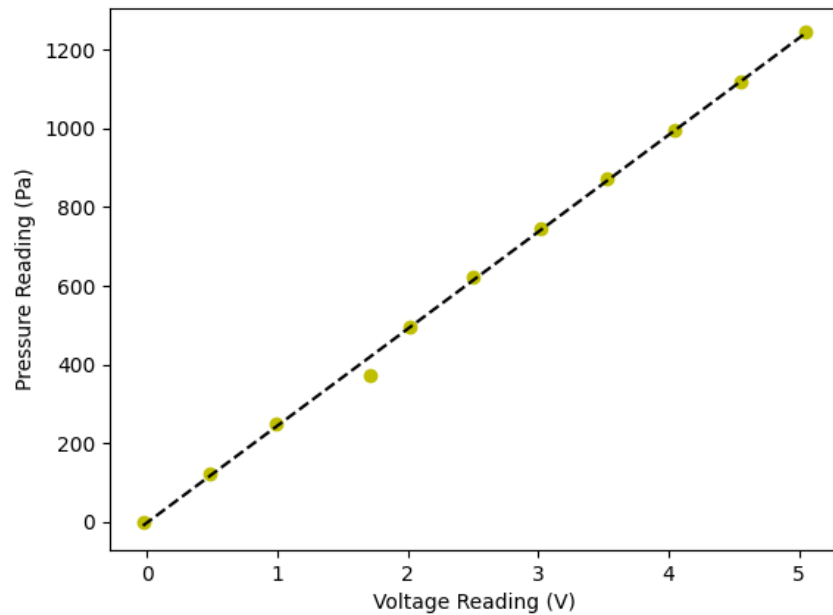


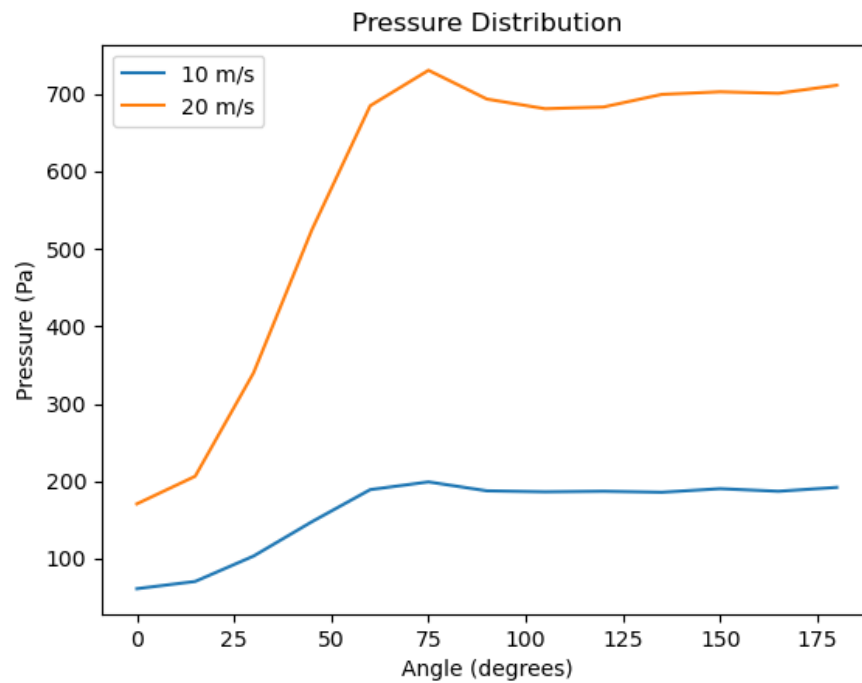
Fluids Lab 3 - Individual Calculations (12" Wind Tunnel)

Kieran Cosgrove

Linear Fit of Voltage to Pressure



Pressure Distribution Graphs



Static pressure voltage measurements along the surface of the cylinder at different angles are converted to pascals using above linear fit, and then plotted against angle.

Pressure Coefficient Sample Calculation

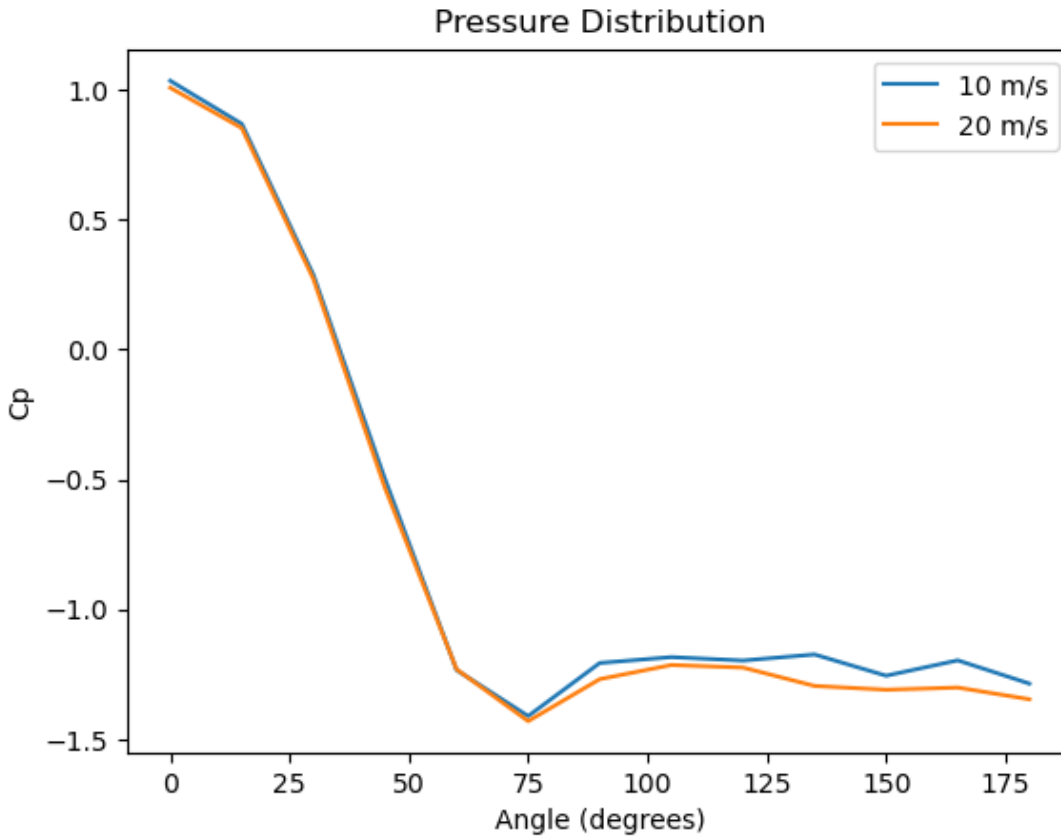
$C_p = (\text{surface pressure} - \text{upstream static pressure}) / (\text{upstream dynamic pressure})$

$C_p @ 0^\circ - 20 \text{ m/s} = (-174.188 \text{ Pa} + 408.098 \text{ Pa}) / 233.91 \text{ Pa}$

$C_p = 1$

$C_p @ 90^\circ - 20 \text{ m/s} = (-701.7288 \text{ Pa} + 408.098 \text{ Pa}) / 233.91 \text{ Pa}$

$C_p = -1.255$



Pressure coefficient determined using the same formula in sample calculation along the surface of the cylinder at different angles and plotted against angle.

Pressure Distribution Sample Integration

Using the integral pictured below with trapezoidal numerical integration as in the spreadsheets below and attached, the drag force was found to be 0.415 N and 1.763 N for 10 m/s and 20 m/s flow speeds, respectively. 'L' and 'D' were converted to metric, the average of $P \cdot \cos(\theta)$ between two positions was used to determine the average height for the numerical integration, times the step size which is the change in theta in degrees. The pressure used was the absolute pressure, which was the absolute pressure of 100.5 kPa minus the gauge pressure.

$$F_D = \int dF_x = \frac{2\pi D}{360} L \int_0^{180} P \cos \theta d\theta$$

Theta (degrees)	10 m/s pressure		10 m/s step force
	gauge (Pa)	absolute (Pa)	
0	61.11763373	100438.88237	150.968866167417
15	70.47666902	100429.52333	140.651091221238
30	103.2332925	100396.76671	120.733467796946
45	147.5655649	100352.43444	92.6024495814621
60	189.0408687	100310.95913	58.1974005962013
75	199.040259	100300.95974	19.850201461799
90	187.5056585	100312.49434	-19.8514686925248
105	186.2331581	100313.76684	-58.2017160978013
120	186.9720293	100313.02797	-92.5858251357138
135	185.6584805	100314.34152	-120.658211639407
150	190.2723207	100309.72768	-140.508572229661
165	186.9309809	100313.06902	-150.782987714277
180	191.9799341	100308.02007	
		Force (N):	0.414695315679921

Theta (degrees)	20 m/s pressure		20 m/s step force
	gauge (Pa)	absolute (Pa)	
0	170.6676047	100329.3323953	150.78441303944
15	206.3468743	100293.6531257	140.39024160448
30	339.8128161	100160.1871839	120.36449439126
45	524.4954334	99975.5045666	92.199708906907
60	684.8234484	99815.1765516	57.899359480796
75	730.6334633	99769.3665367	19.747533840024
90	693.50929	99806.49071	-19.752445983746
105	680.9895279	99819.0104721	-57.914218429157
120	683.2471899	99816.7528101	-92.119673858761
135	699.5023564	99800.4976436	-120.04089587649
150	702.8683253	99797.1316747	-139.78957780295
165	700.9390504	99799.0609496	-150.006376739
180	711.2750377	99788.7249623	
		Force (N):	1.7625625727921

Pressure Coefficient Sample Calculation

$$C_D = (\text{drag force}) / ((\text{dynamic pressure}) * (\text{area}))$$

$$\text{area} = \text{length} * \text{diameter of cylinder}$$

$$C_D @ 20 \text{ m/s} = 1.763 \text{ N} / (229.827 \text{ Pa} * 0.3048 \text{ m} * 0.019 \text{ m})$$

$$C_D = \mathbf{1.325}$$

$$C_D @ 10 \text{ m/s} = 0.415 \text{ N} / (56.438 \text{ Pa} * 0.3048 \text{ m} * 0.019 \text{ m})$$

$$C_D = \mathbf{1.270}$$

Control Volume Graphs

Measured dynamic pressure voltages upstream and downstream of the cylinder are taken, averaged, and converted to pressures using the earlier linear fit. These pressures are then converted into velocities using the dynamic pressure equation given by:

$$\frac{1}{2} \rho U^2$$

These velocities were then graphed against their position vertically upstream and downstream of the cylinder.

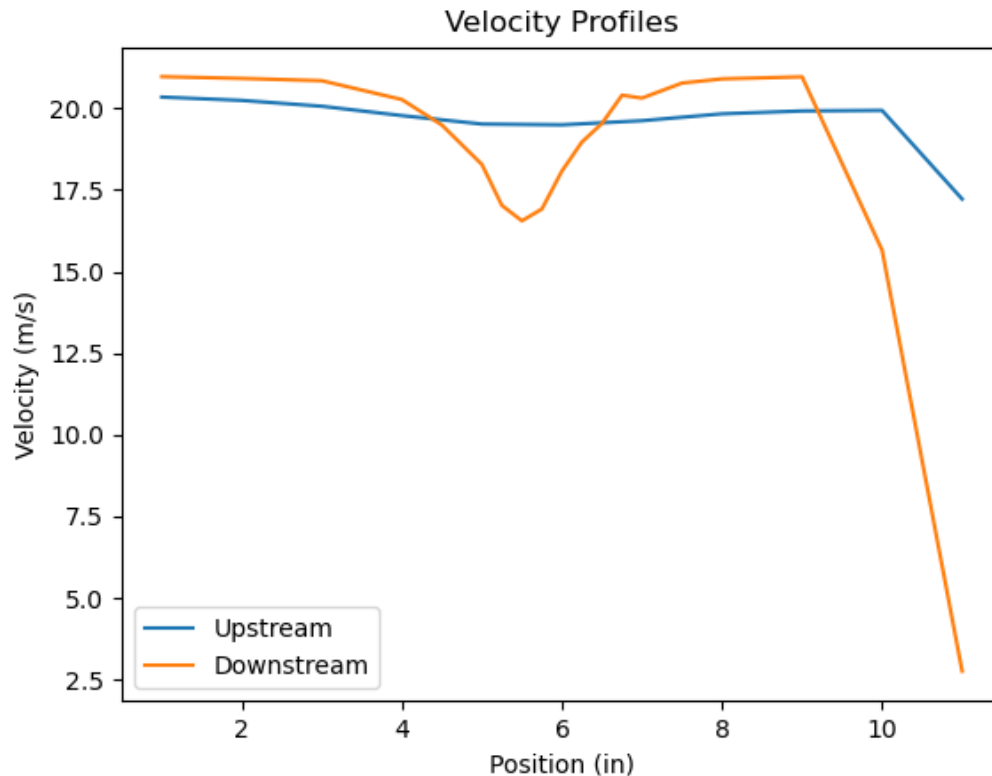
Upstream Velocity Sample Calculation

$$\text{Given } P_{\text{dyn}} = 255.5 \text{ Pa}$$

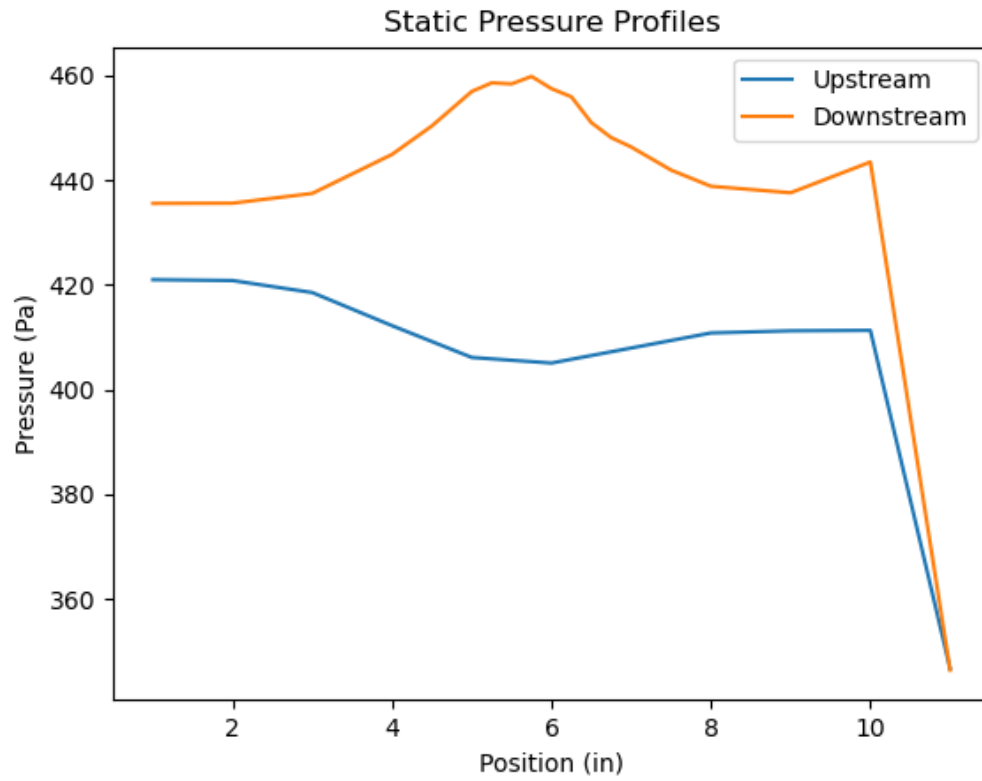
$$P_{\text{dyn}} = \frac{1}{2} * (\text{density}) * (\text{velocity})^2$$

$$255.5 \text{ Pa} = \frac{1}{2} * (1.225 \text{ kg/m}^3) * (\text{velocity})^2$$

$$\text{Velocity} = 20.42 \text{ m/s}$$



The negative gauge pressure (converted from a voltage using the linear fit) is then also mapped against the vertical position upstream and downstream of the cylinder.



Control Volume Integration

Positions and widths were converted to the metric system, velocity comes from velocities found for velocity profile graph. Absolute pressures are used in force from pressure integration, which is found from atmospheric pressure of 100.5 kPa minus gauge pressure. Force from pressure is found from the following:

$$\int_{y_{2,bot}}^{y_{2,top}} p_{0,2} w dy$$

Where 'p' was the average pressure between two positions, 'w' was the width, and 'dy' was the difference in height between two positions. The momentum flow integration was found from the following:

$$\int_{y_{2,bot}}^{y_{2,top}} \rho v_2^2 w dy$$

Where 'p' is the density of air, 'v^2' is the average squared velocity between two positions, 'w' was the width, and 'dy' was the difference in height between two positions. The last two positions were excluded from the control volume analysis due to the drag of the wind tunnel seen in the calculated velocities of 15.66 and 2.76 m/s at these positions.

upstream CV						
position (m)	width (m)	velocity (m/s)	gauge pressure (Pa)	absolute pressure (Pa)	momentum flow in (kg*m/s^2)	pressure force (kg*m/s^2)
0.0254	0.3048	20.34938224	421.030008254117	100078.97	3.9075348599815	774.80401495
0.0508	0.3048	20.24703075	420.865814652602	100079.134	3.8535099765654	774.81354875
0.0762	0.3048	20.067494416	418.567104231395	100081.433	3.7651634025083	774.84691706
0.1016	0.3048	19.781590096	412.245650573077	100087.754	3.6628339174495	774.89490387
0.127	0.3048	19.522412804	406.17048731703	100093.83	3.6100802077607	774.9225519
0.1524	0.3048	19.498367945	405.103228907184	100094.897	3.6291478136723	774.91556044
0.1778	0.3048	19.625128521	407.976616933693	100092.023	3.6920709131806	774.8933149
0.2032	0.3048	19.835722226	410.850004960201	100089.15	3.7480025571879	774.88060316
0.2286	0.3048	19.92337325	411.260488963988	100088.74	ignore b/c drag from tunnel	ignore b/c drag from tunnel
0.254	0.3048	19.940185045	411.342585764746	100088.657	ignore b/c drag from tunnel	ignore b/c drag from tunnel
0.2794	0.3048	17.224464521	346.732403568684	100153.268		
Force (N):					29.868343648306	6198.971415

	downstream CV						
position (in)	position (m)	width (m)	velocity (m/s)	gauge pressure (Pa)	absolute pressure (Pa)	momentum flow out (kg*m/s^2)	pressure force (kg*m/s^2)
1	0.0254	0.3048	20.975371916	435.60219	100064.39781	4.161610708753	774.6904037935
2	0.0508	0.3048	20.920184345	435.643239	100064.35676	4.136822821068	774.6830150962
3	0.0762	0.3048	20.850391816	437.510941	100062.48906	4.010499931904	774.6469455417
4	0.1016	0.3048	20.273493506	444.961226	100055.03877	1.875272452321	387.2985180416
4.5	0.1143	0.3048	19.491492537	450.404244	100049.59576	1.693295988403	387.2752396727
5	0.127	0.3048	18.282844621	456.988407	100043.01159	0.739968711544	193.6296591109
5.25	0.13335	0.3048	17.027334936	458.630343	100041.36966	0.668643948181	193.6282608198
5.5	0.1397	0.3048	16.555880544	458.433311	100041.56669	0.664353736851	193.6270690944
5.75	0.14605	0.3048	16.920732077	459.861795	100040.1382	0.727157663822	193.6279509712
6	0.1524	0.3048	18.08520801	457.522036	100042.47796	0.814511133405	193.6317247682
6.25	0.15875	0.3048	18.973593263	455.962197	100044.0378	0.879460166041	193.6380011885
6.5	0.1651	0.3048	19.541284437	451.036389	100048.96361	0.946315959268	193.6455487825
6.75	0.17145	0.3048	20.405703053	448.163001	100051.837	0.983021100648	193.6500574768
7	0.1778	0.3048	20.318071253	446.377395	100053.6226	2.002389825065	387.3121116556
7.5	0.1905	0.3048	20.777945954	441.964692	100058.03531	2.059878656889	387.3266268706
8	0.2032	0.3048	20.906244479	438.877853	100061.12215	4.157701849542	774.6700173448
9	0.2286	0.3048	20.96962	437.634086	100062.36591	ignore b/c drag from tunnel	
10	0.254	0.3048	15.65712274	443.508112	100056.49189	ignore b/c drag from tunnel	
11	0.2794	0.3048	2.7569752763	346.445065	100153.55494		
					Force (N):	30.52090465371	6196.981150229

Control Volume Sample Drag Calculation

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA = \sum \vec{F}$$

CV integral is 0 because system is steady state - assuming no body forces & inviscid flow

$$F_D = \left[\int_{y_{1,bot}}^{y_{1,top}} \rho v_1^2 w dy - \int_{y_{2,bot}}^{y_{2,top}} \rho v_2^2 w dy \right] + \left[\int_{y_{1,bot}}^{y_{1,top}} p_{0,1} w dy - \int_{y_{2,bot}}^{y_{2,top}} p_{0,2} w dy \right]$$

0 + (total momentum flow delta) = (drag force) - (total force from pressure delta)

(drag force) = (total momentum flow delta) + (total force from pressure delta)

= (29.868 - 30.521) + (6198.971 - 6196.981)

Drag = **1.338 N**

Using the same C_D function as defined above, yields the following for the CV:

$C_D = 1.005$

Which, when compared against the given C_D vs Reynolds' number graph, is almost exactly what should be expected for Reynolds' number for 20 m/s of 5×10^5 .

Python Plotting & Calculation Code:

```
import numpy as np
import matplotlib.pyplot as plt
import json
import csv
from math import cos, pi, radians, sqrt

def main():
    global density
    density = 1.225 # air density

    global lab_data
    lab_data = json.load(open("lab_data.json", "r+"))

    global voltage_to_pressure_func
    voltage_to_pressure_func = step_1(plot = True)

    # surface pressure analysis
```

```

slow_pressure_list, fast_pressure_list = step_2(plot = True)
step_3(slow_pressure_list, fast_pressure_list, plot = True)

slow_drag, fast_drag = 0.415, 1.763 # get values from spreadsheet

# control volume analysis
slow_cd = cd(slow_drag, "slow_speed")
fast_cd = cd(fast_drag, "fast_speed")
print(slow_cd, fast_cd)

up_v, down_v = step_4(plot = True)
up_p, down_p = step_5(plot = True)

# up_v, up_p, down_v, and down_p to be used in spreadsheet to integrate
cv_drag = 1.3377 # get values from spreadsheet

cv_fast_cd = cd(cv_drag, "fast_speed")
print(cv_fast_cd)

# linear fit of calibration data
def step_1(plot = False):
    # converts inH2O to pascals
    inH2O_to_Pa = lambda pressure_list: [x * 248.84 for x in pressure_list]

    y = inH2O_to_Pa(lab_data.get("manometer").get("manometer_height"))
    x = lab_data.get("manometer").get("voltage_readout")
    voltage_to_pressure_func = np.poly1d(np.polyfit(x, y, 1))

    if plot:
        plt.plot(x, y, 'yo', x, voltage_to_pressure_func(x), '--k')
        plt.ylabel("Pressure Reading (Pa)")
        plt.xlabel("Voltage Reading (V)")
        plt.show()

    return voltage_to_pressure_func

# show pressure distribution around cylinder for 2 different velocities
def step_2(plot = False):
    slow_volt = lab_data.get("slow_speed_voltage")
    fast_volt = lab_data.get("fast_speed_voltage")

```



```

    slow_speed_data = [voltage_to_pressure_func(avg(lst)) for lst in
slow_volt]
    fast_speed_data = [voltage_to_pressure_func(avg(lst)) for lst in
fast_volt]
    theta = [i for i in range(0,190,15)]

    if plot:
        plt.plot(theta, slow_speed_data)
        plt.plot(theta, fast_speed_data)
        plt.ylabel("Pressure (Pa)")
        plt.xlabel("Angle (degrees)")
        plt.legend(["10 m/s", "20 m/s"])
        plt.title("Pressure Distribution")
        plt.show()

    return slow_speed_data, fast_speed_data

# show Cp distribution around cylinder for 2 different velocities
def step_3(slow_pressure_list, fast_pressure_list, plot = False):
    up_p = lab_data.get("upstream_pressures")
    def cp(data, speed):
        #  $C_p = (\text{surface pressure} - \text{upstream static pressure}) / (\text{upstream dynamic pressure})$ 
        up_dyn_p =
voltage_to_pressure_func(up_p.get(speed).get("voltage_dyn"))
        up_stat_p =
voltage_to_pressure_func(up_p.get(speed).get("voltage_stat"))
        return (-data + up_stat_p) / up_dyn_p

    slow_cp_data = [cp(x, "slow_speed") for x in slow_pressure_list]
    fast_cp_data = [cp(x, "fast_speed") for x in fast_pressure_list]
    theta = [i for i in range(0,190,15)]

    if plot:
        plt.plot(theta, slow_cp_data)
        plt.plot(theta, fast_cp_data)
        plt.ylabel("Cp")
        plt.xlabel("Angle (degrees)")
        plt.legend(["10 m/s", "20 m/s"])
        plt.title("Pressure Distribution")

```

```

plt.show()

# find velocity profiles up & downstream and plot them against position
def step_4(plot = False):
    #  $p = 1/2 * \text{density} * \text{velocity}^2$  (pressure to velocity func)
    p_to_vel = lambda pressure : sqrt(pressure * 2 / density)

    up_voltages = [avg(data) for data in
lab_data.get("upstream_cv").get("p_dyn")]
    down_voltages = [avg(data) for data in
lab_data.get("downstream_cv").get("p_dyn")]
    up_velocities = [p_to_vel(voltage_to_pressure_func(volt)) for volt in
up_voltages]
    down_velocities = [p_to_vel(voltage_to_pressure_func(volt)) for volt in
down_voltages]

    if plot:
        plt.plot(lab_data.get("upstream_cv").get("positions"),
up_velocities)
        plt.plot(lab_data.get("downstream_cv").get("positions"),
down_velocities)
        plt.ylabel("Velocity (m/s)")
        plt.xlabel("Position (in)")
        plt.legend(["Upstream", "Downstream"])
        plt.title("Velocity Profiles")
        plt.show()

    return up_velocities, down_velocities

# plot position against static pressure
def step_5(plot = False):
    up_voltages = [avg(data) for data in
lab_data.get("upstream_cv").get("p_static")]
    down_voltages = [avg(data) for data in
lab_data.get("downstream_cv").get("p_static")]
    up_pressures = [voltage_to_pressure_func(volt) for volt in up_voltages]
    down_pressures = [voltage_to_pressure_func(volt) for volt in
down_voltages]

    if plot:

```

```

        plt.plot(lab_data.get("upstream_cv").get("positions"),
up_pressures)
        plt.plot(lab_data.get("downstream_cv").get("positions"),
down_pressures)
        plt.ylabel("Pressure (Pa)")
        plt.xlabel("Position (in)")
        plt.legend(["Upstream", "Downstream"])
        plt.title("Static Pressure Profiles")
        plt.show()
    return up_pressures, down_pressures

# # # helper functions # # #

# Cd = drag/(dynamic_pressure * Area)
def cd(drag, speed):
    dynamic_voltage =
lab_data.get("upstream_pressures").get(speed).get("voltage_dyn")
    area = lab_data.get("cylinder_data").get("length") *
lab_data.get("cylinder_data").get("diameter")
    return drag / (voltage_to_pressure_func(dynamic_voltage) * area)

avg = lambda a_list: sum(a_list)/len(a_list)

# # # # # #

if __name__ == "__main__":
    main()

```

Data JSON File:

```

{
    "manometer":
    {
        "manometer_height": [0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0],
"voltage_readout": [-0.03, 0.485, 0.988, 1.710, 2.012, 2.502, 3.018, 3.525, 4.046, 4
.550, 5.054]
    },
    "slow_speed_voltage":
    [

```

```
[0.255, 0.254, 0.256],
[0.293, 0.294, 0.292],
[0.426],
[0.610, 0.607, 0.610, 0.600, 0.603],
[0.782, 0.770, 0.775, 0.771, 0.774],
[0.811, 0.802, 0.832, 0.811, 0.814, 0.820],
[0.772, 0.762, 0.763, 0.781, 0.765, 0.766],
[0.772, 0.750, 0.774, 0.768, 0.762, 0.748, 0.767],
[0.779, 0.775, 0.769, 0.754, 0.753],
[0.763, 0.757, 0.768, 0.758, 0.766, 0.752],
[0.784, 0.787, 0.763, 0.793, 0.770],
[0.767, 0.751, 0.780, 0.750, 0.775, 0.772],
[0.879, 0.757, 0.751, 0.767, 0.781, 0.783]
],
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    [0.702, 0.699, 0.701, 0.700, 0.697],
    [0.844, 0.847, 0.843, 0.841, 0.848, 0.845],
    [1.389, 1.382, 1.385, 1.380, 1.390, 1.385, 1.395],
    [2.133, 2.135, 2.141, 2.133, 2.154, 2.129, 2.130],
    [2.785, 2.763, 2.805, 2.799, 2.785],
    [2.957, 2.983, 2.976, 2.953, 2.998],
    [2.810, 2.816, 2.848, 2.803, 2.789, 2.870],
    [2.805, 2.781, 2.756, 2.784, 2.771, 2.734],
    [2.774, 2.813, 2.754, 2.764, 2.800],
    [2.868, 2.837, 2.832, 2.861, 2.823, 2.861],
    [2.947, 2.784, 2.893, 2.897, 2.848, 2.795],
    [2.862, 2.884, 2.902, 2.803, 2.903, 2.763],
    [2.918, 2.913, 2.832, 2.922, 2.889]
],
"upstream_pressures":
{
    "slow_speed":
    {
        "voltage_dyn":0.236,
        "voltage_stat":0.492
    },
    "fast_speed":
    {
```

```
        "voltage_dyn":0.940,
        "voltage_stat":1.640
    },
    "cylinder_data":
    {
        "length":0.3048,
        "diameter":0.019,
        "note":"IN METERS"
    },
    "upstream_cv":
    {
        "positions":[1,2,3,4,5,6,7,8,9,10,11],
        "p_static":
        [
            [1.719, 1.717, 1.713],
            [1.713, 1.720, 1.714],
            [1.711, 1.709, 1.699],
            [1.680, 1.684, 1.678],
            [1.657, 1.654, 1.657],
            [1.654, 1.653, 1.648],
            [1.663, 1.662, 1.665],
            [1.675, 1.675, 1.675],
            [1.675, 1.677, 1.678],
            [1.676, 1.677, 1.678],
            [1.419, 1.412, 1.413]
        ],
        "p_dyn":
        [
            [1.035, 1.038, 1.037],
            [1.026, 1.027, 1.026],
            [1.012, 1.004, 1.009],
            [0.980, 0.978, 0.982],
            [0.958, 0.954, 0.952],
            [0.952, 0.953, 0.952],
            [0.963, 0.965, 0.966],
            [0.983, 0.985, 0.988],
            [0.994, 0.995, 0.993],
            [0.997, 0.996, 0.994],
            [0.745, 0.742, 0.747]
```

```

    ],
    "downstream_cv":
    {
"positions": [1,2,3,4,4.5,5,5.25,5.5,5.75,6,6.25,6.5,6.75,7,7.5,8,9,10,11],
    "p_static":
    [
        [1.774, 1.780, 1.773, 1.775],
        [1.779, 1.776, 1.772],
        [1.781, 1.787, 1.781, 1.784],
        [1.806, 1.824, 1.810, 1.814],
        [1.832, 1.847, 1.831, 1.828, 1.840],
        [1.858, 1.870, 1.860, 1.856, 1.868, 1.862],
        [1.889, 1.864, 1.866, 1.867, 1.859],
        [1.866, 1.855, 1.863, 1.882, 1.875],
        [1.873, 1.869, 1.883, 1.871],
        [1.861, 1.870, 1.860, 1.854, 1.876, 1.866],
        [1.853, 1.846, 1.849, 1.864, 1.869, 1.868],
        [1.836, 1.843, 1.841, 1.833, 1.841, 1.835],
        [1.824, 1.829, 1.828, 1.831, 1.816, 1.831],
        [1.818, 1.821, 1.822, 1.816],
        [1.796, 1.803, 1.794, 1.801, 1.807, 1.807],
        [1.782, 1.793, 1.790, 1.786, 1.793],
        [1.785, 1.781, 1.786, 1.783],
        [1.802, 1.804, 1.811, 1.812, 1.809],
        [1.417, 1.416, 1.408, 1.413]
    ],
    "p_dyn":
    [
        [1.099, 1.102, 1.103, 1.100],
        [1.098, 1.096, 1.094, 1.093],
        [1.092, 1.091, 1.082, 1.087],
        [1.037, 1.024, 1.039, 1.017, 1.028],
        [0.962, 0.942, 0.974, 0.962, 0.929, 0.941],
        [0.840, 0.879, 0.807, 0.893, 0.826, 0.756, 0.864, 0.840],
        [0.690, 0.770, 0.731, 0.673, 0.789, 0.707, 0.733, 0.730],
        [0.679, 0.652, 0.708, 0.680, 0.704, 0.639, 0.666, 0.780],
        [0.651, 0.722, 0.785, 0.747, 0.686, 0.674, 0.836, 0.650],
        [0.753, 0.808, 0.793, 0.884, 0.835, 0.784, 0.812, 0.893],
    ]

```

```
[0.913, 0.845, 0.919, 0.942, 0.902, 0.874, 0.936, 0.886],  
[0.960, 0.938, 0.974, 0.969, 0.953, 0.941, 0.907, 1.010],  
[0.998, 1.032, 0.982, 0.997, 1.306, 1.001, 1.022, 1.001],  
[1.026, 1.021, 1.038, 1.041, 1.035, 1.040],  
[1.081, 1.069, 1.083, 1.077, 1.082, 1.091],  
[1.097, 1.089, 1.095, 1.097, 1.091],  
[1.095, 1.104, 1.102, 1.103, 1.098],  
[0.634, 0.613, 0.615, 0.604],  
[0.022, 0.043, 0.025, 0.013]  
]  
}  
}
```