SIGGRAPH 2005 Course #6:

Advanced Topics on Clothing Simulation and Animation

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	Kwang-Jin Choi
Lecturers:	**Ronald Fedkiw**
	Dongliang Zhang

A Brief History of the Course

- SIGGRAPH 1998
  - "Cloth and Clothing in Computer Graphics"
  - Organized by House

- SIGGRAPH 2003
  - "Clothing Simulation and Animation"
  - Organized by Breen and Ko

- SIGGRAPH 2005
  - "Advanced Topics on Clothing Simulation and Animation"
  - Organized by Choi and Ko

Course Schedule

1st Session

    1:45~2:00 Introduction (Ko)

    2:00~2:45 Physical Model of Cloth I (Ko)

    2:45~3:30 Physical Model of Cloth II (Choi)

2nd Session

    3:45~4:30 Collision Handling (Fedkiw)

    4:30~5:00 Cloth Design and Applications (Zhang)

    5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

Introduction

How Cloth Simulation Works?

Hyeong-Seok Ko
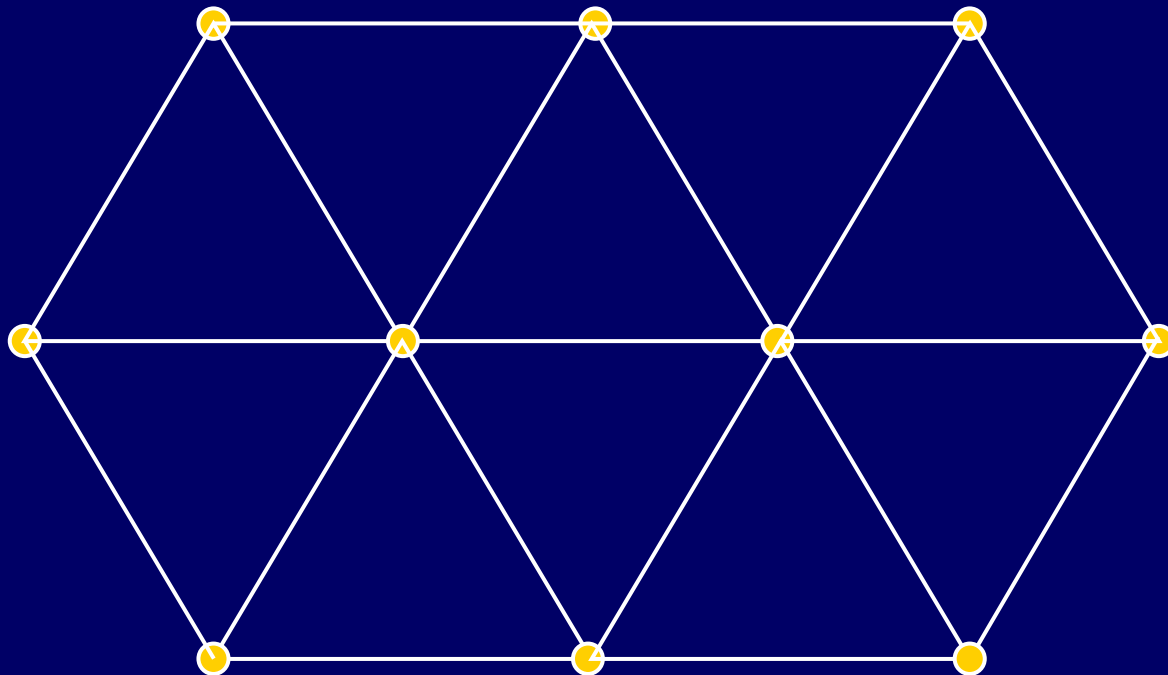Seoul National Univ.
Graphics & Media Lab

How does it Work?

- Finding Governing Equation

- Solving the Equation

- Collision Handling
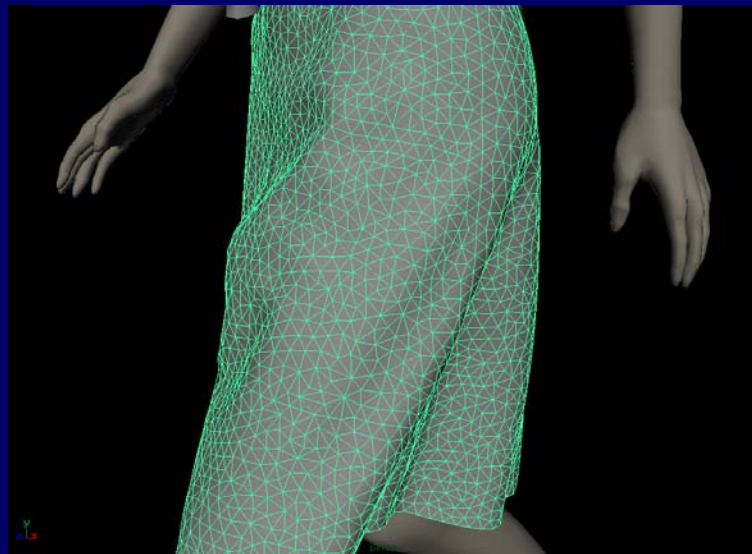
First of All, How to Represent Cloth?

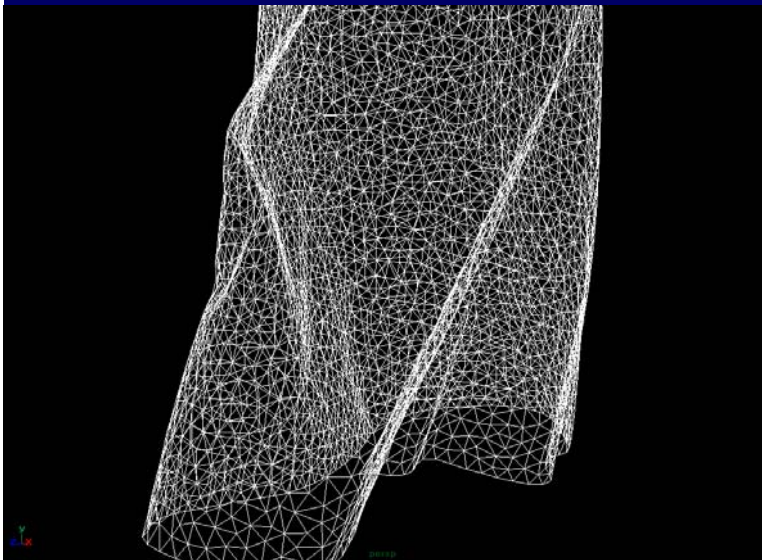## Particles interconnected by springs

Particle-based Cloth Simulation

- Repeat the following:
    1. Find the new position of the particles
    2. Draw the surface from the particles

Finding the Governing Equation
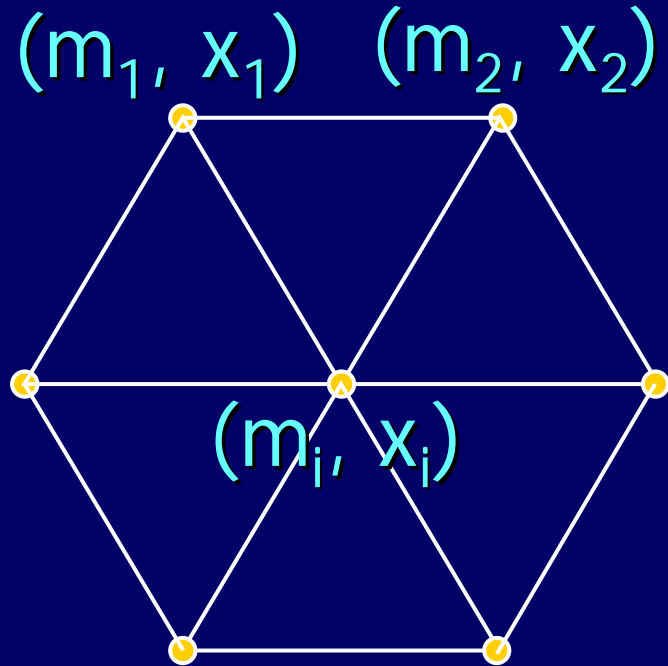
Physically-based Simulation

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$

How this equation was formed?

Newton's Equation (ma=F)

(m_1, x_1) (m_2, x_2)

(m_i, x_i)

$$M_i \ddot{x}_i = \sum \text{forces}$$

$$M_i = \begin{bmatrix} m_i & 0 & 0 \\ 0 & m_i & 0 \\ 0 & 0 & m_i \end{bmatrix}$$

Forces Stem from Deformation

- Deformations store some potential energy
  - Stretch / Shear / Bending
- -(derivative of potential energy) = force
- What are the potential energy functions for each type of deformation?
  - $E_{stretch}$, E_{shear}, $E_{bending}$
  - Further readings
    - David Baraff and Andrew Witkin, "Large Steps in Cloth Simulation", Proceedings of SIGGRAPH 1998, pp. 43—54
    - David Breen, Donald House, and Michael Wozny, "Predicting the drape of woven cloth using interacting particles", Proceedings of SIGGRAPH 1994, pp. 365-372

So, governing eq looks like

$$E = E_{stretch} + E_{shear} + E_{bending}$$

$$M_i \ddot{x}_i = -\left[\frac{\partial E}{\partial x} \right]_{x=x_i}$$

$$= -\left[\frac{\partial E_{\text{stretch}}}{\partial x} + \frac{\partial E_{\text{shear}}}{\partial x} + \frac{\partial E_{\text{bending}}}{\partial x} \right]_{x=x_i}$$

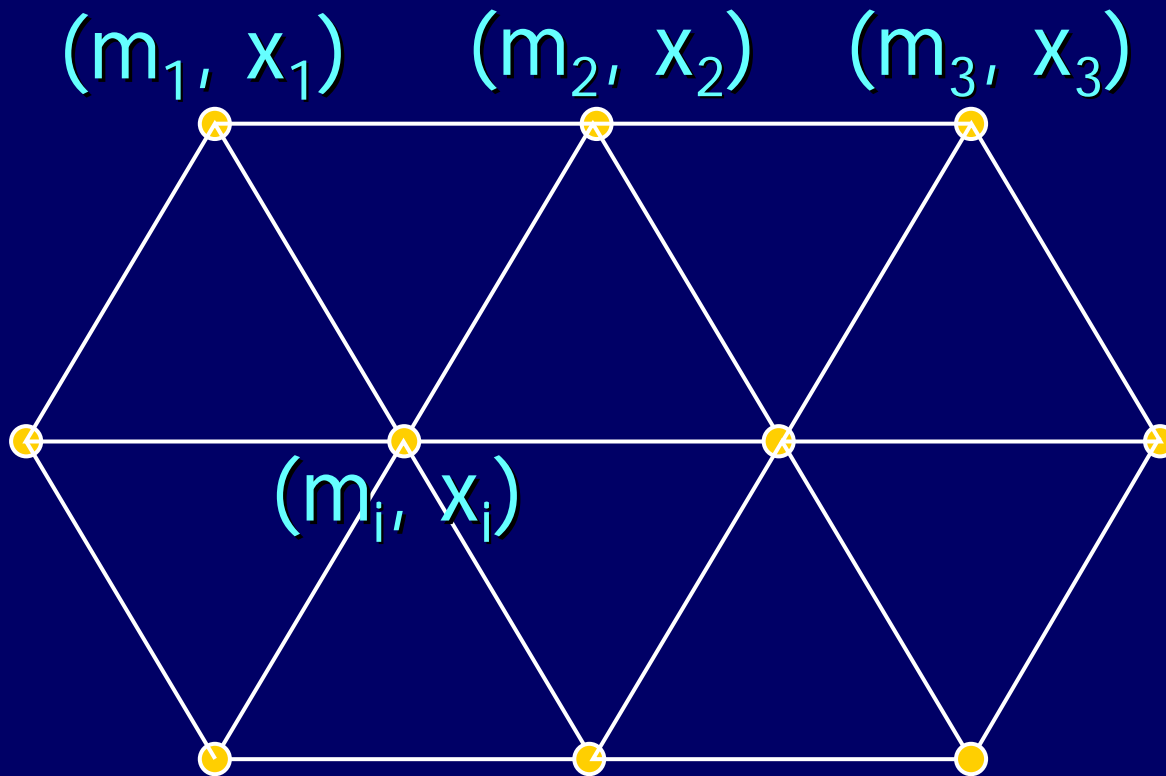# There are also other kind of forces

$$M_i \ddot{x}_i = -\left[\frac{\partial E}{\partial x}\right]_{x=x_i} + F_i$$

Putting all particles into a vector...

(m_1, x_1) (m_2, x_2) (m_3, x_3)

(m_i, x_i)

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

Putting all particles into a vector...

$$M_i \ddot{x}_i = -\left[\frac{\partial E}{\partial x}\right]_{x=x_i} + F_i$$

$$M\ddot{x} = -\frac{\partial E}{\partial x} + F$$

$$\boxed{\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)}$$

Cloth Properties Under Our Control

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$

$$E = E_{stretch} + E_{shear} + E_{bending}$$

- Density
- Stiffness
  - stretch, bending, shear

Solving the Equation

Now, numerically solve
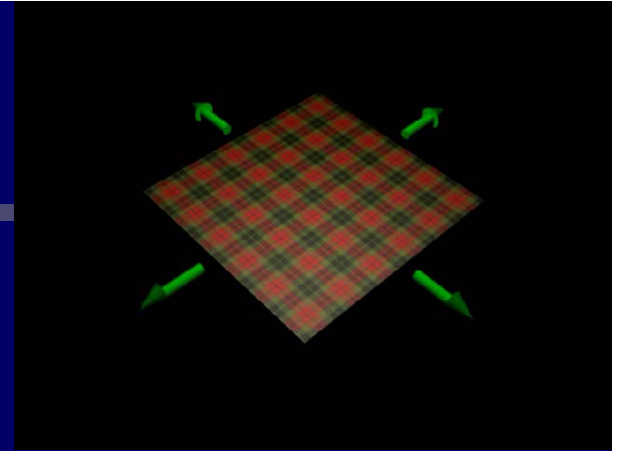
$$\ddot{x}(t) = M^{-1}(-\frac{\partial E}{\partial x} + F)(t)$$

```
while(1) {
  compute the force part at t;
  compute accel, vel, pos;
  t = t + Δt;
}
```

We want to use large Δt

- Animators love large Δt

- Large Δt can cause inaccuracy

- Large Δt can cause a more serious problem

- We must have a stable algorithm

Course Schedule

Finding the Governing Equation
Solving the Equation

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2nd Session

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5:00~5:30 Current State-of-the Art / Challenges Ahead (Ko)

Collision Handling

Collision Handling

- Consists of Detection/Resolution

- Undetected collision can cause a lot of trouble.

- Cloth simulation is abundant of challenging cases for collision detection.

- Difficult problem: Multiple simultaneous collisions.

  - An interesting challenge: simulate garments formed by multiple layers of cloth.

"Collision Handling" by Fedkiw (Session 2)

So, how cloth simulation works?

- Finding Governing Equation

- Solving the Equation

- Collision Handling

Additional Parts

1st Session

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Physical Model of Cloth I

Hyeong-Seok Ko
Seoul National Univ.
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Physical Model of Cloth I

1. Introduction to the immediate buckling model

2. Stability analysis of the immediate buckling model

Introduction to the Immediate Buckling Model

Physical Model of Cloth Material

- Weak resistance to bending

- Strong resistance to tension

- What about to compression?
    - Compression stiffness ≈ Tension stiffness
    - Hookean model (Linear elastic model)
    $$\sigma = E\varepsilon, \ f = k(x - l)$$

**Why Cloth Simulation is Unstable
when Wrinkles are Formed?**

An Experiment

- Two Step Process
  - Resist + Buckle

- How was the Buckling?
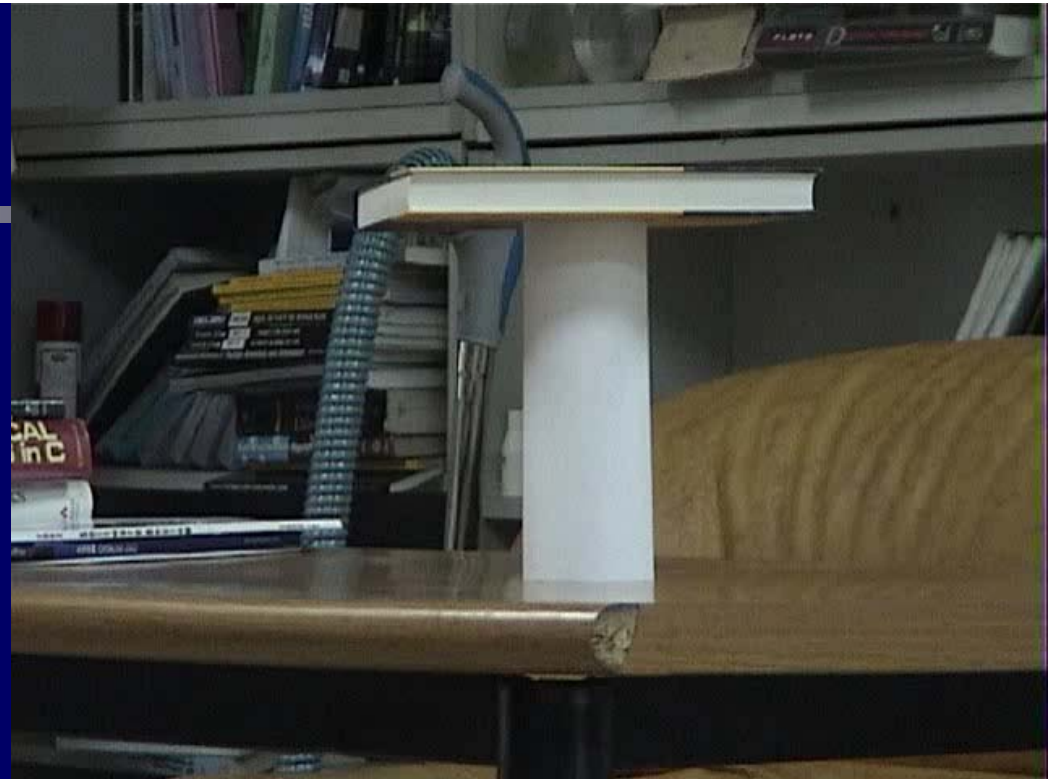 - Explosive/Unstable
  - Why?

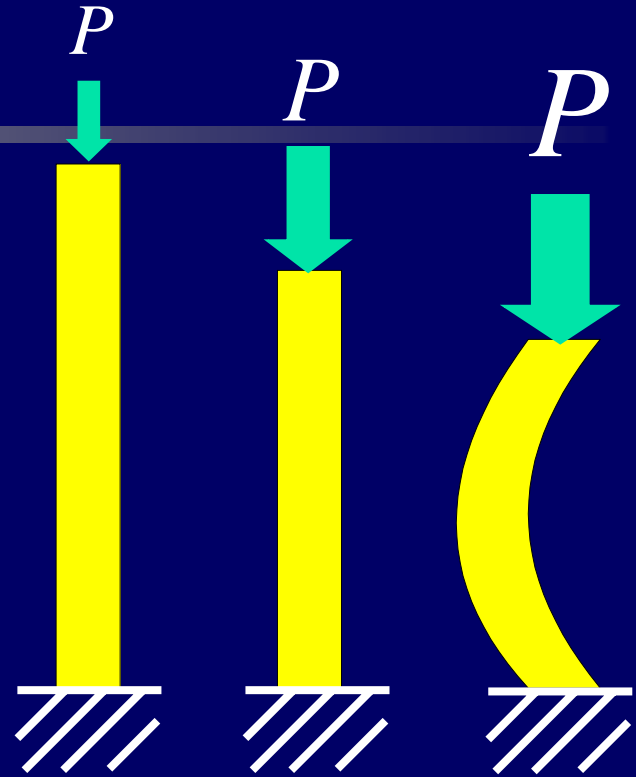| strong resistance to compression	**+**	weak resistance to bending

It is a Physical Instability!

It's not a Numerical Instability
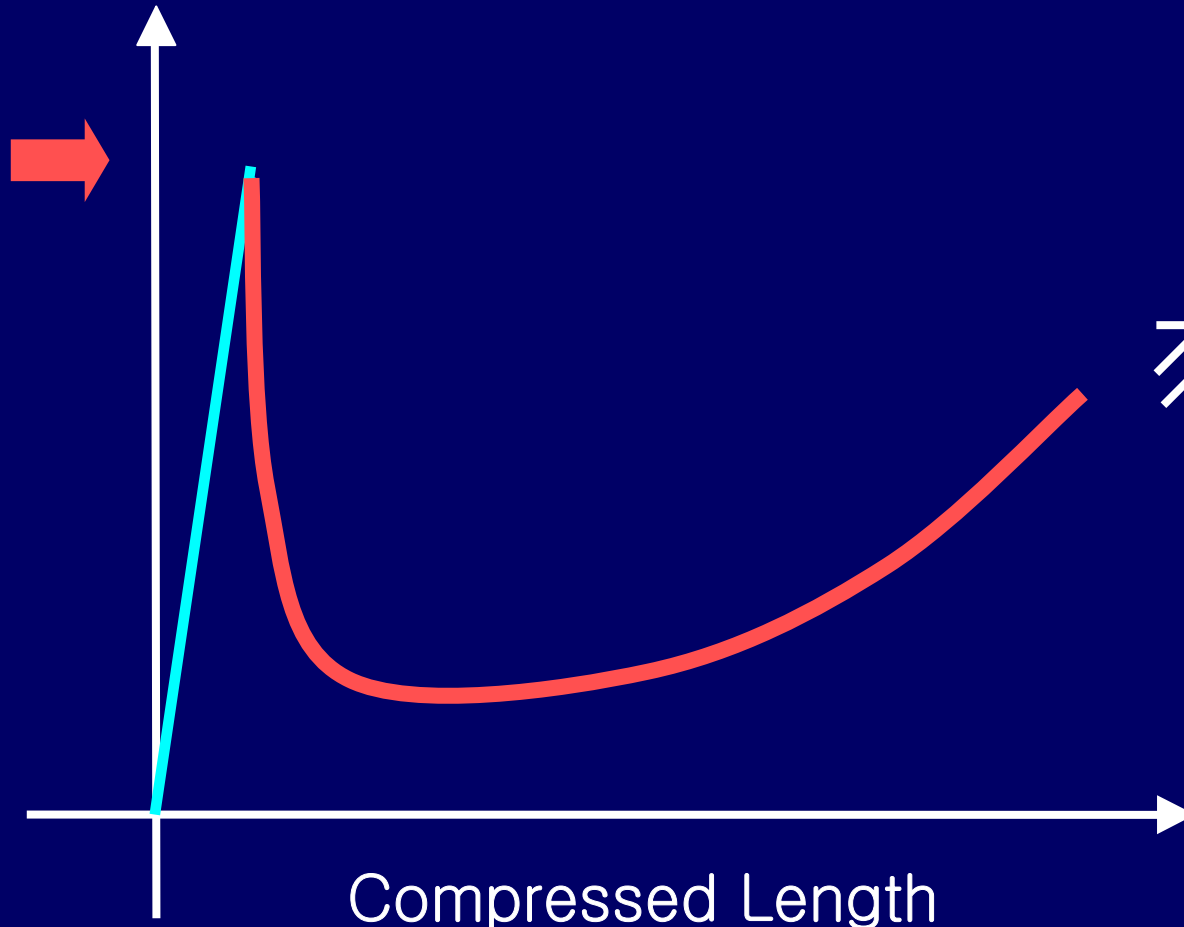
Effect on Implicit Method

$$\left(\frac{\mathbf{I}}{\Delta t^2} + \mathbf{K} \right) \Delta \mathbf{x}_n = \mathbf{f}_{n-1}$$

Ill-conditioned or indefinite system matrix ➔ Divergence!

Stability can be Improved

$$\left(\frac{\mathbf{I}}{\Delta t^2} + \frac{\mathbf{D}}{\Delta t} + \alpha \mathbf{K} \right) \Delta \mathbf{x}_n = \mathbf{f}_{n-1}$$

Reducing time step ➔ Longer comp time

Reducing stiffness ➔ Stretchy material

Adding damping ➔ Non-responsive cloth!
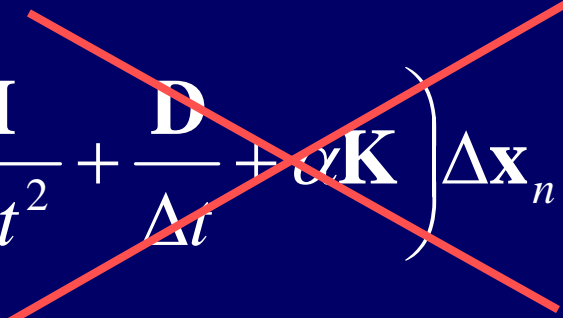
Quest for New Physical Model

Find a Physical Model that <u>just</u> makes K always positive definite!

$$\left(\frac{\mathbf{I}}{\Delta T^2} + \mathbf{K} \right) \Delta \mathbf{x}_n = \mathbf{f}_{n-1}$$

$$\left(\frac{\mathbf{I}}{\Delta t^2} + \frac{\mathbf{D}}{\Delta t} + \alpha \mathbf{K} \right) \Delta \mathbf{x}_n = \mathbf{f}_{n-1}$$

How can we do that?

Compression causes <u>bending</u> rather than <u>shortening</u>
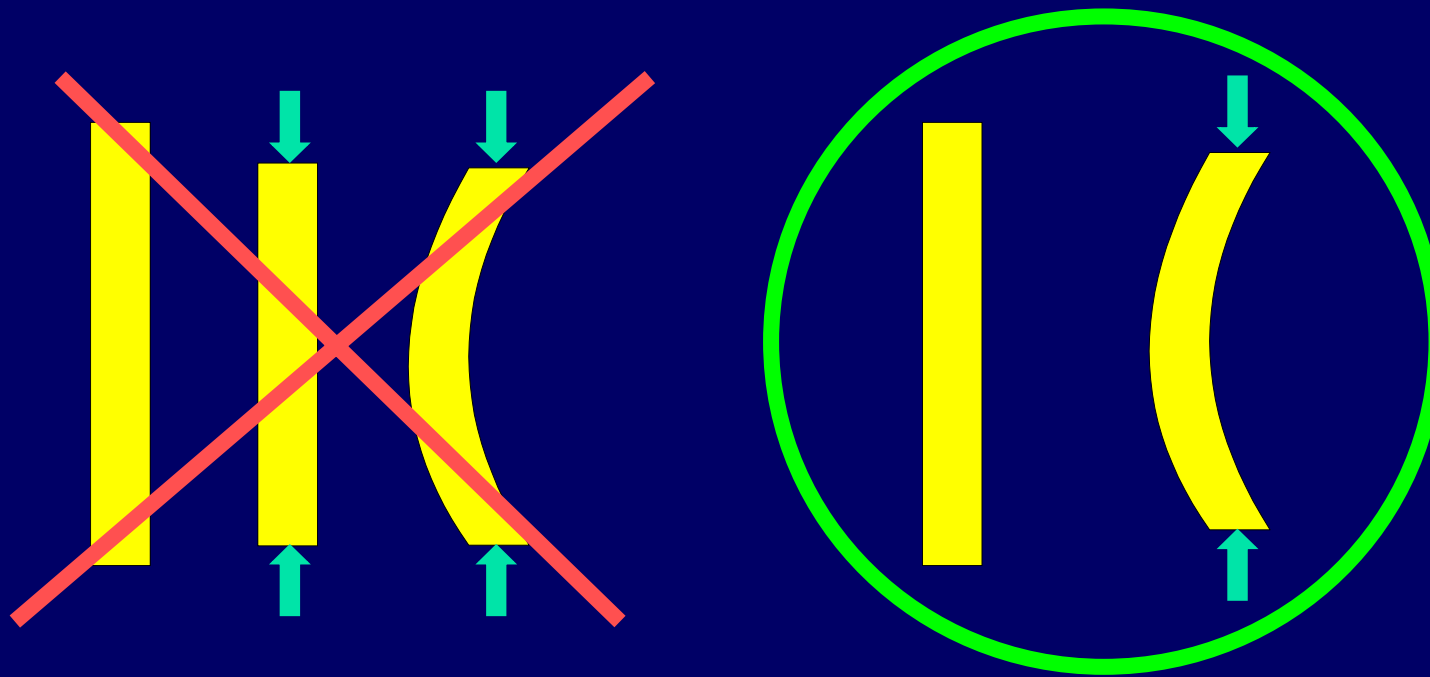
Immediate Buckling Assumption

Experiment
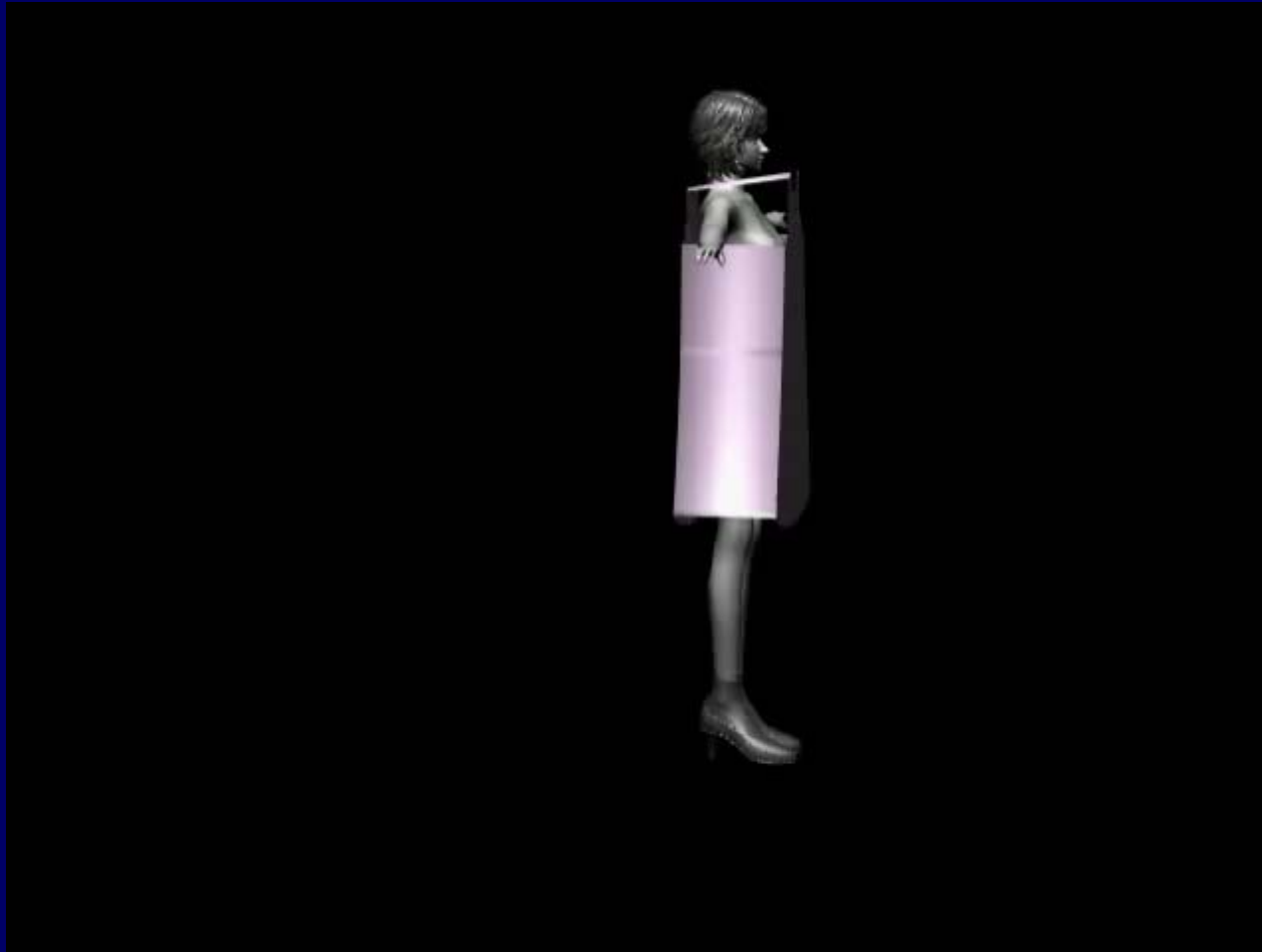
# of particles = 5608, Δt=0.011s, CPU=0.51s

How to implement this idea?

Compression causes <u>bending</u> rather than <u>shortening</u>
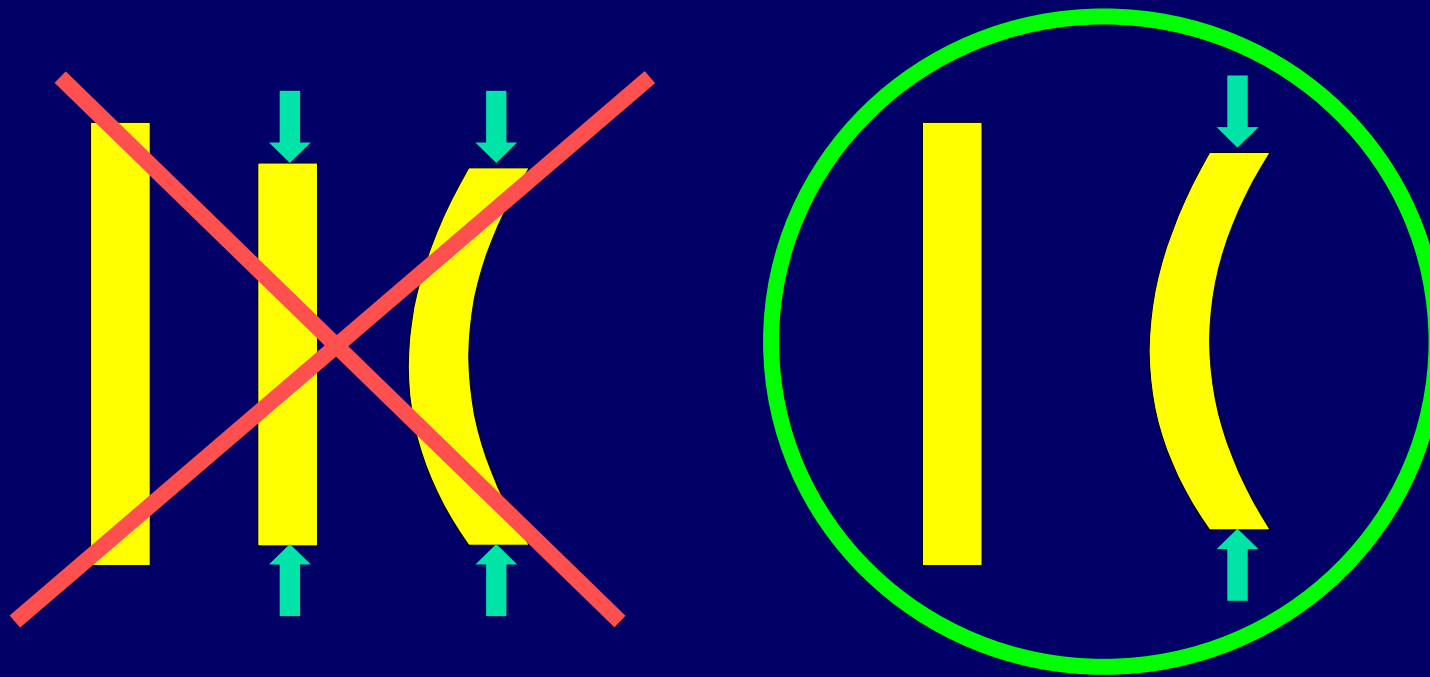
Immediate Buckling Assumption

The Approach We Took

1. Predict the shape at the static equilibrium after buckling

2. Formulate the energy function from the deformed shape

3. Derive the force, Jacobian, etc.

Predicting Static Equilibrium
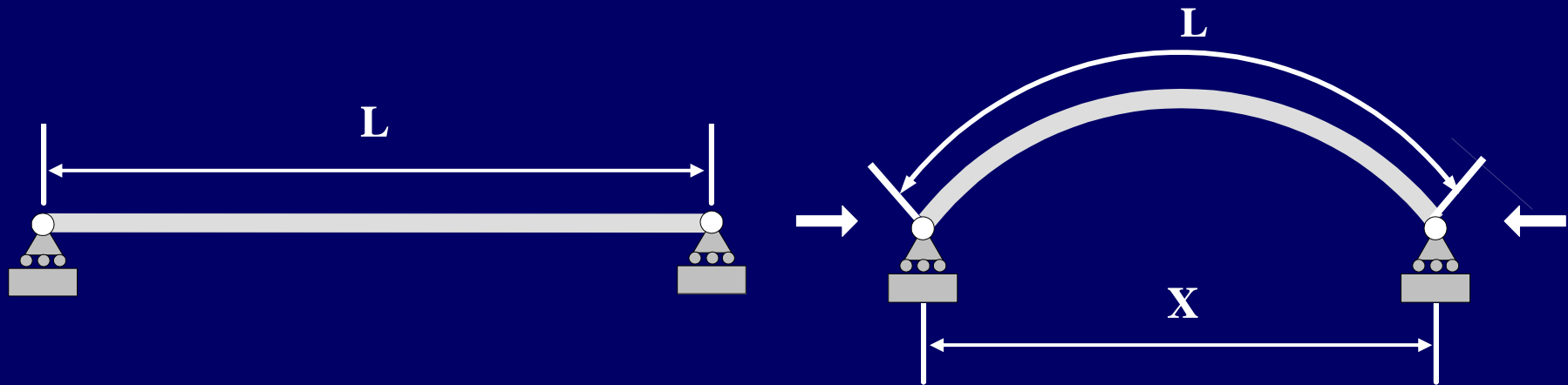
Moment Equilibrium Equation

$$k_b \kappa + Py = 0$$
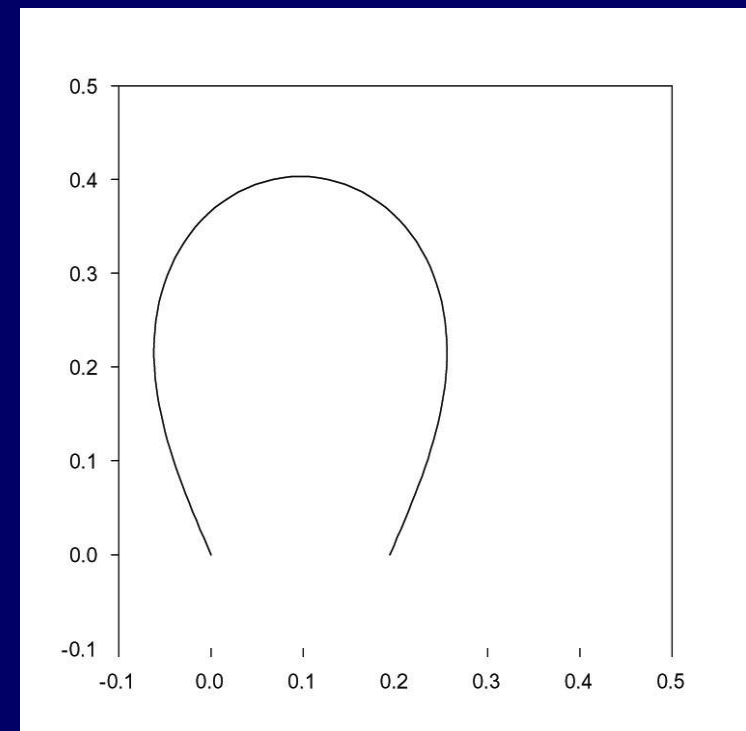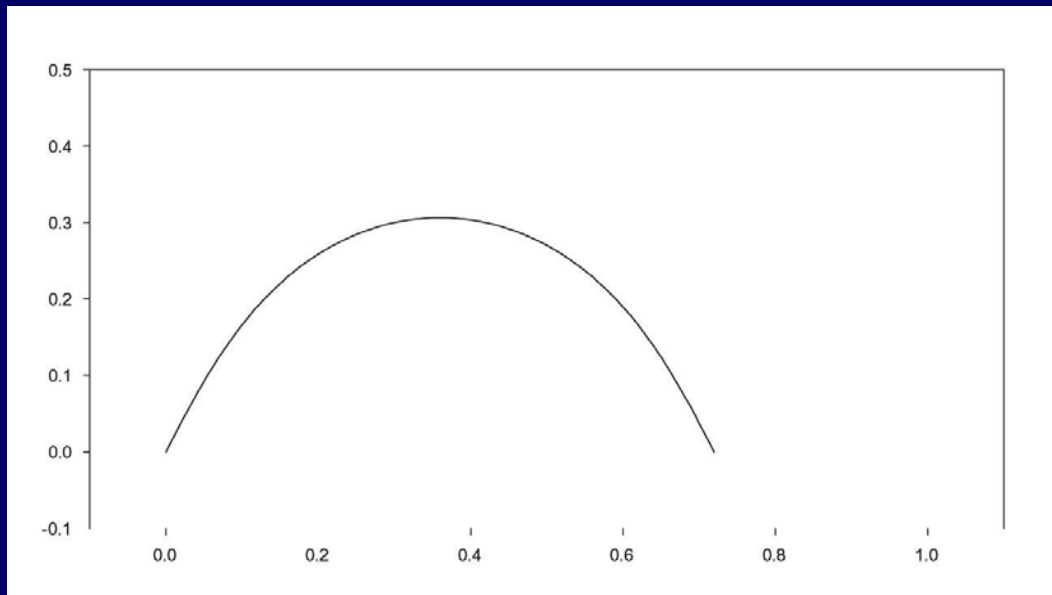
Simplifying Assumptions

- Equilibrium shape = circular arc
- Arc length remains constant
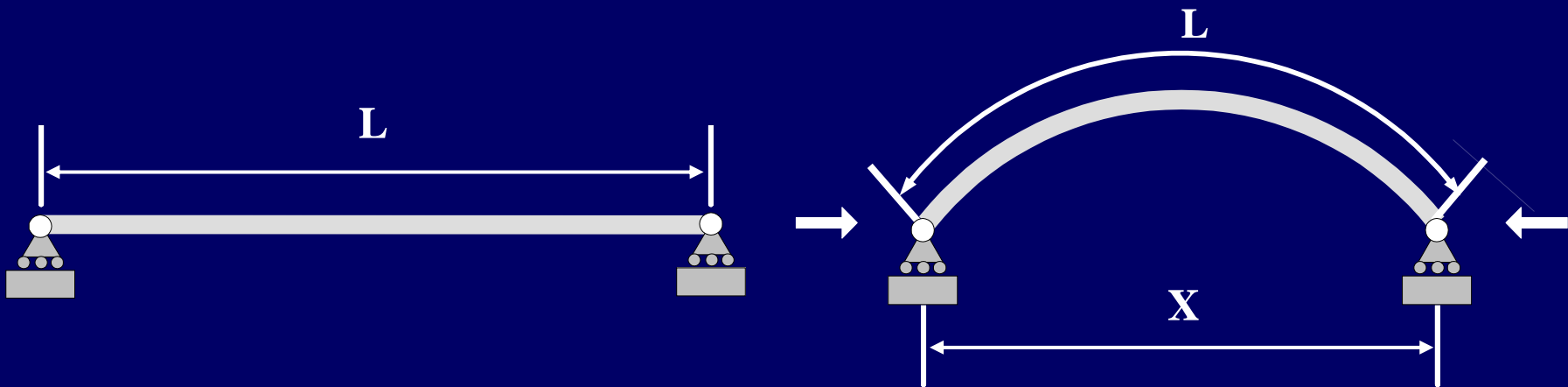
The Energy Function in terms of $x_{ij} = x_j - x_i$

- The energy function

$$E = \frac{1}{2} \int_0^L M\kappa \, dx$$

- Since $M \propto \kappa$ & curvature=constant

$$E = \frac{1}{2} k_b L \kappa^2$$

- Since arc length=constant

$$\kappa = \frac{2}{L} \text{sinc}^{-1}\left(\frac{|x_{ij}|}{L}\right) \qquad E = \frac{1}{2} k_b L \left[\frac{2}{L} \text{sinc}^{-1}\left(\frac{|x_{ij}|}{L}\right)\right]^2$$

Force Derivation

$$\mathbf{f}_i = -\frac{\partial E}{\partial \mathbf{x}_i} = \sum_{j \in \mathrm{N}(i)} f_b\left(\left|x_{ij}\right|\right)\frac{x_{ij}}{\left|x_{ij}\right|},$$

where $\mathrm{N}(i)$ is the set of neighboring particles

Force Graph

New Force-Displacement Curve

Force : P

Compressed Length : L-|x|

P

L

L-|x|

|x|

L
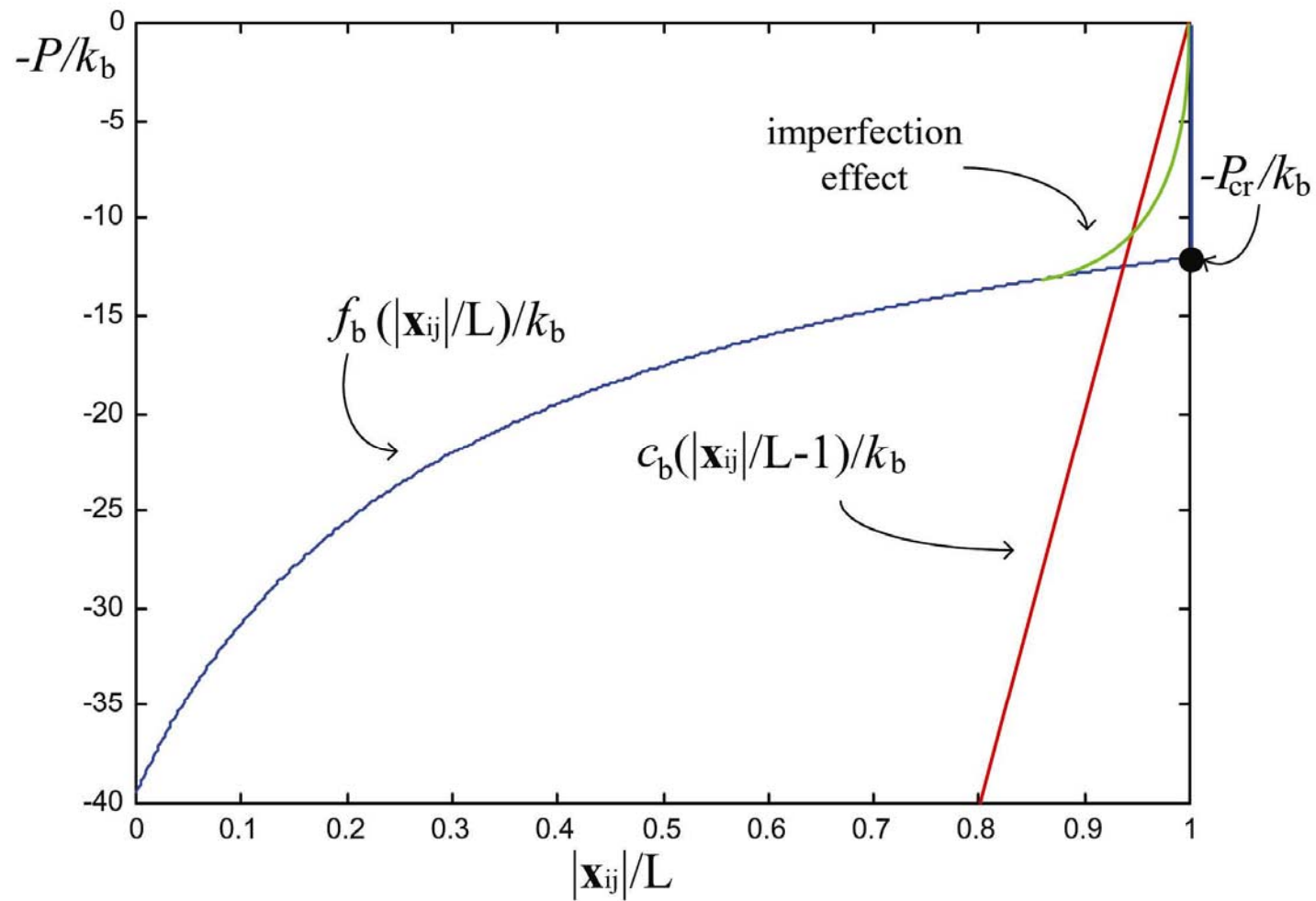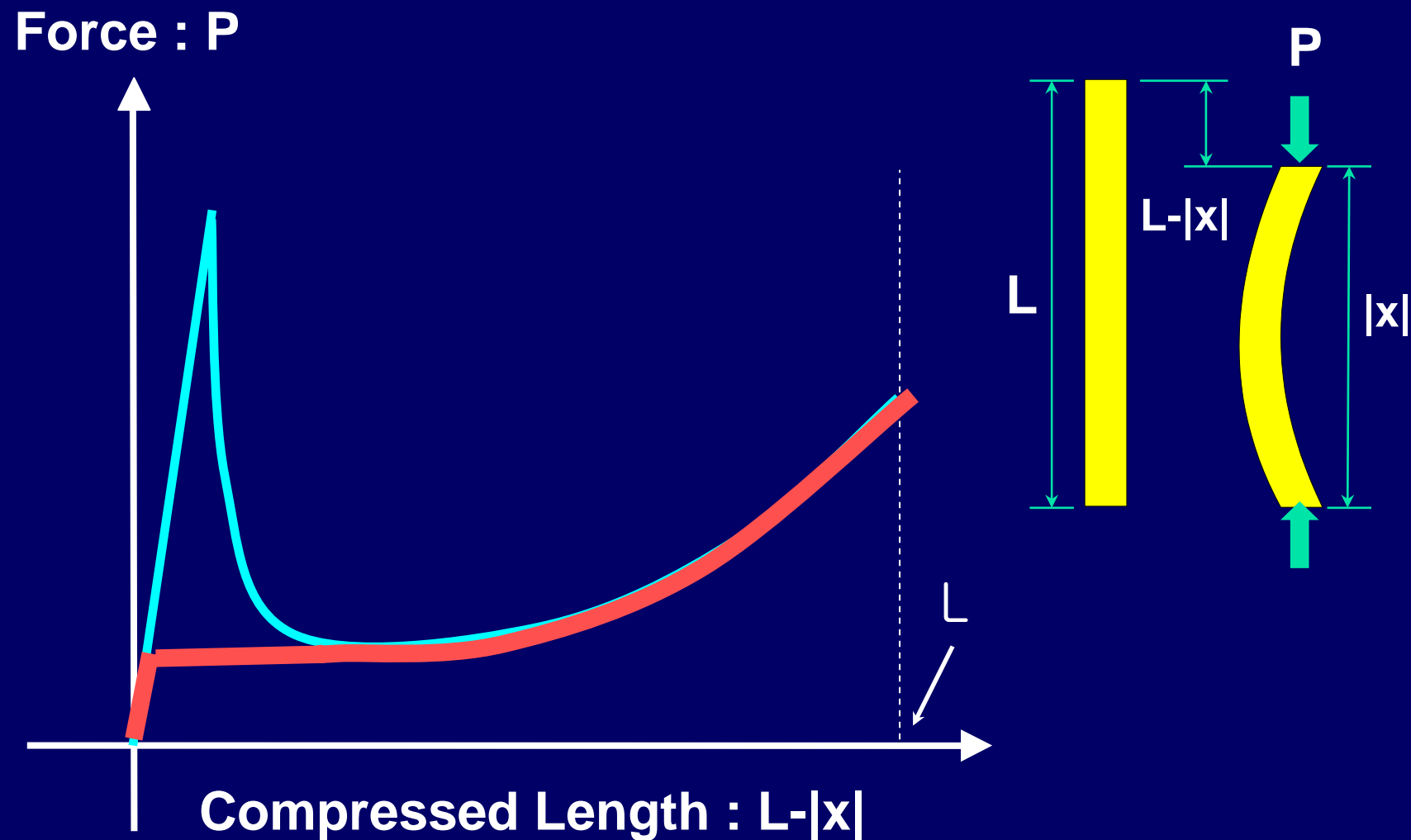
Now, We Can Start the Integration!

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

Haven't we considered Only Compressions?

- ## We use two types of springs
  - Type 1: for stretch
  - Type 2: for compression

$P(i,j)$
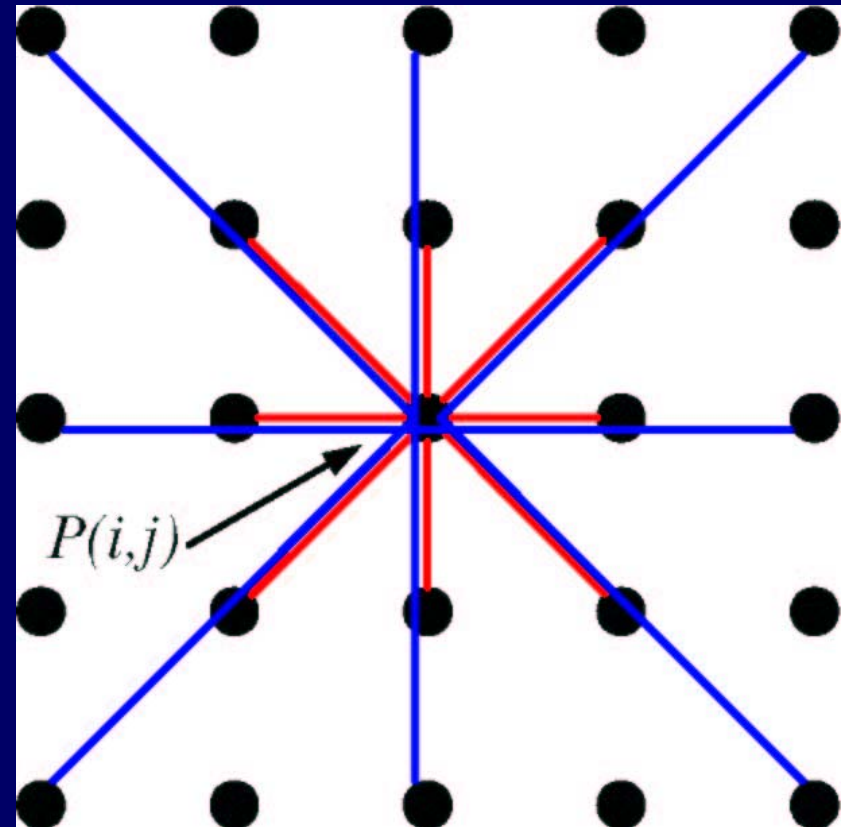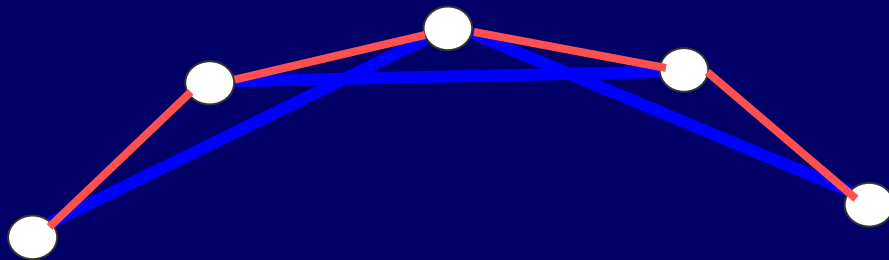
Connection of Springs
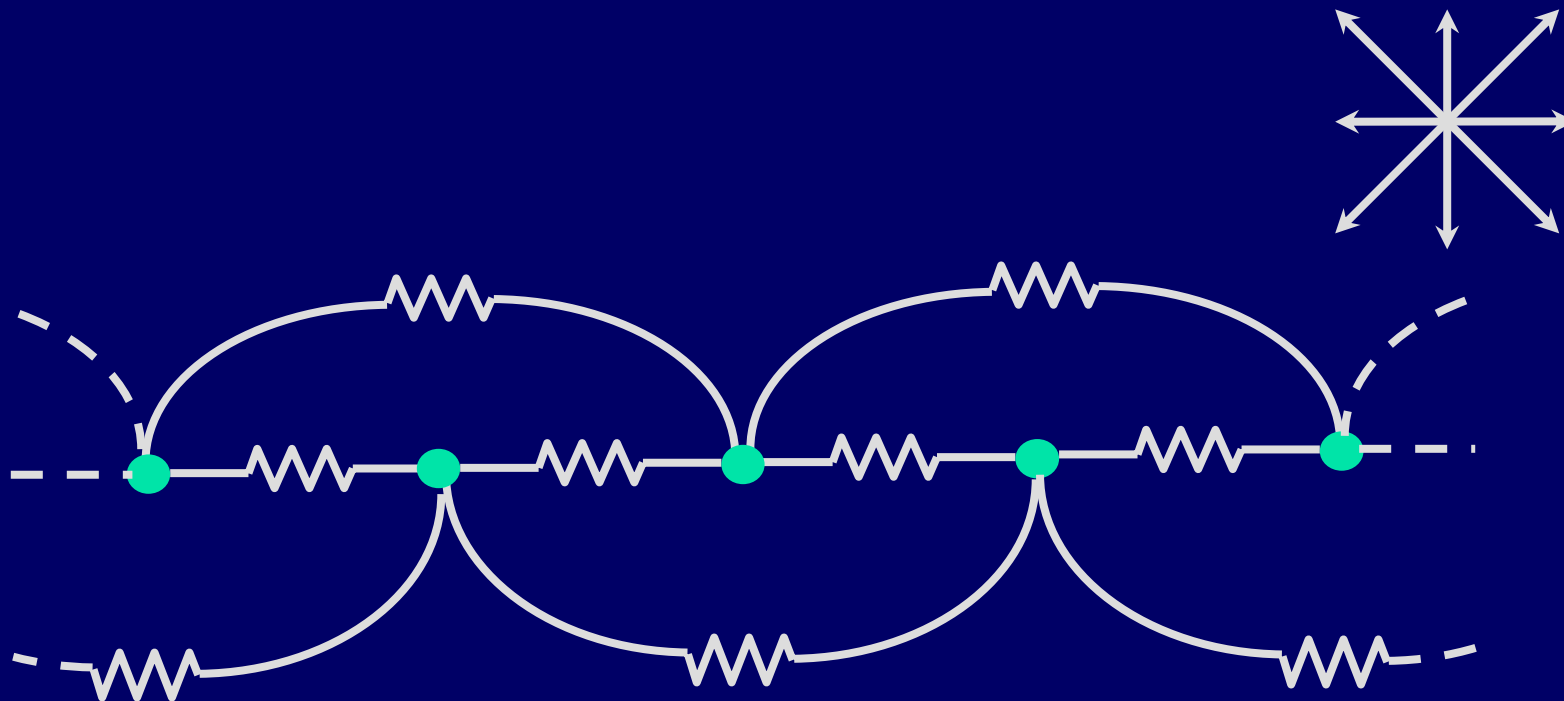
Let's Examine the Nature of This Part

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$

$$\ddot{x} = M^{-1}\left(\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_N \end{bmatrix} + \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_N \end{bmatrix}\right)$$

Let's Examine the Nature of This Part

$$\ddot{x} = M^{-1}(-\frac{\partial E}{\partial x} + F)$$

$$\ddot{x} = M^{-1}\left(\begin{bmatrix} \sum \mathbf{f}_{1j} \\ \sum \mathbf{f}_{2j} \\ \vdots \\ \sum \mathbf{f}_{Nj} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_N \end{bmatrix}\right)$$

Let's Examine the Nature of This Part

$$\ddot{x} = M^{-1}\left(-\frac{\partial E}{\partial x} + F\right)$$

$$\ddot{x} = M^{-1}\left(\begin{bmatrix} \sum \mathbf{f}_{1j} \\ \sum \mathbf{f}_{2j} \\ \vdots \\ \sum \mathbf{f}_{Nj} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \mathbf{F}_N \end{bmatrix}\right) \equiv M^{-1}\mathbf{f}(x, \dot{x})$$

Force Vector Derivation

$$\mathbf{f}_i = -\frac{\partial E}{\partial \mathbf{x}_i} = \sum_{j \in \mathrm{N}(i)} \mathbf{f}_{ij} = \sum_{j \in \mathrm{N}(i)} f_b\left(\left\|x_{ij}\right\|\right)\frac{x_{ij}}{\left|x_{ij}\right|},$$

where N(i) is the set of neighboring particles

HW 1: Derive Eqs 8, 9, and 10 of the paper

What Force Vector Means?

$$\mathbf{f}_{ij} = f_b\left(\left|x_{ij}\right|\right)\frac{x_{ij}}{\left|x_{ij}\right|}$$

- The contribution to move i-th particle when particles i and j are at x_i and x_j, respectively.
- Therefore, the contribution to move j-th particle when particles i and j are at x_i and x_j, respectively, is simply $-f_{ij}$

Implicit Integration of $\ddot{x} = M^{-1}\mathbf{f}(x, \dot{x})$

$$\ddot{x} = M^{-1}\mathbf{f}(x, v)$$

$$\begin{bmatrix} \Delta x \\ \Delta v \end{bmatrix} = h \begin{bmatrix} v_0 + \Delta v \\ M^{-1}\mathbf{f}(x + \Delta x, v + \Delta v) \end{bmatrix}$$

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f_0} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v_0} \right)$$

HW 2: Derive this

Implicit Integration of $\ddot{x} = M^{-1}\mathbf{f}(x, \dot{x})$

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta\mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f_0} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

- For now, forget about the damping term.

$$\left(\mathbf{I} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta\mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f_0} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

Meaning of the Force Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{\partial \sum\limits_{k \in \mathrm{N}} \mathbf{f}_{ik}}{\partial \mathbf{x}_j} = \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}$$

- Other f_{ik} is not a function of x_j.

The rate of change of f_i as x_j changes

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} = -\sum\limits_{j \neq i} \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}$$

- As for each f_{ij}, moving x_i has the same effect as moving x_j in the opposite direction

- The rate of change of f_i as x_i changes

Meaning of the Force Jacobian $\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}$

- The rate of change of f_i as x_j changes
- Scalar reasoning:
  - When x_{ij} is larger than the rest length, it means stretch stiffness.
  - When x_{ij} is shorter than the rest length, it means *buckling stiffness* (our terminology).
  - When x_j is moving orthogonal direction, it means out-of-plane stiffness.

Derivation of Force Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{|x_{ij}|}\left(I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}\right)$$

HW 3: Derive the above

- What is the nature of this 3×3 matrix?
- What is the nature of the global 3n×3n matrix?

The Properties of $\dfrac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}} = A$

$$\frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}} = \begin{bmatrix} u^2 & uv & uw \\ uv & v^2 & vw \\ uw & vw & w^2 \end{bmatrix}, \text{where } (u,v,w) = \frac{x_{ij}}{\|x_{ij}\|}$$

$$\det(\frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}} - \lambda I) = \det \begin{bmatrix} u^2 - \lambda & uv & uw \\ uv & v^2 - \lambda & vw \\ uw & vw & w^2 - \lambda \end{bmatrix} = \lambda^2(1-\lambda)$$

HW 4: Derive it

$\therefore \dfrac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}}$ is positive definite. Eigen Vector for $\lambda = 1$ is $\begin{bmatrix} u \\ v \\ w \end{bmatrix}$

HW 5: Why?

$\dfrac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}}$ functions as a projection operator.

HW 6: Why?

Az produces the projection of z along x_{ij}

The Properties of $I - \dfrac{x_{ij} x_{ij}^{T}}{x_{ij}^{T} x_{ij}} = I - A$
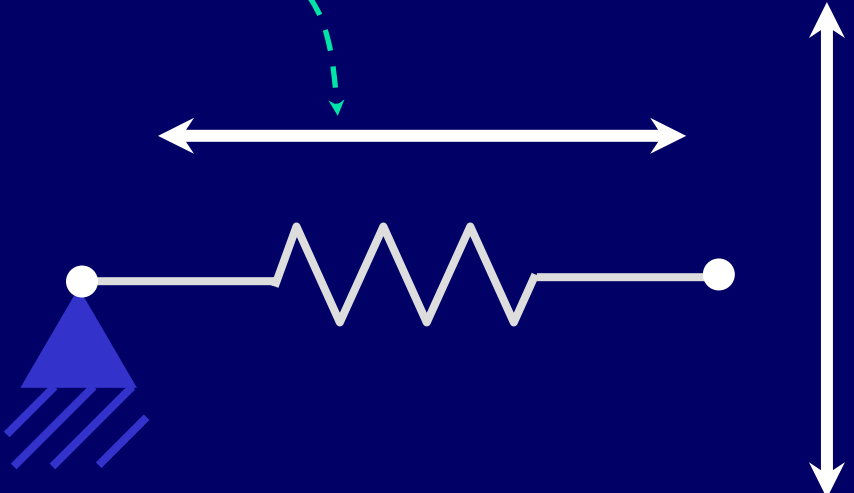
- Eigenvalues are 1, 1, 0, thus pos. def.
  - If $A = E\Lambda E^{T}$, $I-A = EE^{T} - E\Lambda E^{T} = E(I-\Lambda)E^{T}$
  - Thus eigenvalues(I-A) = 1-eigenvalues(A)

- Eigenvectors are orthogonal to (u,v,w)
  - Since the diagonalizing matrix E is common for both I-A and A, we can conclude engenvectors of I-A corresponding to $\lambda=1$ are orthogonal to the engenvector of A corresponding to $\lambda=1$.

- (I-A)z projects z to the engenspace (plane).

HW 7: Why?

When a displacement is multiplied:

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} d\mathbf{x}_j = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} d\mathbf{x}_j + \frac{f_b^*}{|x_{ij}|} (I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}) d\mathbf{x}_j$$

- 1st term: dx_j is projected to x_{ij} dir, and gives the increase (or decrease) of the force in the x_{ij} dir due to dx_j
- 2nd term: dx_j is projected to a plane that is normal to x_{ij} dir, and gives the increase (or dec.) of the force in the plane dir due to dx_j
- $\frac{df_b^*}{d|x_{ij}|}$ = buckling stiffness $\frac{f_b^*}{|x_{ij}|}$ =out-of-pl stiffness

Interpret as Projection Operators

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} d\mathbf{x}_j = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} d\mathbf{x}_j + \frac{f_b^*}{|x_{ij}|} (I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}) d\mathbf{x}_j$$

Nature of the Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{|x_{ij}|} (I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}})$$

- $\frac{df_b^*}{d|x_{ij}|}$ is positive.
- But $\frac{f_b^*}{|x_{ij}|}$ is negative.
- So the sum is not definite
- We omit 2nd term to make it pos. def.

Nature of the Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d\left|x_{ij}\right|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}$$

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- But $\dfrac{f_b^*}{|x_{ij}|}$ is negative.
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- We omit 2nd term to make it pos. def.
- What is the Effect of it?

Nature of the Jacobian

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij}x_{ij}^T}{x_{ij}^T x_{ij}}$$

- $\frac{df_b^*}{d|x_{ij}|}$ is positive.
- But $\frac{f_b^*}{|x_{ij}|}$ is negative.
- So the sum is not definite
- We omit 2nd term to make it pos. def.
- What is the effect of it?
- Is the global matrix negative definite?

Is the Global Matrix Neg. Def.?

- First of all, how does the global matrix look like?

Formation of Global Force Jacobian

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} & \cdots & \dfrac{\partial \mathbf{f}_1}{\partial \mathbf{x}_n} \\[2em] \dfrac{\partial \mathbf{f}_2}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{f}_2}{\partial \mathbf{x}_2} & \cdots & \dfrac{\partial \mathbf{f}_2}{\partial \mathbf{x}_n} \\[1em] \vdots & \vdots & \ddots & \vdots \\[1em] \dfrac{\partial \mathbf{f}_n}{\partial \mathbf{x}_1} & \dfrac{\partial \mathbf{f}_n}{\partial \mathbf{x}_2} & \cdots & \dfrac{\partial \mathbf{f}_n}{\partial \mathbf{x}_n} \end{bmatrix}$$

Formation of Global Jacobian

In this case, we use the rigorous definition.

$$\mathbf{f}_i = \sum_{j \in N} f_b \left(\left| x_{ij} \right| \right) \frac{x_{ij}}{\left| x_{ij} \right|} = \sum_{j \in N} \mathbf{f}_{ij}$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}, \text{ for } j \neq i$$

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} = -\sum \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{x}_j}$$
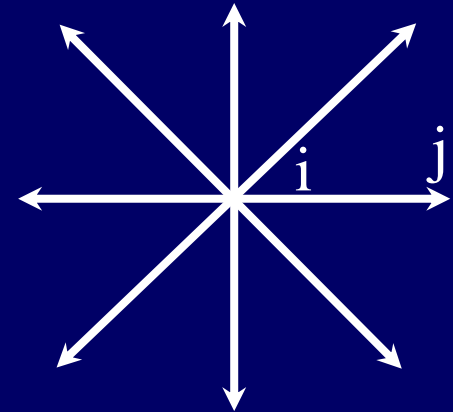
Global Jacobian Looks Like...

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} -\sum_{j \neq 1} \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_j} & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_n} \\ \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_1} & -\sum_{j \neq 2} \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_j} & \cdots & \frac{\partial \mathbf{f}_2}{\partial \mathbf{x}_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_1} & \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_2} & \cdots & -\sum_{j \neq n} \frac{\partial \mathbf{f}_n}{\partial \mathbf{x}_j} \end{bmatrix}$$

- If $\partial f_i / \partial x_j$ is positive definite for all $i \neq j$, is $\partial f / \partial x$ negative definite? HW 8: YES, Prove it

  - Hint: $\partial f_i / \partial x_j$ is symmetric, and $\partial f_i / \partial x_j = \partial f_j / \partial x_i$. Thus $\partial f / \partial x$ is symmetric.

Physical Model of Cloth II

Kwang-Jin Choi

FXGear Inc.

What is the meaning of Omitting the Second Term?

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} + \frac{f_b^*}{|x_{ij}|} (I - \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}})$$

What is the meaning of Omitting the Second Term?

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}}$$

- It makes each $\partial f_i/\partial x_j$ positive definite
- Thus it makes $\partial f/\partial x$ negative definite
- Thus makes the whole system matrix positive definite
- Does it make the algorithm unconditionally stable?

Local Matrix Reasoning

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} d\mathbf{x}_j = \frac{df_b^*}{d|x_{ij}|} \frac{x_{ij} x_{ij}^T}{x_{ij}^T x_{ij}} d\mathbf{x}_j$$

- When particle j is displaced by dx$_j$, only the parallel component of dx$_j$ is counted; the orthogonal components are disregarded.

$d\mathbf{x_j}$ not counted

L

counted
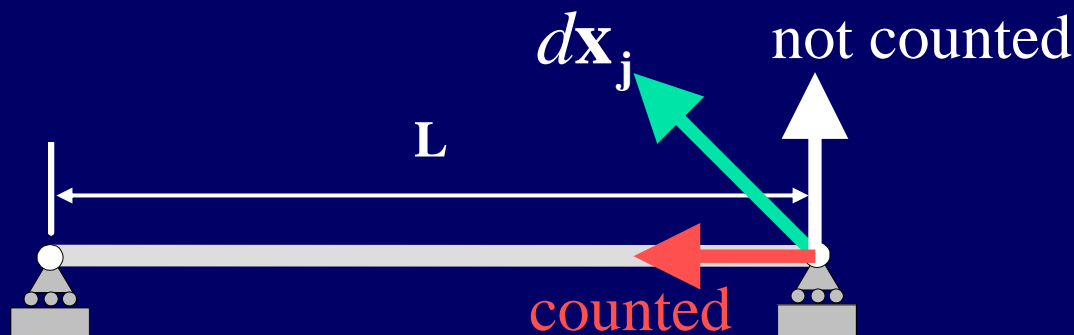
In Global Implicit Integration

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f_0} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

- For now, forget about the damping term.

$$\left(\mathbf{I} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f_0} + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

Looking at the i-th Row

$$LHS = \begin{bmatrix} \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0 & \cdots & -h^2\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{x}_{j1}} & \cdots & \mathbf{I} - h^2\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{x}_i} & \cdots & -h^2\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{x}_{jN}} & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \end{bmatrix} \begin{bmatrix} \Delta \mathbf{v}_1 \\ \Delta \mathbf{v}_2 \\ \vdots \\ \Delta \mathbf{v}_n \end{bmatrix}$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

Comparison with Explicit Method

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

- In RHS, $h \mathbf{M}_i^{-1} \left(\sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$ is the part added due to using implicit integration

  - It is negative quantity since $\partial f_i / \partial x_i$ is dominant

- In LHS, every part except $\Delta \mathbf{v}_i$ is the part added due to using implicit integration

  - It is positive quantity since $\partial f_i / \partial x_i$ is dominant

Implicit Filtering

$$LHS = \Delta\mathbf{v}_i + \left(\sum_{j\neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta\mathbf{v}_i - \sum_{j\neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta\mathbf{v}_j \approx (I + \varepsilon)\Delta\mathbf{v}_i$$

$$RHS = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right) = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} - \varepsilon_{\mathbf{f}} \right)$$

- Compared to explicit method $\Delta\mathbf{v}_i$ is scaled
  - It corresponds to slowing down, thus producing artificial damping effect
- It may also viewed as averaging the terms neighboring particle i
- We can view multiplying the global matrix $\partial f/\partial x$ as a filter.

Decomposition of Displacements

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) (\Delta \mathbf{v}_i^p + \Delta \mathbf{v}_i^o) - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\Delta \mathbf{v}_j^p + \Delta \mathbf{v}_j^o)$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

$$= h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\mathbf{v}_{0j}^p + \mathbf{v}_{0j}^o) \right)$$

If 2nd Term of $\partial \mathbf{f}_i / \partial \mathbf{x}_j$ is Omitted

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) (\Delta \mathbf{v}_i^p + \Delta \mathbf{v}_i^o) - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\Delta \mathbf{v}_j^p + \Delta \mathbf{v}_j^o)$$

$$= \Delta \mathbf{v}_i + \left(\sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i^p - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j^p$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^n h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

$$= h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^n h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} (\mathbf{v}_{0j}^p + \mathbf{v}_{0j}^o) \right) = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^n h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j}^p \right)$$

So, What is the Effect?

- "The force in the orthogonal direction is not affected by implicit filtering."
- Thus, the force in the orthogonal direction is in fact integrated by explicit method

What happens to Stability?

- By the omission, the system matrix is positive definite

- However, since the method is now not fully semi-implicit, stability is not guaranteed

- How large is the orthogonal force?
  - If it is small compared to stretch force, there is little possibility system will diverge

When is the Ortho Force Generated?
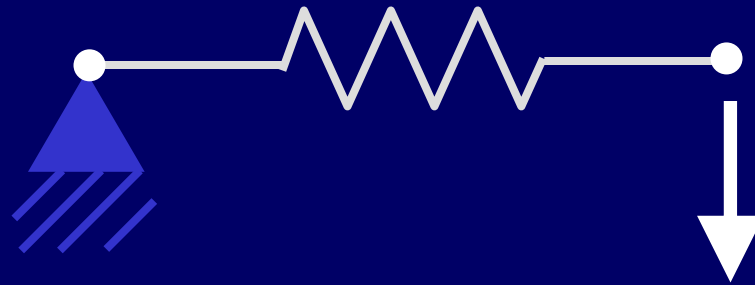
- When one end is fixed, and the other end is pushed
  - e.g, gravity, pushing by finger

More Probable Scenario
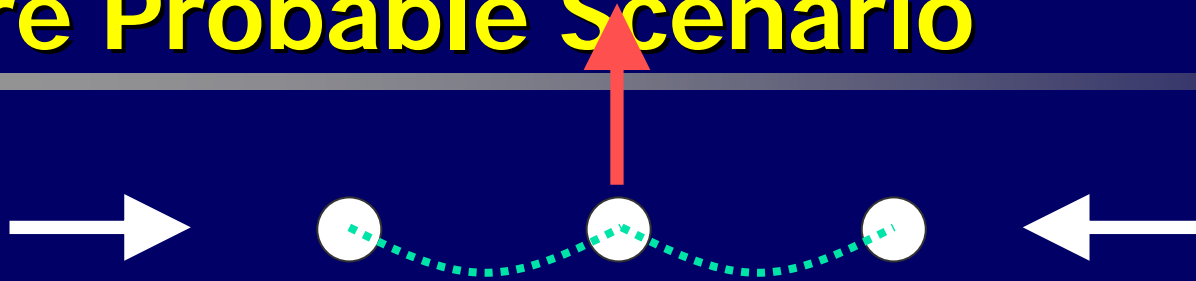
- When a compressive force is applied to two consecutive links

When it becomes Unstable?

- When we simulate a stiff material with high resolution

Now, Consider Damping

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right)$$

Now, Consider Damping

$$\mathbf{f}_i = -k_d(\mathbf{v}_i - \mathbf{v}_j)$$

- Damping force (contribution) exerted on particle i from the interaction with particle j

- Not for stability but to prevent oscillatory responses; We will show it has no effect for linear rigid motion.

- Its derivative

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_j} = k_d \mathbf{I} \qquad \frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_i} = -\sum_{j \in N(i)} \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{v}_{ij}} = -|N(i)| k_d \mathbf{I}$$

Damping in Orthogonal Direction

$$\mathbf{f}_i = -k_d (\mathbf{v}_i - \mathbf{v}_j)$$

- Note that this force does not add any damping to the orthogonal direction.
- The out-of-plane movement of cloth won't be affected by inclusion of this force.
- This is important because damping along out-of-plane direction degrades the realism significantly.

Avoiding Out-of-plane Damping

- Out-of-plane damping can be caused by air resistance, even though it is completely ignored in this work.

- To consider it, we have to model the air resistance. We do not do that.

- But in this work, we avoid out-of-damping being generated in the way we did not intend.

- One possible source unintended out-of-plane damping steps in is through implicit filtering.

Does Our Damping Force Produce Filtering Effect?

$$\mathbf{f}_i = -k_d (\mathbf{v}_i - \mathbf{v}_j)$$

- It does not create any undesirable filtering effect.

- How to show that point?

  - "Our damping force does not create a filtering effect under a semi-implicit formulation when the cloth undergoes a linear rigid motion."

Proof

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2 \mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right)\Delta \mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0 \right)$$

- Look at the i-th row of system eq.
- At this time, we forget about the stiffness part.

Proof

$$\left(\mathbf{I} - h\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - h^2\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)\Delta\mathbf{v} = h\mathbf{M}^{-1}\left(\mathbf{f}_0 + h\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\mathbf{v}_0\right)$$

$$LHS = \begin{bmatrix} \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \\ 0 & \cdots & -h\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{v}_{j1}} & \cdots & \mathbf{I} - h\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{v}_i} & \cdots & -h\mathbf{M}_i^{-1}\dfrac{\partial \mathbf{f}_i}{\partial \mathbf{v}_{jN}} & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots & \vdots \end{bmatrix}\begin{bmatrix} \Delta\mathbf{v}_1 \\ \Delta\mathbf{v}_2 \\ \vdots \\ \Delta\mathbf{v}_n \end{bmatrix}$$

$$= \left(\mathbf{I} + \sum_{j \neq i} h\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_j}\right)\Delta\mathbf{v}_i - \sum_{j \neq i} h\mathbf{M}_i^{-1}\frac{\partial \mathbf{f}_i}{\partial \mathbf{v}_j}\Delta\mathbf{v}_j = \Delta\mathbf{v}_i + \sum_{j \neq i} hk_d\mathbf{M}_i^{-1}(\Delta\mathbf{v}_i - \Delta\mathbf{v}_j)$$

- In linear rigid motion, v_k are identical for all k. Same for Δv_k. So LHS= Δv_i.

- Therefore, our damping force does not cause any filtering effect for linear rigid motion along any direction.

In linear rigid motion, the stiffness term causes no filtering effect, either.

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j$$

$$= \Delta \mathbf{v}_i + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \left(\Delta \mathbf{v}_i - \Delta \mathbf{v}_j \right) = \Delta \mathbf{v}_i$$

- Some might view some filtering effect still remaining in the RHS:

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j} \right)$$

Situation with absolutely no filtering effect

- When cloth is making a linear rigid motion along purely orthogonal direction of particle interaction direction
  - e.g. flat handkerchief is making a rigid motion along the normal direction

$$RHS = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}\mathbf{v}_{0j}\right) = h\mathbf{M}_i^{-1}\left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j}(\mathbf{v}_{0j}^{o})\right) = h\mathbf{M}_i^{-1}\mathbf{f}_{0i}$$

- The above wouldn't be the case if we did not omit the second term of $\partial \mathbf{f}/\partial \mathbf{x}$

- Omit helps to eliminate undesired damping

More General Statement

- When cloth is moving along purely orthogonal direction of particle interaction direction, then stiffness term causes no filtering effect.

- This is the consequence resulting from omitting the 2nd term in $\partial f/\partial x$

$$LHS = \left(\mathbf{I} + \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \right) \Delta \mathbf{v}_i^o - \sum_{j \neq i} h^2 \mathbf{M}_i^{-1} \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \Delta \mathbf{v}_j^o = \Delta \mathbf{v}_i$$

$$RHS = h \mathbf{M}_i^{-1} \left(\mathbf{f}_{0i} + \sum_{j=1}^{n} h \frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} \mathbf{v}_{0j}^o \right) = h \mathbf{M}_i^{-1} \mathbf{f}_{0i}$$

About the Filtering Effect of Damping Term + Stiffness Term

- When cloth is making a linear rigid motion along purely orthogonal direction of particle interaction direction, then neither of damping term or stiffness term generates filtering effect.

- Even in more general situations, our formulation produces very small artificial damping in the out-of-plane direction, which potentially makes the cloth movement look realistic.

New Force-Displacement Curve
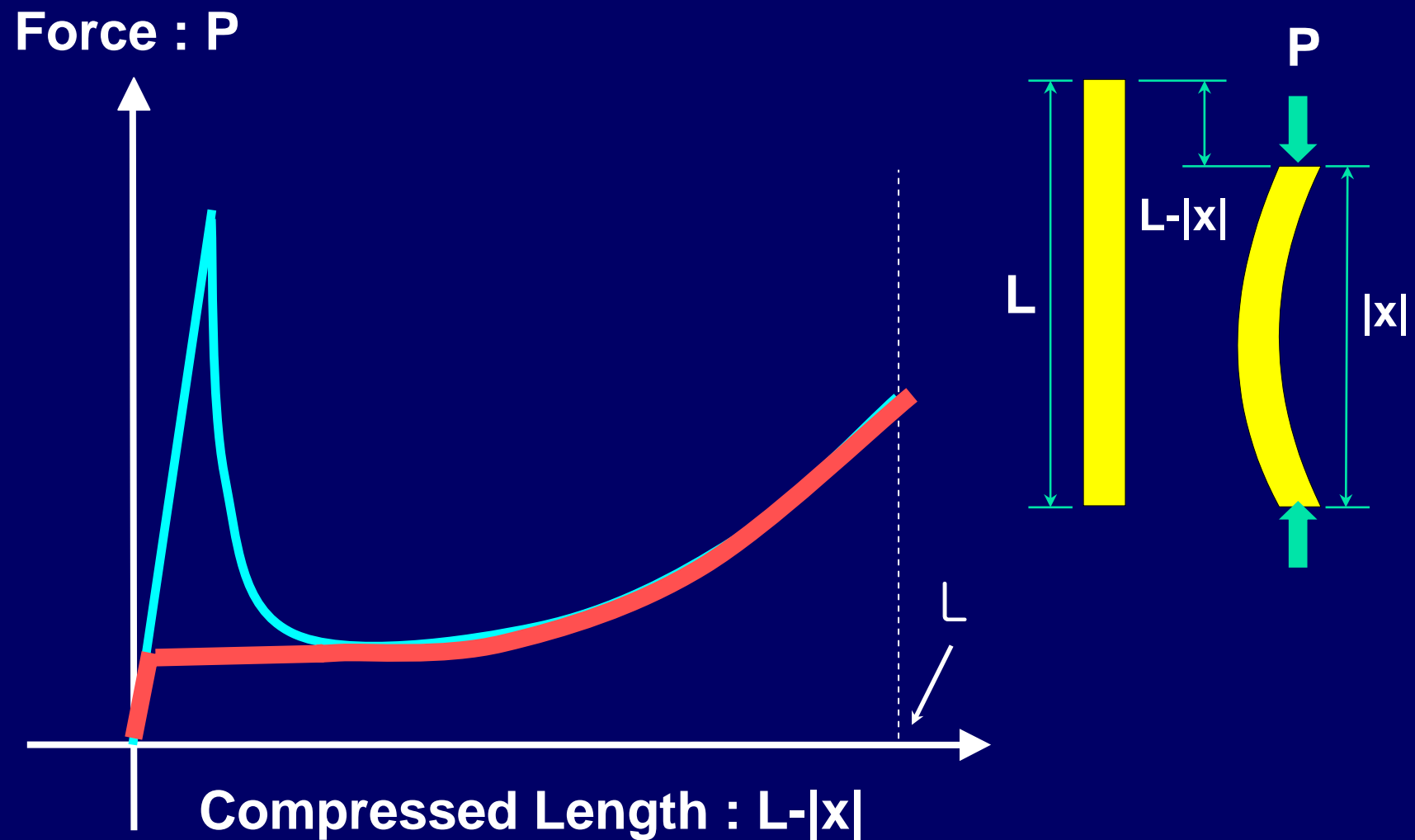
The Stuff We Get

- A model that does not have buckling instability
    - → Large time steps / Coarse meshes

- No Fictitious Damping is Needed
    - → Realistic Cloth

Results

# of particles = 3456, Δt=0.011s, CPU=0.25s

Leaping

# of particles = 5579, Δt=0.011s, CPU=0.47s

Walking
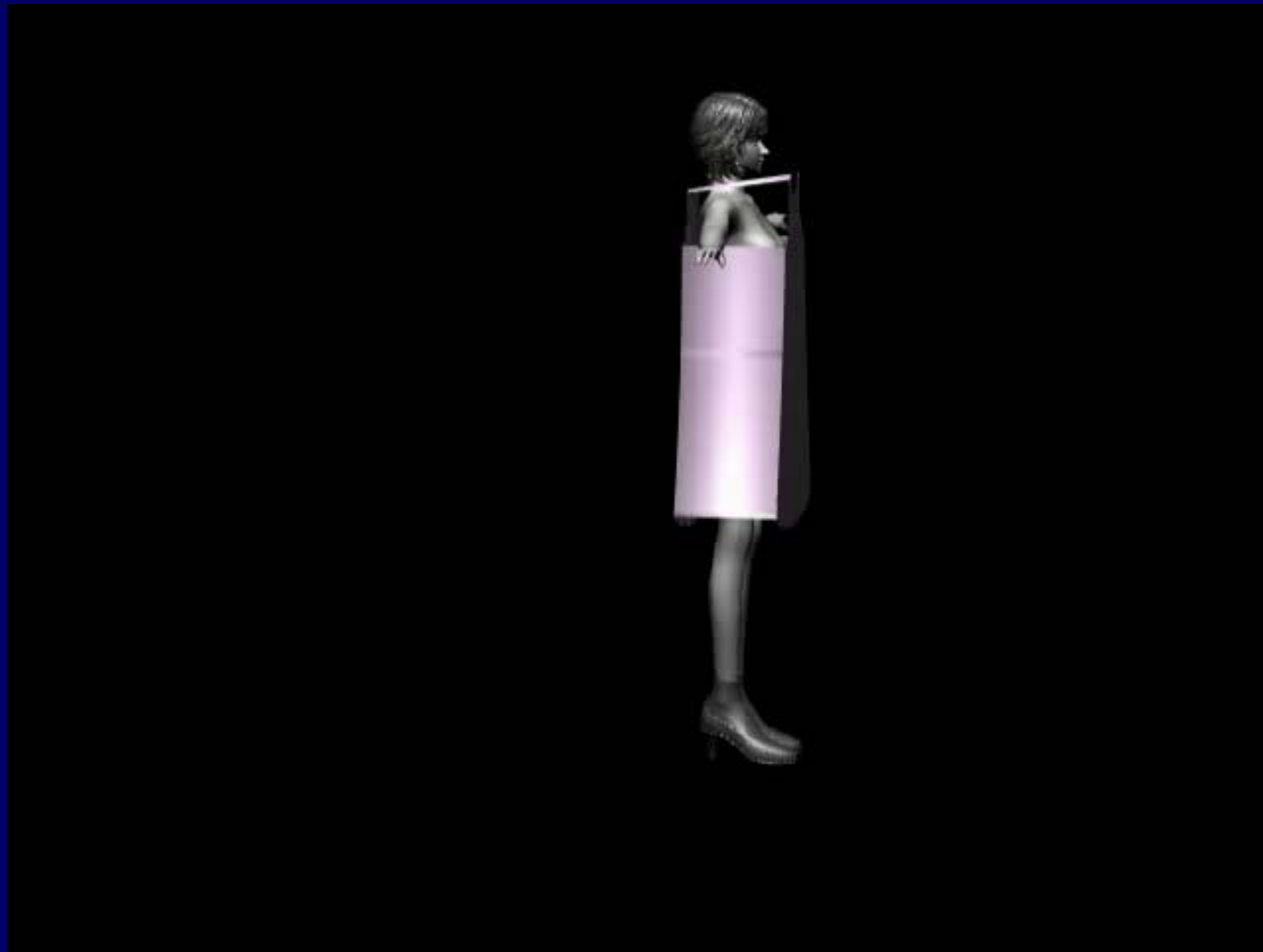
# of particles = 5608, Δt=0.011s, CPU=0.51s

Comparison
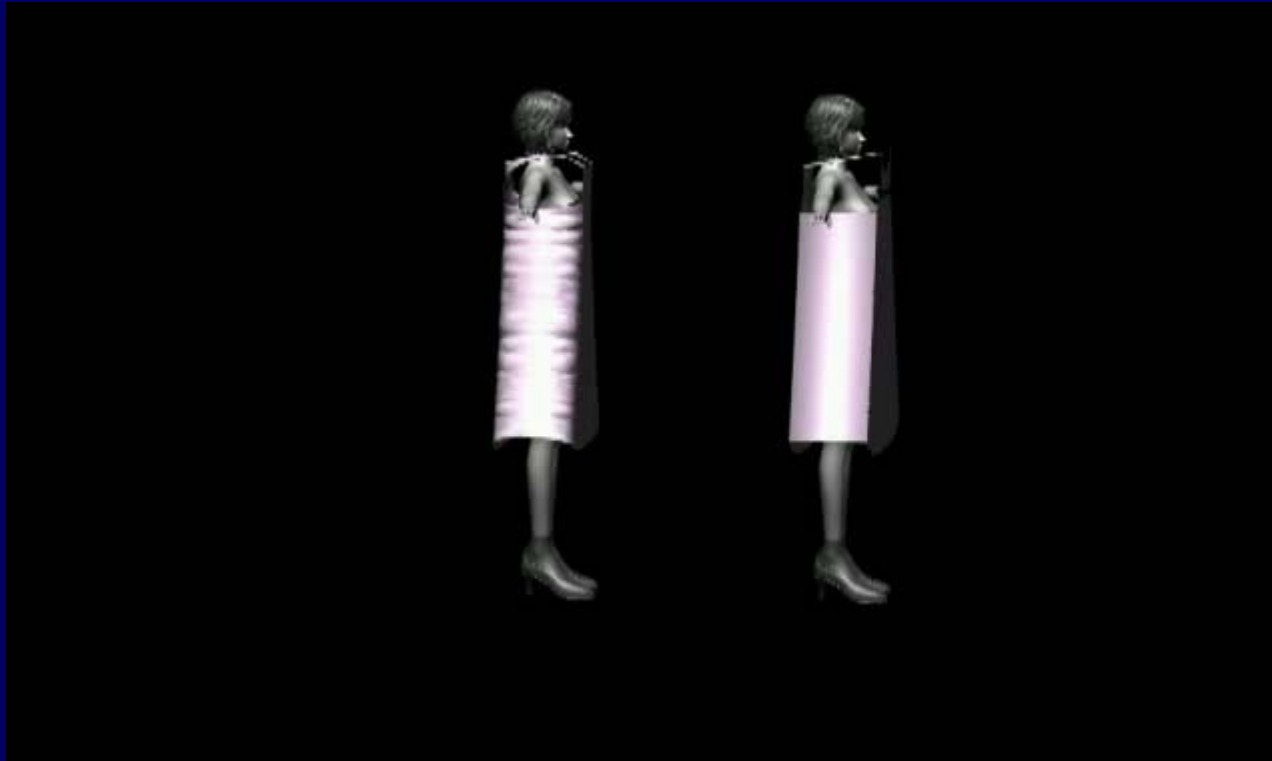
Fictitious damping	No fictitious damping
Adaptive time step size	Fixed time step size
Calculation time : 4 hours	Calculation time : 19 min

Extension to Triangle Mesh

Quad Mesh Implementation

Immediate
Buckling Model
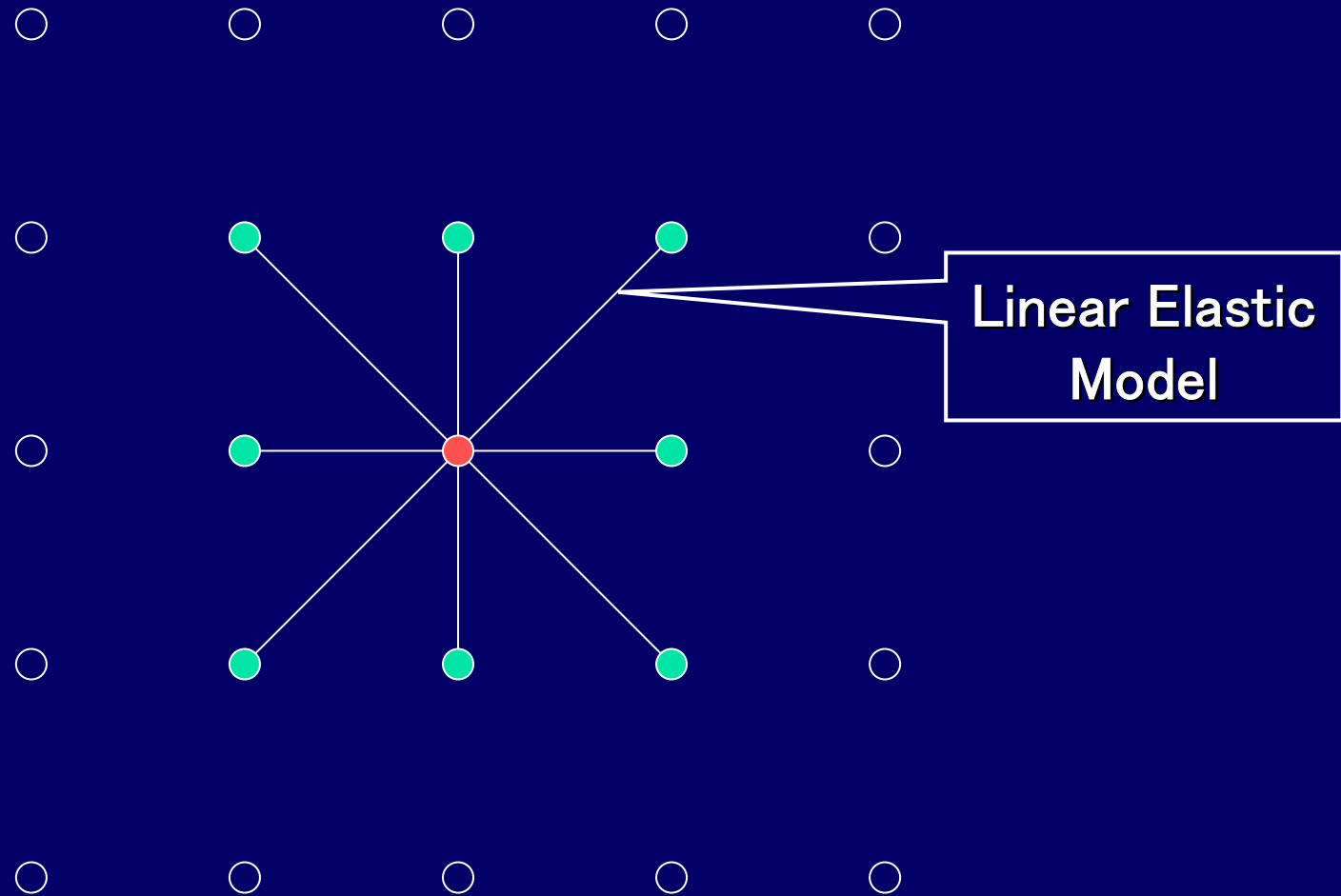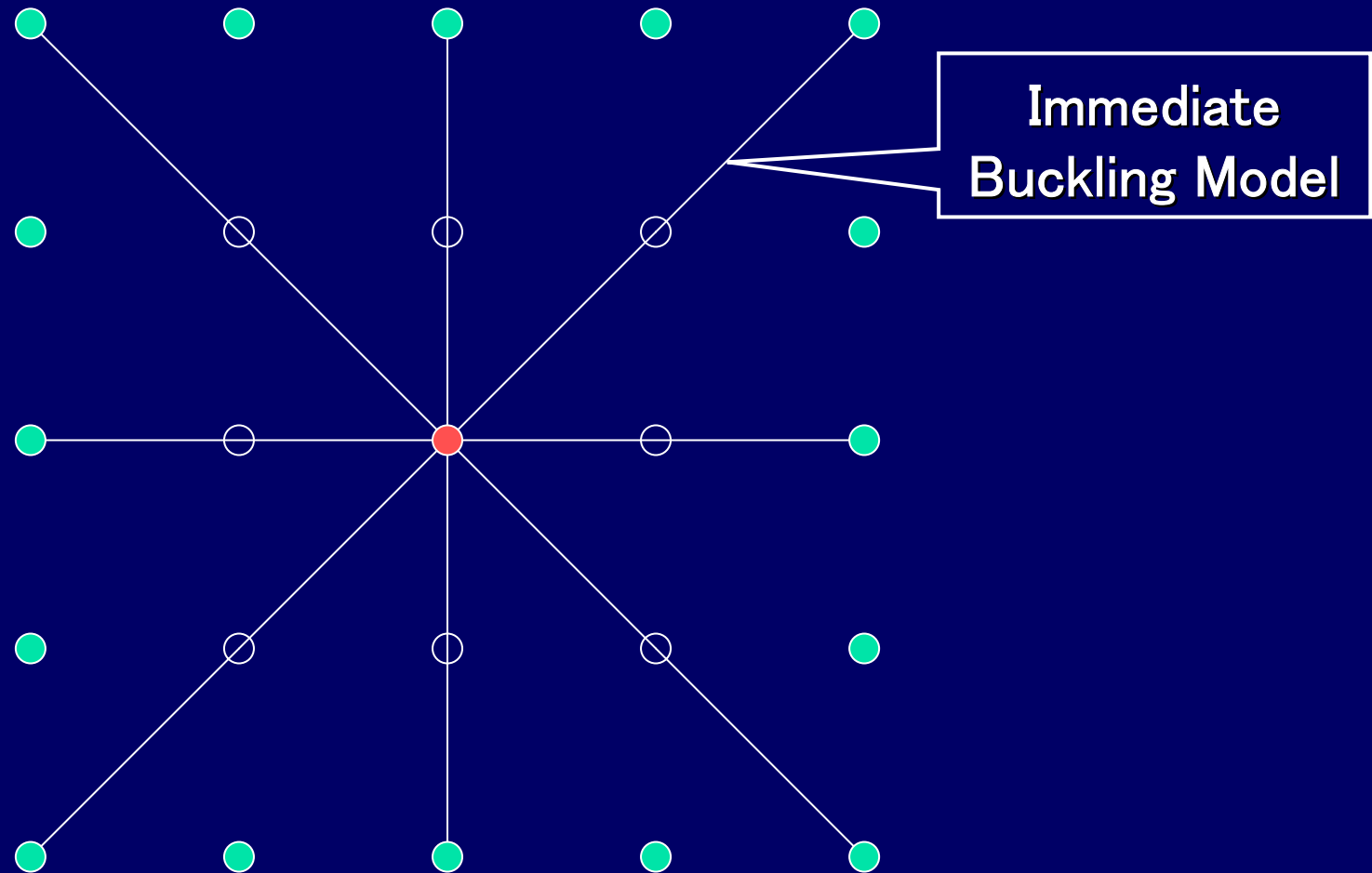
Quad Mesh Implementation
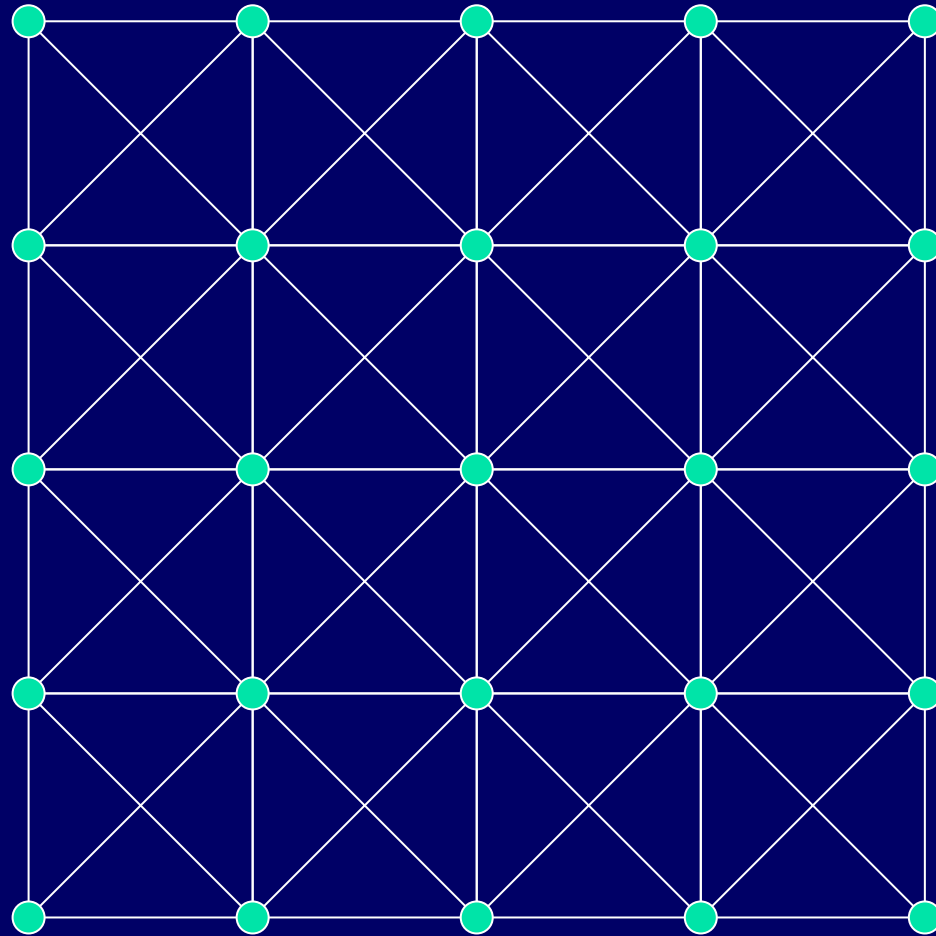
Why Unstructured Triangles?

- Cloth pieces have arbitrary boundary shapes
  - Structured quads : non-uniform resolution/shape
  - Unstructured triangles : almost uniform resolution/shape

Extension to Triangle Mesh

Deform

Undeformed

Deformed
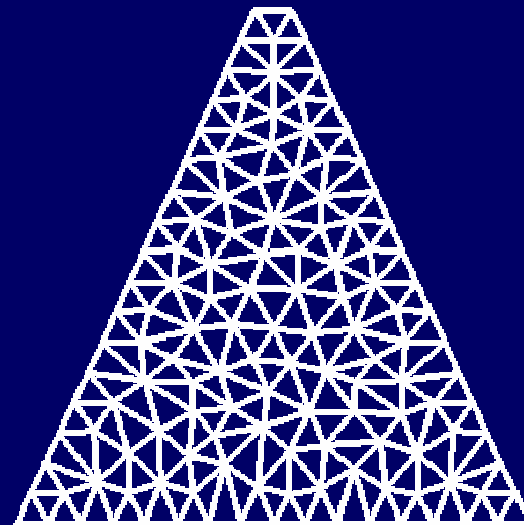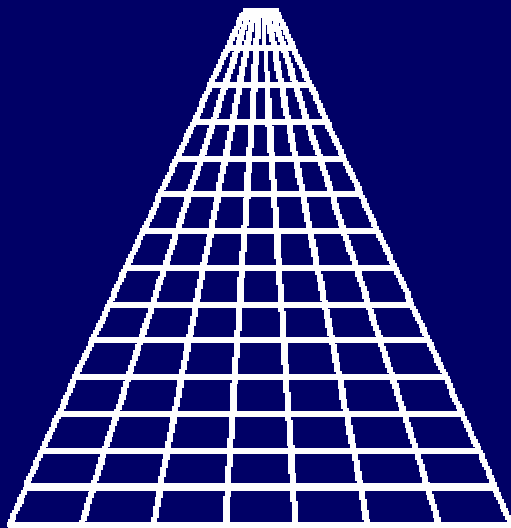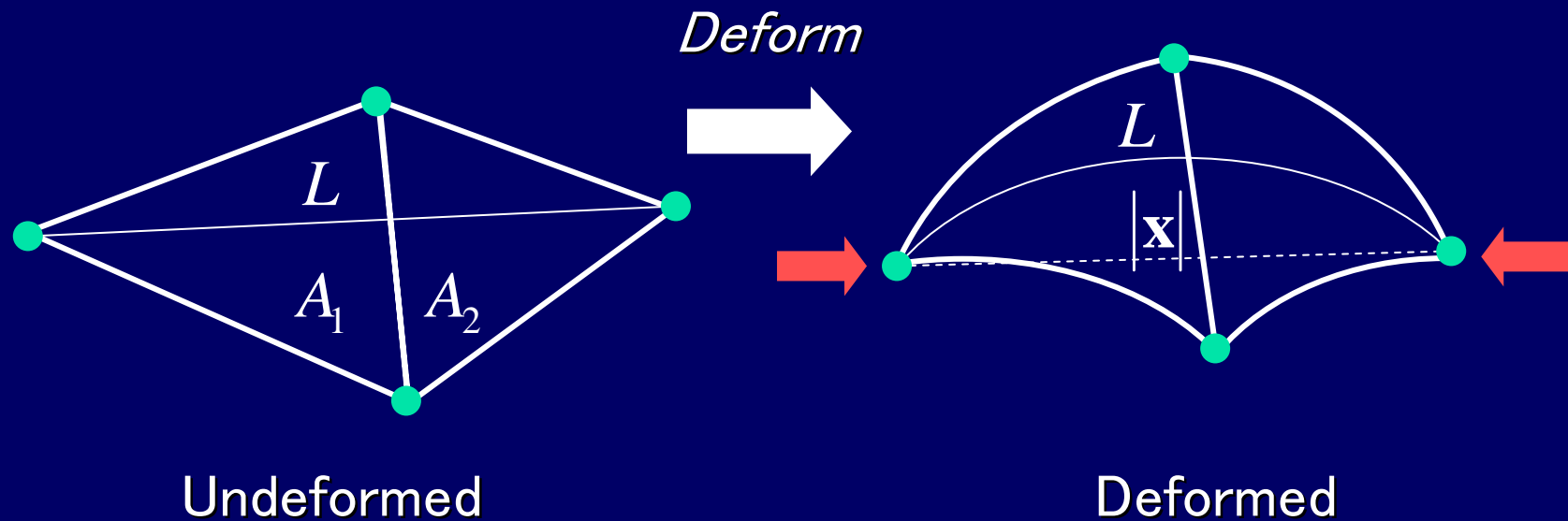
$$\kappa = \frac{2}{L} sinc^{-1}\left(\frac{|x|}{L}\right), \quad sinc(x) = \frac{\sin x}{x}$$

$$E_{bend} = (A_1 + A_2)K_b\kappa^2$$

Extension to Triangle Mesh

Triangle mesh connectivity
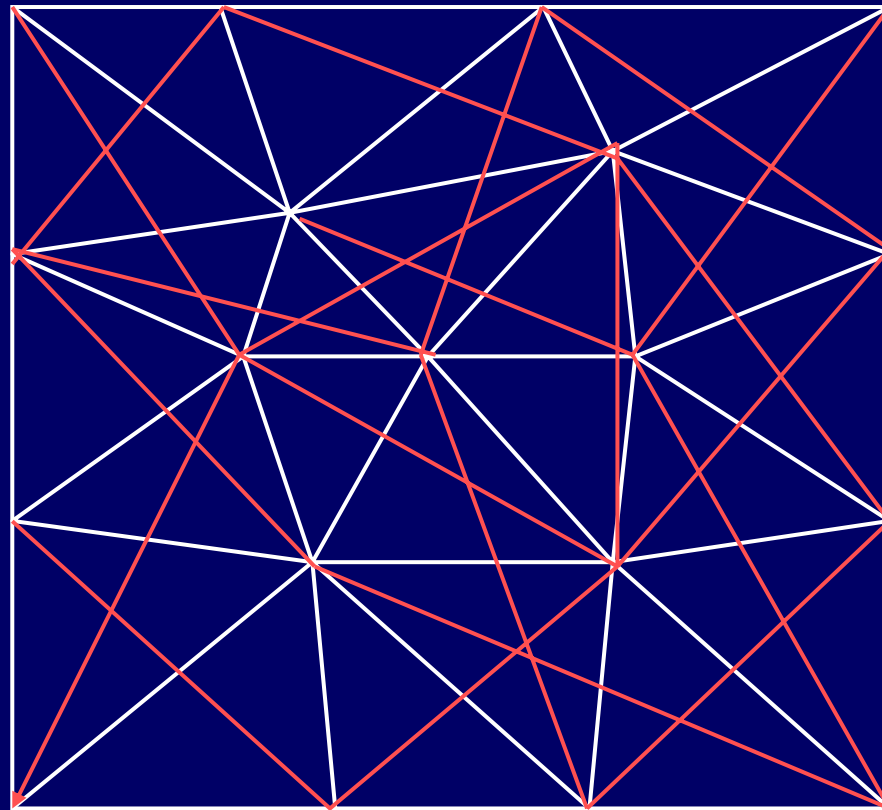
Examples
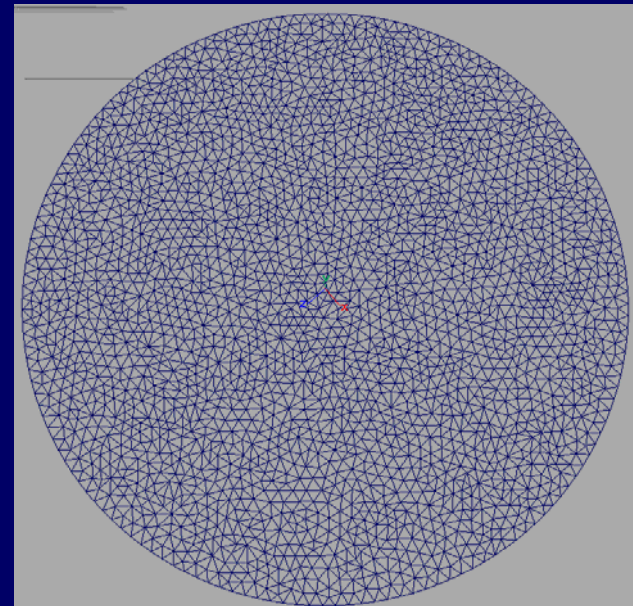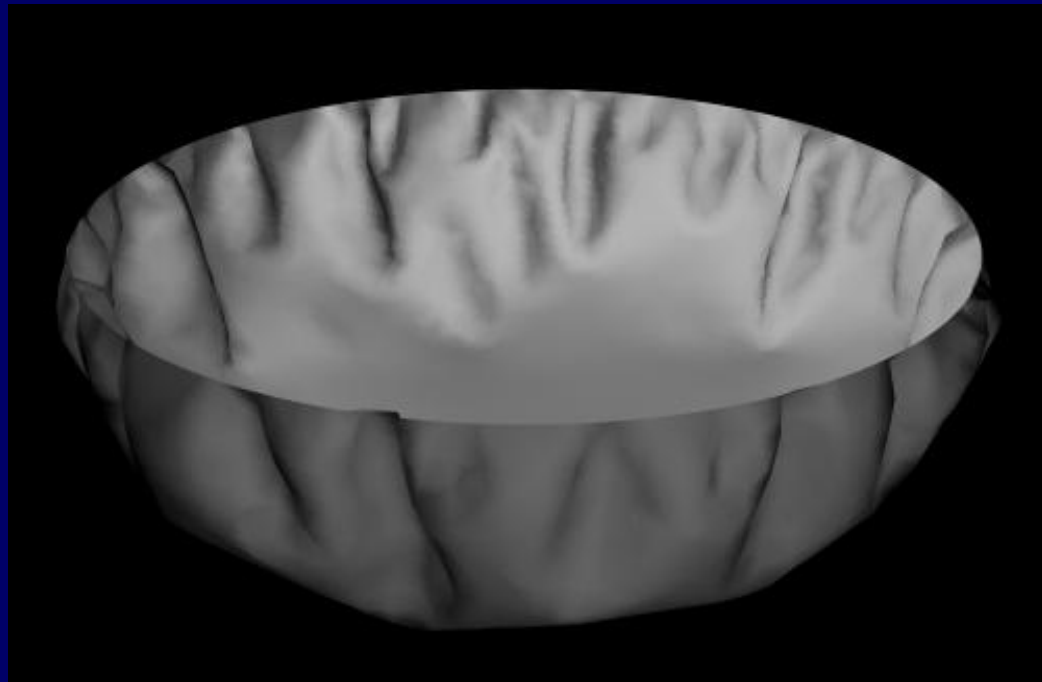
Examples
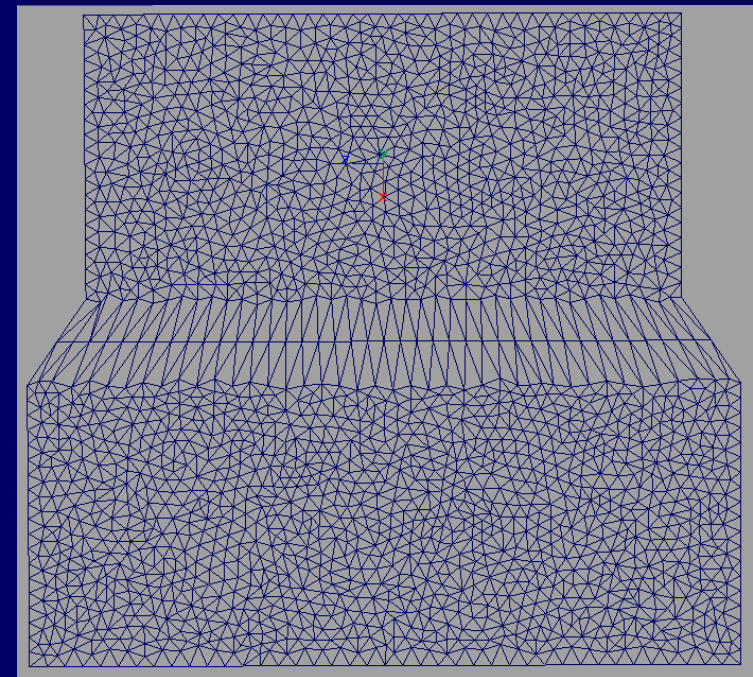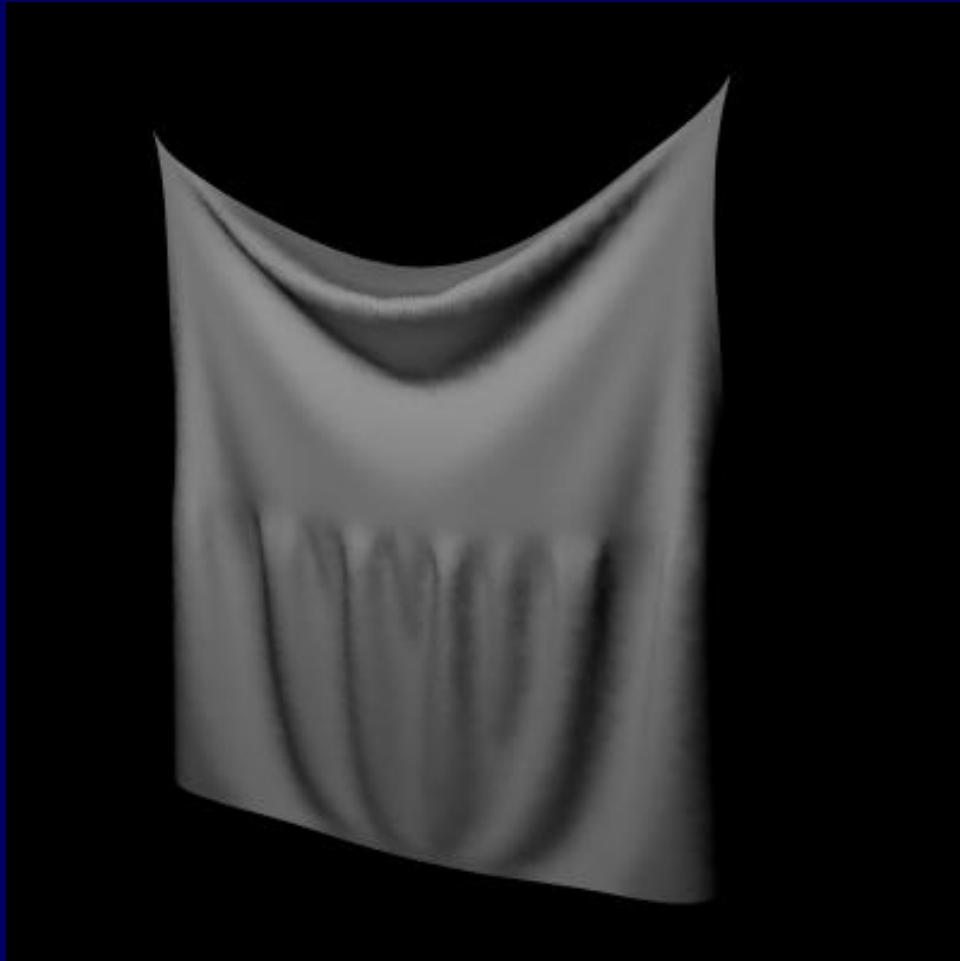
Examples
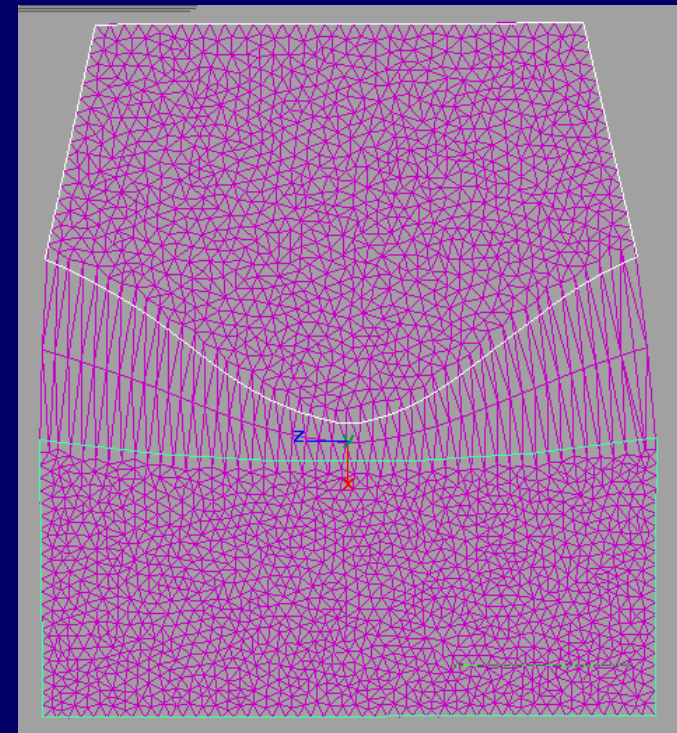
Stable but Responsive Cloth

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Seoul National University

Abstract

We present a semi-implicit cloth simulation technique that is very
stable yet also responsive. The stability of the technique allows
the use of a large fixed time step when simulating all types of fab-
rics and character motions. The animations generated using this
technique are strikingly realistic. Wrinkles form and disappear in a
quite natural way, which is the feature that most distinguishes tex-
tile fabrics from other sheet materials. Significant improvements
in both the stability and realism were made possible by overcom-
ing the *post-buckling instability* as well as the numerical instabil-
ity. The instability caused by buckling arises from a structural in-
stability and therefore cannot be avoided by simply employing a
semi-implicit method. Addition of a damping force may help to
avoid instabilities; however, it can significantly degrade the realism
of the cloth motion. The method presented here uses a particle-
based physical model to handle the instability in the post-buckling
response without introducing any fictitious damping.

Keywords: Deformations, Numerical Analysis, Physically Based
Animation, Physically Based Modeling

CR Categories: I.3.7 [Computer Graphics]: Three-Dimensional
Graphics and Realism—Animation; G.1.7 [Numerical Analysis]:
Ordinary Differential Equations—Convergence and stability, Stiff
equations I.6.5 [Simulation and Modeling]: Model Development

1 Introduction

A normal outfit covers more than 90 percent of the human body.
Therefore, the realistic animation of cloth is imperative if we are to
animate humans to a satisfactory level of detail and realism. Over
the last decade a great deal of research has been dedicated to sim-
ulating cloth motion [Terzopoulos and Fleischer 1988; Carignan
et al. 1992; Breen et al. 1994; Courshesnes et al. 1995; Provot 1995;
Eberhardt et al. 1996; Eischen et al. 1996; Baraff and Witkin 1998;
Desbrun et al. 1999; Volino and Magnenat-Thalmann 2000]. All
of the methods proposed to date boil down to numerically solving
an ordinary differential equation, although they differ in regard to
characteristics such as stability, allowed time step size, etc.

Cloth is characterized by strong resistance to stretch and weak
resistance to bending, which leads to a stiff set of equations and
thus prohibits the use of large time steps. However, cloth simulation
techniques must be stable and fast if they are to be of practical use.

Previous studies have shown that implicit methods are well suited
to solving stiff equations with a reasonable step size, and successful
results have been reported in [Baraff and Witkin 1998; Volino and
Magnenat-Thalmann 2000].

Another property that is crucial to the appearance of fabrics in
motion is their buckling behavior. The buckling of fabrics is the
process by which wrinkles form, and leads to structures such as
those shown in Figure 1. The buckling of textile fabrics has a quite
different nature from solid materials [Amirbayat and Hearle 1989],
thus animation of a cloth would not look natural without having
such property. Nevertheless it has been largely ignored though
its problematic nature (instability and non-linearity) was recently
pointed out by [Eischen et al. 1996] and [Yu et al. 2000]. This pa-
per presents a stable and practical solution to this problem.

Figure 1: Buckling of Real Fabrics

The buckling of a thin material involves a very unstable state,
regardless of whether it is rigid (e.g. aluminum sheet) or flexible
(e.g. fabrics). When a compressive force is applied at the extremes
of a thin material, it initially resists changing shape. As this force is
increased, it eventually reaches the neutral equilibrium, the point at
which an infinitesimal increase or decrease in the force bifurcates
the situation in two radically different directions: increasing the
force leads to an unstable post-buckling response whereas the sys-
tem remains at stable equilibrium if the force is decreased. (Buck-
ling will be described in detail in Section 3.2.) Given that buck-
ling is a ubiquitous characteristic of fabrics, creating natural look-
ing cloth in an animation is very difficult without a stable way to
model this phenomenon.

The instability of the post-buckling response arises from a
structural instability [Bathe 1996], not from the stiff equations.
Therefore, the buckling instability cannot be overcome by simply
employing implicit methods. Some cloth simulation techniques
[Baraff and Witkin 1998; Volino and Magnenat-Thalmann 2000]
avoid this instability by adding damping forces[1]. The damping

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[1]Damping is an important concept in this work. The Implicit method has
an intrinsic damping effect that comes from the formulation itself, which we
refer to as *artificial damping*. This damping is not related to the nature of
the cloth. On the other hand, we refer to damping that is deliberately added
to the formulation to simulate the nature of the cloth as *material intrinsic
damping*. A third kind of damping is sometimes added to enhance numerical
stability. We refer to this as *fictitious damping*. A damping term appearing

forces help to stabilize the physical system, or equivalently, they make the system matrix better conditioned and help to maintain positive definiteness in a semi-implicit formulation. However, the damping forces can significantly degrade the realism of the simulated cloth movement. For example, [Volino and Magnenat-Thalmann 2000] and [Volino and Magnenat-Thalmann 2001] found that the damping forces can lead to systems in which wrinkles will not form on the cloth surface, wrinkles resist disappearing, or even the fabrics resist falling under their own weight.

The artificial damping in implicit methods mainly affects the in-plane deformation of the cloth because the in-plane rigidity is much higher than the bending rigidity. Therefore, although artificial damping is expected to be partially responsible for the degradation of the quality of the out-of-plane movement of the cloth, we conjecture that the degradation in the quality mentioned above arises mainly from fictitious damping. The method we propose in this paper includes artificial damping (since it is an implicit method) and material intrinsic damping, but does *not* include fictitious damping. The need for fictitious damping is avoided through the use of the predicted static post-buckling response as an effective way to handle the instabilities associated with post-buckling situations. Because fictitious damping is not used, our method gives significantly more realistic cloth motion. This represents a significant step forward for the simulation of textile fabrics.

For solid materials, buckling signifies a failure and thus only the mechanics prior to buckling have been studied (e.g. determining the critical load on a column). Even in the study of textiles, there has been no significant result on the buckling process that can be applied to the dynamic simulation of cloth movement. Therefore, instead of physically simulating the unstable post-buckling dynamic response, we solve the instability problem by calculating the deformation energy of the shape at the predicted static equilibrium of the post-buckling state. We treat the numerical instability caused by stiff equations using implicit time stepping. Using the physical model outlined above and implicit time stepping, we could stably integrate the equation of motion with a large fixed time step and without the need for fictitious damping forces.

2 The Physical Model

Before we describe our particle-based physical model of fabrics, we discuss the problems in recently proposed physical models.

2.1 Problems in Previous Physical Models

Cloth is not a homogeneous continuum. Therefore modeling fabrics as a continuum and employing FEM or FDM has several potential drawbacks, a number of which are described in [Amirbayat and Hearle 1989]. One drawback of such methods is that they require a very fine meshing to produce large deformations. For a simulator to be practical in a computer graphics application, however, coarse discretization (about $1 \sim 5$ cm spacing between the nodal points) should be allowed to guarantee reasonable performance. In the analysis of almost incompressible and/or thin materials such as cloth, a continuum formulation with the elements at this scale might produce highly erratic results in the stress and strain [Bathe 1996], regardless of the interpolation order.

Another problem confronting the continuum approach is treating the divergence associated with buckling. [Eischen et al. 1996] used a non-linear shell model for cloth and performed finite element analysis to obtain the drape shape. They had to take special care and use measures such as arc length control to prevent divergence due to the non-linearity of the load-deflection curve or the

in an equation may serve for both material intrinsic damping and fictitious damping.

(a) all the connections for $P(i, j)$

(b) connections among neighboring particles in a particular direction
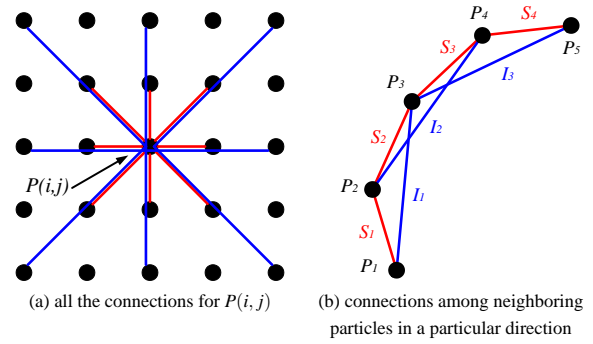
Figure 2: Connectivity of Interacting Particles

indefiniteness of the instantaneous stiffness matrix caused by buckling. [Yu et al. 2000] performed explicit dynamic simulation with fictitious viscous damping to avoid the divergence problem in the static analysis of the drape.

The fabric models in [Baraff and Witkin 1998] and [Courshesnes et al. 1995] are similar and can be understood as systems of connected flat triangles that are treated as a continuum. The in-plane deformation energy (or stress/strain relationship) of each triangle is derived from the continuum mechanics formulation. The bending deformation measure, on the other hand, is based on the angle between adjacent triangles. Therefore the fabric is not treated as a single homogeneous continuum and the bending and in-plane properties are modeled separately. The independent treatment of in-plane and out-of-plane cloth properties allows large bending deformation between triangles regardless of the in-plane rigidity of each triangle. However, the post-buckling instability can be a problem in this model, because each triangle is modeled as an almost incompressible material and the bending rigidity between triangles is very weak. Another problem of this physical model is that the bending characteristics largely depend on the triangulation. Since each triangle has very high in-plane rigidity, deformations tend to develop along the edges of the triangles. This causes a problem in systems comprised of aligned triangles because bending will occur along the edges. This problem can be cured by irregular triangulation, although this remedy introduces artificial flexural rigidity [Courshesnes et al. 1995]. As the triangulation becomes coarser, the artificial flexural rigidity will grow accordingly.

2.2 Using Interacting Particles

In our search for a way to overcome the drawbacks of the physical models outlined above, we found that systems of interacting particles are better suited for generating large deformations and handling the buckling problem. The method presented here draws on the idea that inspired the work of [Breen et al. 1994], who first applied the particle model to the simulation of textile fabrics. However, our particle model is much simpler than that used by Breen et al. and the treatment of compression and bending deformation is quite different from their approach. In this section, we describe the connectivity of the mass points representing the cloth surface. The associated energy functions and their derivatives are presented in the next section.

We approximate a cloth with a quadrilateral mesh of particles; thus each particle can be indexed as $P(i, j)$. Figure 2(a) shows all the connections associated with a given center particle. With the exception of particles at the boundaries, where some connections are broken, every particle has the connectivity shown in Figure 2(a). Two types of particle interaction model are employed, which are

referred to as *type 1* and *type 2*.

The *type 1* interaction model (red lines in Figure 2) is responsible for stretch and shear resistance. For this type of interaction, the particle $P(i, j)$ is connected to $P(i \pm 1, j)$, $P(i, j \pm 1)$, and $P(i \pm 1, j \pm 1)$. Such connections are referred to as sequential connections. The *type 2* interaction model (blue lines in Figure 2) is responsible for flexural and compression resistance. For type 2 interaction, the particles connected to $P(i, j)$ are $P(i \pm 2, j)$, $P(i, j \pm 2)$, and $P(i \pm 2, j \pm 2)$. Note that type 2 connections are made with every other particle, leading us to refer to them as *interlaced* connections. Figure 2(b) illustrates the sequential and interlaced connections in a particular direction, in which $S_1 \sim S_4$ are sequential connections and $I_1 \sim I_3$ are interlaced connections.

3 Energy Functions and Derivatives

In this section we first describe the *type 1* interaction model. We then elaborate on the concept of buckling and its profoundly different meanings in rigid materials and fabrics, after which we describe how this distinction is reflected in our handling of the post-buckling instability. Based on fabric-specific buckling behavior, we formulate the *type 2* interaction model. Finally, we add a material intrinsic damping to those models.

In the description of the energy functions and their derivatives presented below, the distinctive features of our formulation are highlighted by comparing the results of the proposed model with those presented in [Baraff and Witkin 1998] and [Volino and Magnenat-Thalmann 2000].

3.1 Type 1 Interaction

The *type* 1 interaction is represented by a simple linear spring model, for which the energy function for particles i and j is,

$$E = \begin{cases} \frac{1}{2} k_s (|\mathbf{x}_{ij}| - L)^2 & : & |\mathbf{x}_{ij}| \geq L \\ 0 & : & |\mathbf{x}_{ij}| < L \end{cases} \quad (1)$$

where $\mathbf{x}_{ij} = \mathbf{x}_j - \mathbf{x}_i$, L is the natural length, and k_s is the spring constant. Note that this energy function accounts for the tension only. The force acting on particle i due to the deformation between the two particles is,

$$\mathbf{f}_i = -\frac{\partial E}{\partial \mathbf{x}_i} = \begin{cases} k_s(|\mathbf{x}_{ij}| - L) \frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|} & : & |\mathbf{x}_{ij}| \geq L \\ 0 & : & |\mathbf{x}_{ij}| < L \end{cases} \quad (2)$$

The Jacobian matrix of the force vector is

$$\frac{\partial \mathbf{f}_i}{\partial \mathbf{x}_j} = \begin{cases} k_s \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}} + k_s(1 - \frac{L}{|\mathbf{x}_{ij}|})(\mathbf{I} - \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}}) & : & |\mathbf{x}_{ij}| \geq L \\ 0 & : & |\mathbf{x}_{ij}| < L \end{cases} \quad (3)$$

The first term tells us that the stiffness in the direction of the spring interaction is constant, which is an obvious consequence of modeling the interaction using a linear spring. The second term tells us that the stiffness orthogonal to the interaction direction is proportional to $(1 - \frac{L}{|\mathbf{x}_{ij}|})$. If we consider the 2-dimensional structure of the cloth, the direction orthogonal to all interactions corresponds to the out-of-plane direction. When the spring is stretched, in an implicit formulation, the second term plays an important role in stabilizing the spring because it introduces large positive eigenvalues of the system matrix in that direction. (If the same function were to be used for compression, the second term might turn the overall system matrix $(\mathbf{I} - \alpha \partial \mathbf{f}/\partial \mathbf{v} - \beta \partial \mathbf{f}/\partial \mathbf{x})$ indefinite since $k_s(1 - \frac{L}{|\mathbf{x}_{ij}|})$ is negative and can be arbitrarily large regardless of the time step size, as $|\mathbf{x}_{ij}| \to 0$. For this reason we do not use the same function

for compression. Compression is handled by the *type 2* interaction model, which is presented in Section 3.3.)

[Volino and Magnenat-Thalmann 2000] used the same spring model (i.e., $\frac{1}{2} k_s (|\mathbf{x}_{ij}| - L)^2$) for both stretching and compression in running a semi-implicit method. In their formulation, however, the second term of the Jacobian matrix in Equation 3 is omitted for both stretch and compression. They added a damping term to avoid null eigenvalues in the orthogonal direction. For a compressed spring, addition of a damping force in the out-of-plane direction can make the system matrix better conditioned. However, this damping force may generate unnecessarily high resistance to the movement of the cloth.

[Baraff and Witkin 1998] used a linear elastic model for both stretching and compression. Although their formulation looks different from ours, we can make the similarity apparent by converting their formulation to that of the linear spring model. If we define the behavior function (as they refer to it) as $\mathbf{c} = |\mathbf{x}_{ij}| - L$, then the force vector will be same as Equation 2 and consequently the Jacobian $\partial \mathbf{f}/\partial \mathbf{x}$ will also be the same. Therefore, when the spring is compressed, the Jacobian matrix has negative eigenvalues in the orthogonal direction as indicated above. [Baraff and Witkin 1998] reduced the possibility of the system matrix becoming indefinite by the inclusion of a specially designed damping force. In their formulation, the damping force for stretch/compression does not contribute to the out-of-plane motion, but the damping force for bending acts along the out-of-plane direction. This damping force for bending will help to make the system matrix better conditioned at the cost of adding resistance to the out-of-plane movement of the cloth. Unfortunately there is still no guarantee that the damping will make the matrix positive definite.

In our formulation, a highly stiff linear spring is used only for tension. Therefore the system matrix is guaranteed to be positive definite, and no additional damping term is needed to cure the divergence problem. However, the cloth may shrink under compressive load if no compression resistance is included in the model. In our method, the *type 2* interaction model simultaneously accounts for compression and bending, and consequently the buckling problem is handled in a single interaction model.

3.2 Our Solution to the Post-Buckling Instability

As mentioned above, buckling causes serious stability problems in physical simulations of cloth. It is interesting to note that buckling has quite different meanings in rigid materials and textile fabrics. Buckling means a failure in rigid materials, whereas it means success in textile fabrics [Amirbayat and Hearle 1989]. The post-buckling behavior in rigid materials is quite destructive, while the same behavior in textile fabrics naturally evolves into the shapes that are the essence of a fabric's appearance. We refer to [Amirbayat and Hearle 1989] for this fabric-specific property. Although there are clear distinctions between the buckling behavior of fabrics and rigid materials, they are not reflected in most existing cloth simulation techniques. Since our solution to the post-buckling instability is based on the distinguished feature of fabric buckling, we clarify the concept of buckling in this section.

First, we analyze the buckling of a rigid material in terms of solid mechanics. We then contrast this phenomenon with the buckling of textile fabrics. Consider the idealized rigid column shown in Figure 3(a), which consists of two rigid bars connected at point C by a rotational spring of stiffness k_θ. In this configuration, the bending resistance is condensed at point C. Now, suppose an axially compressive force P is applied at point A. Since the rotational spring resists bending, the two bars remain straight at equilibrium (Figure 3(b)). Now, suppose that the structure under force P is disturbed by an external force that causes a small lateral movement of point C (Figure 3(c)). The compressive force P will try to increase the
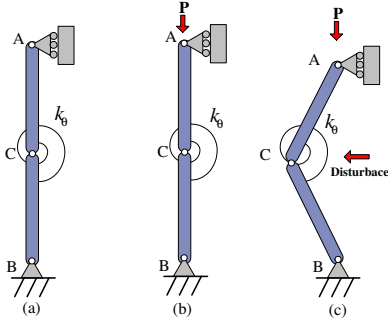
Figure 3: Column Buckling

(a) before buckling   (b) after buckling

Figure 4: Simplified Structure for Type 2 Interaction

(a) P/k_b=23.50   (b) P/k_b=35.25

Figure 5: Numerical Solutions of Moment Equilibrium Equation

lateral displacement, while the rotational spring will try to restore the system to the original straight position. Suppose that the disturbance is removed at this moment. If P is relatively small, the bars will return to the straight position (i.e. the structure is stable). However, if P is relatively large the lateral displacement will become larger and larger (i.e. the structure is unstable), and the structure will eventually collapse by lateral buckling. The magnitude of the axial force at which the structure bifurcates into either the stable or unstable condition by application of an infinitesimal increase or decrease in force is known as the critical load and is denoted by P_{cr}. At the critical load, the deflection of the structure is mathematically arbitrary. Once the axial load exceeds P_{cr} the structure collapses and the original shape cannot be recovered, regardless of the degree to which the load exceeds P_{cr}, and no matter how quickly the load is reduced back to a value less than or equal to P_{cr}.

Now, let us consider the behavior when a fabric buckles. As for rigid materials, a textile fabric will buckle when subjected to an axial force greater than the critical load. When it buckles, it exhibits an unstable post-buckling response similar to that found in rigid materials. In contrast to rigid materials, however, fabrics do not break or collapse. Instead, they quickly pass the unstable state and reach a stable equilibrium (a smoothly bent shape). Moreover, the bent shape tends to return to the original straight shape when the axial load is removed [Amirbayat and Hearle 1989].

As described above, textile fabrics pass through an unstable state when they buckle. The simulation of this unstable post-buckling response requires special care if divergence problems are to be avoided. Once the material goes into the unstable post-buckling state, the deflection increases even when the load decreases. In other words, the stiffness of the material in the buckling direction is instantaneously negative. Especially in semi-implicit methods where the internal force is explicitly predicted with derivatives [Baraff and Witkin 1998; Volino and Magnenat-Thalmann 2000], this structural instability makes the system matrix extremely ill-conditioned or indefinite, and a large time step often leads to divergence. There have been a number of efforts to avoid this post-buckling instability in the analysis of cloth deformation. For example, [Eischen et al. 1996] used an adaptive arc-length control of the load-deflection curve, and [Yu et al. 2000] employed an explicit method with fictitious damping to avoid divergence.

We solve the structural instability problem by predicting the static post-buckling response. The approach developed here is based on the above observations regarding fabric-specific behavior. The concept underlying our approach is that since the fabric quickly passes the unstable post-buckling state to reach a stable equilibrium, it has little chance to get into the unstable post-buckling state at the discrete time steps of the simulation. Thus we assume that the fabric is not in the unstable post-buckling state at any time step. Then, in calculating the internal force at each time step, we can evaluate the deformation energy in the area where the cloth buckles
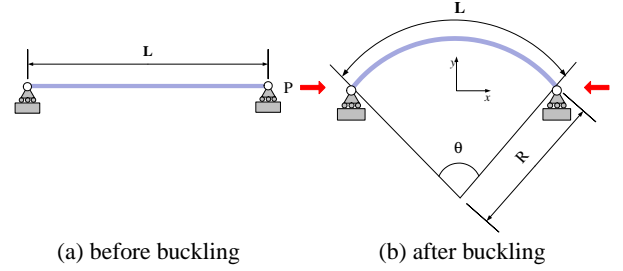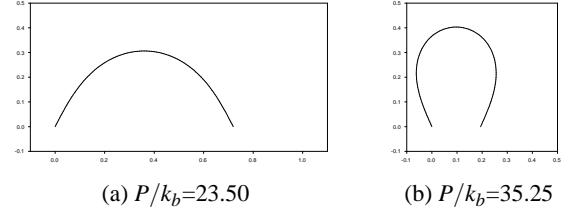
from the locally estimated deformed shape, which corresponds to the shape at the static equilibrium after buckling. The details of this procedure are presented in the next section. Unless the time step is miniscule, the loss of accuracy resulting from the approximated buckling response will be much less than the accuracy loss (and instabilities) resulting from the plain implicit time stepping. The approximated response model relieves the burden of *simulating* the unstable post-buckling dynamic response. According to our simulations, the approximated response model generates very realistic cloth motion with significantly improved stability. Wrinkle formation was quite natural. The simulation could be performed with a large step size.

3.3 Type 2 Interaction

The *type 2* interaction model is responsible for the post-buckling response created by compressive and bending forces. We predict the shape of the fabric after buckling and calculate the deformation energy from the deformed shape as described in the previous section.

The beam structure shown in Figure 4(a) approximates the region between two particles. Prior to buckling, the structure is a straight beam of length L.

After the structure buckles under a compressive load (Figure 4(b)), it will eventually reach a stable equilibrium structure. To predict the equilibrium shape, we use the moment equilibrium equation under the pinned ends condition[Gere 2001], which is given by

$$k_b \kappa + Py = 0, \qquad (4)$$

where k_b is the flexural rigidity, κ is the curvature, P is the compressive load, and y is the deflection. Because we are modeling systems with large deflections, we cannot use the approximation $\kappa = y''$. Using the exact expression for the curvature[2], we obtained several numerical solutions corresponding to different values of k_b and P. Two of these solutions are shown in Figure 5. The results show

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[2]For a more general moment equilibrium equation for the analysis of fabric buckling, see [Kang et al. 2001].

that the shape after buckling is close to a circular arc even when the deformation is large. Therefore, we approximated the equilibrium shape as a circular arc with constant arc length. As an alternative, we could have constructed a table of the numerical solutions of the moment equation at various values of $\frac{P}{k_b}$ for more accurate analysis. This was not undertaken because the results produced using the circular arc assumption were quite realistic. The bending deformation energy can be calculated from the estimated shape using the relation:

$$E = \frac{1}{2}\int_0^L \mathbf{M}\kappa dx \qquad (5)$$

where $\mathbf{M}$ is the bending moment and κ is the curvature. Taking into account the linear relationship between the curvature and bending moment, and the constant curvature through the structure, the integral yields the solution:

$$E = \frac{1}{2}k_b L\kappa^2, \qquad (6)$$

where k_b is the flexural rigidity. Since the arc length is assumed to be the same as the initial straight length L, the curvature κ can be expressed solely in terms of the distance $|\mathbf{x}_{ij}|$ between the two extremities.

$$\kappa = \frac{2}{L}\text{sinc}^{-1}(\frac{|\mathbf{x}_{ij}|}{L}), \qquad (7)$$

where $\text{sinc}(x) = \frac{\sin x}{x}$.

The force vector is derived as,

$$\mathbf{f}_i = k_b\kappa L\frac{d\kappa}{d|\mathbf{x}_{ij}|}\frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|} = k_b\kappa L\left(\frac{d|\mathbf{x}_{ij}|}{d\kappa}\right)^{-1}\frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|} \qquad (8)$$

$$= k_b\kappa^2(\cos\frac{\kappa L}{2} - \text{sinc}(\frac{\kappa L}{2}))^{-1}\frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|} \qquad (9)$$

$$\equiv f_b(|\mathbf{x}_{ij}|)\frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|} \qquad (10)$$

The blue curve in Figure 6 depicts the dependence of f_b on the distance between particles (approximated with a fifth order polynomial function). The unit of each axis in this graph is made dimensionless. The system shows the following behavior. When the compression force P is initially applied (top right corner of Figure 6), the structure remains straight until the load reaches the buckling load P_{cr}. However, in real systems geometric imperfections in the structure cause the fabric to start to buckle at the onset of loading, giving an actual curve (the green curve in the graph) that exhibits finite deflection even at small magnitudes of the compression force, and asymptotically approaches f_b as compressive force increases [Gere 2001]. To model this characteristic, we used the function f_b^* in our final implementation:

$$f_b^* = \begin{cases} c_b(|\mathbf{x}_{ij}|-L) & : \quad f_b < c_b(|\mathbf{x}_{ij}|-L) \\ f_b & : \quad \text{otherwise} \end{cases} \qquad (11)$$

where c_b is a constant of our choice, usually assigned a value comparable to k_s. Although we could have used a higher order function to model the deflection at small values of the compression force, we found no significant difference in the results obtained using higher order functions and those obtained with the linear function $c_b(|\mathbf{x}_{ij}|-L)$ (red curve in Figure 6).

The Jacobian matrix of the force vector is derived as

$$\frac{\partial\mathbf{f}_i}{\partial\mathbf{x}_j} = \frac{df_b^*}{d|\mathbf{x}_{ij}|}\frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}} + \frac{f_b^*}{|\mathbf{x}_{ij}|}(\mathbf{I} - \frac{\mathbf{x}_{ij}\mathbf{x}_{ij}^T}{\mathbf{x}_{ij}^T\mathbf{x}_{ij}}). \qquad (12)$$

In the above equation, $\frac{df_b^*}{d|\mathbf{x}_{ij}|}$ is always positive but $\frac{f_b^*}{|\mathbf{x}_{ij}|}$ is always negative, creating the possibility that the second term could turn
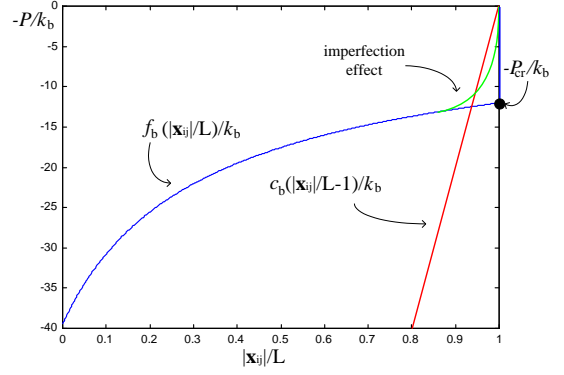

Figure 6: Nondimensionalized Curves of P vs. $|\mathbf{x}_{ij}|$

the system matrix indefinite. To guarantee the positive definiteness of the system matrix, we dropped this term and thus the force in the orthogonal direction is not affected by implicit filtering. Although this force is not filtered, there is little possibility that the system will diverge even when a large time step is employed because the magnitude of this force is very small compared to the stretch force, and it is always finite. Note that in a model where the repulsive and contractive forces are equally strong, dropping the orthogonal term can make the system unstable under a large time step unless the bending resistance is of comparable strength.

The above discussion of the type 2 interaction model highlights the necessity of interlacing the type 2 connections in the manner shown in Figure 2(b). If only sequential connections were used, the global shape could be bent without increasing the bending energy provided that each local connection maintained the initial distance.

3.4 Damping

The physical model described above is quite stable; thus, there is no need for additional energy dissipative terms to stabilize the numerical procedure. However, we do need to consider the intrinsic damping property of fabrics. Without an appropriate (material intrinsic) damping term, the simulated fabric can exhibit large unrealistic in-plane oscillations. To include this type of damping, we added a simple linear damper along the direction of interaction.

The damping force exerted on particle i from the interaction with particle j is given by,

$$\mathbf{f}_i = -k_d(\mathbf{v}_i - \mathbf{v}_j) \qquad (13)$$

and the Jacobian matrix is simply expressed as,

$$\frac{\partial\mathbf{f}_i}{\partial\mathbf{v}_j} = k_d\mathbf{I}. \qquad (14)$$

Note that the force term in Equation 13 does not add any damping to the orthogonal direction. This is important because the most interesting fabric deformation occurs in the out-of-plane direction.

The above force term does not create a filtering effect under a semi-implicit formulation when the cloth undergoes a linear rigid motion (i.e., when the velocity vectors of all the particles are identical). This is the case because for such a motion the 3×3 block-wise row sums of the matrix $\frac{\partial\mathbf{f}}{\partial\mathbf{v}}$ are 3×3 zero matrices from the above equations, and the rigid motion vector $\mathbf{v}_{rigid}$ is an eigenvector of $\frac{\partial\mathbf{f}}{\partial\mathbf{v}}$ with zero eigenvalue. Therefore, there is no implicit filtering effect caused by $\frac{\partial\mathbf{f}}{\partial\mathbf{v}}$. Even when the $\frac{\partial\mathbf{f}}{\partial\mathbf{x}}$ terms in Equations 3 and 12 are

included in the system equation, there will be no implicit filtering to the rigid motion, provided that all the interacting directions of particle i are orthogonal to $\mathbf{v}_i$ for all i and that the cloth is not stretched, since the $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ terms in Equations 3 and 12 also produce null vectors when multiplied by $\mathbf{v}_{rigid}$. Therefore, the motion to the orthogonal direction would not be filtered under a semi-implicit formulation.

In more general situations, even though the above condition is not met, our formulation has very small artificial damping in the out-of-plane direction, which potentially makes the cloth movement look more realistic.

4   Numerical Integration

We use semi-implicit integration with a second-order backward difference formula (BDF). The k-th order BDF is defined as,

$$\frac{d}{dt} = \frac{1}{\Delta t} \sum_{q=1}^{k} \frac{1}{q} (\Delta^{-1})^q, \tag{15}$$

where

$$\Delta^{-1}\mathbf{x} = \mathbf{x}^{n+1} - \mathbf{x}^n.$$

For $k = 2$, the discretization of $\dot{\mathbf{x}}$ becomes,

$$\dot{\mathbf{x}} = \frac{1}{\Delta t}(\frac{3}{2}\mathbf{x}^{n+1} - 2\mathbf{x}^n + \frac{1}{2}\mathbf{x}^{n-1}). \tag{16}$$

Considering both performance and accuracy, we chose second order BDF for the semi-implicit formulation. The second order BDF creates less artificial damping than the first order BDF, but is equally stable.

The state equation of motion

$$\left(\begin{array}{c} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{array} \right) = \left(\begin{array}{c} \mathbf{v} \\ \mathbf{M}^{-1}\mathbf{f} \end{array} \right) \tag{17}$$

is discretized with the second order BDF as

$$\frac{1}{\Delta t} \left(\begin{array}{c} \frac{3}{2}\mathbf{x}^{n+1} - 2\mathbf{x}^n + \frac{1}{2}\mathbf{x}^{n-1} \\ \frac{3}{2}\mathbf{v}^{n+1} - 2\mathbf{v}^n + \frac{1}{2}\mathbf{v}^{n-1} \end{array} \right) = \left(\begin{array}{c} \mathbf{v}^{n+1} \\ \mathbf{M}^{-1}\mathbf{f}^{n+1} \end{array} \right). \tag{18}$$

The nonlinear term $\mathbf{f}^{n+1}$ in the above equation is replaced with

$$\mathbf{f}^{n+1} = \mathbf{f}^n + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}^{n+1} - \mathbf{x}^n) + \frac{\partial \mathbf{f}}{\partial \mathbf{v}}(\mathbf{v}^{n+1} - \mathbf{v}^n). \tag{19}$$

By combining Equations 18 and 19, we can obtain a linear system rearranged either for $\Delta^{-1}\mathbf{x}$ or $\Delta^{-1}\mathbf{v}$. If we rearrange the linear system for $\Delta^{-1}\mathbf{x}$, the equation becomes,

$$(\mathbf{I} - \Delta t \frac{2}{3}\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}} - \Delta t^2 \frac{4}{9}\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{x}})(\mathbf{x}^{n+1} - \mathbf{x}^n)$$
$$= \frac{1}{3}(\mathbf{x}^n - \mathbf{x}^{n-1}) + \frac{\Delta t}{9}(8\mathbf{v}^n - 2\mathbf{v}^{n-1})$$
$$+ \frac{4\Delta t^2}{9}\mathbf{M}^{-1}(\mathbf{f}^n - \frac{\partial \mathbf{f}}{\partial \mathbf{v}}\mathbf{v}^n) - \frac{2\Delta t}{9}\mathbf{M}^{-1}\frac{\partial \mathbf{f}}{\partial \mathbf{v}}(\mathbf{x}^n - \mathbf{x}^{n-1}). \tag{20}$$

The linear system of Equation 20 is sparse and generally unbanded. We solve this system using a preconditioned conjugate gradient method. We used a 3×3 block diagonal matrix for the preconditioner, which showed an improvement of approximately 20% over the diagonal preconditioner. In addition, we assessed other preconditioners such as IC and ILU but found no performance gain though the number of iterations decreased.

5   Collision Handling

Collision detection and response model is not a contribution of this paper. In this section, we briefly describe how we handled collisions in our implementation.

To detect collisions we use a voxel-based collision detection algorithm similar to that proposed by [Zhang and Yuen 2000]. After voxelizing the space in which the cloth is enclosed, we register each cloth particle and solid triangle to the corresponding voxels based on their spatial coordinates. Then, we independently perform collision detection for each voxel. This voxelization method locates the possible collisions very efficiently and shows nearly linear performance.

We detect the cloth-solid collision by checking the particle-triangle pairs to determine if particles are beneath the solid surface. To avoid missing pairs near the voxel boundaries, the triangles are redundantly registered to the nearby voxels. When a collision is detected, the particle's next displacement along the normal direction of the colliding surface is determined, and this constraint is enforced using the invariant method in the conjugate gradient iteration proposed in [Baraff and Witkin 1998]. For the tangential direction, we add a frictional force that is proportional both to the constraint force and to the velocity difference between the solid surface and the particle in contact.

To test for self-collision, we check the particle-particle pairs. If the particles are too close, we simply add a repulsive proximity force between the colliding particles. The Jacobian matrix of this force is made to have null eigenvalues in the directions orthogonal to the repelling direction as in the case of the *type 2* interaction model in Section 3.3.

6   Results

This section reports the results from several simulations. The animations of these simulations can be found at `http://graphics.snu.ac.kr/∼kjchoi/cloth.htm`

Table 1 summarizes the performance of our algorithm on a Pentium3-550 machine. In this table, the CPU sec/frame field cites the total CPU time required to carry out all of the steps (i.e., collision detection, linear system setup, conjugate gradient iteration, etc.) required to produce one frame of 30 Hz animation. For all the simulations, the collision detection time was less than 20% of the total CPU time. The mesh resolutions of the clothes in the animations were about $L = 1 \sim 2cm$.

For the simulations involving human motions (Animations 1∼4), the time step was fixed to $\Delta t = 1/90s$ throughout the animation; thus the simulator produced one frame of 30 Hz animation with three time steps. The simulations were stable despite the use of a fixed time step. All attempts to use a time step greater than $1/90s$ encountered collision handling failures before the stability limit was reached.

In Animation 1(a), the character is wearing a one-piece made of a thin fabric. The nature of the fabric was controlled by assigning a small value to the bending rigidity. The character walks at a normal pace without any fast movements. Nevertheless the cloth motion is quite responsive; the wrinkle details delicately form and disappear. However, we considered the cloth motion in Animation 1(a) to be more responsive than would be expected for a real cloth. To produce a more fabric-like motion, we increased the bending rigidity and intrinsic damping, and reduced the frictional force. The result was Animation 1(b). Animations 2 and 3 contain more vigorous character motions, which created very dynamic movement of the cloth and wrinkles. In Animation 4 the character is wearing jeans. The jeans fabric was modeled by assigning it a high bending rigidity and a high resistance to buckling. The animation produced the

Table 1: Performance Summary

animation		# particles of cloth	# triangles of solid	CPU sec/ frame	time step (s)
1(a)		5608	13802	6.13	0.011
1(b)		5579	13802	5.86	0.011
2		5579	15308	5.68	0.011
3	top	3396	15308	2.98	0.011
	skirt	3456	15308	3.01	0.011
4	top	3294	14324	3.25	0.011
	pants	6624	14324	7.01	0.011
5		2601	0	0.18	0.2
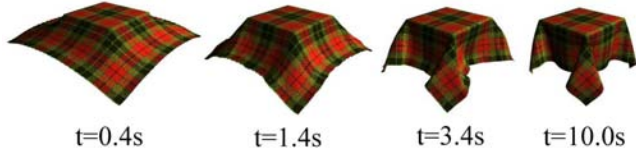
Figure 7: Snapshots from Animation 5 : Stability Test ($\Delta t = 0.2s$)

buckled shapes near the knees and ankles quite well, which are frequently observed in real jeans. Several snapshots taken from each of the animations 1∼4 are shown in Figure 8.

Animation 5 was designed purely to determine the maximum time step that could be used in our cloth animation technique; it was not intended as a realistic animation. The simulation modeled a square of fabric draped over a solid box, as shown in Figure 7. To exclude the collision detection problem we did not explicitly include the box; Instead, we simply constrained the movement of the sub-square region of the fabric. Additionally, we disabled both self-collision and solid-collision. Under the above conditions, we verified that the algorithm runs stably with time step sizes up to *100s*, although the resulting animation was very choppy and it required hundreds of time steps for the fabric to settle down to the final shape. Such a large time step is not meaningful and cannot possibly generate realistic animation because the derivatives have no significance after 100 seconds. A marginally acceptable animation with self-collision enabled was obtained with $\Delta t = 0.2s$. Several snapshots taken during this animation are shown in Figure 7.

Although there is no established method or system for validating the dynamic motion of cloth, our technique produced animations that are visually quite believable. It is noteworthy that these animations were obtained using a reasonable, practical amount of computation.

7 Conclusion

The groundbreaking work of [Baraff and Witkin 1998] on implicit time stepping greatly reduced the computational cost of integrating the stiff equations used in simulating textile fabrics, and thereby provided a practical solution for animating clothed characters. However, the phenomenon of buckling, which is another source of instability and a crucial property in cloth deformation, has been largely ignored until now. If the buckling problem cannot be handled appropriately, natural cloth animation would require enormous (almost impractical) amount of computation.

This paper represents the first report of a stable and practical method to handle the post-buckling instability without introducing a damping force into the dynamic simulation. The proposed method was shown to produce very realistic motion of clothes made from a

range of fabric types using a uniform time step size. In particular, the power of the new method was shown in the animation of a light and thin cloth where sensitive response of cloth is required, which was very difficult to produce using the previous methods.

The tremendously increased stability of our algorithm allowed the simulation of the motion of cloth with time steps of 0.2 seconds or longer which could not be achieved in previous methods.

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Figure 8: Snapshots from Animations 1~4. Each animation corresponding to each row shows different materials with different parameters.

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