University of Nottingham

School of Mathematical Sciences

G11ACF **MATLAB**

2015-16: Assessed Coursework 2 **Hand in by 5pm, Friday 18th March 2016**

In this Coursework you will be studying the coupled ordinary differential equations

$$\frac{dx}{dt} = a + x^n y - (b+1)x + A\cos\omega t, \tag{1}$$

$$\frac{dy}{dt} = bx - x^n y, \tag{2}$$

subject to the initial conditions $x = y = 0$ when $t = 0$.
Here a, b, n, A and ω are parameters that you will choose in various ways in the questions below.

1. In this part of the question, $n = A = 0$. Note that the equations are linear in this case.

   (a) *Using pen and paper:*

   i. Show that by eliminating y between (1) and (2) you can obtain the second order ordinary differential equation
   $$\frac{d^2x}{dt^2} + (b+2)\frac{dx}{dt} + x = a, \tag{3}$$
   which has constant coefficients. [5]

   ii. What is dx/dt when $t = 0$? [3]

   iii. Show that the complementary function for (3) is
   $$x(t) = B_+ e^{m_+ t} + B_- e^{m_- t},$$
   where
   $$m_\pm = \frac{1}{2}\left\{-b - 2 \pm \sqrt{b(b+4)}\right\}$$
   and $B_\pm$ are constants. Hence find the solution of (3) subject to the given initial conditions. [8]

   iv. If $a > 0$, find U, the set of values of b for which $x \to a$ as $t \to \infty$. Find the subset of U for which $x - a$ oscillates infinitely often as $t \to \infty$? [4]

   (b) Write a MATLAB function (call it `Eulersol`), which takes as input values of a, b, N and dt and uses Euler's method (see Video Lecture 8) with timestep dt to find an approximate solution of (1) and (2) for $0 \le t \le t_{max}$, with $t_{max} = Ndt$. Your function should return vectors $\mathbf{x}$ and $\mathbf{y}$ that contain this approximate solution and a vector $\mathbf{t}$ that contains the times at which the solution is calculated. [5]

   Now write a MATLAB script that successively does the following:

   (c) Uses the symbolic mathematics capabilities of MATLAB (see Video Lecture 9) to find the exact, analytical solution of (1) and (2) with $A = 0$ (subject to $x(0) = y(0) = 0$) for arbitrary a and b. Call this solution S. [10]

   (d) Simplifies S as far as possible, and displays the simplified expression for $x(t)$. [3]

   (e) Subtracts the exact solution that you found in part (a) from the solution S that MATLAB has found and shows that this simplifies to zero (verifying the accuracy of your pen and paper solution). [5]

(f) Uses the function `Eulersol` from part (b) with timestep $dt = 0.2$ to find an approximate solution when $a = 1$, $b = -1$, for $0 \le t \le 10$. [8]

(g) Plots on the same figure the exact solution for $x(t)$, which you obtained in part (c), and the solution that you obtained using Euler's method, which you obtained in part (f). Comment briefly on what you learn from this figure. [9]

2. In this part of the question, $n = 2$ and $A \ge 0$.

(a) Write a MATLAB function, `solplot`, that uses `ode45` to calculate an approximate solution of (1) and (2) for $0 \le t \le t_{max}$ subject to the initial conditions $x = y = 0$ when $t = 0$.

The function `solplot` should take a, b, A, ω, t_0 and t_{max} as inputs and have no outputs.

The function should plot a graph of y as a function of x for $t_0 \le t \le t_{max}$.

The plot should have labelled axes and a title that states the values of a, b, A and ω. [20]

Write a MATLAB script that:

(b) Calls the function `idscript` (available on the G11ACF Moodle page) to convert your student ID number into a set of values of a, b and ω unique to you. Specifically, you need the line of code `[a,b,w] = idscript(IDnumber)`. [2]

(c) Uses `solplot` to demonstrate how the solution behaves for various values of A with $0 \le A \le 1$ by producing at least five illustrative plots, using $t_0 = 500$ and $t_{max} = 1000$. [15]

Hint: You should extensively investigate how the solution depends on A, but just show enough plots in your script to illustrate different types of behaviour.

(d) Comment briefly on what these plots tell you about the behaviour of the solution for $0 \le A \le 1$. [3]

- *Whenever you are asked to use* `ode45`*, use the default accuracy settings (as shown in Video Lecture 8). Any comments that you are asked for should be included in your MATLAB scripts. Label all plots appropriately.*

- *For each script you have written, open it in the MATLAB editor, go to the* `PUBLISH` *tab and click on* `Publish`*. This will produce an html file that includes your script and any outputs it produces, including plots. You should hand in a print-out of each of these html files, along with any functions you write that are used in the scripts. These functions should be clearly set out, with comment statements.*

- *Please staple your work to a completed yellow coursework submission sheet, date-stamp it and hand it in using the boxes near the entrance of the Mathematics Building.*