This review homework is intended to help reinforce your familiarity with discrete mathematics concepts and notations in addition to testing your ability to write proofs.

Note that all written homework assignments are to be placed in the dropbox labeled "Com S 311" in Atanasoff Hall.

Below are some definitions that are used throughout this homework assignment.

natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$.

integers $\mathbb{Z}$ denotes the set of integers and $\mathbb{Z}^+$ denotes the set of positive integers $\{1, 2, 3, \ldots\}$

reals $\mathbb{R}$ denotes the set of real numbers.

divides relation For integers $a, b \in \mathbb{N}$, we say a *divides* b, (and we write $a|b$) if b can be divided by a with no remainder. Also, we define $a|\mathbb{N} = \{b \in \mathbb{N} \text{ such that } a|b\}$

difference For sets A, B, let $A - B = \{x \text{ such that } x \in A \text{ and } x \notin B\}$

one to one A function $f : A \to B$ is *one to one* iff for every two elements $x, y \in A$, if $f(x) = f(y)$, then $x = y$.

onto A function $f : A \to B$ is *onto* iff for every $b \in B$ there exists an $a \in A$ such that $f(a) = b$.

equivalence relation A relation $R \subseteq A \times A$ is an equivalence relation iff it has the following properties. For all $a, b, c \in A$:

1. *reflexive*: aRa
2. *symmetric*: $aRb \implies bRa$
3. *transitive*: aRb and $bRc \implies aRc$

Problem 1

Prove:

a) $12|\mathbb{N} \subseteq 3|\mathbb{N}$

b) $35|\mathbb{N} = 5|\mathbb{N} \cap 7|\mathbb{N}$

c) $20|\mathbb{N} \not\subseteq 3|\mathbb{N}$

Problem 2

For arbitrary sets A, B, prove:

a) $A \cup B = B \iff A \subseteq B$

b) $A \cap B = B \iff B \subseteq A$

c) $A - (A - B) \subseteq B$

And prove there exists sets A, B such that:

d) $B \nsubseteq A - (A - B)$

Problem 3

Give an example of a function $f : \mathbb{Z} \to \mathbb{N}$ that is both one-to-one and onto.

Problem 4

Let $f : \mathbb{Z} \to \mathbb{Z}$ be a function defined as $f(x) = 3x + 7$. Prove:

a) f is one to one

b) f is NOT onto

Problem 5

Let $\sim$ be a relation over the real numbers such that for $a, b \in \mathbb{R}$, $a \sim b$ if and only if $a - b \in \mathbb{Z}$. Prove that $\sim$ is an equivalence relation.

Problem 6

Now use induction to prove the following:

a) For all $n \in \mathbb{Z}^+$,
$$1 + 3 + 5 + \cdots + 2n - 1 = n^2$$

b) For all $n \in \mathbb{Z}^+$,
$$3^n > 2^n$$

c) For all $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

d) For all $n \in \mathbb{Z}^+$,

$$n^3 + 2n \text{ is divisible by } 3$$

Bonus Problem: Skittles

A friend of yours challenges you to a game skittles. The game requires two piles each containing exactly N skittles. On a player's turn, the player removes some (non-zero) number of skittles from exactly one of the piles. The player that takes the last skittle, wins!

Your friend decides to go first. Describe a strategy that ensures that you will always win. Prove its correctness using induction.