Lab #3 Report

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Abstract

The objectives of this lab are to gain experience using wind tunnels and common measurement instruments, measure pressure distribution around the cylinder, measure velocity and pressure distributions upstream and in the wake of a cylinder to find the drag force on the cylinder using integral equations. Using the obtained data, we then compared the drag in terms of dimensionless parameters with published data. The drag coefficient for both measurement methods were similar but not the same. The drag coefficient found by integrating the measured pressure on the surface of the cylinder was higher than the drag coefficient found through the analysis of a control volume. When the data was compared to the typical drag coefficient data for cylinders in cross-flow as a function of Reynolds number, the Reynolds number we calculated agreed with the predicted drag coefficient data, thus validating our calculated drag coefficient. The pressure coefficient plots we calculated looked similar to the laminar boundary conditions plot on the pressure coefficient distributions around a cylinder in a cross-flow graph that was provided in the handout.

Background

Bernoulli's Equation was used in this experiment to assist in determining pressure and velocity measurements. The equation provides the relationship among the differences of pressure, velocity, and elevation between two points.

$$(p_1/\gamma) + ((V_1^2)/2g) + z_1 = (p_2/\gamma) + ((V_2^2)/2g) + z_2 \qquad \text{Eq. 1}$$

The z_1 and z_2 terms are negligible because they are both at similar heights. At the tip of the Pitot-static probe, there is no moving flow and the pressure at this point is equal to the static pressure plus the dynamic pressure and is called the stagnation pressure.

$$p_2 = p_1 + \tfrac{1}{2}\rho_{air}(V_1^2) \qquad \text{Eq. 2}$$

Where p_2 is the stagnation pressure, p_1 is the static pressure, and $\frac{1}{2}\rho_{air}V_1^{\ 2}$ is the dynamic pressure. The point where stagnation occurs is also where the highest pressure is achieved. The static pressure is the absolute pressure and is measured when the fluid is at rest relative to the measurement. The dynamic pressure is the pressure due to velocity, which is caused by the motion of air in this situation. The dynamic pressure is equivalent to half the product of the density of air and velocity squared. Lastly, the total pressure was considered the same as the stagnation pressure.

The Pitot-static probe is an instrument used to determine fluid velocity through the measurement of static and stagnation pressure. The stagnation pressure, the pressure due to the velocity and the static pressure, is measured by a tube whose inlet faces the flow. It is shown in Eq. 2 and in Figure 1 at point 2. The static pressure is measured by a tube whose inlet faces perpendicular to the flow. The dynamic pressure is then found by subtracting the static pressure from the stagnation pressure. The dynamic pressure is then used to find the velocity of the fluid. A typical Pitot-static tube is shown in Figure 1:

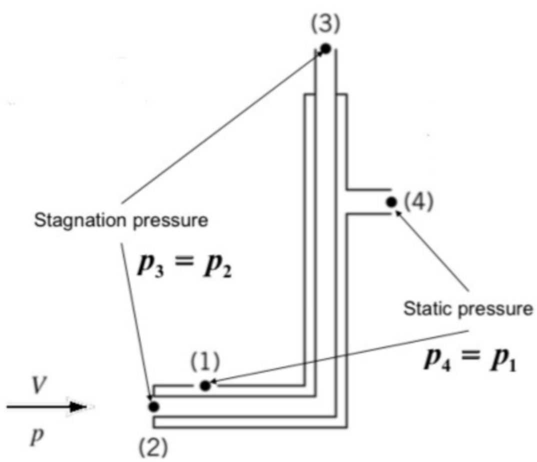


Figure 1: An example of a Pitot-static tube system that measures the difference between stagnation and static pressure in order to find dynamic pressure.

Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid. The three most common types of drag are parasitic drag, lift-induced drag, and wave drag. In this lab, we are dealing with parasitic drag which is the drag resulting from the pressure gradient around the surface of the cylinder. The drag force is representative of the normal pressure forces on the cylinder, but neglects drag from tangential, viscous shear forces. The total drag force can be calculated by integrating pressure with respect to area of the cylinder and the angle of the cylinder's surface at the points where it is measured and is shown to be:

$$F_d = ((2\pi DL)/360) \int_0^{180} p(\theta)cos(\theta)d\theta \qquad \text{Eq. 3}$$

Equation 3 integrates the differential pressures at each angle around the cylinder. θ is the angle in relation to the flow direction, D is the cylinder's diameter, and L is the cylinders length. The integration was performed using the trapezoidal method below.

$$\int_0^{180} p(\theta)cos(\theta)d\theta \approx \sum_{i=1}^{N-1} (((p_i(\theta)cos(\theta_i)) + (p_{i+1}(\theta)cos(\theta_{i+1})))/2) * (\theta_{i+1} - \theta_i) \qquad \text{Eq. 4}$$

The $p(\theta)$ terms are the static pressure measurements we obtained from the pressure transducer at each angle θ. The summation of the results of each iteration multiplied by the term outside of the integral in Eq. 3 will result in the drag force on the cylinder. The drag coefficient, C_d, is characterized as the dimensionless parameter for drag force. The drag coefficient is defined below in Eq. 5.

$$C_d = F_d/((1/2)(\rho_{air})(V^2)A) \qquad \text{Eq. 5}$$

The drag coefficient is directly proportional to the drag force, and inversely proportional to the product of the dynamic pressure and the cross-sectional area of the cylinder. Using the information we have obtained, we can also present pressure distribution in terms of a pressure coefficient:

$$C_p = \frac{p-p_o}{\frac{1}{2}\rho_{air}U^2} \qquad \text{Eq. 6}$$

Where p is the absolute surface pressure, p_o is the upstream absolute static pressure, ρ_{air} is the density of air, and U is the upstream velocity in the wind tunnel.

The second way drag force can be calculated is using the control volume momentum equation. For this equation, the change in momentum is equal to the pressure and drag forces and because the flow was relatively steady, we can use the conservation of linear momentum equation.

$$F_d = P_{Dynamic,\ upstream}A - P_{Dynamic,\ downstream}A + P_{Static,\ downstream}A - P_{Static,\ upstream}A \qquad \text{Eq. 7}$$

$$P_{Dynamic} = \rho_{air} L \sum_{i=1}^{N-1} (((V_n^{\,2}) + (V_{n-1}^{\,2}))/2) * (h_n - h_{n-1}) \qquad \text{Eq. 8}$$

$$P_{Static} = L \sum_{i=1}^{N-1} ((P_{static,\ n} + P_{static,\ n-1})/2) * (h_n - h_{n-1}) \qquad \text{Eq. 9}$$

Equation 7 is the conservation of linear momentum equation and the summation of forces represent the drag force. For both the dynamic and static equations, Eq. (8) and Eq. (9), ρ_{air} is the density of air, L is the length of the cylinder, V is the velocity, P is the static pressure, and h is the vertical position of the Pitot-static probe. In order to use an appropriate control volume for this analysis, the control volume must be an area in the wind tunnel where there are no boundary effects. The control volume is far away from the walls and towards the center of the wind tunnel in this lab.

Experimental Setup

<u>Overview</u>

In this lab, we used two different sized winds tunnels, 12” and 24”, to measure the drag force on a cylinder model. The pressure transducer measures voltage inside the wind tunnel, and to obtain accurate pressure readings, we first calibrated the transducer to a manometer. We then wanted to measure the pressure at various points on the surface of the cylinder inside the wind tunnels at both 10 m/s and 20 m/s. Rotating the cylinder, we took measurements at every 15 degrees between 0 and 180 degrees from the pressure tap in the cylinder. From this, we filled in the static pressure (P) in the coefficient of pressure formula. To obtain the upstream static pressure (P0) and velocity (U) for the control volume analysis described in the background, we then measured the pressure profile upstream and downstream of the cylinder using a pitot-static probe.

<u>Calibration</u>

The first step was to calibrate the pressure transducer, which was used to make pressure measurements throughout this experiment. There is a differential pressure transducer inside the instrument case for the wind tunnel. The "A" and "B" pressure ports are input pressures for the two sides of this differential pressure transducer, and we first set these ports to 0 V at atmospheric pressure so it would read gage pressure. We then connected manometer tubing to the B port at the back of the pressure transducer. Using the inclined manometer as our reference source, we squeezed a rubber bulb, causing the manometer fluid to go down, and took measurements from 0 to 5 inches of water and its corresponding voltages to calibrate the transducers later through a line of best fit.

Static Pressure on Cylinder

In part 2 of the lab, pressure distributions measurements were made along the perimeter of the cylinder. The probe was traversed to a position upstream of the cylinder and aligned with the center of the test section height-wise. The total pressure tube was placed in port A of the pressure transducer and static pressure from the pitot static probe to port B. Matching the pressure transducer readout with the calculated dynamic pressures from the pre-lab, we set the wind tunnel speed to first 10 m/s to take measurements and then 20 m/s by adjusting the frequency using the wind tunnel controller. This setting causes the pressure transducer to readout upstream dynamic pressure. We then unplug the line connected to port A, and the transducers read out the vacuum upstream static pressure.

Once these values were obtained, we set the speed back to 10 m/s and disconnected the pressure lines from the pressure transducer. Then we took the static pressure line from the cylinder model and connected it to port B, reading the static pressure on the surface of the cylinder. Beginning at 0 degrees, one student rotated the cylinder in increments of 15 degrees and the other took the measurements from the pressure transducer until 180 degrees. If the values on the transducer fluctuated significantly, we recorded multiple values so we could average them later. This process was repeated for 20 m/s.

Control Volume & Measurement Procedure

In the third part of the lab, upstream and downstream static and dynamic pressure measurements were made to perform control volume analysis. First the wind tunnel velocity was set to 20 m/s, and one student moved the pitot-static probe to a position as far upstream of the cylinder as possible and recorded its location from the traverse position indicator. Another student connected the total pressure line from the pitot static probe to port A of the transducer and the static line from the probe to port B, recording the dynamic pressure. The total pressure line was then disconnected, reading the static pressure. The transducer was then lowered at increments defined by the data collection tables, repeating the above procedure until measurements of static pressure and dynamic pressure have been made at all positions upstream of the cylinder. The pitot-static probe was then moved about two feet downstream of the cylinder. Repeating the producer above, we recorded the static and dynamic pressures at all positions downstream of the cylinder. Like in part two, if the transducer read fluctuating values, we took multiple measurements to calculate the average later.

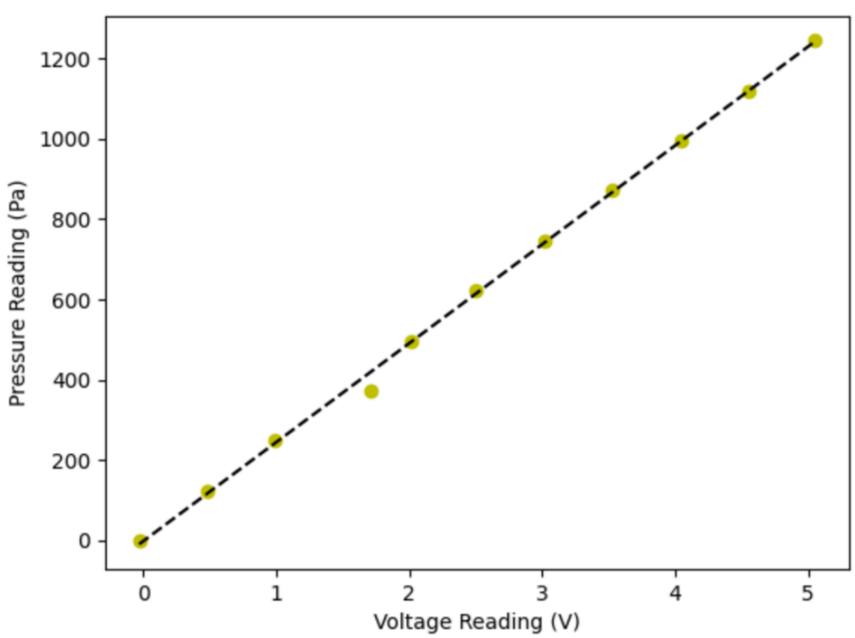
Results/Discussion

The following results are completed using data taken from the 12” wind tunnel. Plotted in yellow is the recorded pressure reading mapped to voltage reading. Given this set of points, a linear fit was found to be:

$$y = 246.29 * x - 1.686$$

This was plotted on the same graph as the pressure readings to visualize the variance from this equation of individual points.

Linear Fit of Voltage to Pressure



Using the variance of this fit, the standard error and uncertainty was calculated (as shown in the appendix) to be:

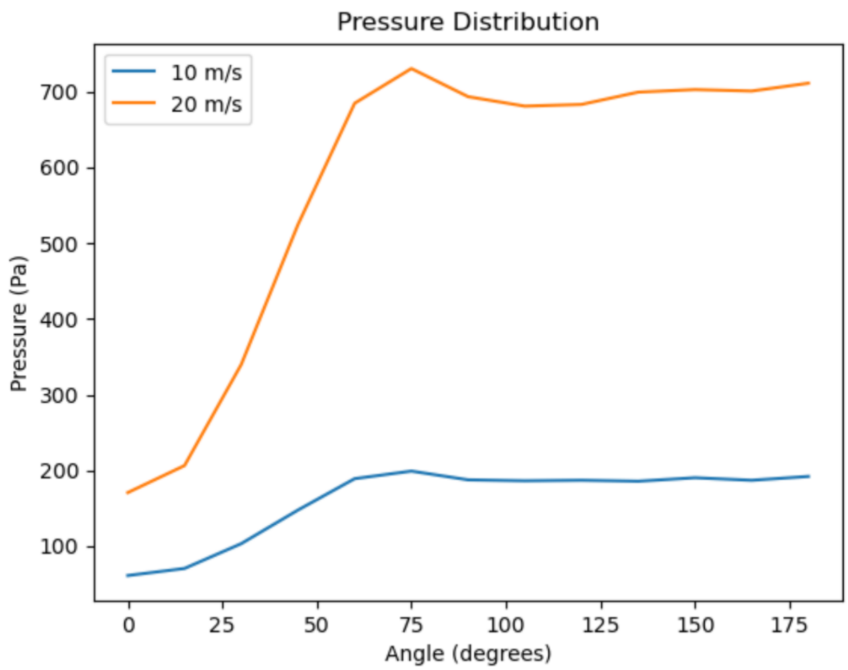
$$\pm\ \mathbf{0.1549\ inH_2O = \pm\ 38.55\ Pa}$$

This value is greater than the estimated uncertainty value of 0.01 inH2O the manometer because of how difficult it was to obtain an exact measurement on the manometer. The manometer fluid did not remain steady due to a leak, causing human error while measuring and increasing the magnitude of individual deviations (δi).

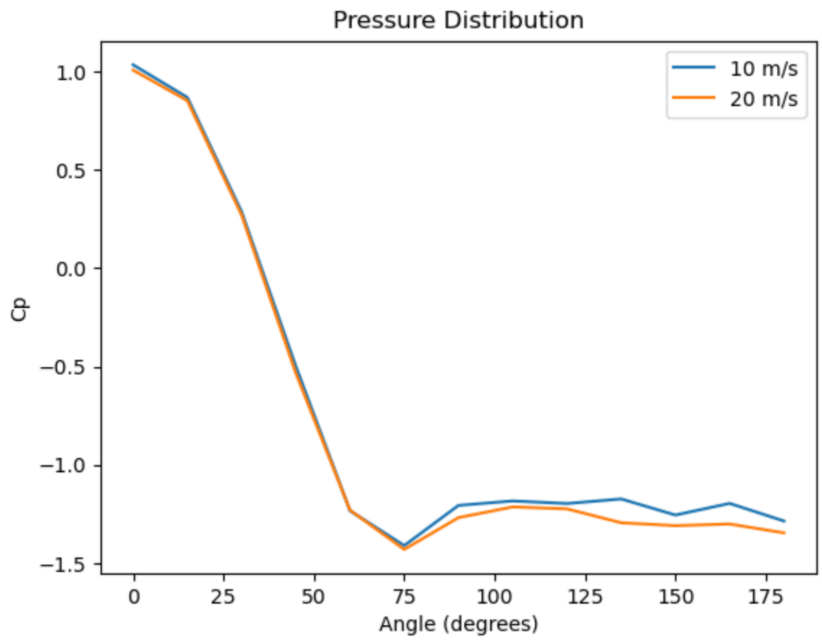
With the pressure uncertainty calculated, the uncertainty of velocity can be calculated. This calculation is shown in the appendix, and comes out to be:

$$\pm\ \mathbf{3.082\ m/s}$$

The negative measured gauge pressure is then plotted against the angle of measurement along the cylinder to show the pressure distribution along the cylinder.



The negative measured gauge pressure is then converted to the non-dimensional form C_p for comparison of results between the two cylinders, once again showing the pressure distribution along the cylinder.



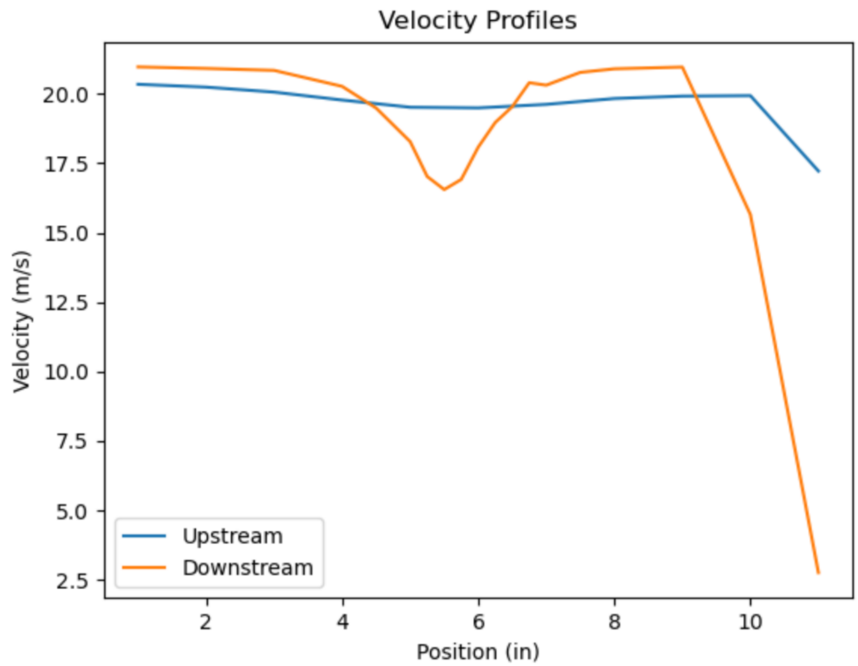
Using this pressure distribution around the cylinder, the total drag and drag coefficient C_D are calculated (with sample calculations in the appendix) and shown in the table below:

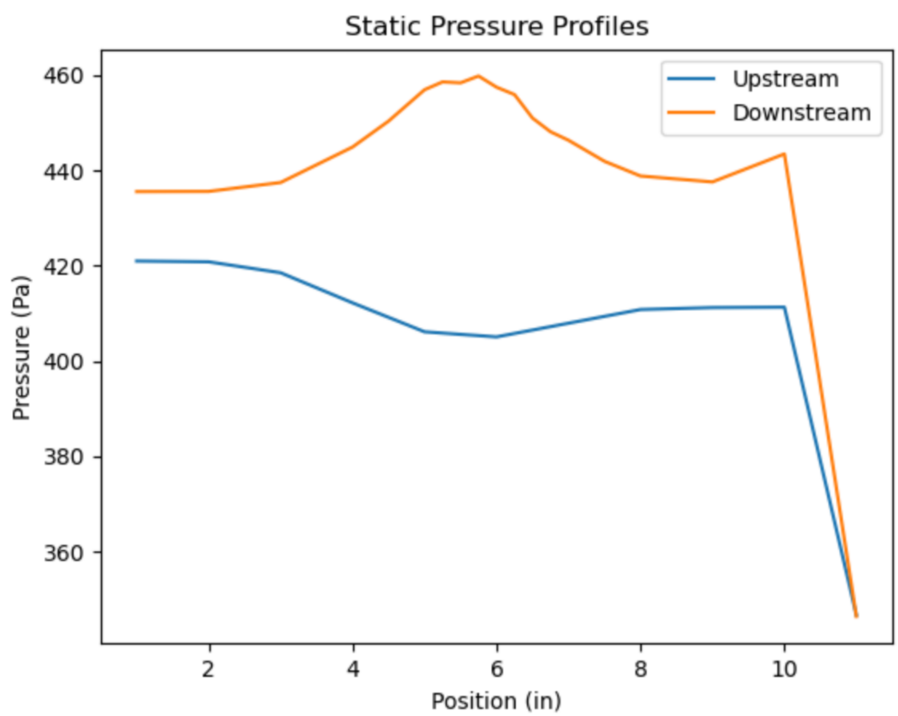
12” Wind Tunnel Results of Pressure Distribution

Velocity	**20 m/s**	**10 m/s**
Force drag (N)	1.763	0.415
C_D	1.325 ± 0.131	1.270 ± 0.867

The two velocities have differing pressure curves in the first pressure distribution graph, as expected since a higher velocity will create a larger stagnation pressure. However, when their relative ratios to their velocity is taken into account as C_P does in the second graph, the curves collapse and become almost identical. The two curves didn't line up perfectly in the C_P graph likely due to viscosity being dependent on velocity of the air.

Next a control volume analysis was used to calculate drag and C_D values for the 20 m/s velocity. This analysis uses the upstream and downstream static pressure and velocity profiles, and are plotted in the below graphs:





Based on the velocity graph, the heights for the control volume analysis were chosen to be from positions 1 to 9 inches positions due to the influence of drag of the wind tunnel on the air in positions 10 through 11 inches. The horizontal positions were approximately 2.83 inches upstream of the cylinder and 10.5 inches downstream of the cylinder. After the region for the control volume was set, the necessary conservation of momentum equation was needed. The base equation is:

$$\frac{\partial}{\partial t}\int_{CV} V\rho dV' + \int_{CS} V\rho V \bar{n} dA = \sum \vec{F}$$

The first integral goes to zero because the wind tunnel measurements are assumed to be steady state. The second integral totals momentum flow delta across the control volume, and comes out to the first integral below. The second integral below comes from the integration of the force caused by static pressure, coming from the right side of the original equation above, and is then moved to the other side of the equation below:

$$F_D = \left[\int_{A_1} \rho v_1^2 dA_1 - \int_{A_2} \rho v_2^2 dA_2\right] + \left[\int_{A_1} p_{0,1} dA_1 - \int_{A_2} p_{0,2} dA_2\right]$$

Using numerical integration in excel, the drag force is found using the equation above. Also, the drag coefficient can be found using the same method shown in the appendix for the surface pressure distribution drag. These values are shown below:

12” Wind Tunnel Results of Control Volume

F drag (N)	1.338
C_D	1.005 ± 0.168

To double check our calculated C_D, comparing them against theoretical values was needed. This was done through calculating the Reynold’s number for the flow, which has a theoretical drag coefficient. The calculated Reynold’s number were $5*10^5$ and $2.5*10^5$ for 20 m/s and 10 m/s, respectively. For each found Reynold number, the expected C_D values were 0.9 and 1.3, respectively. Comparing this against the empirically found C_D values shows that they are close to their expected values, as the C_D for 20 m/s and 10 m/s were 1.325 and 1.270, respectively. The C_D value found from the control volume analysis was also close to the expected value of 0.9, with a value of 1.005.

Conclusions

We used two different methods to find the pressure distribution and drag forces in this experiment. One method used to find the drag force was by integrating pressure with respect to area around the cylinder, and the other was using the conservation of linear momentum equation. Because the experiment involved a steady flow and a controlled volume, we were able to calculate the pressure and drag forces as equal to the change in the linear momentum of the flow. From the measurements obtained by the moment conservation analysis, we produced velocity profile and static pressure plots similar to the expected results. The pressure distribution plot we created was similar to the plot given in the handout as well. The drag coefficients have their theoretical values within their range of uncertainties for most measured cases in the lab. This lab gave us hands on experience in collecting real data and using fundamental equations and techniques to use that data to calculate useful fluid mechanic information. The lab deepened our understanding of the equations we use and how they apply in real world settings. It could be improved by streamlining the data collecting process and making it less cumbersome.

Appendix

<u>Member Contributions</u>

Kieran Cosgrove

- Results/Discussion formatting and answers
- Cd & Cp uncertainty calculations

Ryan Lee

- Background
- Abstract

Erin Collins

- Methods
- Pressure Transducer & Velocity Uncertainty

Justin Mader

- Conclusion
- Checked with Rubric

<u>Sample Calculations</u>

Pressure Transducer Uncertainty

$$s_{xy} = \left[\frac{(\sum \delta_i^2)}{(N-2)}\right]^{1/2}$$

δ **=** difference between measured manometer pressure & pressure obtained by transducer

N = number of calibration point

δ_i = [(0-0.03) , (.5 - 0.485) , (1- 0.988), (1.5-1.710) , (2-2.012) ,(2.5 - 2.502) , (3 - 3.018) ,(3.5- 3.525), (4 - 4.046) , (4.5 - 4.550 , (5 - 5.054)]

N = 11

$$S_{xy} = \left[\frac{(.03^2)+(.015^2)+(.012^2)+(.21^2)+(.012^2)+(.002^2)+(.018^2)+(.025^2)+(.046^2)+(.05^2)+(.054^2)}{11-2}\right]^{1/2} = 0.077458$$

δP = ± 2Sxy

δ**P = 0.1549 in H2O = 38.55 Pa**

Upstream Velocity & Uncertainty

Given P_{dyn} = 255.5 Pa

P_{dyn} = 1⁄2 * (density) * (velocity) ^{2}

255.5 Pa = 1⁄2 * (1.225 kg/m3) * (velocity) ^{2}

Velocity = 20.42 m/s

Don’t include density uncertainty

$$P_{dyn} = 1/2\rho U^2$$

$$dP_{dyn} = \rho * U * dU$$

$$dU = dP_{dyn}/(\rho * U)$$

$$\delta U_{P\,dyn} = dU * \delta P_{dyn} \div dP_{dyn}$$

$$\delta U_{P\,dyn} = dP_{dyn}/(\rho * U) * \delta P_{dyn} \div \partial P_{dyn}$$

$$\delta U = \delta U_{P\ dyn} = (255.5/(1.225*20.42))*(38.5494)/255.5$$

$$\delta U = \mathbf{1.541\ m/s}$$

Pressure Coefficient & Uncertainty (90° for 20 m/s)

Cp = (surface pressure - upstream static pressure) / (upstream dynamic pressure)

Cp @ 90°- 20 m/s = (-701.7288 Pa + 408.098 Pa) / 233.91 Pa

C_P = -1.255

<u>Uncertainty</u>

$$C_P = (P_{stat} - P_{surf})/(P_{DYN}) \qquad \delta P_{stat} = \delta P_{surf} = \delta P_{DYN} = \delta P$$

$$\delta C_P = \left[\left(\frac{dC_P}{dP_{stat}}\delta P_{stat}\right)^2 + \left(\frac{dC_P}{\partial P_{surf}}\delta P_{surf}\right)^2 + \left(\frac{dC_P}{dP_{DYN}}\delta P_{DYN}\right)^2\right]^{1/2}$$

$$\delta C_P = \left[\left(\frac{\delta P}{P_{DYN}}\right)^2 + \left(\frac{-\delta P}{P_{DYN}}\right)^2 + \left(\frac{-(P_{stat} - P_{surf})\,\delta P}{(P_{DYN})^2}\right)^2\right]^{1/2} = \frac{\delta P}{P_{DYN}}\left[2 + (P_{surf} - P_{stat})^2\right]^{1/2}$$

$$\delta C_P = \delta P/P_{dyn} * (2 + ((P_{stag} - P_{stat})/P_{dyn})^2)^{1/2}$$

$$\delta C_P = 38.55 / 233.91 * (2 + ((701.7288 - 408.098)/233.91)^2)^{1/2}$$

$$\delta C_P = \mathbf{0.312}$$

Drag Force from Pressure Distribution

Integrals in excel

Drag Coefficient from Pressure Distribution & Uncertainty

C_D = (drag force) / ((dynamic pressure) * (area))

area = length * diameter of cylinder

C_D @ 20 m/s = 1.763 N / (229.827 Pa * 0.3048 m * 0.019 m)

C_D = 1.325

<u>Uncertainty</u>

not required, no uncertainty

$$C_D = \frac{F_{Drag}}{P_{dyn} \cdot A} \qquad \delta C_D = \left[\left(\frac{dC_D}{dF} \delta F \right)^2 + \left(\frac{dC_D}{dP} \delta P \right)^2 + \left(\frac{dC_D}{dA} \delta A \right)^2 \right]^{1/2}$$

$$\delta C_D = \frac{dC_D}{dP} \delta P \qquad \frac{dC_D}{dP} = -\frac{F_{Drag}}{P^2 A}$$

$$\delta C_D = -\frac{F \delta P}{P^2 A} = \frac{-1.763 \cdot 38.55}{(229.827)^2 \cdot (.3048 \cdot .019)} = 0.1305$$

Drag Force from Control Volume (integrals in excel)

$$F_D = \left[\int_{A_1} \rho v_1^2 dA_1 - \int_{A_2} \rho v_2^2 dA_2 \right] + \left[\int_{A_1} p_{0,1} dA_1 - \int_{A_2} p_{0,2} dA_2 \right]$$

0 + (total momentum flow delta) = (drag force) - (total force from pressure delta)

(drag force) = (total momentum flow delta) + (total force from pressure delta)

= (29.868 - 30.521) + (6198.971 - 6196.981)

Drag = 1.338 N

Drag Coefficient from Control Volume & Uncertainty

C_D = (drag force) / ((dynamic pressure) * (area))

area = length * diameter of cylinder

C_D @ 20 m/s = 1.338 N / (229.827 Pa * 0.3048 m * 0.019 m)

$\mathbf{C_D = 1.005}$

<u>Uncertainty</u>

Using previously determined δC_D equation

$$\delta C_D = -F * \delta P / (P^2 A)$$

$$\delta C_D = 1.338 * 38.55 / (229.827^2 * (.3048 * .019))$$

$$\delta C_D = \mathbf{0.168}$$