University of Nottingham

School of Mathematical Sciences

G11ACF **MATLAB**

2015-16: Assessed Coursework 1 **Hand in by 5pm on Friday December 4th 2015**
The marks available for each section below are shown in square brackets.

1. In this part, you will investigate a method of finding an approximate solution of the equation

$$x = ke^{-x}, \tag{1}$$

where x is a real, positive, scalar variable and k a positive, real parameter.

(a) Prove, without using MATLAB, that equation (1) has a unique solution. [4]

(b) Plot a graph of both $y = x$ and $y = ke^{-x}$ on the same axes in MATLAB to illustrate this when $k = 1$. [4]

(c) Define a sequence of numbers $\{x_i\} = x_0, x_1, x_2, \ldots$ using

$$x_{n+1} = ke^{-x_n}.$$

Write a MATLAB function (call it `Q1c`) that takes k, x_0, δ and n_{max} as inputs and calculates the sequence x_i.

Your function should return the single value x_n as its output when $|x_n - x_{n-1}| < \delta$ or $n = n_{max}$, whichever occurs first.

Hint: Use a `while` loop.

Your function should also plot x_i as a function of i and log-log plot $|x_i - x_{i-1}|$ as a function of i in the same MATLAB Figure (use the command `subplot`). [20]

(d) Explain how your function `Q1c` can be used to find a solution of equation (1), and illustrate this by running your code with $k = 0.5$, $x_0 = 1$, $\delta = 10^{-14}$ and $n_{max} = 1000$. [5]

(e) Write a function (call it `Q1e`) that plots a graph of the solution of equation (1) as a function of k for $0 \leq k < 2.65$ by repeatedly calling the function `Q1c`. [10]

(f) Use `Q1c` to explore what happens for $2.65 \leq k \leq 2.75$. Produce some representative plots and describe what you see. [10]

2. In this part, you will investigate a method of finding an approximate solution of the equation

$$x = ke^x, \tag{2}$$

where x is a real, positive, scalar variable and k a positive, real parameter.

(a) Prove, without using MATLAB, that equation (2) has

- two solutions in $x \geq 0$ if $0 \leq k < k^*$,
- a unique solution in $x \geq 0$ if $k = k^*$,
- no solutions in $x \geq 0$ if $k > k^*$,

where k^* is a positive real number that you should determine. [6]

(b) Plot three separate graphs of both $y = x$ and $y = ke^x$ on the same axes in a MATLAB Figure to illustrate what happens in each of these three cases, using suitable values of k. [6]

(c) For the case $k = 0.3 < k^*$, with $x_0 = 1$, $\delta = 10^{-14}$ and $n_{max} = 1000$, use a suitably modified version of `Q1c` (call it `Q2c`) to find an approximate solution of equation (2). Which of the two solutions does it return? How does the output of `Q2c` vary if you change x_0? [10]

3. In this part, we will rearrange (2) to obtain

$$x = \log\left(|x|/k\right) \tag{3}$$

and assume that $k > 0$.

(a) Repeat part 2(c) for equation (3). [10]

(b) Investigate what happens when $k = 0.5 > k^*$, and discuss what you find. [15]

Your answers to parts 1(a) and 2(a) can be handwritten, word processed or included in your scripts as comments.

The MATLAB functions `Q1c` *and* `Q1e` *should be clearly set out, with comment statements, and printed out. You do not need to print out* `Q2c` *or any MATLAB functions that you write for part 3.*

For each question, write a MATLAB script that performs the required actions.

Add to these scripts comments that answer the questions .

For each script you have written, open it in the MATLAB editor, go to the `PUBLISH` *tab and click on* `Publish`*. This will produce an html file that includes your script and any outputs it produces, including plots. You should hand in a print-out of each of these html files.*

Please staple your work to a completed yellow coursework submission sheet, date-stamp it and hand it in using the boxes near the entrance of the Mathematics Building.