AoS

If We Observe Everything, Where Does Probability Come In?

> How often do we expect to see seven or more separate homicide incidents in England and Wales in a single day?

When extreme events happen in close succession, such as multiple plane crashes or natural disasters, there is a natural propensity to feel they are in some sense linked. It then becomes important to work out just how unusual such events are, and the following example shows how we can make such a call.

To assess how rare a ‘cluster’ of at least seven homicides in a day might be, we can examine data for the three years (1,095 days) between April 2014 and March 2016, in which there were 1,545 homicide incidents in England and Wales, an average of 1,545/1,095 = 1.41 per day.[fn7] Over this period there were no days with seven or more incidents,

$i = 1, \ldots, 1095$ (n)

X_i : NUMBER OF HOMICIDES ON A DAY i

x_i : RECORDED OBSERVATION ON DAY i

$$\frac{1}{n} \sum_i x_i = 1.41$$

0	ObservedFigures	259
1	ObservedFigures	387
2	ObservedFigures	261
3	ObservedFigures	131
4	ObservedFigures	40
5	ObservedFigures	13
6	ObservedFigures	3
7 or more	ObservedFigures	0

AoS

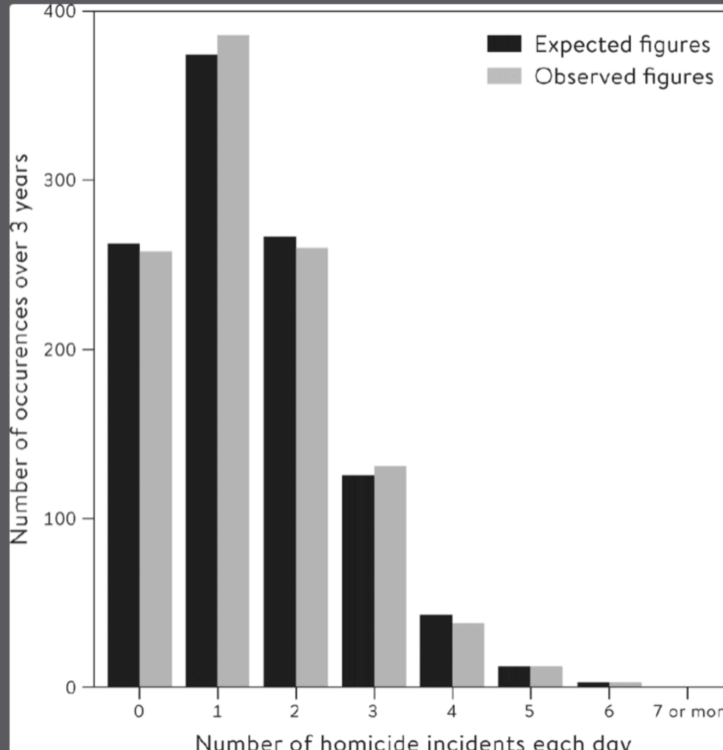

Figure 8.5
Observed and expected (assuming a Poisson distribution) daily number of recorded homicide incidents, 2014 to 2016, England and Wales.[3]

BLACK

POISSON PROB. FOR EACH OUTCOME

TIMES $n = 1{,}094$

$n \cdot P_X(x)$

WITH PARAMETER 1.41

- CONSIDER THE FOLLOWING SITUATION
  - 3 YEARS OF CURRENT GOVERNMENT
  - GVMT PROMISED CONTROL VIOLENT CRIME
  - ONE DAY AFTER 3 YEARS 7 HOMICIDES HAPPEN
    ↳ OPPOSITION & PRESS CLAIM THAT IS A SIGN OF THE BREAKDOWN OF GOVN POLICY ON CRIME

$$P_x(7)$$

$$P_x(7) + P_x(8) + \cdots$$

$$\sum_{i=0}^{\infty} p_x(i) = 1$$

$$\sum_{i=7}^{\infty} p_x(i) = 1 - p_x(0) - p_x(1) - \cdots - p_x(6)$$

$$= 0.0007$$

$$n \times 0.0007 \geq 1$$

EXPECT

- WHAT IS PROBABILITY & WHAT IS USEFUL FOR

  - MATHEMATICAL <u>INVENTION</u>, LIKE CALCULUS, WHOSE VALUE LIES IN GIVING <u>GOOD</u> MODELS FOR <u>DATA</u>

    ANALOGY : DIFFERENTIAL CALCULUS

- DATA ARE TREATED AS UNCERTAIN OUTCOMES WITH POTENTIAL VALUES

- PROBABILITY, AS A MATHEMATICAL THEORY, IS <u>RICH</u> & <u>POWERFUL</u> & <u>USEFUL</u>

  ↳ 3 AXIOMS

- NUMBER OF OTHER WAYS TO 'DEFINE'
PROBABILITY

A.S

is as if probability is some 'virtual' quantity, which we can put a number on, but can never directly measure.

Even more worrying is to ask the rather obvious question: what does probability mean anyway? What's a good definition? This may seem pedantic, but the philosophy of probability is both a gripping topic in itself and also has a major role in practical applications of statistics.

Don't expect a neat consensus from the 'experts'. They may agree on the mathematics of probability, but philosophers and statisticians have come up with all sorts of different ideas for what these elusive numbers actually mean, and argue intensively over them. Some popular suggestions include:

HIGH SCHOOL

- *Classical probability*: This is what we are taught in school, based on the symmetries of coins, dice, packs of cards, and so on, and can be defined as, 'The ratio of the number of outcomes favouring the event divided by the total number of possible outcomes, assuming the outcomes are all equally likely.' For example, the probability of throwing a 'one' on a balanced die is 1/6, since there are six faces. But this definition is somewhat circular, as we need to have a definition of 'equally likely'.
- *'Enumerative' probability*:[fn4] Suppose there are three white socks and four black socks in a drawer, and we take a sock at random, what is the probability of drawing a white sock? It is 3/7, obtained by enumerating the opportunities. Many of us have had to suffer questions like this in school, and it is essentially an extension of the classical idea discussed above, requiring the idea of a 'random choice' from a physical set of objects. We have been using this idea extensively already, when describing a data-point as being picked at random from a population.
- *'Long-run frequency' probability*: This is based on the proportion of times an event occurs in an infinite sequence of identical experiments, exactly as we found when we simulated the Chevalier's games. This may be reasonable (at least theoretically) for infinitely repeatable events, but what about unique occasions such as horse-racing, or tomorrow's weather? In

fact almost any realistic situation is not, even in principle, infinitely repeatable.

- *Propensity or 'chance'*: This is the idea that there is some objective tendency of the situation to produce an event. This is superficially attractive – if you were an all-knowing being, maybe you could say there was a particular probability of your bus arriving soon, or of being hit by a car today. But it seems to provide no basis for we mortals to estimate this rather metaphysical 'true chance'.
- *Subjective or 'personal' probability*: This is a specific person's judgement about a specific occasion, based on their current knowledge, and is roughly interpreted in terms of the betting odds (for small stakes) that they would find reasonable. So if I will be given £1 if I can juggle three balls for five minutes, and I am willing to offer a (non-repayable) 60p stake on the bet, then my probability for that event is 0.6.

BETTING MARKETS

Different 'experts' have their own preference among these alternatives, but personally I prefer the final interpretation – subjective probability. This

PDF version of the entry
Interpretations of Probability
https://plato.stanford.edu/archives/win2023/entries/probability-interpret/
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Interpretations of Probability

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> *Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means.*
>
> —Bertrand Russell, 1929 Lecture
> (cited in Bell 1945, 587)

One regularly reads and hears probabilistic claims like the following:

- The Democrats will probably win the next election.
- The coin is just as likely to land heads as tails.
- There's a 30% chance of rain tomorrow.
- The probability that a radium atom decays in one year is roughly 0.0004.

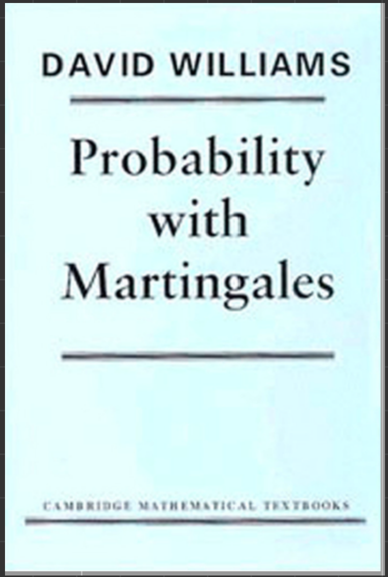
But what do these statements mean? This may be understood as a metaphysical question about what kinds of things are probabilities, or more generally as a question about what makes probability statements true or false. At a first pass, various *interpretations of probability* answer this question, one way or another.

However, there is also a stricter usage: an 'interpretation' *of a formal theory* provides meanings for its primitive symbols or terms, with an eye to turning its axioms and theorems into true statements about some subject. In the case of probability, Kolmogorov's axiomatization (which we will see shortly) is the usual formal theory, and the so-called 'interpretations of probability' usually interpret *it*. That axiomatization introduces a function 'P' that has certain formal properties. We may then ask 'What is P?'. Several of the views that we will discuss also answer this question, one way or another.

II.—BERTRAND RUSSELL ON PROBABILITY

By Harold Jeffreys

The analysis of the processes of acquirement of knowledge given in Russell's recent book *Human Knowledge* is the fullest I have seen. He remarks "The apparent publicity of our



Let $\mathcal{A}$ be the set of all maps $\alpha : \mathbf{N} \to \mathbf{N}$ such that $\alpha(1) < \alpha(2) < \ldots$. For $\alpha \in \mathcal{A}$, let

$$F_\alpha = \left\{\omega : \frac{\sharp(k \le n : \omega_{\alpha(k)} = H)}{n} \to \frac{1}{2}\right\}$$

the 'truth set of the Strong Law for the subsequence α'. Then, of course, we have $\mathsf{P}(F_\alpha) = 1, \forall \alpha \in \mathcal{A}$.

Exercise. Prove that

$$\bigcap_{\alpha \in \mathcal{A}} F_\alpha = \emptyset.$$

(*Hint.* For any given ω, find an α ... .)

The moral is that the concept of 'almost surely' gives us (i) absolute precision, but also (ii) enough flexibility to avoid the self-contradictions into which those innocent of measure theory too easily fall. (Of course, since philosophers are pompous where we are precise, they are thought to think deeply ... .)

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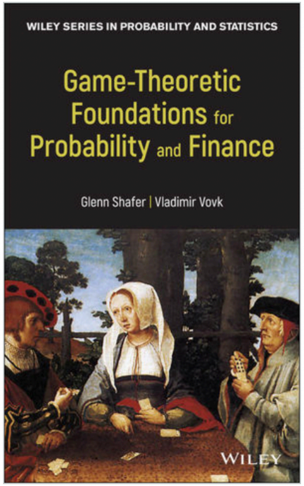

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