

Stat 602 Homework 2

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Due 3/4/2019

Problem 1

Problem 4.3

A table of RMSEs for 8-fold cross-validation can be found in Table 1.

Table 1: 8-fold CV Model RMSEs for Problem 4.3

Model	8-fold CV RMSE
None	36759.42
Size	27351.2
Fireplace	28847.33
Bsmt Bath	35733.8
Land	33588.19
Size, Fireplace	23360.18
Size, Bsmt Bath	26550.65
Size, Land	25116.52
Fireplace, Bsmt Bath	28200.39
Fireplace, Land	27952.19
Bsmt Bath, Land	32093.66
Size, Fireplace, Bsmt Bath	24187.06
Size, Fireplace, Land	22249.03
Fireplace, Bsmt Bath, Land	26548.87
Size, Fireplace, Bsmt Bath, Land	21419.63

The lowest RMSE appears to be for the full model which includes Size, Fireplace, Bsmt Bath, and Land.

Problem 4.4

Part a

```
## k-Nearest Neighbors
##
## 146 samples
##   9 predictor
##   2 classes: '1', '2'
##
## Pre-processing: centered (9), scaled (9)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 131, 132, 131, 131, 132, 132, ...
## Resampling results across tuning parameters:
##
##     k    Accuracy   Kappa
##     1    0.8066667  0.6146085
##     2    0.8200000  0.6402192
##     3    0.7933333  0.5864147
```

```

##   4  0.7857143  0.5720579
##   5  0.7804762  0.5634280
##   6  0.7671429  0.5340682
##   7  0.7600000  0.5221561
##   8  0.7947619  0.5908508
##   9  0.7476190  0.4952573
##  10  0.7538095  0.5105635
##  11  0.7676190  0.5386198
##  12  0.7533333  0.5086489
##  13  0.7542857  0.5125185
##  14  0.7542857  0.5115930
##  15  0.7328571  0.4689609
##  16  0.7538095  0.5110334
##  17  0.7476190  0.4987705
##  18  0.7342857  0.4738164
##  19  0.7342857  0.4744563
##  20  0.7619048  0.5280193
##
## Accuracy was used to select the optimal model using the largest value.
## The final value used for the model was k = 2.

```

Based on 10-fold cross-validation, the accuracy is estimated to be highest when $k = 4$.

Part b

The classification rule $\hat{y} = 2I[t(x) \geq k/2] + I[t(x) < k/2]$ is equivalent to $\hat{y} = 2I[t(x)/k \geq 1/2] + I[t(x)/k < 1/2]$. Here, $t(x)/k$ is an estimate of $E[\text{type} = 2|x] = P(\text{type} = 2|x)$. Thus, this is directly an estimate of a posterior class probability which accounts for the prior class probabilities. This means that no modification of kNN needs to be made to account for differing prior probabilities.

Problem 5.1

The following are decompositions using the matrix

$$X = \begin{bmatrix} 2 & 4 & 7 & 2 \\ 4 & 3 & 5 & 5 \\ 3 & 4 & 6 & 1 \\ 5 & 2 & 4 & 2 \\ 1 & 3 & 4 & 4 \end{bmatrix}$$

Part a

The QR decomposition of X is:

$$Q = \begin{bmatrix} -0.27 & 0.57 & 0.77 & 0.04 \\ -0.54 & -0.07 & -0.13 & 0.56 \\ -0.4 & 0.37 & -0.35 & -0.71 \\ -0.67 & -0.5 & 0.1 & -0.13 \\ -0.13 & 0.53 & -0.5 & 0.4 \end{bmatrix}, \quad R = \begin{bmatrix} -7.42 & -6.07 & -10.25 & -5.53 \\ 0 & 4.15 & 5.99 & 2.28 \\ 0 & 0 & 1.06 & -1.23 \\ 0 & 0 & 0 & 3.57 \end{bmatrix}$$

Thus, the basis for $C(X)$ will be the columns of the matrix Q .

The Singular Value Decomposition of X is:

$$U = \begin{bmatrix} -0.5 & 0.53 & 0.16 & -0.66 \\ -0.5 & -0.59 & 0.13 & -0.11 \\ -0.46 & 0.49 & -0.25 & 0.65 \\ -0.39 & -0.33 & -0.68 & -0.09 \\ -0.37 & -0.17 & 0.65 & 0.35 \end{bmatrix}, \quad D = \begin{bmatrix} 16.58 & 0 & 0 & 0 \\ 0 & 3.78 & 0 & 0 \\ 0 & 0 & 3.38 & 0 \\ 0 & 0 & 0 & 0.55 \end{bmatrix},$$

$$V = \begin{bmatrix} -0.4 & -0.43 & -0.8 & 0.12 \\ -0.44 & 0.3 & 0.18 & 0.83 \\ -0.71 & 0.45 & 0.04 & -0.54 \\ -0.38 & -0.72 & 0.57 & -0.06 \end{bmatrix}$$

Thus, the basis for $C(X)$ will be the columns of the matrix U .

Part b

We can find the eigen (spectral) decomposition of $X'X$ by calculating the eigenvalues, D^2 , and the eigenvectors, which will be the columns of V :

$$D^2 = \begin{bmatrix} 274.94 \\ 14.32 \\ 11.44 \\ 0.3 \end{bmatrix}, \quad V = \begin{bmatrix} -0.4 & -0.43 & -0.8 & 0.12 \\ -0.44 & 0.3 & 0.18 & 0.83 \\ -0.71 & 0.45 & 0.04 & -0.54 \\ -0.38 & -0.72 & 0.57 & -0.06 \end{bmatrix}$$

We can find the eigen (spectral) decomposition of XX' by calculating the eigenvalues, D^2 , and the eigenvectors, which will be the columns of U (rather than V as for $X'X$):

$$D^2 = \begin{bmatrix} 274.94 \\ 14.32 \\ 11.44 \\ 0.3 \end{bmatrix}, \quad V = \begin{bmatrix} -0.5 & 0.53 & 0.16 & -0.66 \\ -0.5 & -0.59 & 0.13 & -0.11 \\ -0.46 & 0.49 & -0.25 & 0.65 \\ -0.39 & -0.33 & -0.68 & -0.09 \\ -0.37 & -0.17 & 0.65 & 0.35 \end{bmatrix}$$

Part c

The best $rank = 1$ approximation to X will be $X^{*1} = U_1 \text{diag}(d_1)V'_1$, where U_1 is a matrix with the first column of U , and V_1 is a matrix with the first column of V :

$$X^{*1} = U_1 \text{diag}(d_1)V'_1 = \begin{bmatrix} -0.5 \\ -0.5 \\ -0.46 \\ -0.39 \\ -0.37 \end{bmatrix} \times [16.58] \times [-0.4 \quad -0.44 \quad -0.71 \quad -0.38]$$

$$= \begin{bmatrix} 3.35 & 3.61 & 5.89 & 3.11 \\ 3.38 & 3.64 & 5.94 & 3.14 \\ 3.08 & 3.31 & 5.4 & 2.85 \\ 2.63 & 2.83 & 4.62 & 2.43 \\ 2.45 & 2.64 & 4.31 & 2.27 \end{bmatrix}$$

The best $rank = 2$ approximation to X will be $X^{*2} = U_2 \text{diag}(d_1, d_2)V'_2$, where U_2 is a matrix with the first two columns of U , and V_2 is a matrix with the first two columns of V :

$$\begin{aligned}
X^{*2} &= U_2 \text{diag}(d_1, d_2) V'_2 = \begin{bmatrix} -0.5 & 0.53 \\ -0.5 & -0.59 \\ -0.46 & 0.49 \\ -0.39 & -0.33 \\ -0.37 & -0.17 \end{bmatrix} \times \begin{bmatrix} 16.58 & 0 \\ 0 & 3.78 \end{bmatrix} \times \begin{bmatrix} -0.4 & -0.44 & -0.71 & -0.38 \\ -0.43 & 0.3 & 0.45 & -0.72 \end{bmatrix} \\
&= \begin{bmatrix} 2.49 & 4.2 & 6.78 & 1.66 \\ 4.35 & 2.97 & 4.95 & 4.75 \\ 2.28 & 3.86 & 6.23 & 1.51 \\ 3.16 & 2.46 & 4.07 & 3.33 \\ 2.73 & 2.44 & 4.02 & 2.75 \end{bmatrix}
\end{aligned}$$

Part d

Note that

$$\tilde{X} = \begin{bmatrix} -1 & 0.8 & 1.8 & -0.8 \\ 1 & -0.2 & -0.2 & 2.2 \\ 0 & 0.8 & 0.8 & -1.8 \\ 2 & -1.2 & -1.2 & -0.8 \\ -2 & -0.2 & -1.2 & 1.2 \end{bmatrix}$$

The Singular Value Decomposition of \tilde{X} is:

$$\begin{aligned}
\tilde{U} &= \begin{bmatrix} -0.57 & 0.17 & 0.29 & -0.6 \\ 0.51 & 0.11 & 0.7 & 0.21 \\ -0.5 & -0.25 & -0.09 & 0.69 \\ 0.34 & -0.69 & -0.32 & -0.33 \\ 0.22 & 0.65 & -0.57 & 0.03 \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} 3.83 & 0 & 0 & 0 \\ 0 & 3.38 & 0 & 0 \\ 0 & 0 & 2.01 & 0 \\ 0 & 0 & 0 & 0.48 \end{bmatrix} \\
\tilde{V} &= \begin{bmatrix} 0.34 & -0.81 & 0.45 & 0.18 \\ -0.37 & 0.18 & 0.26 & 0.88 \\ -0.58 & 0.03 & 0.68 & -0.45 \\ 0.64 & 0.56 & 0.52 & 0 \end{bmatrix}
\end{aligned}$$

Therefore, the principal component directions will be the columns of \tilde{V} , the principal components will be the inner products $z_j = \langle x_i, v_j \rangle$, the columns of:

$$z = \begin{bmatrix} -2.19 & 0.56 & 0.57 & -0.29 \\ 1.95 & 0.39 & 1.4 & 0.1 \\ -1.91 & -0.84 & -0.18 & 0.33 \\ 1.3 & -2.32 & -0.65 & -0.16 \\ 0.85 & 2.21 & -1.14 & 0.02 \end{bmatrix}$$

The “loadings” of the first principal component are the first column of the matrix \tilde{V} ,

$$\tilde{V}_1 = \begin{bmatrix} 0.34 \\ -0.37 \\ -0.58 \\ 0.64 \end{bmatrix}$$

Part e

The best $rank = 1$ approximation to \tilde{X} will be $\tilde{X}^{*1} = \tilde{U}_1 diag(\tilde{d}_1) \tilde{V}_1'$, where \tilde{U}_1 is a matrix with first column of \tilde{U} and \tilde{V}_1 is a matrix with the first column of \tilde{V} :

$$\begin{aligned}\tilde{X}^{*1} &= \tilde{U}_1 diag(\tilde{d}_1) \tilde{V}_1' = \begin{bmatrix} -0.57 \\ 0.51 \\ -0.5 \\ 0.34 \\ 0.22 \end{bmatrix} \times [3.83] \times [0.34 \quad -0.37 \quad -0.58 \quad 0.64] \\ &= \begin{bmatrix} -0.75 & 0.81 & 1.26 & -1.41 \\ 0.67 & -0.72 & -1.12 & 1.26 \\ -0.66 & 0.71 & 1.1 & -1.23 \\ 0.45 & -0.48 & -0.75 & 0.84 \\ 0.29 & -0.31 & -0.49 & 0.55 \end{bmatrix}\end{aligned}$$

The best $rank = 2$ approximation to \tilde{X} will be $\tilde{X}^{*2} = \tilde{U}_2 diag(\tilde{d}_1, \tilde{d}_2) \tilde{V}_2'$, where \tilde{U}_2 is a matrix with the first two columns of \tilde{U} , and \tilde{V}_2 is a matrix with the first two columns of \tilde{V} :

$$\begin{aligned}\tilde{X}^{*2} &= \tilde{U}_2 diag(\tilde{d}_1, \tilde{d}_2) \tilde{V}_2' = \begin{bmatrix} -0.57 & 0.17 \\ 0.51 & 0.11 \\ -0.5 & -0.25 \\ 0.34 & -0.69 \\ 0.22 & 0.65 \end{bmatrix} \times [3.83 \quad 0] \times [0.34 \quad -0.37 \quad -0.58 \quad 0.64] \\ &= \begin{bmatrix} -1.2 & 0.91 & 1.28 & -1.1 \\ 0.36 & -0.65 & -1.11 & 1.47 \\ 0.02 & 0.56 & 1.07 & -1.71 \\ 2.32 & -0.89 & -0.83 & -0.46 \\ -1.49 & 0.08 & -0.41 & 1.79 \end{bmatrix}\end{aligned}$$

Part f

Note that since here we are dealing with a sample covariance matrix, we have a symmetric non-negative definite matrix, so an eigen analysis will also yield all the information we need for a singular value decomposition, and thus we can easily find the best 1 and 2 component approximations.

We begin with the matrix:

$$\frac{1}{5} \tilde{X}' \tilde{X} = \begin{bmatrix} 2 & -0.6 & -0.4 & -0.2 \\ -0.6 & 0.56 & 0.76 & -0.36 \\ -0.4 & 0.76 & 1.36 & -0.76 \\ -0.2 & -0.36 & -0.76 & 2.16 \end{bmatrix}$$

An eigen decomposition yields:

$$\text{Eigenvalues } D = \begin{bmatrix} 2.94 \\ 2.29 \\ 0.81 \\ 0.05 \end{bmatrix}, \quad \text{Eigenvectors } U = V = \begin{bmatrix} 0.34 & 0.81 & -0.45 & -0.18 \\ -0.37 & -0.18 & -0.26 & -0.88 \\ -0.58 & -0.03 & -0.68 & 0.45 \\ 0.64 & -0.56 & -0.52 & 0 \end{bmatrix}$$

Thus, a best 1 component approximation will be of the form:

$$\begin{aligned} \frac{1}{5}\tilde{X}'\tilde{X}^{*1} &= \tilde{U}_1 \text{diag}(\tilde{d}_1) \tilde{V}_1' = \begin{bmatrix} 0.34 \\ -0.37 \\ -0.58 \\ 0.64 \end{bmatrix} \times [2.94] \times [0.34 \quad -0.37 \quad -0.58 \quad 0.64] \\ &= \begin{bmatrix} 0.35 & -0.37 & -0.58 & 0.65 \\ -0.37 & 0.4 & 0.62 & -0.7 \\ -0.58 & 0.62 & 0.97 & -1.09 \\ 0.65 & -0.7 & -1.09 & 1.22 \end{bmatrix} \end{aligned}$$

A best 2 component approximation will be of the form:

$$\begin{aligned} \frac{1}{5}\tilde{X}'\tilde{X}^{*2} &= \tilde{U}_2 \text{diag}(\tilde{d}_1, \tilde{d}_2) \tilde{V}_2' = \begin{bmatrix} 0.34 & 0.81 \\ -0.37 & -0.18 \\ -0.58 & -0.03 \\ 0.64 & -0.56 \end{bmatrix} \times [2.94 \quad 0] \times [0.34 \quad -0.37 \quad -0.58 \quad 0.64] \\ &= \begin{bmatrix} 1.84 & -0.7 & -0.64 & -0.39 \\ -0.7 & 0.47 & 0.64 & -0.47 \\ -0.64 & 0.64 & 0.97 & -1.05 \\ -0.39 & -0.47 & -1.05 & 1.94 \end{bmatrix} \end{aligned}$$

Part d - standardized

Note that

$$\tilde{\tilde{X}} = \begin{bmatrix} -0.63 & 0.96 & 1.38 & -0.49 \\ 0.63 & -0.24 & -0.15 & 1.34 \\ 0 & 0.96 & 0.61 & -1.1 \\ 1.26 & -1.43 & -0.92 & -0.49 \\ -1.26 & -0.24 & -0.92 & 0.73 \end{bmatrix}$$

The Singular Value Decomposition of $\tilde{\tilde{X}}$ is:

$$\begin{aligned} \tilde{\tilde{U}} &= \begin{bmatrix} -0.6 & 0 & 0.14 & -0.65 \\ 0.3 & 0.22 & 0.79 & 0.2 \\ -0.44 & -0.34 & -0.16 & 0.68 \\ 0.58 & -0.58 & -0.24 & -0.27 \\ 0.16 & 0.7 & -0.53 & 0.04 \end{bmatrix}, \quad \tilde{\tilde{D}} = \begin{bmatrix} 3.04 & 0 & 0 & 0 \\ 0 & 2.14 & 0 & 0 \\ 0 & 0 & 1.39 & 0 \\ 0 & 0 & 0 & 0.49 \end{bmatrix} \\ \tilde{\tilde{V}} &= \begin{bmatrix} 0.36 & -0.69 & 0.56 & 0.29 \\ -0.63 & 0.13 & 0.19 & 0.74 \\ -0.6 & -0.17 & 0.49 & -0.61 \\ 0.33 & 0.69 & 0.64 & 0 \end{bmatrix} \end{aligned}$$

Therefore, the principal component directions will be the columns of $\tilde{\tilde{V}}$, the principal components will be the inner products $z_j = \langle x_i, v_j \rangle$, the columns of:

$$z = \begin{bmatrix} -1.82 & -0.01 & 0.19 & -0.32 \\ 0.92 & 0.48 & 1.09 & 0.1 \\ -1.34 & -0.73 & -0.22 & 0.33 \\ 1.75 & -1.24 & -0.33 & -0.13 \\ 0.49 & 1.5 & -0.73 & 0.02 \end{bmatrix}$$

The “loadings” of the first principal component are the first column of the matrix \tilde{V} ,

$$\tilde{V}_1 = \begin{bmatrix} 0.36 \\ -0.63 \\ -0.6 \\ 0.33 \end{bmatrix}$$

Part e - standardized

The best $rank = 1$ approximation to \tilde{X} will be $\tilde{X}^{*1} = \tilde{U}_1 diag(\tilde{d}_1) \tilde{V}_1'$, where \tilde{U}_1 is a matrix with first column of \tilde{U} and \tilde{V}_1 is a matrix with the first column of \tilde{V} :

$$\begin{aligned} \tilde{X}^{*1} &= \tilde{U}_1 diag(\tilde{d}_1) \tilde{V}_1' = \begin{bmatrix} -0.6 \\ 0.3 \\ -0.44 \\ 0.58 \\ 0.16 \end{bmatrix} \times [3.04] \times [0.36 \quad -0.63 \quad -0.6 \quad 0.33] \\ &= \begin{bmatrix} -0.65 & 1.15 & 1.09 & -0.61 \\ 0.33 & -0.58 & -0.55 & 0.31 \\ -0.48 & 0.85 & 0.8 & -0.45 \\ 0.63 & -1.11 & -1.05 & 0.58 \\ 0.18 & -0.31 & -0.29 & 0.16 \end{bmatrix} \end{aligned}$$

The best $rank = 2$ approximation to \tilde{X} will be $\tilde{X}^{*2} = \tilde{U}_2 diag(\tilde{d}_1, \tilde{d}_2) \tilde{V}_2'$, where \tilde{U}_2 is a matrix with the first two columns of \tilde{U} , and \tilde{V}_2 is a matrix with the first two columns of \tilde{V} :

$$\begin{aligned} \tilde{X}^{*2} &= \tilde{U}_2 diag(\tilde{d}_1, \tilde{d}_2) \tilde{V}_2' = \begin{bmatrix} -0.6 & 0 \\ 0.3 & 0.22 \\ -0.44 & -0.34 \\ 0.58 & -0.58 \\ 0.16 & 0.7 \end{bmatrix} \times \begin{bmatrix} 3.04 & 0 \\ 0 & 2.14 \end{bmatrix} \times \begin{bmatrix} 0.36 & -0.63 & -0.6 & 0.33 \\ -0.69 & 0.13 & -0.17 & 0.69 \end{bmatrix} \\ &= \begin{bmatrix} -0.65 & 1.15 & 1.09 & -0.61 \\ 0 & -0.52 & -0.63 & 0.64 \\ 0.03 & 0.75 & 0.93 & -0.95 \\ 1.49 & -1.27 & -0.84 & -0.27 \\ -0.86 & -0.12 & -0.55 & 1.2 \end{bmatrix} \end{aligned}$$

Part f - standardized

Note that since here we are dealing with a sample covariance matrix, we have a symmetric non-negative definite matrix, so an eigen analysis will also yield all the information we need for a singular value decomposition, and thus we can easily find the best 1 and 2 component approximations.

We begin with the matrix:

$$\frac{1}{5} \tilde{X}' \tilde{X} = \begin{bmatrix} 0.8 & -0.45 & -0.19 & -0.08 \\ -0.45 & 0.8 & 0.7 & -0.26 \\ -0.19 & 0.7 & 0.8 & -0.35 \\ -0.08 & -0.26 & -0.35 & 0.8 \end{bmatrix}$$

An eigen decomposition yields:

$$\text{Eigenvalues} = D = \begin{bmatrix} 1.85 \\ 0.91 \\ 0.39 \\ 0.05 \end{bmatrix}, \quad \text{Eigenvectors} = U = V = \begin{bmatrix} 0.36 & -0.69 & -0.56 & 0.29 \\ -0.63 & 0.13 & -0.19 & 0.74 \\ -0.6 & -0.17 & -0.49 & -0.61 \\ 0.33 & 0.69 & -0.64 & 0 \end{bmatrix}$$

Thus, a best 1 component approximation will be of the form:

$$\begin{aligned} \frac{1}{5} \tilde{\tilde{X}}' \tilde{\tilde{X}}^{*1} &= \tilde{\tilde{U}}_1 \text{diag}(\tilde{\tilde{d}}_1) \tilde{\tilde{V}}_1' = \begin{bmatrix} 0.36 \\ -0.63 \\ -0.6 \\ 0.33 \end{bmatrix} \times [1.85] \times [0.36 \quad -0.63 \quad -0.6 \quad 0.33] \\ &= \begin{bmatrix} 0.24 & -0.42 & -0.4 & 0.22 \\ -0.42 & 0.74 & 0.7 & -0.39 \\ -0.4 & 0.7 & 0.66 & -0.37 \\ 0.22 & -0.39 & -0.37 & 0.21 \end{bmatrix} \end{aligned}$$

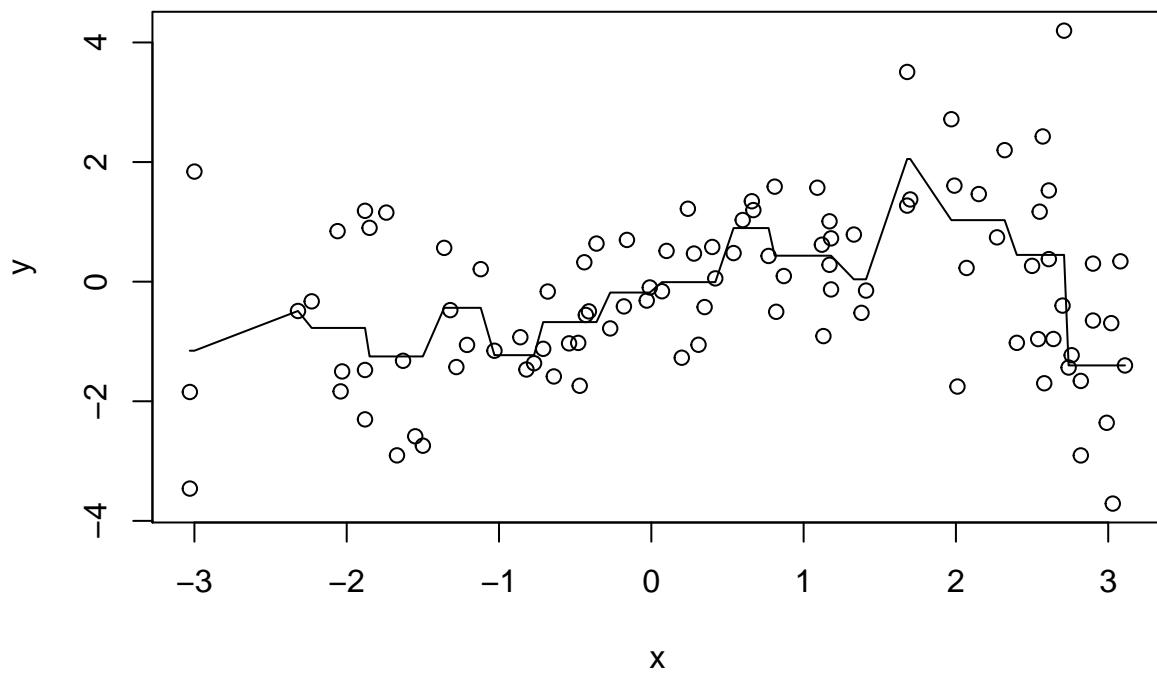
A best 2 component approximation will be of the form:

$$\begin{aligned} \frac{1}{5} \tilde{\tilde{X}}' \tilde{\tilde{X}}^{*2} &= \tilde{\tilde{U}}_2 \text{diag}(\tilde{\tilde{d}}_1, \tilde{\tilde{d}}_2) \tilde{\tilde{V}}_2' = \begin{bmatrix} 0.36 & -0.69 \\ -0.63 & 0.13 \\ -0.6 & -0.17 \\ 0.33 & 0.69 \end{bmatrix} \times \begin{bmatrix} 1.85 & 0 \\ 0 & 0.91 \end{bmatrix} \times \begin{bmatrix} 0.36 & -0.63 & -0.6 & 0.33 \\ -0.69 & 0.13 & -0.17 & 0.69 \end{bmatrix} \\ &= \begin{bmatrix} 0.68 & -0.5 & -0.29 & -0.21 \\ -0.5 & 0.76 & 0.68 & -0.31 \\ -0.29 & 0.68 & 0.69 & -0.48 \\ -0.21 & -0.31 & -0.48 & 0.64 \end{bmatrix} \end{aligned}$$

Problem 2

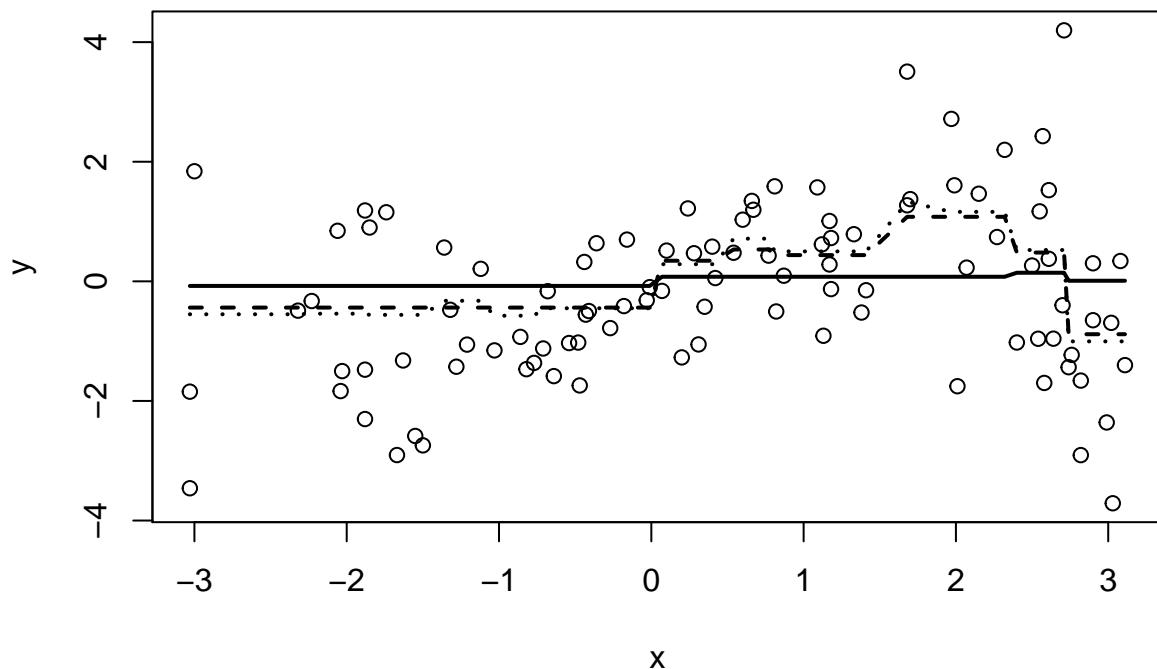
Part a

A scatterplot of the data and the fitted values from the OLS model are shown below.

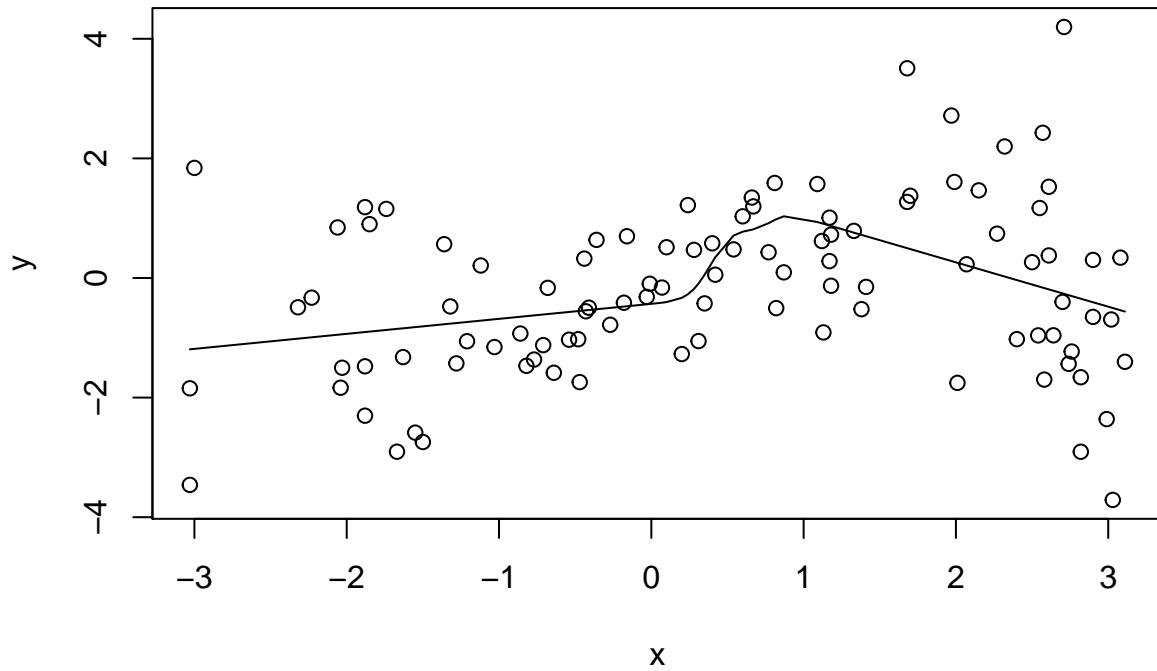


Part b

The solid line in the plot below gives (approximately) the fitted values when there are 2 nonzero coefficients, the dashed line are fitted values when there are 4 nonzero coefficients, and the dotted line are fitted values for 8 nonzero coefficients.



Problem 3



Problem 4 Note

Part a

```

## [1] "Matrix H"
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
## [1,]    1  0.0 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## [2,]    1  0.1 0.001 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## [3,]    1  0.2 0.008 0.001 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## [4,]    1  0.3 0.027 0.008 0.001 0.000 0.000 0.000 0.000 0.000 0.000
## [5,]    1  0.4 0.064 0.027 0.008 0.001 0.000 0.000 0.000 0.000 0.000
## [6,]    1  0.5 0.125 0.064 0.027 0.008 0.001 0.000 0.000 0.000 0.000
## [7,]    1  0.6 0.216 0.125 0.064 0.027 0.008 0.001 0.000 0.000 0.000
## [8,]    1  0.7 0.343 0.216 0.125 0.064 0.027 0.008 0.001 0.000 0.000
## [9,]    1  0.8 0.512 0.343 0.216 0.125 0.064 0.027 0.008 0.001 0.000
## [10,]   1  0.9 0.729 0.512 0.343 0.216 0.125 0.064 0.027 0.008 0.001
## [11,]   1  1.0 1.010 0.738 0.520 0.350 0.222 0.130 0.068 0.030 0.010

## [1] "Matrix Omega"
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
## [1,]    0  0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## [2,]    0  0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000
## [3,]    0  0 4.704 3.744 2.820 1.968 1.224 0.624 0.204 0.000 0.048
## [4,]    0  0 3.744 3.024 2.310 1.632 1.026 0.528 0.174 0.000 0.042
## [5,]    0  0 2.820 2.310 1.800 1.296 0.828 0.432 0.144 0.000 0.036
## [6,]    0  0 1.968 1.632 1.296 0.960 0.630 0.336 0.114 0.000 0.030
## [7,]    0  0 1.224 1.026 0.828 0.630 0.432 0.240 0.084 0.000 0.024
## [8,]    0  0 0.624 0.528 0.432 0.336 0.240 0.144 0.054 0.000 0.018
## [9,]    0  0 0.204 0.174 0.144 0.114 0.084 0.054 0.024 0.000 0.012

```

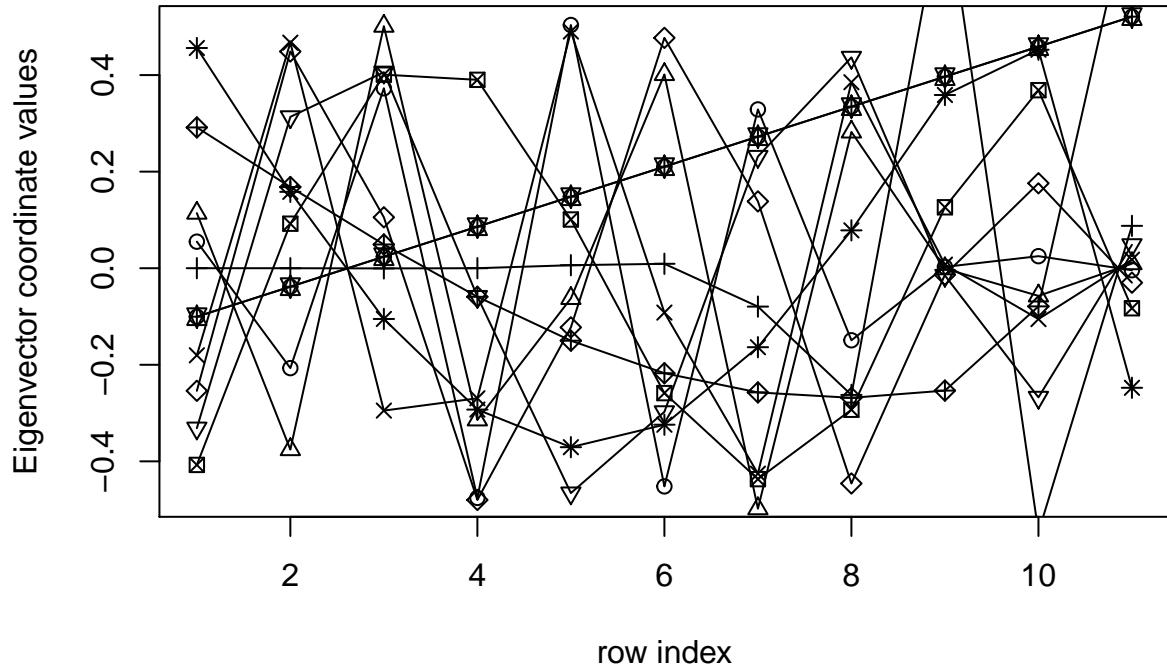
```

## [10,]    0    0 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.006
## [11,]    0    0 0.048 0.042 0.036 0.030 0.024 0.018 0.012 0.006 0.000
## [1] "Matrix K"

##          [,1]          [,2]          [,3]          [,4]          [,5]
## [1,] 1607.6951262 -3646.1707571 2.584683e+03 -692.561356 185.56240
## [2,] -3646.1707571  9877.0245425 -9.508098e+03 4155.368138 -1113.37438
## [3,] 2584.6830284 -9508.0981702 1.403239e+04 -10621.472553 4453.49753
## [4,] -692.5613564 4155.3681382 -1.062147e+04 14330.522073 -10700.61574
## [5,] 185.5623971 -1113.3743825 4.453498e+03 -10700.615737 14348.96542
## [6,] -49.6882320 298.1293918 -1.192518e+03 4471.940877 -10695.24594
## [7,] 13.1905308 -79.1431848 3.165727e+02 -1187.147772 4432.01835
## [8,] -3.0738912 18.4433474 -7.377339e+01 276.650211 -1032.82746
## [9,] 0.1774497 -1.0646979 4.258792e+00 -15.970469 59.62308
## [10,] 0.2222482 -1.3334891 5.333956e+00 -20.002336 74.67539
## [11,] -0.0365436 0.2192616 -8.770464e-01 3.288924 -12.27865
##          [,6]          [,7]          [,8]          [,9]          [,10]
## [1,] -49.68823 13.19053 -3.073891 0.1774497 0.2222482
## [2,] 298.12939 -79.14318 18.443347 -1.0646979 -1.3334891
## [3,] -1192.51757 316.57274 -73.773390 4.2587917 5.3339563
## [4,] 4471.94088 -1187.14777 276.650211 -15.9704689 -20.0023361
## [5,] -10695.24594 4432.01835 -1032.827456 59.6230838 74.6753880
## [6,] 14309.04289 -10540.92563 3854.659612 -222.5218665 -278.6992159
## [7,] -10540.92563 13731.68415 -8385.810993 830.4643821 1040.1214758
## [8,] 3854.65961 -8385.81099 5688.584360 2900.6643383 -3881.7866872
## [9,] -222.52187 830.46438 2900.664338 -8602.1481175 6020.8472547
## [10,] -278.69922 1040.12148 -3881.786687 6020.8472547 -3504.2529772
## [11,] 45.82568 -171.02406 638.270548 -974.3301494 544.8743825
##          [,11]
## [1,] -0.0365436
## [2,] 0.2192616
## [3,] -0.8770464
## [4,] 3.2889242
## [5,] -12.2786502
## [6,] 45.8256766
## [7,] -171.0240560
## [8,] 638.2705476
## [9,] -974.3301494
## [10,] 544.8743825
## [11,] -73.9323467

```

Part b



Part c

Problem 5

Part a

In order to compute and plot effective degrees of freedom, we need to compute the matrix

$$L_\lambda = \begin{bmatrix} l'(x_1) \\ \vdots \\ l'(x_{11}) \end{bmatrix}$$

where $l'(x_i) = (1, x_i)(\mathbf{B}'\mathbf{W}(x_i)\mathbf{B})^{-1}\mathbf{B}'\mathbf{W}(x_i)$, where

$$\mathbf{B} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_{11} \end{bmatrix}, \quad \mathbf{W} = \text{diag}(K_\lambda(x_i, x_1), \dots, K_\lambda(x_i, x_N))$$

A plot can be seen in Figure 1. The corresponding table for specific degrees of freedom can be seen in Table 2.

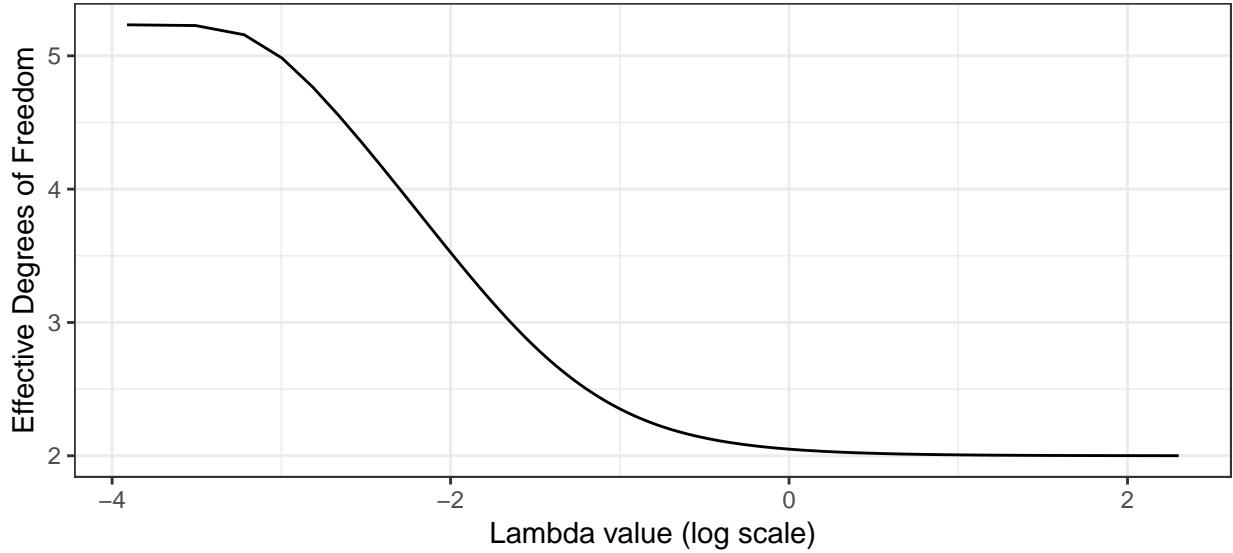


Figure 1: A plot of effective degrees of freedom as a function of Lambda values for Problem 5.

Table 2: Values of λ for effective degrees of freedom.

Effective DF	λ value
2.5	0.2997197
3	0.1898298
4	0.0999199
5	0.04997

Part b

First, note that we are working with $\lambda = 0.0999$, and our L_λ matrix is:

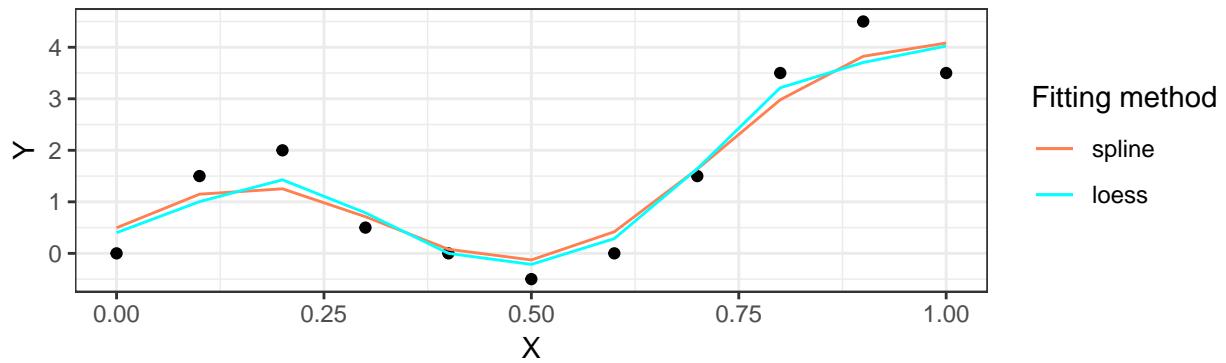
```
##      [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9]  [,10]  [,11]
## [1,]  0.97  0.06 -0.03  0.00  0.00  0.00  0.00  0.00  0.00   0.00  0.00
## [2,]  0.31  0.43  0.22  0.04  0.00  0.00  0.00  0.00  0.00   0.00  0.00
## [3,] -0.03  0.35  0.43  0.21  0.04  0.00  0.00  0.00  0.00   0.00  0.00
## [4,] -0.15  0.16  0.40  0.39  0.17  0.03  0.00  0.00  0.00   0.00  0.00
## [5,] -0.19  0.03  0.25  0.40  0.35  0.15  0.03  0.00  0.00   0.00  0.00
## [6,] -0.20 -0.04  0.13  0.28  0.38  0.31  0.12  0.02  0.00   0.00  0.00
## [7,] -0.19 -0.07  0.05  0.17  0.29  0.35  0.27  0.11  0.02  0.00   0.00
## [8,] -0.18 -0.09  0.01  0.10  0.20  0.29  0.33  0.24  0.09  0.02  0.00
## [9,] -0.17 -0.10 -0.02  0.06  0.13  0.21  0.28  0.30  0.22  0.08  0.01
## [10,] -0.17 -0.10 -0.04  0.02  0.09  0.15  0.21  0.27  0.29  0.20  0.07
## [11,] -0.17 -0.11 -0.06  0.00  0.06  0.11  0.17  0.23  0.28  0.29  0.20
```

Problem 6

Part a

The plots for this problem can be seen in Figure 2.

Approximately 5 DF



Approximately 9 DF

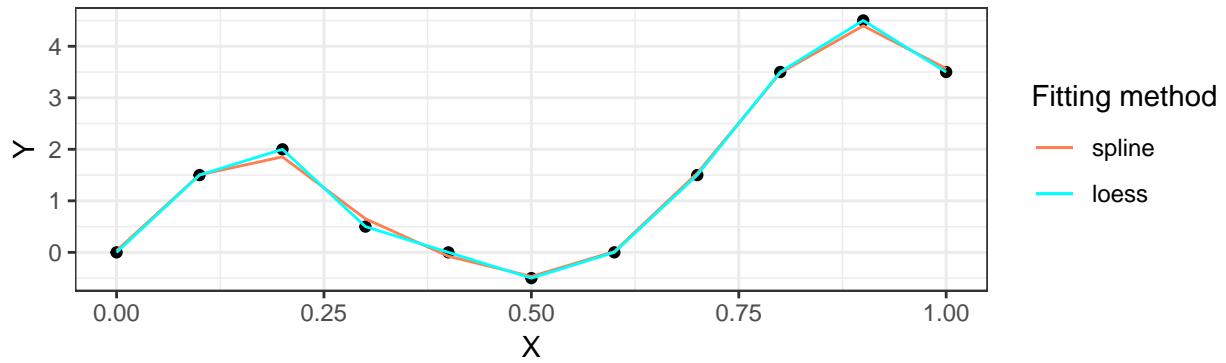


Figure 2: Plots for effective degrees of freedom 5 (top) and 9 (bottom) for Problem 6(a).

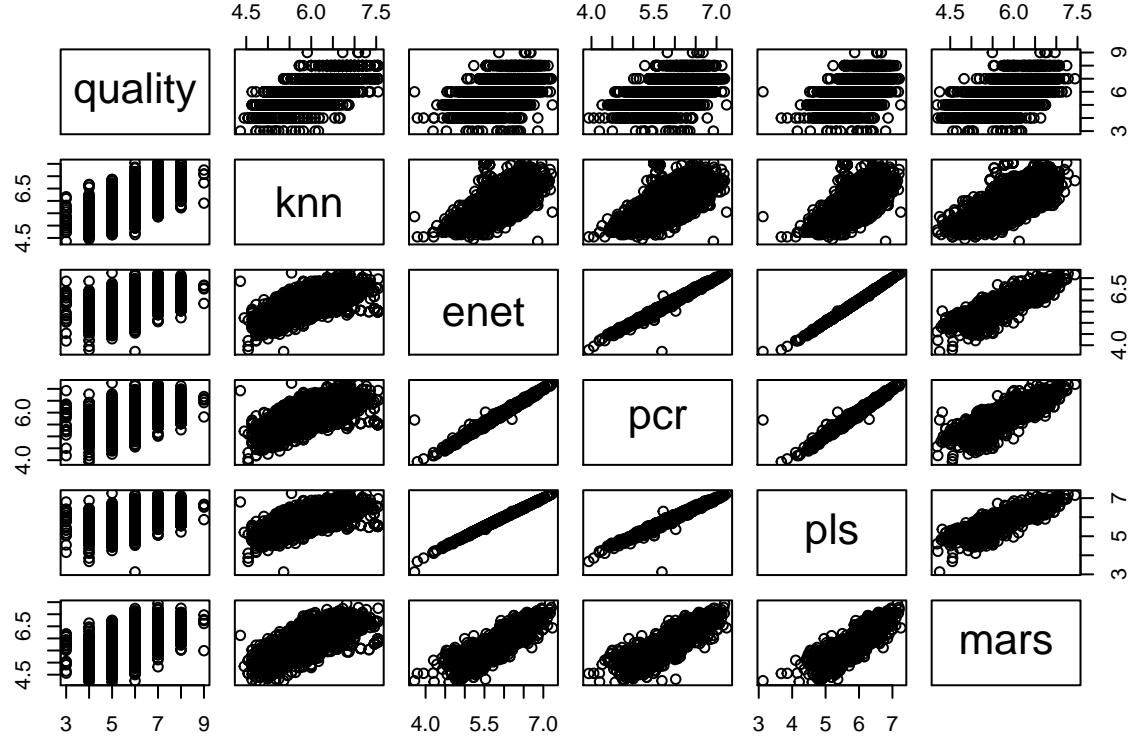


Figure 3: A scatterplot matrix for the five different prediction methods in Problem 7.

Problem 7

A scatterplot matrix for the five different methods and the response can be seen in Figure 3.

A correlation matrix can be seen below:

```
##          quality      knn      enet      pcr      pls      mars
## quality 1.0000000 0.6940572 0.5305748 0.5251641 0.5307829 0.5751068
## knn     0.6940572 1.0000000 0.7089463 0.7054792 0.7077081 0.7519074
## enet    0.5305748 0.7089463 1.0000000 0.9937171 0.9992582 0.8934130
## pcr    0.5251641 0.7054792 0.9937171 1.0000000 0.9895513 0.8778578
## pls    0.5307829 0.7077081 0.9992582 0.9895513 1.0000000 0.8939416
## mars   0.5751068 0.7519074 0.8934130 0.8778578 0.8939416 1.0000000
```

Problem 8 Note

Part a

Part b

Part c

Part d