

DISCUSSION LOG

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February 11, 2019

THEORY

In our cloth simulation, we follow the procedure written by Rohmer et al.. We consider to begin with the *Deformation gradient* \mathbf{F} , which is defined by

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \quad (1)$$

where the \mathbf{x} denotes the deformed vector and the \mathbf{X} denotes the reference vector.

Since we have no further information about the mapping from \mathbf{X} to \mathbf{x} , we use the vector $(\mathbf{u}_1, \mathbf{u}_2)$ and $(\overline{\mathbf{u}}_1, \overline{\mathbf{u}}_2)$ to approximate the Deformation gradient, which is defined as

$$\mathbf{F} = [\mathbf{u}_1, \mathbf{u}_2] [\overline{\mathbf{u}}_1, \overline{\mathbf{u}}_2]^{-1}, \quad (2)$$

Attention should be paid especially:

- eq.(2) characrizе only the 2D deformation of each triangle.
- in Rohmer et al. \mathbf{F} is symbolised as \mathbf{T} .

We provide here our code preceed with concrete data:

- $faces(i, j)$ is the j th vertex of the i th triangle, here we choose the $face(0, 0)$, $face(0, 1)$, $face(0, 2)$ as examples and calculate the $VecT$ and $VecR$ for $face(0, 0)$.

$$VecT = e_{l_1} VertT_1 - e_{l_1} VertT_2; e_{l_1} VertT_1 - e_{l_1} VertT_3 \quad (3)$$

$$VecR = e_{l_1} VertR_1 - e_{l_1} VertR_2; e_{l_1} VertR_1 - e_{l_1} VertR_3 \quad (4)$$

$$VecT = [0.771842 \quad -0.0144887 \quad 6.39045] - [0.780121 \quad -0.0186188 \quad 6.37318] \quad (5)$$

$$[0.771842 \quad -0.0144887 \quad 6.39045] - [0.737177 \quad -0.00912791 \quad 6.39292] \quad (6)$$

$$VecR = [0.759919 \quad -0.015194 \quad 6.38401] - [0.767822 \quad -0.0212492 \quad 6.3669]; \quad (7)$$

$$[0.759919 \quad -0.015194 \quad 6.38401] - [0.726977 \quad -0.00749985 \quad 6.38692] \quad (8)$$

$$(9)$$

such that

cloth_vec

$$VecT = [-0.0079 \ 0.0061 \ 0.0171 \ 0.0329 \ -0.0077 \ -0.0029]; \quad (10)$$

$$VecR = [-0.0083 \ 0.0041 \ 0.0173 \ 0.0347 \ -0.0054 \ -0.0025]; \quad (11)$$

$$(12)$$

where the first 3 entries of $VecR$ is the vector \mathbf{u}_1 and the last 3 entries of $VecR$ is the vector \mathbf{u}_2 . $VecT$ analogously.

cloth_eig_2D

we use here *Eigen :: Map* to transform the vector *VecT* and *VecR* to 2×2 2D deformation gradient \mathbf{F} .

$$\mathbf{F} = \begin{bmatrix} -0.0083 & 0.0347 \\ 0.0041 & -0.0054 \end{bmatrix} \begin{bmatrix} -0.0079 & 0.0329 \\ 0.0061 & -0.0077 \end{bmatrix}^{-1} \quad (13)$$

hence the \mathbf{F} has a rotation information \mathbf{R} and a stretch information \mathbf{U} ,

$$\mathbf{F} = \mathbf{R}\mathbf{U}. \quad (14)$$

we use $\mathbf{F}^T \mathbf{F}$ to eliminate the rotation information to obtain $\det \mathbf{R} = 1$

$$\mathbf{F}^T \mathbf{F} = (\mathbf{R}\mathbf{U})^T \mathbf{R}\mathbf{U} \quad (15)$$

$$= \mathbf{U}^T \mathbf{R}^T \mathbf{R} \mathbf{U} \quad (16)$$

$$= \mathbf{U}^2 \quad (17)$$

$$= \mathbf{C} \quad (18)$$

and using decomposition, if we have the form

$$\mathbf{C} = \lambda_1^2 \mathbf{v}_1 \mathbf{v}_1^T + \lambda_2^2 \mathbf{v}_2 \mathbf{v}_2^T \quad (19)$$

then we could obtain the \mathbf{U} by applying

$$\mathbf{U} = \lambda_1 \mathbf{v}_1 \mathbf{v}_1^T + \lambda_2 \mathbf{v}_2 \mathbf{v}_2^T \quad (20)$$

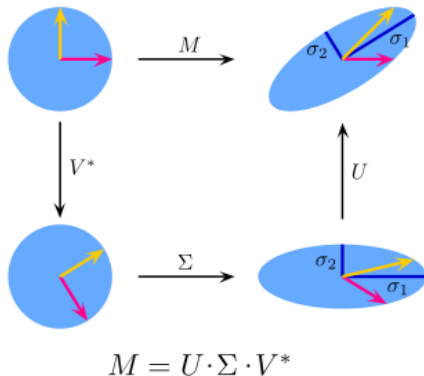


Figure: Singular Value Decomposition SVD

the main drawback of Rohmer et al. is that this is only applied for 2D problem, thus we need a new algorithm, which can also take the consideration for the vertical deformation. Therefore, the *Kabisch Algorithm* is introduced.

Kabsch Algorithm

Let \mathbf{P} denotes the points set of template frame, and \mathbf{Q} denotes the points set of reference frame. The *optimal rotation matrix* \mathbf{R} can be calculated as

$$\mathbf{R} = \mathbf{V} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{bmatrix} \mathbf{U}^T \quad (21)$$

where \mathbf{V} and \mathbf{U} and the *SVD* of *cross-covariance matrix* \mathbf{H} , which is determined by

Theorem (cross-covariance matrix)

$$\mathbf{H} = \mathbf{P}^T \mathbf{Q} \quad (22)$$

and $d = \det \mathbf{V} \mathbf{U}^T$

The cross-covariance matrix describe the covariance of one process with the other at pairs of time points and measure of similarity. We use therefore the cross-covariance matrix \mathbf{H} to approximate our deformation gradient \mathbf{F} .

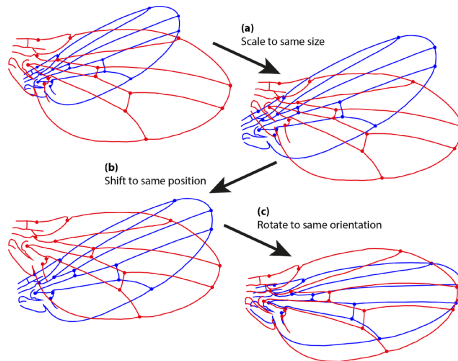


Figure: Procrustes superimposition

Let i be the all index set of the nearest vertice of vertex l in the template frame and \bar{i} be the all index set of the nearest vertice of vertex l in the template frame. Process define

$$\mathbf{P} = \begin{bmatrix} \vec{x}_1 & \vec{y}_1 & \vec{z}_1 \\ \vec{x}_2 & \vec{y}_2 & \vec{z}_2 \\ \vdots & \vdots & \vdots \\ \vec{x}_N & \vec{y}_N & \vec{z}_N \end{bmatrix} \quad 1, 2, \dots, N \in i \quad (23)$$

$$\mathbf{Q} = \begin{bmatrix} \vec{\bar{x}}_1 & \vec{\bar{y}}_1 & \vec{\bar{z}}_1 \\ \vec{\bar{x}}_2 & \vec{\bar{y}}_2 & \vec{\bar{z}}_2 \\ \vdots & \vdots & \vdots \\ \vec{\bar{x}}_N & \vec{\bar{y}}_N & \vec{\bar{z}}_N \end{bmatrix} \quad 1, 2, \dots, N \in \bar{i} \quad (24)$$

and we apply to obtain the deformation gradient \mathbf{F} of vertex l

$$\mathbf{F} = \mathbf{P}^T \mathbf{Q} \quad (25)$$

from 0-2 EigNorm1 8759 NaN, Problrm:: 26278 has -0 eigenvalues, can't be squire. So I use a if(isnan) and define idx=0.
add lambda1,2,3 for all, which is in debug

save the backside of clothes using 1-2, 1-3, 1-4. NOt much differences from colormap, need a new color map???

save lambda1 for 1,40,74, they are different, but slightly.