l_1 + l_2 + l_3 + l_4 + \cdots\ l_N$$

and the square of linear distance covered,

$$D^2 = |l_1|^2 + |l_2|^2 + |l_3|^2 + \cdots\cdots\ |l_N|^2 + 2(l_1 \cdot l_2 + l_1 \cdot l_3 + \cdots\cdots)$$

will be zero as it is sum over many vector dot products

and its expectation is

$$\langle D^2 \rangle = N l^2$$

or,

$$\langle D^2 \rangle^{1/2} = D = \sqrt{N}\, l$$

so, total time taken for photon to emerge from centre to surface,

$$\tau_{rw} \approx \frac{l}{c}\frac{r_\odot^2}{l^2} = \frac{r_\odot^2}{lc} = \frac{(7 \times 10^{10}\,\text{cm})^2}{10^{-1}\,\text{cm} \times 3 \times 10^{10}\,\text{cm}\,s^{-1}}$$

$$= 1.6 \times 10^{12}\,\text{sec}$$

$$\approx 52000 \text{ years}$$

Each step requires $\tau = l/c$ and distance of $r_\odot$ requires $N = \frac{r_\odot^2}{l^2}$ steps