hw2_errors

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Consider the Gram-Schmidt process using the L_2 inner product $\int f(x)g(x)dx$ on functions $f_c(x) = e^{-\frac{(c-x)^2}{2}}$ for $c = x_1, x_2, x_3$. Define

$$u_1(x) = f_{x_1}(x) / \left[\int f^2_{x_1}(x)dx\right]^{1/2} \tag{1}$$

$$= e^{-\frac{(x_1-x)^2}{2}} / \left[\int e^{-(x_1-x)^2}dx\right]^{1/2} \tag{2}$$

$$= \pi^{1/4}e^{-\frac{(x_1-x)^2}{2}}. \tag{3}$$

Next, define

$$u_2(x) = \frac{f_{x_2}(x) - \int f_{x_2}(x)u_1(x)dxu_1(x)}{\left[\int (f_{x_2}(x) - \int f_{x_2}(x)u_1(x)dxu_1(x))^2\right]^{1/2}} \tag{4}$$

$$= \frac{f_{x_2}(x) - (\pi^{1/4}e^{\frac{1}{4}(x_1+x_2)^2 - 2x_1x_2}) \int e^{-\frac{1}{2*1/2}(x-1/2(x_1+x_2)^2)}dx * u_1(x)}{\left[\int (f_{x_2}(x) - (\pi^{1/4}e^{\frac{1}{4}(x_1+x_2)^2 - 2x_1x_2}) \int e^{-\frac{1}{2*1/2}(x-1/2(x_1+x_2)^2)}dx * u_1(x))^2\right]^{1/2}} \tag{5}$$

$$= \frac{f_{x_2}(x) - (\pi^{1/4}e^{\frac{1}{4}(x_1+x_2)^2 - 2x_1x_2})\pi^{1/2}\pi^{1/4}e^{-\frac{(x_1-x)^2}{2}}}{\left[\int (f_{x_2}(x) - (\pi^{1/4}e^{\frac{1}{4}(x_1+x_2)^2 - 2x_1x_2})\pi^{1/2}\pi^{1/4}e^{-\frac{(x_1-x)^2}{2}})^2\right]^{1/2}} \tag{6}$$

$$= \frac{e^{-(x_2-x)^2}/2}{\pi^{1/2} - 2q(x_1,x_2)\sqrt{2\pi} + q(x_1,x_2)}, \tag{7}$$

where $q(c_1, c_2) = \pi e^{\frac{1}{4}(c_1+c_2)^2 - 2c_1c_2}$.

Following the same procedure for the third function $u_3(x)$, we get

$$u_3(x) = \frac{f_{x_3}(x) - \int f_{x_3}(x)u_2(x)dx * u_2(x) - \int f_{x_3}(x)u_1(x)dx * u_1(x)}{\left[\int (f_{x_3}(x) - \int f_{x_3}(x)u_2(x)dx * u_2(x) - \int f_{x_3}(x)u_1(x)dx * u_1(x))^2\right]^{1/2}} \tag{8}$$

$$= \frac{f_{x_3}(x) - \int f_{x_3}(x)u_2(x)dx * u_2(x) - \int f_{x_3}(x)u_1(x)dx * u_1(x)}{v(x_1,x_2,x_3)} \tag{9}$$

$$= \frac{f_{x_3}(x) - \frac{\pi^{-1/2}q(x_2,x_3)e^{-(x_2-x)^2/2}}{\pi^{1/2}+2q(x_2,x_3)\sqrt{2\pi}+q(x_2,x_3)} - 1(x_1,x_3)e^{-(x_1-x)^2/2}}{v(x_1,x_2,x_3)}, \tag{10}$$

where $v(x_1,x_2,x_3) = \pi^{1/2} + \left(\frac{q(x_2,x_3)}{\pi^{1/2}+2q(x_2,x_3)\sqrt{2\pi}+q(x_2,x_3)}\right)^2 + q(x_1,x_3)^2\pi^{1/2} - 2\frac{\pi^{-1/2}q(x_2,x_3)q(x_1,x_3)}{\pi^{1/2}+2q(x_2,x_3)\sqrt{2\pi}+q(x_2,x_3)}\pi^{-1/2}q(x_1,x_2) - 2\frac{\pi^{-1/2}q(x_2,x_3)}{\pi^{1/2}+2q(x_2,x_3)\sqrt{2\pi}+q(x_2,x_3)}\pi^{-1/2}q(x_2,x_3) - 2q(x_1,x_3)\pi^{-1/2}q(x_1,x_3)$.

If we consider the kernel inner product, the inner product expressions are much nicer. Using the kernel inner product, we wind up with

$$u_1(x) = f_{x_1}(x) \tag{11}$$

$$u_2(x) = \frac{f_{x_2}(x) - K(x_1, x_2)u_1(x)}{|1 - K(x_1, x_2)|} \tag{12}$$

$$u_3(x) = \frac{f_{x_3}(x) - \langle f_{x_3}(x), u_2(x)\rangle u_2(x) - \langle f_{x_3}(x), u_1(x)\rangle u_1(x)}{\langle f_{x_3}(x) - \langle f_{x_3}(x), u_2(x)\rangle u_2(x) - \langle f_{x_3}(x), u_1(x)\rangle u_1(x), f_{x_3}(x) - \langle f_{x_3}(x), u_2(x)\rangle u_2(x) - \langle f_{x_3}(x), u_1(x)\rangle u_1(x)\rangle} \tag{13}$$