# Variability Study Paper - Statistical Methodology

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## Working Backwards

### Conclusion

We have developed an extension of the traditional Gauge R&R framework that maintains the traditional mixed-effects modeling assumptions and results structure - still working on a better way to say this - for spatially dependent measurement data as well as pairwise similarity metric measurement data.

### New methodology that gets us to that conclusion

- Subsampling spatially dependent data to account for dependence and allow for assumption of independence when modeling
- Accounting for differences across spatial locations by including location interaction with study factors
- Reframing of study factors as paired levels within paired data context

### Underlying methodology to get us to our adaptations

Traditional three-factor Gauge R&R Model:

- Parts
- Operators
- Devices

Some fixed, unknown process mean $\mu$, and random effects for each study factor. Also state assumptions (independence, variance components).

Summary values obtained from random effects modeling results: $\sigma_{repeatability}, \sigma_{reproducibility}$.

### Background about our data types needed

Forensic firearms analysis poses the "pairwise" question: did a pair of objects originate from the same source or a different source?

Data used to complete that comparison: two-dimensional signature data, represents striation patterns engraved by gun barrel.

Two-dimensional signature data is captured through a microscopic measurement system. Operators and microscopes are varying parts of measurement process.

There are data processing steps which extract a two-dimensional signature from a three-dimensional data object. Processing steps held constant throughout study.

How pattern is engraved on each bullet also introduces variability; bullet of origin represents "parts".

Pairwise similarity structure results in data on $[0, 1]$. Measuring reproducibility of similarity scores across bullets, operators and machines using pre-defined algorithm applied to pairs of extracted signature data.

### Background about Gauge R&R

Gauge R&R framework is typically used to evaluate a measurement system, often used in engineering.

Uses traditional mixed-effects models. Quantifying the magnitude of random effect variance components is of interest.

Innovations in microscopy have increased the use of 3D microscopy to capture surface topographies of physical objects.

Traditional microscopy R&R uses single measured response value, directly applying traditional R&R framework.

Repeatability and reproducibility of the captured *objects* is often more relevant than microscope measurement repeatability of a singular response value. Why? Operator staging differences and their interaction with individual microscopes.

## Resulting Outline

### Introduction/Background

Gauge R&R study framework is used to evaluate a measurement system or measurement process. The goal is to quantify the variability in resulting measurement data when measurements are taken under identical environmental conditions and when environmental conditions are varied.

Uses traditional mixed-effects models. Quantifying magnitude of random effect variance components is the goal.

Innovations in microscopy have increased use of 3D microscopy to capture surface topographies of physical objects. R&R framework is defined for singular measurement values.

Quantifying repeatability and reproducibility of more complex topographical structures captured in 3D microscopy is of interest. We propose an extended R&R framework for modeling that maintains the same modeling assumptions and resulting model structure, but accounts for more complex data structures.