Prelab 7

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1. Damping ratio can be found for a 2nd order ODE using an exponential decay envelope on the displacement. Using the slope of the log decrement plot damping ratio is calculated:

$$\beta = \left[\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

2. Undamped natural frequency is the frequency that is just dependent on the mass and stiffness of the system, and it oscillates back and forth forever since there is no damping. The damped natural frequency is the actual measurable frequency, as damping will exist in the real world. Using an unforced response, the damped natural frequency can be found by allowing it to naturally oscillate back and forth. Below is the relationship between damped and undamped natural frequency. Assumptions made include assuming all damping occurs within the material and not including the air and connected structure, as otherwise the damped frequency would not always be the same.

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}},$$

3. An estimate for mass can be found by measuring the dimensions of the beam and multiplying by the density. The stiffness can be found using the beam stiffness equation with the dimensions of the beam.
4. The first mode would be equal to the following:
   W = a1*(E *I/ui/L^4)^(1/2) =
   A1 = 3.52
   E =1e7 psi
   I = bh^3/12 = 1*(1/8)^3/12 = 0.000163 in^4
   Ui = W/gL = 0.0975437*(1/8*1) = 0.0122 lb/in
   L = 10in
   Natural frequency = 12.88 rad/s **-> 2 Hz**

V. Uniform Beams
(Transverse or bending vibrations)

The same general formula holds for all the following cases,

$$\omega_n = a_n\sqrt{\frac{EI}{\mu_1 l^4}} \tag{29}$$

where $EI$ is the bending stiffness of the section, $l$ is the length of the beam, $\mu_1$ is the mass per unit length $= W/gl$, and $a_n$ is a numerical constant, different for each case and listed below