$p(m|N,\theta)$ is a Binomial distribution, $P(\theta|a,b)$ is a beta distribution

We want to prove the Beta-Binomial conjugation

pf.

$$p(\theta|m) = \frac{p(m|N,\theta)\cdot p(\theta|a,b)}{p(m)} \propto \binom{N}{m}\theta^{m}(1-\theta)^{N-m}\cdot\theta^{a-1}\cdot(1-\theta)^{b-1}\cdot\frac{1}{\beta(a,b)}$$

$$\propto \theta^{m+a-1}(1-\theta)^{N-m+b-1}$$

$\because$ $p(\theta|m)$ is a valid probability distribution and integrates to 1.

We need a scaling factor $C$ so that $\int_0^1 C\,\theta^{m+a-1}(1-\theta)^{N-m+b-1}\,d\theta = 1$

It turns out that if $C$ is $\frac{1}{\beta(m+a,N-m+b)}$, $\int_0^1 C\,\theta^{m+a-1}(1-\theta)^{N-m+b-1}\,d\theta = 1$

$\therefore$ $p(\theta|m) = \frac{1}{\beta(m+a,N-m+b)}\theta^{m+a-1}(1-\theta)^{N-m+b}$ $\rightsquigarrow$ a Beta distribution