6CCS3AIN & 7CCSMAIN, 2018, Tutorial 05 Answers (Version 1.0)

1. (a) We have the scenario:

j

		L	R
i	U	3 3	4 2
	D	1 1	2 4

i. In that initial scenario, R dominates L for j, since j gets a bigger payoff for R, no matter what i plays.

ii. Since L is dominated, we can delete that option, to give a reduced scenario:

j

		R
i	U	4 2
	D	2 4

Given this reduction, D now dominates U for i, so the scenario reduces to:

j

		R
i	D	2 4

and we know what each agent will do.

iii. Consider each of the outcomes in turn.
Start with (D, R). If j changes strategy, it will get 1 rather than 2. If i changes strategy, it will get 2 rather than 4, so (D, R) is a Nash equilibrium.
Now consider (U, L). If j changes strategy to R, its payoff will go up to 4. So (U, L) is not a Nash equilibrium.
Similar reasoning applies to (D, L) and (U, R).

iv. For Pareto optimality, the only outcome that is **not** a Pareto optimal state is (D, L). For every other state, moving to a different outcome makes one agent worse off.

v. There are three outcomes with a social welfare of 6: (U, L), (U, R), and (D, R), which is the maximum social welfare of any outcome.

(b) This time the game is

j

		L	R
i	U	-1 -1	2 1
	D	1 2	-1 -1

i. There are no dominated strategies — no strategy is better for either i or j no matter what the other does.

ii. No simplification is possible.

iii. Consider each outcome in turn.
Start with (U, R). If j changes strategy, it gets -1 rather than 2. If i changes strategy, it gets -1 rather than 1. So (U, R) is a Nash equilibrium.
Similarly, (D, L) is a Nash equilibrium.
Now consider (D, R). Both i and j can individually improve their payoff to 1 if they change strategy. So (D, R) is not a Nash equilibrium.
Similarly for (U, L).

iv. Both (U, R) and (D, L) are Pareto optimal since moving away from either will make at least one agent worse off.

v. Both (U, R) and (D, L) maximise social welfare.

(c) We have:

			j		
		L		R	
	U		3		1
i		3		1	
	D		4		2
		2		4	

i. In this scenario, L dominates R for j.

ii. We can reduce the scenario to:

		j	
		L	
	U		3
i		3	
	D		4
		2	

and U now dominates for i, so the scenario reduces to:

		j	
		L	
i	U		3
		3	

iii. (U, L) is a Nash equilibrium since if j changes strategy its payoff will drop from 3 to 1 and if i changes strategy, its payoff will change from 3 to 2.
(D, R) is not a Nash equilbrium because j can change strategy and increase its payoff to 4.
Similarly, (U, R) and (D, R) are not Nash equilibria.

iv. The only outcome that is not Pareto optimal is (U, R). For all the others, there is no way to improve the payoff to one agent without reducing it for another.

v. Every outcome except (U, R) maximises social welfare: all the other outcomes have a social welfare of 6.

2. The Stag Hunt scenario is:

			j		
		R		S	
	R		2		1
i		2		3	
	S		3		4
		1		4	

What are the Nash equilibria?

Well, (R, R) is a Nash equilibrium. Even though both agents can get a better payoff than the 2 they get in (R, R), they cannot get that payoff by changing strategy on their own. In other words, if i switches to S while j plays R, then the stag will escape, and i's payoff will drop to 1. Similarly, if j plays S while i sticks to R.

The reasoning about (R, R) tells us that (R, S) is not a Nash equilibrium because i can improve its payoff by changing to S, and j can improve its payoff by changing to R. Similarly (S, R) is not a Nash equilibrium.

Finally, (S, S) is a Nash equilibrium because neither agent can do better by playing R.

(S, S) is the only Pareto optimal solution. For every other outcome, switching to (S, S) will improve the outcome for both agents.

The difference between the Stag Hunt and the Prisoner's Dilemma is that the greatest utility is available in the "cooperate" outcome where both players work together to get the stag. This is in contrast to the Prisoner's Dilemma where the greatest utility is available for playing "defect" when the other player cooperates. This allows (S, S) to be a Nash equilibrium,and become the "obvious" solution for both players.

3. Chicken has as normal-form game is:

			j		
		S		J	
	S		1		2
i		1		4	
	J		4		3
		2		3	

(S, S) is not a Nash equilibrium — j can improve its payoff to 2 by swicthing to J, and so can i. So both drivers staying in the car is not a Nash eqilibrium (as we might hope).

(S, J) is a Nash eqilibrium. If i tries to move away, it will get a ayoff of 3 rather than 4, and if j tries ot move away it will get a payoff of 1 rather than 2. Similarly, $J, S)$ is a Nash equilibrim. This is the essence of why Chicken is a hard game to play — assuming you buy into the fundamentally stupid idea of the game in the first place — your best strategy of playing is: if I think you will stay in the car, I should jump out, while if I think you will jump out of the car, I should stay in.

(J, J) is not a Nash equilibroum since eithe rplayer can improve its payoff by switching to S.

Every outcome except (S, S) is Pareto optimal.

Compared with Prisoner's dilemma, the maian difference is that the worst payout for both players occurs when they both play S ("defect" in Prisoner's dillema). That makes the outcomes in which players pick different strategies Nash equilibria.

4. No solution will be provided for the computational part, but you can check your solution against the answers above.
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