Variability Study Paper - Statistical Methodology

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Resulting Outline

Gauge R&R for complex data structures.

Introduction/Background

Gauge R&R study framework is used to evaluate a measurement system or measurement process. The goal is to quantify the variability in resulting measurement data when measurements are taken under identical environmental conditions and when environmental conditions are varied.

Uses traditional mixed-effects models. Quantifying magnitude of random effect variance components is the goal.

Innovations in microscopy have increased use of 3D microscopy to capture surface topographies of physical objects. R&R framework is defined for singular measurement values.

Quantifying repeatability and reproducibility of more complex topographical structures captured in 3D microscopy is of interest.

We propose an extended R&R framework for modeling that maintains the same modeling assumptions and resulting model structure, but accounts for more complex data structures.

This paragraph will talk about forensic firearms analysis and that automated methods have two data structures not typical for R&R studies. Talk about the pairwise nature of comparison.

Here I will describe that two-dimensional signature data is captured through a microscopic measurement system. Operators and microscopes are varying parts of measurement process. Processing steps are present to get 2D sig., but are held constant in this case.

Define bullets as parts we are going to measure. (We will focus on individual lands rather than a pooled model for this case.)

Pairwise similarity structure results in data on $[0, 1]$. Measuring reproducibility of similarity scores across bullets, operators and machines using pre-defined algorithm applied to pairs of extracted signature data.

Methodology

Traditional three-factor Gauge R&R Model:

- Parts
- Operators
- Devices

Some fixed, unknown process mean μ, and random effects for each study factor. Also state assumptions (independence, variance components).

Summary values obtained from random effects modeling results: $\sigma_{repeatability}, \sigma_{reproducibility}$.

Model adaptation, dependent data

Describe dependent data structure of signatures, $(\mathbf{X}, \mathbf{Z})$.

Here discuss lack of independence assumption and autocorrelation structure of signatures due to striations.

Here lay out the subsampling approach.

Next lay out including location interaction with study factors.

Model adaptation, pairwise data

Describe pairwise data structure of similarity scores.

Here discuss relationship of paired study factors.

Here discuss structure of same-source scores as opposed to different-source scores.

Reframing of study factors as paired levels within paired data context

Data Collected

Repeated data collected for operators and machines, with 5 repetition rounds.

Signatures extracted using identical processing steps. Pairwise scores calculated for each pair within the set.

Results

Signature Results

Pairwise Results

Conclusion

We have developed an extension of the traditional Gauge R&R framework that maintains the traditional mixed-effects modeling assumptions and results structure - still working on a better way to say this - for spatially dependent measurement data as well as pairwise similarity metric measurement data.

Working Backwards

Conclusion

Gauge R&R framework adapted to complex data types.

New methodology that gets us to that conclusion

- Subsampling spatially dependent data to account for dependence and allow for assumption of independence when modeling
- Accounting for differences across spatial locations by including location interaction with study factors
- Reframing of study factors as paired levels within paired data context

Underlying methodology to get us to our adaptations

Traditional three-factor Gauge R&R Model:

- Parts
- Operators
- Devices

Some fixed, unknown process mean μ, and random effects for each study factor. Also state assumptions (independence, variance components).

Summary values obtained from random effects modeling results: $\sigma_{repeatability}, \sigma_{reproducibility}$.

Background about our data types needed

Forensic firearms analysis poses the "pairwise" question: did a pair of objects originate from the same source or a different source?

Data used to complete that comparison: two-dimensional signature data, represents striation patterns engraved by gun barrel.

Two-dimensional signature data is captured through a microscopic measurement system. Operators and microscopes are varying parts of measurement process.

There are data processing steps which extract a two-dimensional signature from a three-dimensional data object. Processing steps held constant throughout study.

How pattern is engraved on each bullet also introduces variability; bullet of origin represents "parts".

Pairwise similarity structure results in data on $[0, 1]$. Measuring reproducibility of similarity scores across bullets, operators and machines using pre-defined algorithm applied to pairs of extracted signature data.

Background about Gauge R&R

Gauge R&R framework is typically used to evaluate a measurement system, often used in engineering.

Uses traditional mixed-effects models. Quantifying the magnitude of random effect variance components is of interest.

Innovations in microscopy have increased the use of 3D microscopy to capture surface topographies of physical objects.

Traditional microscopy R&R uses single measured response value, directly applying traditional R&R framework.

Repeatability and reproducibility of the captured *objects* is often more relevant than microscope measurement repeatability of a singular response value. Why? Operator staging differences and their interaction with individual microscopes.

Reframing

Central Focus: Measurement systems, and resulting data, have become more complex as the field of data science has grown. Traditional measurement systems analysis methods, such as Gauge R&R, work for univariate measurement data.

Introduction

- **Intro to Gauge R&R**
  - Goal of GR&R is to quantify the repeatability and reproducibility of a measurement system

  - Defined for, and typically used for, one-dimensional measurement response.

  - Example: length of screw.

  - When considering any two objects, we consider the distance between those objects and whether that metric of difference falls within an acceptable range of variability for a system/process/etc.
  - Example: difference in length of two screws

- **Intro to Data Science/Pipelines**
  - Data science has produced more complex data which often captures more than a univariate response measuring an object

  - Example: We could consider more aspects than just the length of the screw

  - Data science pipelines: collection of complex data, applying actions to that data, goal of some result
    * Usually involves more complex data structures

    * Need to adapt the definition of distance between two objects, and how we apply statistical models to quantify variability in measurement

- **Intro to Our Data**
  - We use as a motivating example data produced for automated forensic firearms comparison.
  - Firearm barrels engrave striation patterns on fired bullets

  - Forensic question: same source or different source?

  - In order to answer the forensic question, we need to first measure the objects, and secondly compare the objects using a similarity metric.

  - Measurement systems analysis applies in two ways:
    * First, to the measurement of the objects themselves (translation of objects into data).
    * Secondly, to the "measurement" that is the similarity metric.

  - Measuring the object: 3D microscopy and capturing the striation patterns engraved on bullets
    * Results in a set of measured values, not a singular measurement.

    * Define a distance between two sets of measured values

  - Similarity metric between two striation pattern measurements
    * Results in a singular response value

          ∗ What does a distance between two similarity metrics represent

Collected Data

Not sure if this should go here or after methodology?

Study factors included to study measurement process: bullets, operators, microscopes.

Methodology

- Traditional 3-factor Gauge R&R model

- Signature data as complex data structure
  - To quantify variability of measurements in signatures in general resulting from measurement process

  - Has a natural structure

  - Subsampling to equidistant locations, then model measurement variability at sampled locations

  - Present model

  - Discuss phased approach?

- Pairwise similarity scores
  - Nature of response as univariate measurement

  - Model aspects are *paired*, so need a new grouping structure for statistical models

  - Present model

Results

- Window size results? This could go in methodology. To show consistency of results

- Signature-level results

- Pairwise-level results

Conclusions

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