Photonic Qubits Lab Exercise: Lab Manual

at DTU Physics' Teaching Lab "Nanoteket"

In this exercise, you will perform basic quantum optics experiments that demonstrate the particle nature of light, entanglement, and encoding of qubits into the polarisation of photons.

You will use an educational setup from QuTools consisting of a laser, non-linear crystals, mirrors, waveplates, polarisers, fibres and photon counters.

Please read through these instructions before starting the exercises.

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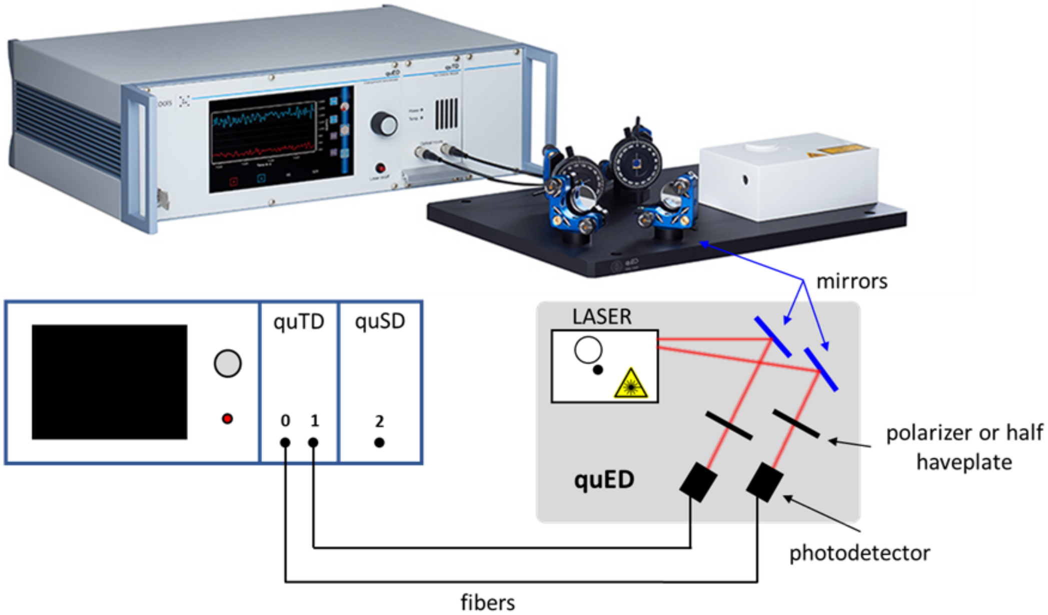
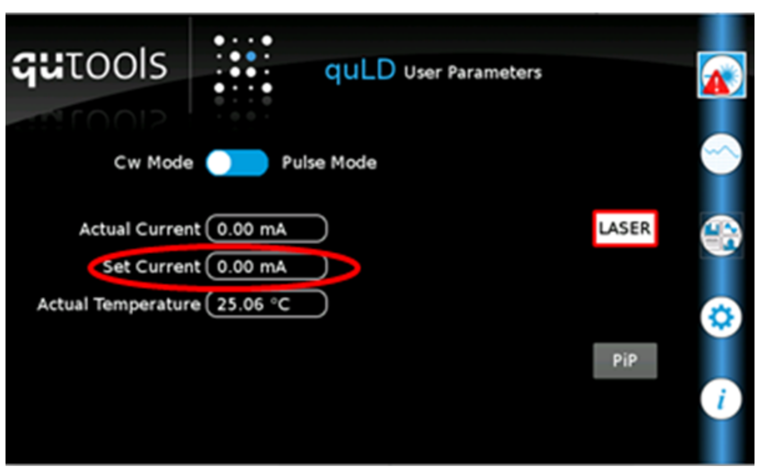
Gentle reminders

- Don't shoot a visible light to the single-photon detectors. These detectors are very sensitive to light and can be damaged when exposed to a strong light.
- Don't touch the optics. The surfaces of the fibre collimators and mirrors are prone to the oil stains and they are very hard to clean.
- Be careful with fibre tips. Be sure not to hit the tips on the metal parts of the connector. You are very welcome to take a cut-off fibre to practise connecting, and proceed to the setup once you are ready
- Don't panic if the coincidence disappears or decreases too much - ask the teacher for help. Turn the knobs of the collimators and mirrors gently during the optimization of counting rates.
- During the alignment optimization, you can turn the knob to tilt/flip the mirror and then use the same tilt/flip knob on the collimator to max the counting rate. If the counting rate is better than before, then you know the tweak is in the correct direction.

Handling of the quED set-up

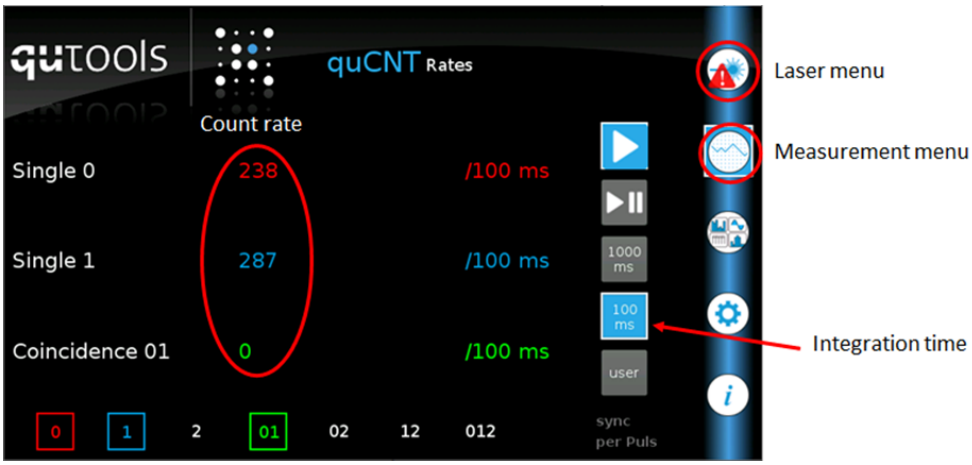


- Check if the quCR is well plugged to the sector and if the white box is linked with the quCR by the DB9 cable.
- Connect both photodetectors to optical inputs 0 and 1, thanks to the single-mode optical fibres
- Set both polarisers to 45 degrees.
- Turn on the quCR using the key.
- Set the current you will send to the laser (the value is written on the plastic box of the laser). Then turn on the laser by pushing the red button on the quCR.



- Switch the interface to the measurement menu and set the integration time to 100ms. You should see a count rate number displayed on the screen.
- Optimise the coupling to both detectors by turning the knobs to obtain the maximum count rate on single 1 and 0. Do it carefully and keep in mind where the initial knobs positions were.

- By rotating both polarisers observe the evolution of count rate for single detection and coincidences.
- Using a small paper block the output of the laser (white box). Observe the count on the 3 channels. What do you observe? How can you explain this?
- Remove the piece of paper. When rotating both polarisers you can see the coincidence count rate vary but it never reaches 0, what could be a reason for that?
- For Exercise 3 you will need to use a 2 quarter wave plate. These wave plates can be rotated the same as the polarisers can be. Find the angle that corresponds to the neutral axis of both your quarter wave plates. Keep it in mind, you will need it later.



Exercise 1: Particle nature of photons, Hanbury-Brown and Twiss experiment

Theory

Showing the particle nature of photons is a tricky experiment. Indeed, an obvious way would be to use a "single" photon and show that it cannot split up as a wave. By using a beam splitter and two detectors this experiment could be easy.

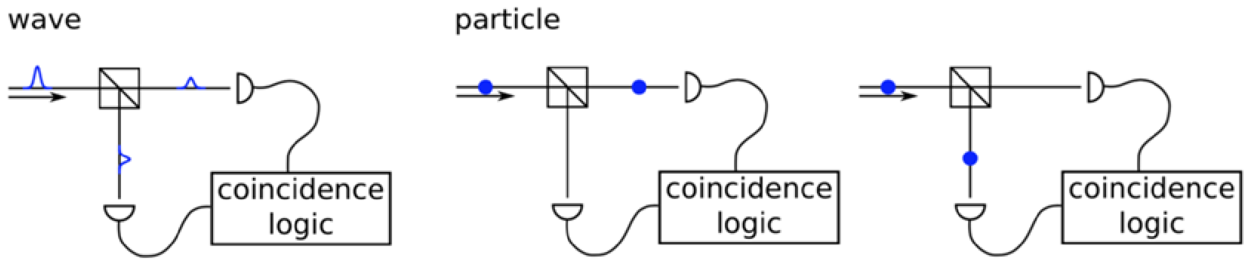


Figure 1: Beam splitter action on light

However, we do not have a real single photon source here. The spontaneous parametric down conversion (SPDC) source like the one we use in the quED set-up does not act as a true single photon source but a so-called "heralded" single photon source. It generates pairs of photons. Because of energy and momentum conservation, there are always pairs produced, so when we see one photon in one arm, we know that there is one photon in the other arm. To show this, we use one arm of the quED as the trigger (heralding the existence of the photon in the other arm) and do single photon experiments with the second arm. Experimentally, we only look at coincidences between these arms. In terms of measurements, a high output (1) results if both of the inputs to the gate are the same. If at least one of inputs is low (0), a low output (0) results.

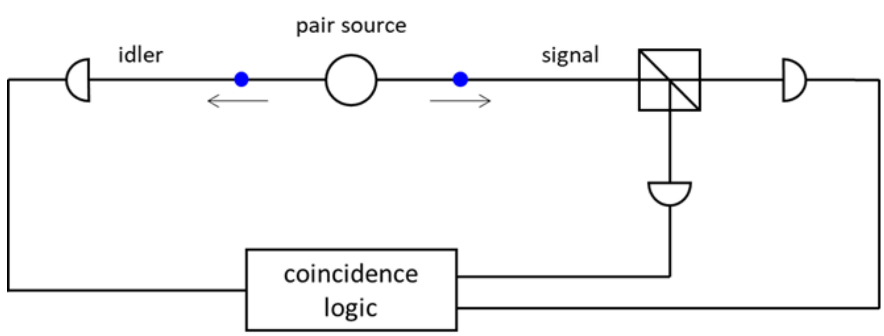


Figure 2: Simplified set-up of the experiment

Set-up

Steps to perform the experiment of HBT:

- "Deactivate the second BBO crystal by rotating it in the same position of the first one by rotating the half-wave plate." (this action should be carried out by the teacher).
- Place the fibred beam splitter between the first photodetector and the port 0 and 1 of the quED.
- Measure the coincidence counts. What do you observe ?

- Remove the fibre plugged in the port 1 and plug here the fibre coming from the second photo detector. Observe the coincidence counts.
- Unplugged the port 0 and plug into it the second beam splitter output. What do you observe as coincidences counts ?

In this case, we look at two-fold coincidences between the trigger and the detectors after the beam splitter. These are the heralded single photon events. If the photon could split up (or if we did not have heralded single photons), we would see a lot of 2-fold coincidences between the 2 outputs of the beam splitter. This is not the case. Why are there still some coincidences between the output of the beam splitter and the second photodetector? Do you have an explanation?

Exercise 2: Polarisation of single photons

In this exercise you will need to fill the following tables with the measurement you are going to make.

- By rotating the polarisers, measure the values displayed by the quED and fill the 3 following tables.

Single 0 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

Single 1 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

Coincidences 01 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

- What can you say about the photon you are observing ? Can you describe the state of the photons ?
- Call a teacher to rotate the half-wave plate inside the white box.
- Once again, by rotating the polarisers, measure the values displayed by the quED and fill the 3 following tables.

Single 0 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

Single 1 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

Coincidences 01 count rate				
Polariser angles	0°	45°	90°	-45°
0°				
45°				
90°				
-45°				

- What can you say about the photon you are observing ? Can you describe the state of the photons?
- Compare the new values with the precedent. What has changed after rotating the half wave plate ? Explain what you see and try to give an explanation.

Exercise 3a: Qubit tomography

In this exercise we will implement a one-qubit tomography: using a set of measurements to make a best guess about the qubit's quantum state.

To make such a best guess we expand the density matrix of the qubit on the Pauli basis:

$$\rho = \frac{1}{2}\left(I + x\sigma_x + y\sigma_y + z\sigma_z\right).$$

Its coefficients $x,\ y,\ z$ can be determined by noticing that $z = Tr(\rho\sigma_z)$ and similar for $x,\ y$. $Tr(\rho\sigma_z)$ is linked to the likelihood of obtaining the outcome +1 when we measure σ_z: let the number of photons registered detected at $|0\rangle$ (+1 eigenstate of σ_z) to be N_0 and the number of photons registered detected at $|1\rangle$ (-1 eigenstate of σ_z) to be N_1, then:

$$Tr(\rho\sigma_z) = \frac{N_0 - N_1}{N_0 + N_1}$$

1. We project qubit B on the horizontal polarisation, and do tomography on qubit A:

Tomography of qubit A | qubit B projected to $\vert\mathbf{0}\rangle = \mathbf{H}$ – polarisation						
Pauli operators	**Measurement Eigenstates**	**Polariser and waveplate angles**			**Coincidence 01**	**Expectations of Pauli operators**
		Polariser B	**QWP A**	**Polariser A**		
$\langle\sigma_z\rangle$	$+1 : \vert 0\rangle$	0°	0°	0°	$N_0 =$	$z =$
	$-1 : \vert 1\rangle$		0°	90°	$N_1 =$	
$\langle\sigma_x\rangle$	$+1 : \vert +\rangle$		45°	45°	$N_+ =$	$x =$
	$-1 : \vert -\rangle$		45°	-45°	$N_- =$	
$\langle\sigma_y\rangle$	$+1 : \vert +i\rangle$		0°	45°	$N_{+i} =$	$y =$
	$-1 : \vert -i\rangle$		0°	-45°	$N_{-i} =$	

2. We again do tomography on qubit A, but this time project qubit B on diagonal polarisation:

Tomography of qubit A | qubit B projected to $\vert+\rangle = \mathbf{D}$ – polarisation						
Pauli operators	**Measurement Eigenstates**	**Polariser and waveplate angles**			**Coincidence 01**	**Expectations of Pauli operators**
		Polariser B	**QWP A**	**Polariser A**		
$\langle\sigma_z\rangle$	$+1 : \vert 0\rangle$	0°	0°	0°	$N_0 =$	$z =$
	$-1 : \vert 1\rangle$		0°	90°	$N_1 =$	
$\langle\sigma_x\rangle$	$+1 : \vert +\rangle$		45°	45°	$N_+ =$	$x =$
	$-1 : \vert -\rangle$		45°	-45°	$N_- =$	
$\langle\sigma_y\rangle$	$+1 : \vert +i\rangle$		0°	45°	$N_{+i} =$	$y =$
	$-1 : \vert -i\rangle$		0°	-45°	$N_{-i} =$	

- Do you see the expectations change between the two experiments?
- From the expectations of the Pauli operators $(x,\ y,\ z)$, write the density matrix using the explicit definition of the Pauli operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- What is the length of the vector $(x,\ y,\ z)$ in both cases? Do you see a difference?

Exercise 3b:

In both of the experiments, the state of qubit A after projecting qubit B can be calculated using this formula:

$$|\Psi_A\rangle = \langle \pi_B | \Psi_{AB} \rangle,$$

where $|\Psi_{AB}\rangle$ is the two-qubit state before projecting the qubit B and $\langle \pi_B|$ is the basis to which the qubit B is projected. $\langle \pi_B|$ is the conjugate transpose of $|\pi_B\rangle$.

By inserting the initial state and the basis of projection to the above formula, we know in the ideal case, the final state in the two experiments will be:

$$|\Psi_{A,1}\rangle = \langle 0_B | \Psi_{AB} \rangle = |0\rangle,$$
$$|\Psi_{A,2}\rangle = \langle +_B | \Psi_{AB} \rangle = |+\rangle.$$

- Could you calculate what is the density matrix of these two states?
- Do they look similar with what you observed in the experiment?

Further, the "similarity" of your experimental state determined by tomography and the target state can be calculated by the fidelity:

$$F_{(1,2)} = \langle \Psi_{A(1,2)} | \rho | \Psi_{A(1,2)} \rangle$$

- Could you calculate the fidelity of the obtained states in the two tomography experiments, with respect to the target state? In which experiment do you get a higher fidelity?

Exercise 3c:

Insert a quarter-wave plate in path B and prepare an arbitrary state pointing in some non-trivial direction on the Bloch sphere. Now do a tomography of this state.

<table>
<tr><th colspan="7">Tomography of qubit A | qubit B projected to random polarisation using QWP</th></tr>
<tr><th rowspan="2">Pauli operators</th><th rowspan="2">Measurement Eigenstates</th><th colspan="3">Polariser and waveplate angles</th><th rowspan="2">Coincidence 01</th><th rowspan="2">Expectations of Pauli operators</th></tr>
<tr><th>Polariser B</th><th>QWP A</th><th>Polariser A</th></tr>
<tr><td rowspan="2">$\langle\sigma_z\rangle$</td><td>$+1 : \|0\rangle$</td><td rowspan="6">0°</td><td>0°</td><td>0°</td><td>$N_0 =$</td><td rowspan="2">$z =$</td></tr>
<tr><td>$-1 : \|1\rangle$</td><td>0°</td><td>90°</td><td>$N_1 =$</td></tr>
<tr><td rowspan="2">$\langle\sigma_x\rangle$</td><td>$+1 : \|+\rangle$</td><td>45°</td><td>45°</td><td>$N_+ =$</td><td rowspan="2">$x =$</td></tr>
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<tr><td rowspan="2">$\langle\sigma_y\rangle$</td><td>$+1 : \|+i\rangle$</td><td>0°</td><td>45°</td><td>$N_{+i} =$</td><td rowspan="2">$y =$</td></tr>
<tr><td>$-1 : \|-i\rangle$</td><td>0°</td><td>-45°</td><td>$N_{-i} =$</td></tr>
</table>

Useful Links

- Entanglement of photons by the QuTools set-up: `https://qutools.com/qued/photonsource`
- Light polarisation: https://emanim.szialab.org/index.html
- Tomography calculator on Kwiat's group website:
  http://tomography.web.engr.illinois.edu/TomographyDemo.php
- Tomography calculator tutorial:
  http://research.physics.illinois.edu/QI/Photonics/tomography-files/tutorial_for_tomography2019.pdf
  https://youtu.be/-T_GiHE4VGg
- Polarisation: https://www.wikiwand.com/en/Polarization_(physics)
- Jones representation of polarisation: https://www.wikiwand.com/en/Jones_vector
- Polariser: https://www.wikiwand.com/en/Polarizer
- Waveplate: https://www.wikiwand.com/en/Wave_plate
- Circular polarisation: https://www.wikiwand.com/en/Circular_polarization
- Linear polarisation: https://www.wikiwand.com/en/Linear_polarization
- SPDC: https://www.wikiwand.com/en/Spontaneous_parametric_down-conversion
- Quantum Flytrap (virtual lab): https://lab.quantumflytrap.com