"Is there a Universal Algorithm which can solve all Mathematical problems?"… attempts to find one failed …

… so perhaps there isn't one..

… can we prove there is no universal algorithm?

… we need to be able to define an algorithm precisely so as to prove properties of algorithms

a formalism of algorithms should be…

- precise and unambiguous
- simple
- general

Formalisms for Algorithms

By the 1930s the emphasis was on formalising algorithms

Alan Turing, at Cambridge, devised an abstract machine now called a Turing Machine to define/represent algorithms

Alonso Church, at Princeton, devised the Lambda Calculus which formalises algorithms as functions (… not discussed in the course).

neither knew of the other's work in progress … both published in 1936

the demonstrated equivalence of their formalisms strengthened both their claims to validity, expressed as the Church-Turing Thesis.

Model of computation: Turing Machines

- precise (yes)
- simple (yes)
- general ?

We will see the definition next, just take it as an abstraction

the Church-Turing Thesis:

"a problem can be solved by an algorithm iff it can be solved by a Turing Machine"

Turing Machines implement algorithms

all algorithmically solvable problems can be solved by a Turing Machine or:

“a function is computable iff it can be solved by a Turing Machine”

“ an algorithm is what a Turing Machine implements”

This is a **Thesis not Theorem!**

i) because we cannot prove this … with a counter example we could disprove it (but this has not been done).

ii) we can show supporting evidence for the validity of the thesis

accepted working hypothesis

yes, but which evidence?

- large sets of Turing-Computable functions

  (many examples...no counter-examples)

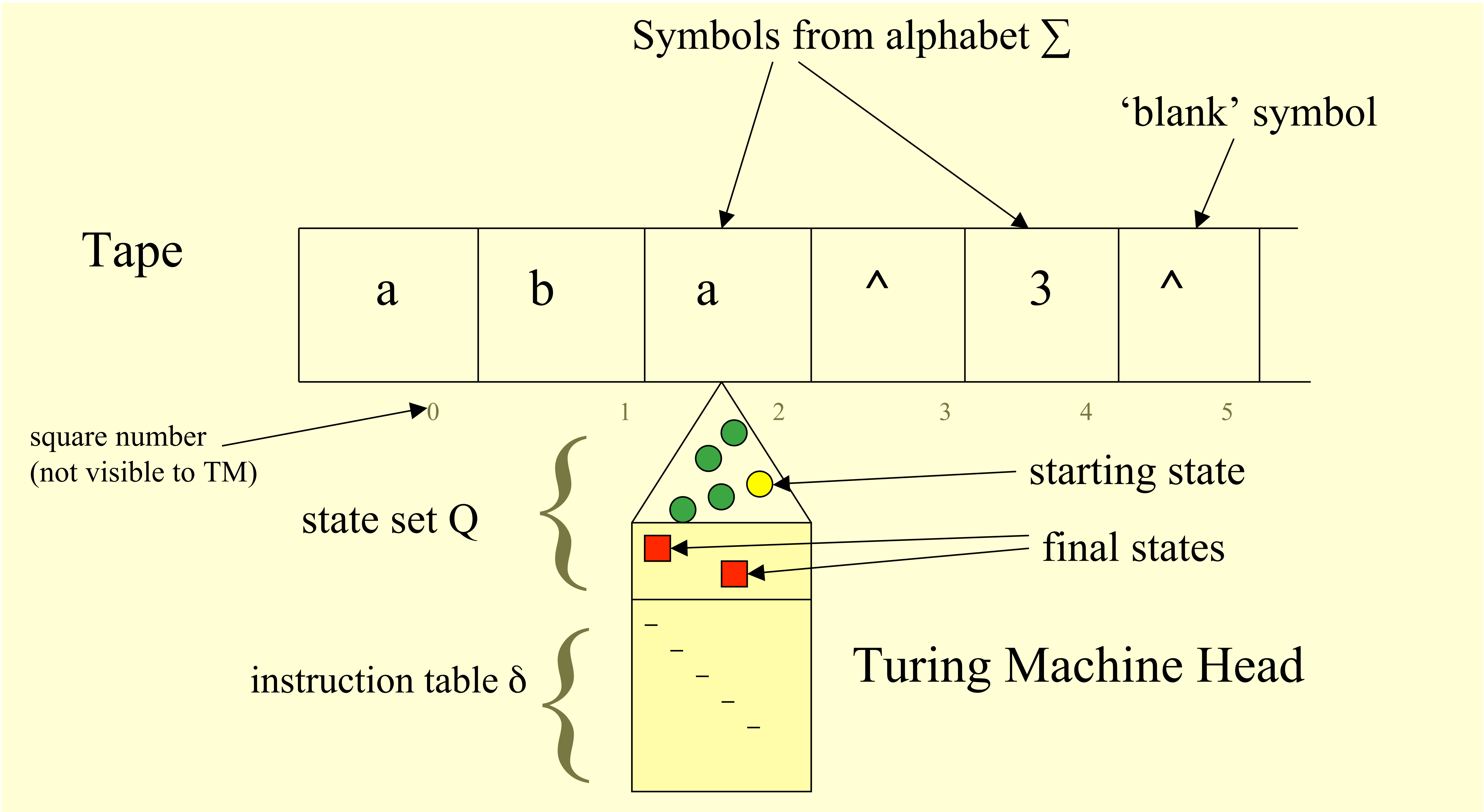
- equivalent to other formalisms for algorithms

  (Church's lambda calculus and others)

- intuitive - any detailed algorithm for calculation can be implemented by a Turing Machine

(via Turing Machine implementation of mechanical methods)

Turing Machine



A Turing Machine Run

Start:
- in initial state
- **head** over square 0
- finite number of non-blank symbols at start of tape rest of tape blank - contains ^

A run is a step-by-step computation:

- reads symbol on current square
- writes a symbol from alphabet ∑ to current square
- moves left 1 square, right 1 square or does not move
- enters a new state

..according to what ???

..the Instruction Table:

depending on:
- current state
- symbol in current square

the table gives
- symbol to write
- direction to move
- new state

the Instruction Table, δ, is the program of the Turing Machine

also called the δ-function:

$$\delta(\text{current-state}, \text{current-symbol}) = (\text{new-state}, \text{new-symbol}, \text{move})$$

when does it stop? (does it stop?)

a) it reaches a final, or halting state:

- *the TM stops (halts) and succeeds.*
- *the output is the tape contents from square 0 up to (but not including) the first ^*

or b) the pair (current-state,current-symbol) is not in the instruction table ("no applicable instruction"):

- *the TM halts and fails*
- *the output is undefined*

or c) the head tries to move left from square 0:

- *the TM halts and fails*
- *the output is undefined*

OR… the TM may not halt … it loops or runs forever

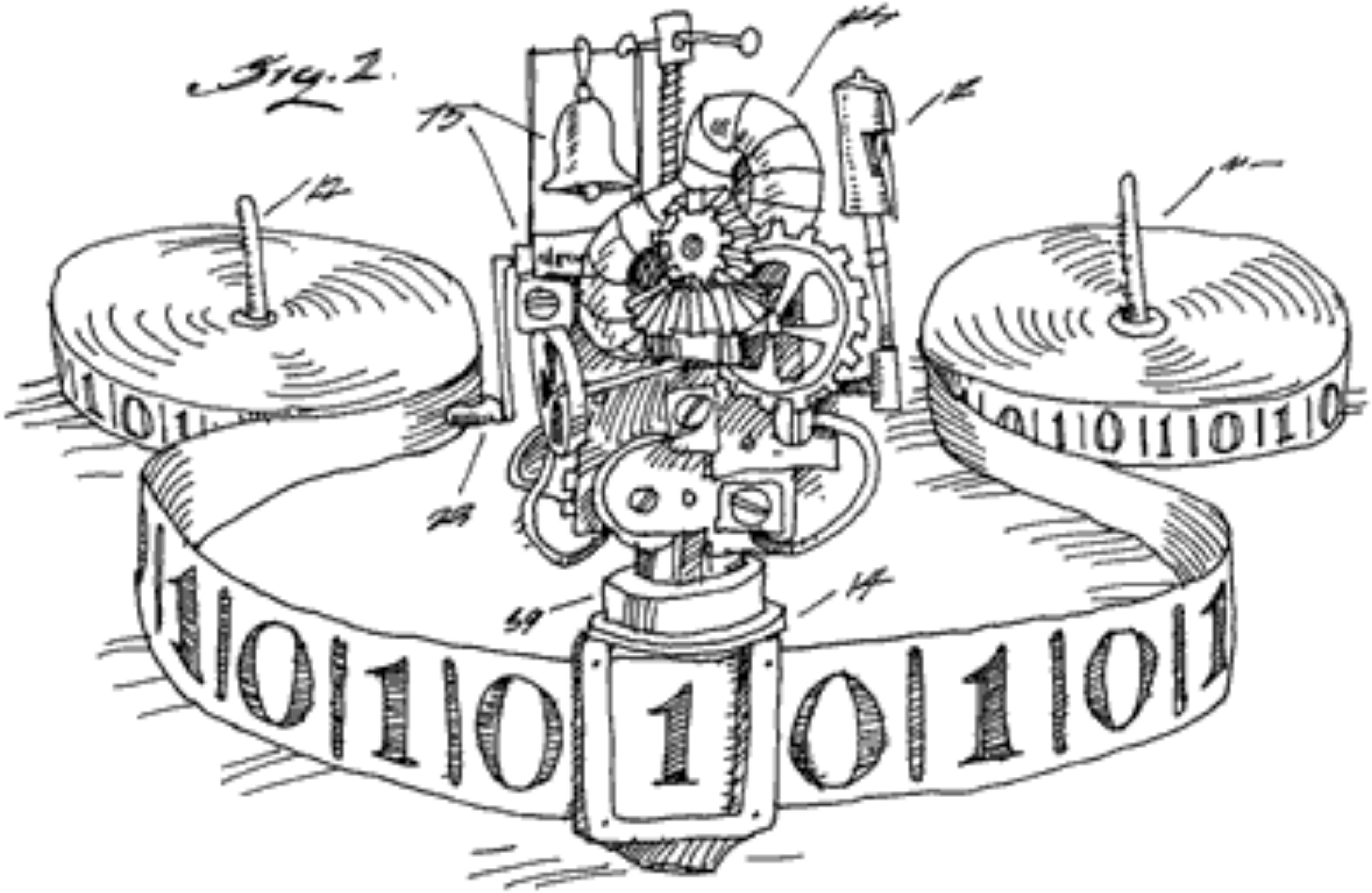


Fig. 1

Modern Computer are equivalent to Turing Machines