Attitude Description Sets: A Cheatsheet

The Attitude Description Problem

Parameter Sets

Directional Cosine Matrix

Addition/Subtraction

Differential Kinematic Relation

Principle Rotation Vector

Euler Angles

Quaternions/Euler Parameters

Addition/Subtraction

Addition: $[FN(\beta)] = [FB(\beta'')]\,[BN(\beta')]$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{bmatrix} \beta_0'' & -\beta_1'' & -\beta_2'' & -\beta_3'' \\ \beta_1'' & \beta_0'' & \beta_3'' & -\beta_2'' \\ \beta_2'' & -\beta_3'' & \beta_0'' & \beta_1'' \\ \beta_3'' & \beta_2'' & -\beta_1'' & \beta_0'' \end{bmatrix} \begin{pmatrix} \beta_0' \\ \beta_1' \\ \beta_2' \\ \beta_3' \end{pmatrix}$$

$$= \begin{bmatrix} \beta_0' & -\beta_1' & -\beta_2' & -\beta_3' \\ \beta_1' & \beta_0' & \beta_3' & -\beta_2' \\ \beta_2' & -\beta_3' & \beta_0' & \beta_1' \\ \beta_3' & \beta_2' & -\beta_1' & \beta_0' \end{bmatrix} \begin{pmatrix} \beta_0'' \\ \beta_1'' \\ \beta_2'' \\ \beta_3'' \end{pmatrix}$$

Subtraction: $[BN(\beta')] = \left[FB(\beta'')^T\right][FN(\beta)]$

$$\begin{pmatrix} \beta_0' \\ \beta_1' \\ \beta_2' \\ \beta_3' \end{pmatrix} = \begin{bmatrix} \beta_0'' & \beta_1'' & \beta_2'' & \beta_3'' \\ -\beta_1'' & \beta_0'' & -\beta_3'' & \beta_2'' \\ -\beta_2'' & \beta_3'' & \beta_0'' & -\beta_1'' \\ -\beta_3'' & -\beta_2'' & \beta_1'' & \beta_0'' \end{bmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

Differential Kinematic Relation

$$\begin{pmatrix} \dot{\beta}_0 \\ \dot{\beta}_1 \\ \dot{\beta}_2 \\ \dot{\beta}_3 \end{pmatrix} = \frac{1}{2} \begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

Gibbs Vector/Classical Rodriguez Parameters

Definition

Euler Parameter relations:
$q_i = \frac{\beta_i}{\beta_0}$
inversely,
$\beta_0 = \frac{1}{1+\mathbf{q}^T\mathbf{q}}; \beta_i = \frac{q_i}{1+\mathbf{q}^T\mathbf{q}}$
Principle Rotation Vector relations:
$\sigma = tan\left(\frac{\Phi}{2}\right)\hat{\mathbf{e}}$;
$\sigma \approx \left(\frac{\Phi}{2}\right)\hat{\mathbf{e}}$

$$[C] = \frac{1}{1+\mathbf{q}^T\mathbf{q}} \begin{bmatrix} 1+q_1^2-q_2^2-q_3^2 & 2(q_1q_2+q_3) & 2(q_1q_3-q_2) \\ 2(q_1q_2-q_3) & 1-q_1^2+q_2^2-q_3^2 & 2(q_2q_3+q_1) \\ 2(q_1q_3+q_2) & 2(q_2q_3-q_1) & 1-q_1^2-q_2^2+q_3^2 \end{bmatrix}$$

$$= \frac{1}{1+\mathbf{q}^T\mathbf{q}}\left(\left(1-\mathbf{q}^T\mathbf{q}\right)[I_{3x3}] + 2\mathbf{q}\mathbf{q}^T - 2\,[\tilde{\mathbf{q}}]\right)$$

Modified Rodriguez Parameters

Definition

Euler Parameter relations:
$\sigma_i = \frac{\beta_i}{1+\beta_0}$
inversely,
$\beta_0 = \frac{1-\sigma^2}{1+\sigma^T\sigma}; \beta_i = \frac{2\sigma_i}{1+\sigma^T\sigma}$
Principle Rotation Vector relations:
$\sigma = tan\left(\frac{\Phi}{4}\right)\hat{\mathbf{e}}$;
$\sigma \approx \left(\frac{\Phi}{4}\right)\hat{\mathbf{e}}$
Classical Rodriguez Parameter relations:
$\mathbf{q} = \frac{2\sigma}{1-\sigma^2}; \sigma = \frac{\mathbf{q}}{1+\sqrt{1+q^2}}$

Addition/Subtraction

Addition:
$[FN(\beta)] = [FB(\beta'')]\,[BN(\beta')]$
$\sigma = \frac{(1-|\sigma'|^2)\sigma''+(1-|\sigma''|^2)\sigma'-2\sigma''\times\sigma'}{1+|\sigma'|^2|\sigma''|^2-2\sigma'\cdot\sigma''}$
Subtraction:
$[BN(\beta')] = \left[FB(\beta'')^T\right][FN(\beta)]$
$\sigma'' = \frac{(1+|\sigma'|^2)\sigma-(1-|\sigma|^2)\sigma'+2\sigma\times\sigma'}{1+|\sigma'|^2|\sigma|^2+2\sigma'\cdot\sigma}$

Differential Kinematic Relation

$\dot{\sigma} =$

$$\frac{1}{4}\begin{bmatrix} 1-\sigma^2+2\sigma_1^2 & 2(\sigma_1\sigma_2-\sigma_3) & 2(\sigma_1\sigma_3+\sigma_2) \\ 2(\sigma_1\sigma_2+\sigma_3) & 1-\sigma^2+2\sigma_2^2 & 2(\sigma_2\sigma_3-\sigma_1) \\ 2(\sigma_1\sigma_3-\sigma_2) & 2(\sigma_2\sigma_3+\sigma_1) & 1-\sigma^2+2\sigma_3^2 \end{bmatrix}\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

Resources