DISCUSSION LOG

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THEORY

In our cloth simulation, we follow the precedure written by Rohmer et al.. We consider to begin with the *Defomation gradient* **F**, which is defined by

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \tag{1}$$

where the **x** denotes the deformed vector and the **X** denotes the reference vector.

Since we have no further information about the mapping from **X** to **x**, we use the vector $\left(\mathbf{u_1}, \mathbf{u_2}\right)$ and $\left(\overline{\mathbf{u_1}}, \overline{\mathbf{u_2}}\right)$ to approximate the Defomation gradient, which is defined as

$$\mathbf{F} = \left[\mathbf{u_1}, \mathbf{u_2}\right]\left[\overline{\mathbf{u_1}}, \overline{\mathbf{u_2}}\right]^{-1}, \tag{2}$$

Attention should be paid especially:

- eq.(2) characrize only the 2D deformation of each triangle.
- in Rohmer et al. **F** is symbolised as **T**.

KW13

We provide here our code preceed with concrete data:

- $faces(i, j)$ is the jth vertex of the ith triangle, here we choose the $face(0, 0)$, $face(0, 1)$, $face(0, 2)$ as examples and calculate the $VecT$ and $VecR$ for $face(0, 0)$.

cloth_vec

$$VecT = el_1 VertT_1 - el_1 VertT_2; el_1 VertT_1 - el_1 VertT_3 \tag{3}$$
$$VecR = el_1 VertR_1 - el_1 VertR_2; el_1 VertR_1 - el_1 VertR_3 \tag{4}$$

cloth_vec

$$VecT = [0.771842 \ -0.0144887 \ 6.39045] - [0.780121 \ -0.0186188 \ 6.37318] \tag{5}$$
$$[0.771842 \ -0.0144887 \ 6.39045] - [0.737177 \ -0.00912791 \ 6.3929 \tag{6}$$
$$VecR = [0.759919 \ -0.015194 \ 6.38401] - [0.767822 \ -0.0212492 \ 6.3669]; \tag{7}$$
$$[0.7599190.015194 \ 6.38401] - [0.726977 \ -0.00749985 \ 6.38692] \tag{8}$$
$$\tag{9}$$

such that

cloth_vec

$$
\begin{aligned}
VecT =& [-0.0079\ 0.0061\ 0.0171\ 0.0329\ -0.0077\ -0.0029]; && (10)\\
VecR =& [-0.0083\ 0.0041\ 0.0173\ 0.0347\ -0.0054\ -0.0025]; && (11)\\
& && (12)
\end{aligned}
$$

where the first 3 entries of $VecR$ is the vector $\mathbf{u_1}$ and the last 3 entries of $VecR$ is the vector $\mathbf{u_2}$. $VecT$ analogiously.

cloth_eig_2D

we use here $Eigen :: Map$ to transform the vector $VecT$ and $VecR$ to $2 * 2$ 2D deformation gradient $\mathbf{F}$.

$$\mathbf{F} = \begin{bmatrix} -0.0083 & 0.0347 \\ 0.0041 & -0.0054 \end{bmatrix} \begin{bmatrix} -0.0079 & 0.0329 \\ 0.0061 & -0.0077 \end{bmatrix}^{-1} \tag{13}$$

hence the $\mathbf{F}$ has a rotation information $\mathbf{R}$ and a stretch information $\mathbf{U}$,

$$\mathbf{F} = \mathbf{R}\mathbf{U}. \tag{14}$$

we use $\mathbf{F}^T\mathbf{F}$ to eliminate the ratotion information to obtain $\det \mathbf{R} = 1$

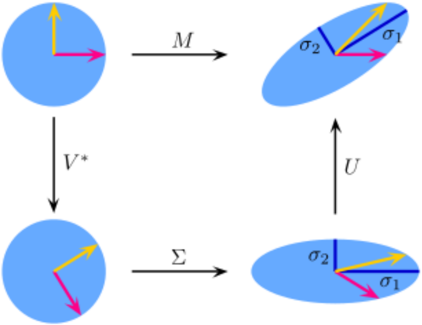
$$\begin{aligned} \mathbf{F}^T\mathbf{F} =& \left(\mathbf{R}\mathbf{U}\right)^T\mathbf{R}\mathbf{U} && (15) \\ =& \mathbf{U}^T\mathbf{R}^T\mathbf{R}\mathbf{U} && (16) \\ =& \mathbf{U}^2 && (17) \\ =& \mathbf{C} && (18) \end{aligned}$$

and using decomposition, if we have the form

$$\mathbf{C} = \lambda_1^2 \mathbf{v}_1 \mathbf{v}_1^T + \lambda_2^2 \mathbf{v}_2 \mathbf{v}_2^T \tag{19}$$

then we could obtain the $\mathbf{U}$ by applying

$$\mathbf{U} = \lambda_1 \mathbf{v}_1 \mathbf{v}_1^T + \lambda_2 \mathbf{v}_2 \mathbf{v}_2^T \tag{20}$$



$$M = U \cdot \Sigma \cdot V^*$$

Figure: Singular Value Decomposition SVD

the main backdraw of Rohmer et al. is that this is only applied for 2D problem, thus we need a new algorithm, which can also take the consideration for the vertical deformation. Therefore, the *Kabisch Algorithm* is introduced.

Kabisch Algorithm

Let $\mathbf{P}$ denotes the points set of template frame, and $\mathbf{Q}$ denotes the points set of reference frame. The *optimal rotation matrix* $\mathbf{R}$ can be calculted as

$$\mathbf{R} = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{bmatrix} \mathbf{U}^T \tag{21}$$

where $\mathbf{V}$ and $\mathbf{U}$ and the *SVD* of *cross-covariance matrix* $\mathbf{H}$, which is determined by

Theorem (cross-covariance matrix)

$$\mathbf{H} = \mathbf{P}^T\mathbf{Q} \tag{22}$$

and $d = \det \mathbf{V}\mathbf{U}^T$

The cross-covariance matrix discribe the covariance of one process with the other at pairs of time points and measure of similarity. We use therefore the cross-covariance matrix **H** to approximate our deformation gradient **F**.

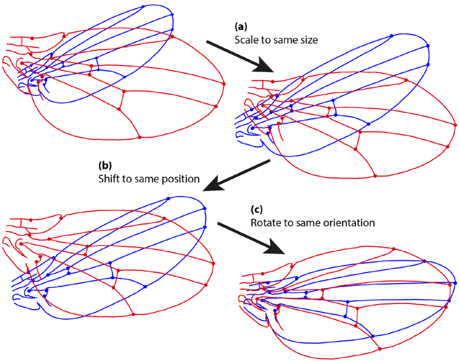


Figure: Procrustes superimposition

Let i be the all index set of the nearst vertice of vertex I in the template frame and $\bar{i}$ be the all index set of the nearst vertice of vertex I in the template frame. Process define

$$\mathbf{P} = \begin{bmatrix} \vec{x}_1 & \vec{y}_1 & \vec{z}_1 \\ \vec{x}_2 & \vec{y}_2 & \vec{z}_2 \\ & \vdots & \\ \vec{x}_N & \vec{y}_N & \vec{z}_N \end{bmatrix} \quad 1, 2, \ldots, N \in i \tag{23}$$

$$\mathbf{Q} = \begin{bmatrix} \bar{\vec{x}}_1 & \bar{\vec{y}}_1 & \bar{\vec{z}}_1 \\ \bar{\vec{x}}_2 & \bar{\vec{y}}_2 & \bar{\vec{z}}_2 \\ & \vdots & \\ \bar{\vec{x}}_N & \bar{\vec{y}}_N & \bar{\vec{z}}_N \end{bmatrix} \quad 1, 2, \ldots, N \in \bar{i} \tag{24}$$

and we apply to obtain the deformation gradient $\mathbf{F}$ of vertex I

$$\mathbf{F} = \mathbf{P}^T\mathbf{Q} \tag{25}$$

KW13

from 0-2 EigNorm1 8759 NaN, Problrm:: 26278 has -0 eigenvalues, can't be squre. So I use a if(isnan) and define idx=0.
add lambda1,2,3 for all, which is in debug