# Seamless Cloth Simulation with High Geometric Fidelity

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**Abstract.** Despite a large volume of studies, a current particle dynamic system results plausible deformation of draping, folding, wrinkling, stretching, etc., but realistic fold patterns generated from body-pose based variations and motion ill pose the problem. In this paper, a new approach based on continuum mechanic is proposed, which

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## 1 Introduction

## 2

### 2.1 Deformation Gradient

Let $\kappa$ be a reference configuration and $\Gamma$ be an arbitrary configuration of body $\mathcal{B}$. Then the mapping

$$\mathbf{x} = \Gamma_\kappa(\mathbf{X}, t) \tag{1}$$

is called the *deformation* of body $\mathcal{B}$ from $\kappa$ to $\Gamma$.

The *deformation gradient* $\mathbf{F}$ of $\Gamma$ relative to $\kappa$, is defined by

$$\mathbf{F} = \nabla_{\mathbf{X}} \Gamma_\kappa = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \begin{bmatrix} \frac{\partial x}{\partial X} & \frac{\partial x}{\partial Y} & \frac{\partial x}{\partial Z} \\ \frac{\partial y}{\partial X} & \frac{\partial y}{\partial Y} & \frac{\partial y}{\partial Z} \\ \frac{\partial z}{\partial X} & \frac{\partial z}{\partial Y} & \frac{\partial z}{\partial Z} \end{bmatrix}, \tag{2}$$

where the coordinates of $\mathbf{X}$ are called the *template coordinates* and the coordinates of $\mathbf{x}$ are called the *reference coordinates*. Moreover, the Eq. (2) is a measure of local deformation of the body. Hence the mapping $\Gamma_\kappa$ is one-to-one and onto, $\mathbf{F}$ is non-singular.

Applied by *decomposition theorem* on the deformation gradient [**?**], there exist two positive definite symmetric tensor $\mathbf{U}$ and $\mathbf{V}$, and an orthogonal tensor $\mathbf{R}$, uniquely determined by $\mathbf{F}$, such that

$$\mathbf{F} = \mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}, \tag{3}$$

where the $\mathbf{U}$ and $\mathbf{V}$ denote the *right* and *left stretch tensor*, and $\mathbf{R}$ denotes the local *rotation tensor*. As a consequence of Eq. (3), we could obtain the pure stretches by applying

$$\mathbf{U}^2 = \mathbf{F}^T\mathbf{F} = \mathbf{C} \quad \text{and} \quad \mathbf{V}^2 = \mathbf{F}\mathbf{F}^T = \mathbf{B}, \tag{4}$$

where the $\mathbf{C}$ and $\mathbf{B}$ are called *right* and *left Cauchy-Green strain tensor*.

Since $\mathbf{C}$ can be diagonalized as

$$\mathbf{C} = \lambda_1^2 \mathbf{e}_1 \mathbf{e}_1^T + \lambda_2^2 \mathbf{e}_2 \mathbf{e}_2^T + \lambda_3^2 \mathbf{e}_3 \mathbf{e}_3^T, \tag{5}$$

we could calculated the $\mathbf{U}$ by applying

$$\mathbf{U} = \lambda_1 \mathbf{e}_1 \mathbf{e}_1^T + \lambda_2 \mathbf{e}_2 \mathbf{e}_2 + \lambda_3 \mathbf{e}_3 \mathbf{e}_3^T. \tag{6}$$

The $\lambda_1 < \lambda_2 < \lambda_3$ are real, positive eigenvalues and the corresponding $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are unit eigenvectors, defining main stretch or shrinkage directions [**?**].

Following by [**?**], the *wrinkle vector field* $\mathbf{v}$ describes the rate of shrinkage per unit of length wherever shrinkage occurs and its direction is orthogonal to the main shrinkage direction, which is given by

$$\mathbf{v} = \max(1 - \lambda_1, 0)\mathbf{e}_2. \tag{7}$$

### 2.2 Velocity Gradient

Whereas the deformation gradient measures the local deformation, the *spacial velocity gradient* $\mathbf{L}$ describes the rate of deformation, given by

$$\mathbf{L} = grad\boldsymbol{v} = \dot{\mathbf{F}}\mathbf{F}^{-1}, \tag{8}$$

where $\boldsymbol{v}$ denotes the *velocity* of the material point $\mathbf{X}$, and $\dot{\mathbf{F}}$ denotes the material time derivative of deformation gradient $\mathbf{F}$.

Analogically, we apply the polar decomposition on the spacial velocity gradient $\mathbf{L}$[**?**], and obtain

$$\mathbf{L} = \mathbf{D} + \mathbf{W}. \tag{9}$$

Thus, the tensor $\mathbf{L}$ could be decomposed into its symmetric part $\mathbf{D}$ and skew-symmetric part $\mathbf{W}$

$$\mathbf{D} = \frac{1}{2}(\mathbf{L} + \mathbf{L}^T), \tag{10}$$

$$\mathbf{W} = \frac{1}{2}(\mathbf{L} - \mathbf{L}^T). \tag{11}$$

where $\mathbf{D}$ is called the *rate of strain tensor*, and $\mathbf{W}$ is called the *rate of rotation tensor*.

Assuming d$\mathbf{x}$ is a material line element in the current configuration, the rate of change of its length $\dot{\epsilon}_{ii}$ and angle $\dot{\gamma}_{ij}$ is measured by means of $\mathbf{D}$[**?**].

$$\dot{\epsilon}_{ii} = \mathbf{e}_i \mathbf{D} \mathbf{e}_i = D_{ii} \tag{12}$$

$$\dot{\gamma}_{ij} = 2\mathbf{e}_i \mathbf{D} \mathbf{e}_j = D_{ij} \tag{13}$$

where $D_{ij}$ are the $ij$-entries of $\mathbf{D}$. Additionally, if $\mathbf{D}$ has the same base $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ as in the eq.(6), then

$$D_{11} = \frac{\dot{\lambda}_1}{\lambda_1} \tag{14}$$

$$D_{22} = \frac{\dot{\lambda}_2}{\lambda_3} \tag{15}$$

$$D_{33} = \frac{\dot{\lambda}_2}{\lambda_3} \tag{16}$$

Given the deformation gradients at the beginning $F_t$ and end $F_{t+\Delta t}$ of a time step, we use *forward difference* [**?**] to calculate the derivation of deformation gradient and take the inverse of the later one.

### 2.3 Cross-covariance Matrix

Let $\Phi$ denote the points set in the template frame, and $\Psi$ denote the points set in the reference frame. Each set of points can be described as

$$\Phi = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ \vdots & \vdots & \vdots \\ X_N & Y_N & Z_N \end{bmatrix} \quad \text{and} \quad \Psi = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots & \vdots & \vdots \\ x_N & y_N & z_N \end{bmatrix}, \tag{17}$$

where $N$ is the number of the points in the points set.

A point $\bar{\mathbf{j}}$ is called the *neighboring point* of point $\mathbf{j}$, when $\bar{\mathbf{j}}$ and $\mathbf{j}$ are connected by a same edge. If only the data set without any further information is given, we shall use the *kd-tree* [**?**] as a data structure to search for the neighboring points in no structural points set.

Let $\overline{\Phi} \subset \Phi$ define the points set of the neighboring points of $\mathbf{X}_i$ in the template frame, and $\overline{\Psi} \subset \Psi$ define the points set of the neighboring points of $\mathbf{x}_i$ in the reference frame. Therefore, we could construct a $M \times 3$ matrix $\mathbf{P}_i$ for each point $\mathbf{X}_i \in \Phi$, as well as a matrix $\mathbf{Q}_i$ for each point $\mathbf{x}_i \in \Psi$,

$$\mathbf{P}_i = \begin{bmatrix} X_1 - X_i & Y_1 - Y_i & Z_1 - Z_i \\ X_2 - X_i & Y_2 - Y_i & Z_2 - Z_i \\ \vdots & \vdots & \vdots \\ X_M - X_i & Y_M - Y_i & Z_M - Z_i \end{bmatrix} \quad \text{and} \quad \mathbf{Q}_i = \begin{bmatrix} x_1 - x_i & y_1 - y_i & z_1 - z_i \\ x_2 - x_i & y_2 - y_i & z_2 - z_i \\ \vdots & \vdots & \vdots \\ x_M - x_i & y_M - y_i & z_M - z_i \end{bmatrix}, \tag{18}$$

where $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_M \in \overline{\Phi}$ and $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_M \in \overline{\Psi}$, and $M$ is the number of the points in the neighboring points set. The matrices $\mathbf{P}_i$ and $\mathbf{Q}_i$ represent the distribution of local shapes near the point $\mathbf{X}_i$ and $\mathbf{x}_i$.

Proceeding to define

$$\mathbf{H} = \mathbf{P}^T \mathbf{Q}. \tag{19}$$

$\mathbf{H}$ is called the *cross-covariance matrix*, which is a measure of similarity of $\mathbf{P}$ and $\mathbf{Q}$ [**?**].

The matrix $\mathbf{H}$ is derived from the *orthogonal Procrustes problem*[**?**]. It is defined as the least-squares problem of transforming a given matrix $\mathbf{P}$ into a given matrix $\mathbf{Q}$ by an orthogonal transformation matrix $\mathbf{R}$, such that the sums of squares of the residual matrix $\mathbf{E} = \mathbf{\Omega P} - \mathbf{Q}$ is a minimum

$$\mathbf{R} = \arg\min_{\mathbf{\Omega}} \|\mathbf{\Omega P} - \mathbf{Q}\|_F \quad \text{subject to} \quad \mathbf{\Omega}^T \mathbf{\Omega} = \mathbf{I}, \tag{20}$$

where $\|\cdot\|_F$ is the Frobenius norm. Moreover, it can be shown[**?**] that this problem is equivalent to find the nearest orthogonal matrix $\mathbf{R}$ to a given matrix $\mathbf{H} = \mathbf{P}^T\mathbf{Q}$, which most closely maps $\mathbf{P}$ to $\mathbf{Q}$. Thus, we could use $\mathbf{H}$ to approximate the deformation gradient $\mathbf{F}$ in Eq. (2)

$$\widetilde{\mathbf{F}} = \mathbf{H}. \tag{21}$$

Applied by Eq. (3) (4) (6), we could obtain the approximated stretch tensor $\widetilde{\mathbf{U}}$ and its eigenvalues $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3$.

## 3 Animation of Wrinkle Deformation Field

### 3.1 Structural Points Sets Using 1st Neighboring Vertices

Given a structual points sets, we follow the procudures described in the Sec. 2.3 to calculate the smallest eigenvalues $\lambda_1$ for every vertices.

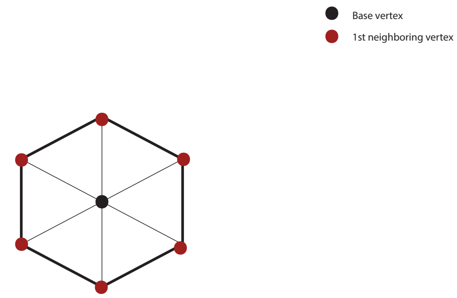


**Fig. 1.** The 1st neighboring vertices of base vertex.

The procedure to evaluate the deformation using 1st neighboring vertices:

begin
  for $i := 1$ to $N$ do — N is the amount of all vertices in template configuration
    $\mathbf{P}_i$ and $\mathbf{Q}_i$; — calculate the matrices $\mathbf{P}_i$ *and* $\mathbf{Q}_i$
    $\mathbf{H_i} = \mathbf{P_i}^T \mathbf{Q_i} = \widetilde{\mathbf{F}}$; — calculate the cross-covariance Matrix $\mathbf{H_i}$
    $\widetilde{\mathbf{C}}_\mathbf{i} = \widetilde{\mathbf{F}}_\mathbf{i}^T \widetilde{\mathbf{F}}_\mathbf{i}$; — calculate the Cauchy-Green strain tensor $\widetilde{\mathbf{C}}_\mathbf{i}$
    $\widetilde{\mathbf{C}} = \widetilde{\lambda}_1^2 \mathbf{e}_1 \mathbf{e}_1^T + \widetilde{\lambda}_2^2 \mathbf{e}_2 \mathbf{e}_2^T + \widetilde{\lambda}_3^2 \mathbf{e}_3 \mathbf{e}_3^T$; — decomposite the Cauchy-Green strain tensor
    $\widetilde{\mathbf{U}} = \widetilde{\lambda}_1 \mathbf{e}_1 \mathbf{e}_1^T + \widetilde{\lambda}_2 \mathbf{e}_2 \mathbf{e}_2^T + \widetilde{\lambda}_3 \mathbf{e}_3 \mathbf{e}_3^T$. — calculate the stretch tensor $\widetilde{\mathbf{U}}_\mathbf{i}$
end

### 3.2 Structural Points Sets Using 2nd Neighboring Vertices

### 3.3 Non-structural Points Sets Using Binary Search Tree

## 4 Conclusion