DYNAMIC SYSTEMS AND CONTROLS
ME 344—Fall 2021

Simulation Project 1
Due 20 October 2021, Submit Online
25 POINTS

Problem Description

This problem asks you to simulate the motion of a mass $m = 1$ kg sliding on a rigid rod (i.e. one that does not flex or bend) and constrained by a linear spring with stiffness $k = 20$ N/m of free length $L_0 = 0.5$ m that is attached to a pinned joint that is found a perpendicular distance of h from the rigid rod. The distance h will differ based on the cases considered below as will the damping coefficients and models of damping. This problem is illustrative of several aspects of dynamic systems modeling because one can observe both linear and nonlinear behavior depending on the selection of system parameters and initial conditions. When solved correctly, the cases below will show some responses of general interest, which should provide you with a better understanding of the physical behavior of these systems.

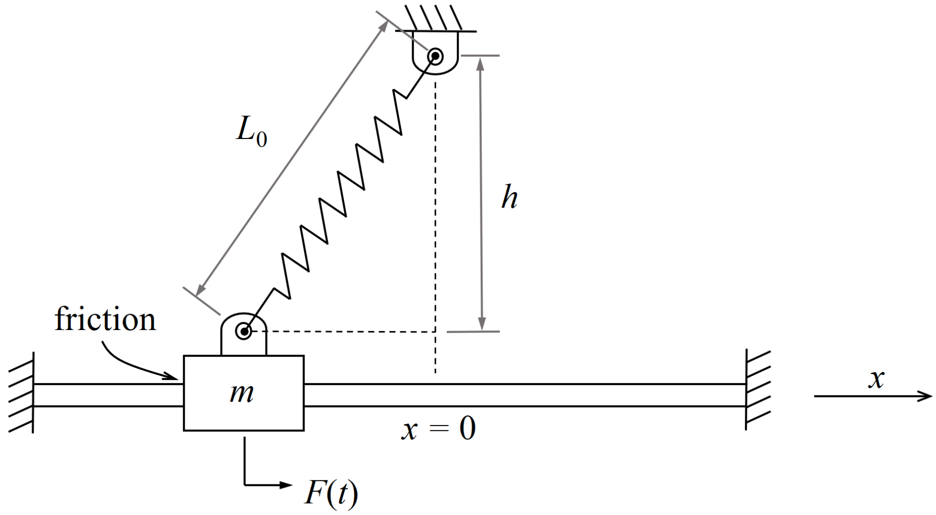

Figure 1: Mass-spring on slider with friction. All motion occurs in x-direction.

A. Bond Graph and Equations of Motion:

(10 points) The first step in analyzing this system will be to derive the equations of motion. Do this by determining a bond graph for the system. Use this bond graph to determine the equations of motion in state-space form which describe the system with $x(t)$ being one of the state variables. Your results from this step will be used for your simulation. Note that since the motion of the mass is along x but isn't parallel to the spring compression, you will need to include a transformer in your BG. The transformer modulus will be depdendent on the

angle θ. This type of transformer is sometimes referred to as a "modulated transformer," and denoted as MTF, but you can write it as $\tilde{m}(\theta)$.

B. Simulation:

Based on your bond graph analysis, use MATLAB to generate a simulation for the cases listed below. Each case will be simulated from $t = 0$ s to $t = 30$ s with a time step of 0.01 seconds. *Include your Matlab scripts in the file you submit for grading.*

Case 1; Small initial velocity, viscous friction: $h = 1.05L_0$, $b = 0.5$ N·s/m. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 0.1$ m/s.

Case 2; Small initial velocity, Coulomb friction: $h = 1.05L_0$, $\mu = 0.1$. Note that the force of Coulomb friction is given by $F_\mathrm{f} = \mu N \mathrm{sgn}(v)$, where N is the normal force and $\mathrm{sgn}(v)$ is known as the signum operator which provides the sign of the velocity. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 0.1$ m/s.

Case 3; Large initial velocity, viscous friction: $h = 1.05L_0$, $b = 0.5$ N·s/m. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 7.5$ m/s.

Case 4; Large initial velocity, Coulomb friction: $h = 1.05L_0$, $\mu = 0.1$. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 7.5$ m/s.

Case 5; Small initial velocity, viscous friction: $h = 0.95L_0$, $b = 0.5$ N·s/m. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 0.1$ m/s.

Case 6; Small initial velocity, Coulomb friction: $h = 0.95L_0$, $\mu = 0.1$. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 0.1$ m/s.

Case 7; Large initial velocity, viscous friction: $h = 0.95L_0$, $b = 0.5$ N·s/m. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 7.5$ m/s.

Case 8; Large initial velocity, Coulomb friction: $h = 0.95L_0$, $\mu = 0.1$. Initial conditions: $x(0) = 0$ m, $\dot{x}(0) = 7.5$ m/s.

B.1: (5 points) Plot the position, $x(t)$, as a function of time for Cases 1–8. Please plot the pairs of Cases 1 and 2, 3 and 4, 5 and 6, and 7 and 8 on the same plots.

B.2: (5 points) Answer the following questions based on your results for Cases 1 – 4.

1. During the entire time range, $t \in [0 \;\; 30]$ s, what is the maximum excursion $x(t)$ attained for Cases 1 and 2? Provide a physical rationale for why these two cases appear to show very different behavior?

2. For cases 2, 3, and 4, the amplitude of the displacement reduces as time increases. Why is this? Based on your simulation, what do you expect the final position of the mass will be when it comes to rest for these four cases? Provide your answer in a table.

3. Which case oscillates at a higher frequency (i.e. a higher number of zero crossings for the same amount of time)? The system parameters (mass, spring constant, damping, etc) don't change, so what is the physical reason for this?

B.3: (5 points) Answer the following questions based on your results for Cases 5 — 8.

1. When the system ultimately comes to rest for Cases 5 – 8, what is the position of the mass? Provide your answer in a table.
2. Cases 7 and 8 have the same initial conditions, but will come to rest at different positions. Describe what is happening to the system in order for this to occur.