Games

Non-cooperative multi-agent systems

Games

Many problems can be modelled as games: multiple agents with (possibly competing) interests:

- Chess
- Go
- Multiple agents competing for limited resources

Games and agents acting in them can be described by:

- **Actions** available to each agent in various stages of a game
- **Utility** (payoff) functions, one for each agent. They assign a real number to every possible outcome of the game (typically terminal states). [If the utility functions are the same the agents become cooperative.]
- **Strategy** functions, one for each agent. They determine what action must be taken in each state. [Typically the goal is to find a strategy that maximises the utility for one agent or for a group of agents.]

Example: rock paper scissors

		Bob		
		rock	*paper*	*scissors*
	rock	$0, 0$	$-1, 1$	$1, -1$
Alice	*paper*	$1, -1$	$0, 0$	$-1, 1$
	scissors	$-1, 1$	$1, -1$	$0{,}0$

- The game is not turn-based; it is a *simultaneous action game*.
- The games has 9 possible outcomes.
- The table shows two utility functions (Alice's and Bob's).

Game Trees

In *perfect-information games* (where agents can see which states they are in) a **game tree** is a finite tree where the nodes are states and the arcs correspond to actions by the agents. In particular:

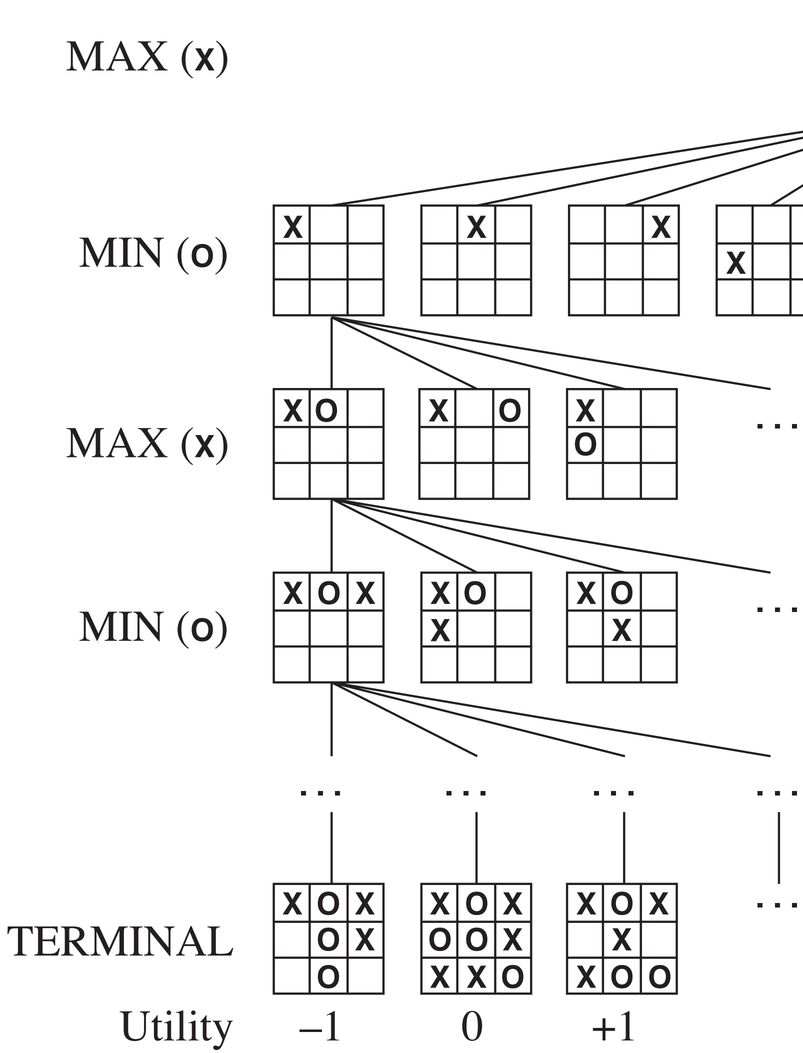
- Each internal node is labeled with an agent (or with nature). The agent is said to control the node.
- Each arc out of a node labeled with agent *i* corresponds to an action for agent *i*.
- Each internal node labeled with **nature** has a probability distribution over its children.
- The leaves represent final outcomes and are labeled with a utility foreach agent.

Perfect-information zero-sum turn-based games

Properties:

- Typically two agents
- They take turn to play
- There is no chance involved.
- The state of the game is fully observable by all agents
- Utility (payoff) values for each agent are the opposite of those of the other. Also called **zero sum**; the sum of the reward and penalty is constant.
- Because they are opposite of each other, we use one value and assume that one player tries to maximise the utility, the other minimise.
- This creates adversary.

Example: noughts and crosses



- A game tree can be seen as a contingency tree containing all possibilities
- The root is the current state
- An actual play is a path from the root to one of the leaf nodes

An optimal strategy: Min-Max function

Designed to find the best move at each stage:

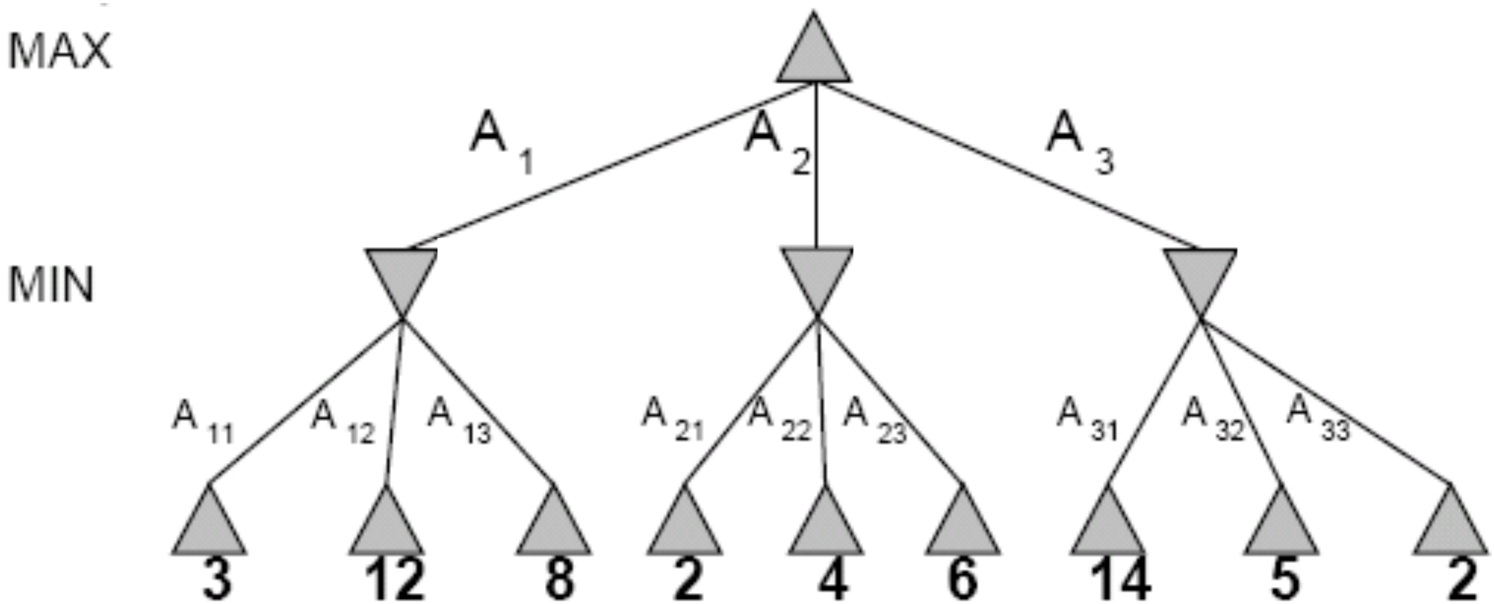
1. Generate the whole game tree, down to the leaves.

2. Apply utility (payoff) function to each leaf.

3. Back-up values from leaves through branch nodes:
   - a Max node computes the Max of its child values
   - a Min node computes the Min of its child values

4. At root:
   - If it's a **max node**, choose the move leading to the child with highest value.
   - If it's a **min node**, choose the move leading to the child with lowest value.

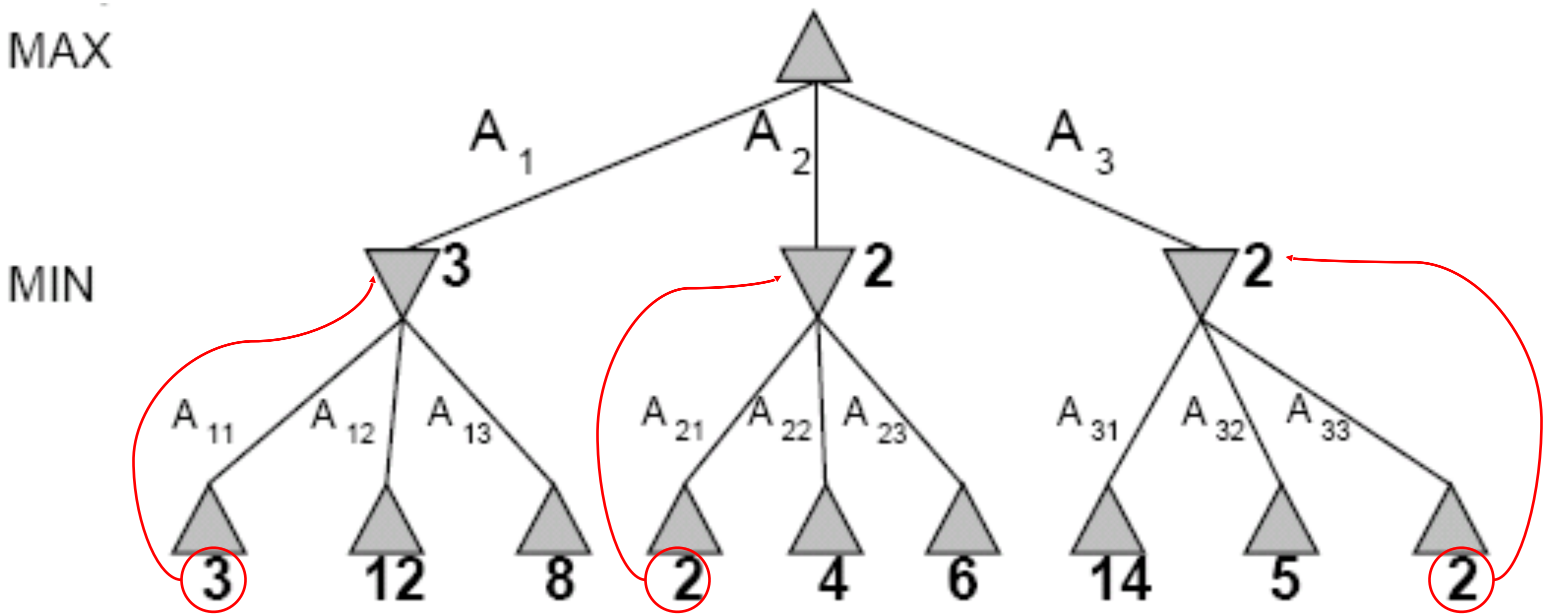
Example: a two-ply game tree



- Two symbols (triangles):
    - Max nodes pointing up
    - Min nodes pointing down

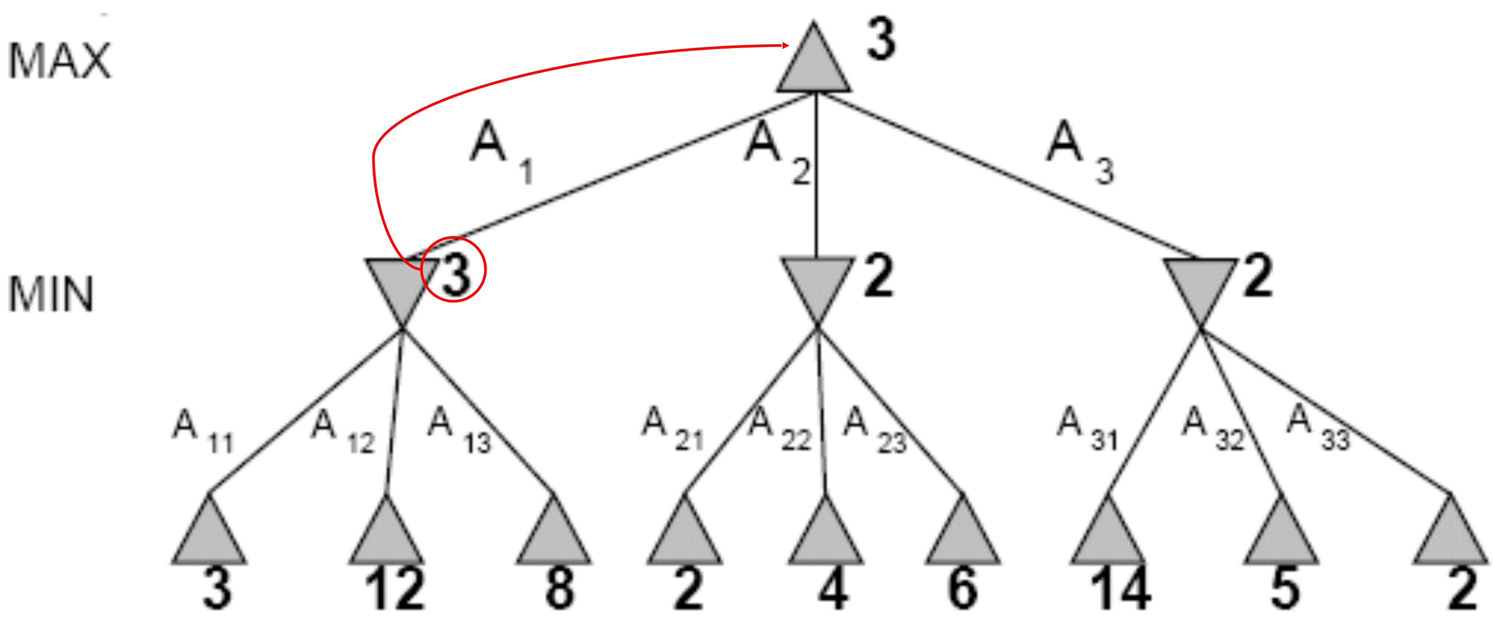
- Leaf (terminal) nodes have a utility (payoff) value.

Example: A Two-Ply Game Tree



- The utility of other nodes are determined by applying MinMax algorithm.

Example: A Two-Ply Game Tree



- The best action for Max is A1
- The best response from Min is A11.

MinMax Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
  inputs: *state*, current state in game

Assuming the root node is Max

  return the *a* in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(*a*, *state*))

---

function MAX-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for *a*, *s* in SUCCESSORS(*state*) **do** $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for *a*, *s* in SUCCESSORS(*state*) **do** $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

Functions used in the algorithm

Actions(state)

Returns the set of legal moves in the given state *state*.

Result(action, state)

Returns a state that is the result of applying action *action* on state *state*.

Successors(state)

Returns a list of pairs of action-state that can be reached from the given *state*.

Terminal-Test(state)

Is the game finished? True if finished, false otherwise.

Utility(state)

Gives numerical value of the terminal state *state*.

Game Tree Size

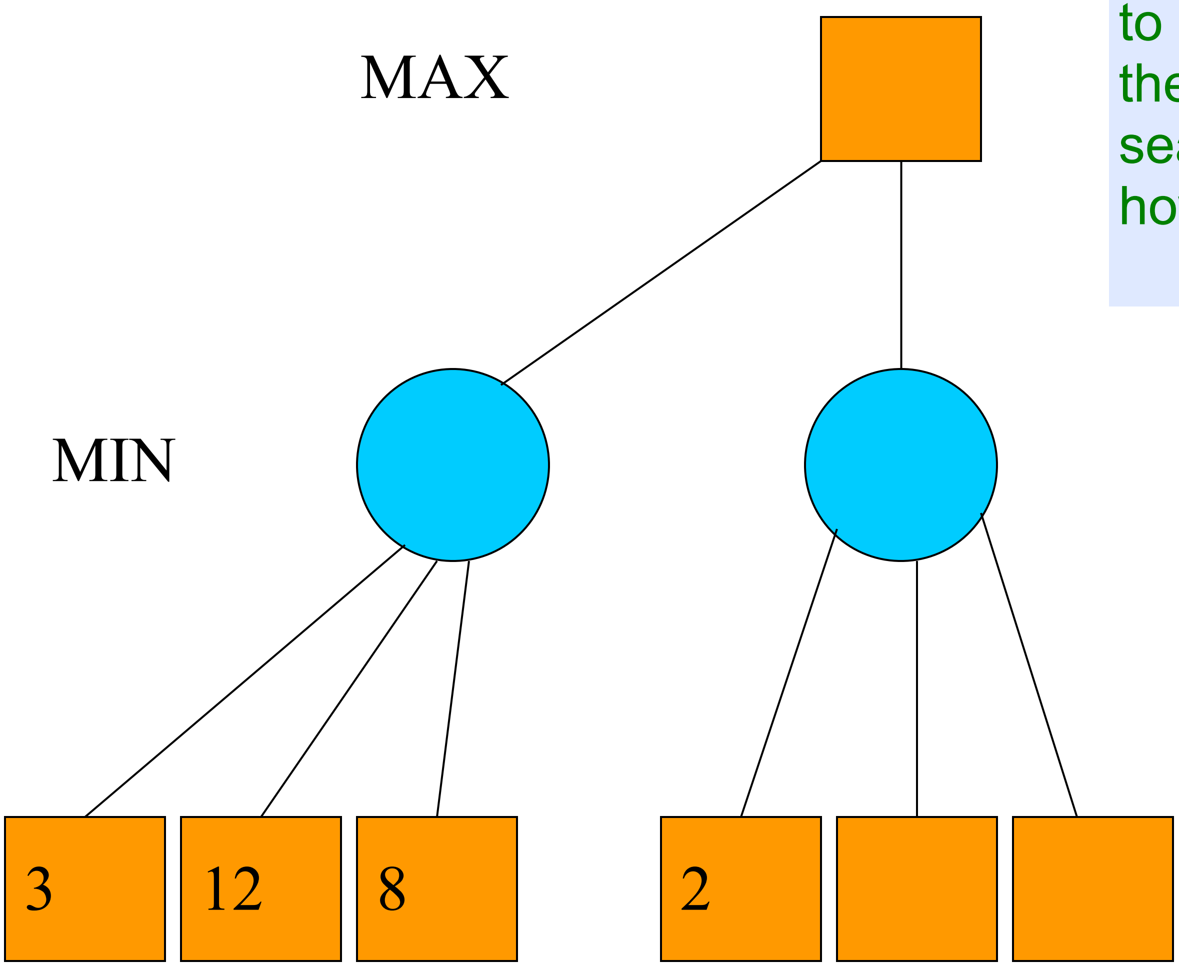
- Tic-Tac-Toe
  - $b \approx 5$ legal actions per state on average
  - total of 9 moves, $d = 9$
  - $b^d \approx 5^9 = 1{,}953{,}125$
  - → **Searching the entire tree quite reasonable**

- Chess
  - $b \approx 35$ (approximate average branching factor)
  - $d \approx 100$ (depth of game tree for a “typical” game)
  - $b^d \approx 35^{100} \approx 10^{154}$ nodes!!
  - → **Searching the entire tree completely infeasible**

How to reduce the search space

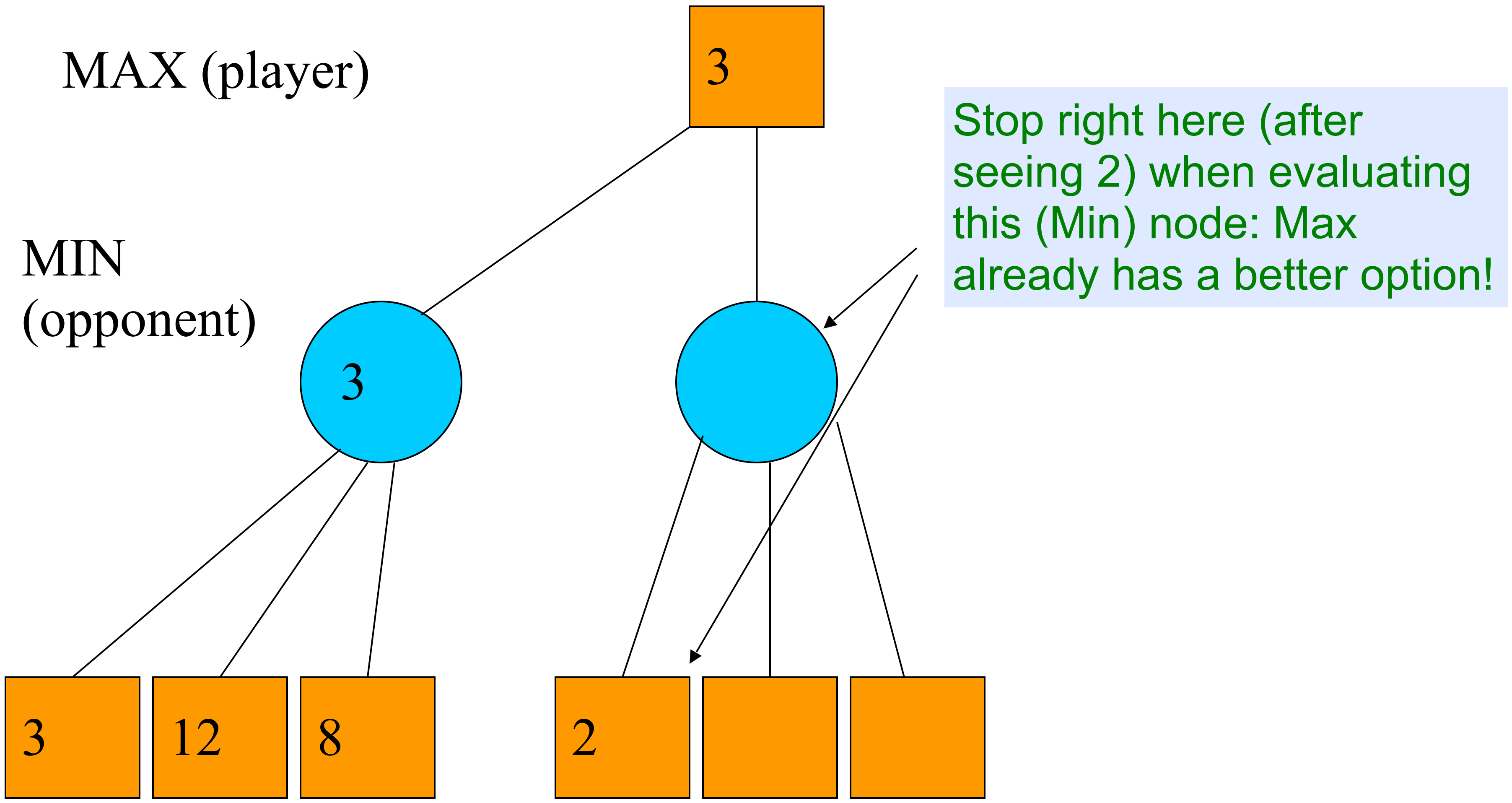
- Pruning the game tree
- Heuristic evaluations of states
- Table lookup instead of search (for opening and closing situations for example)

Pruning: The Idea

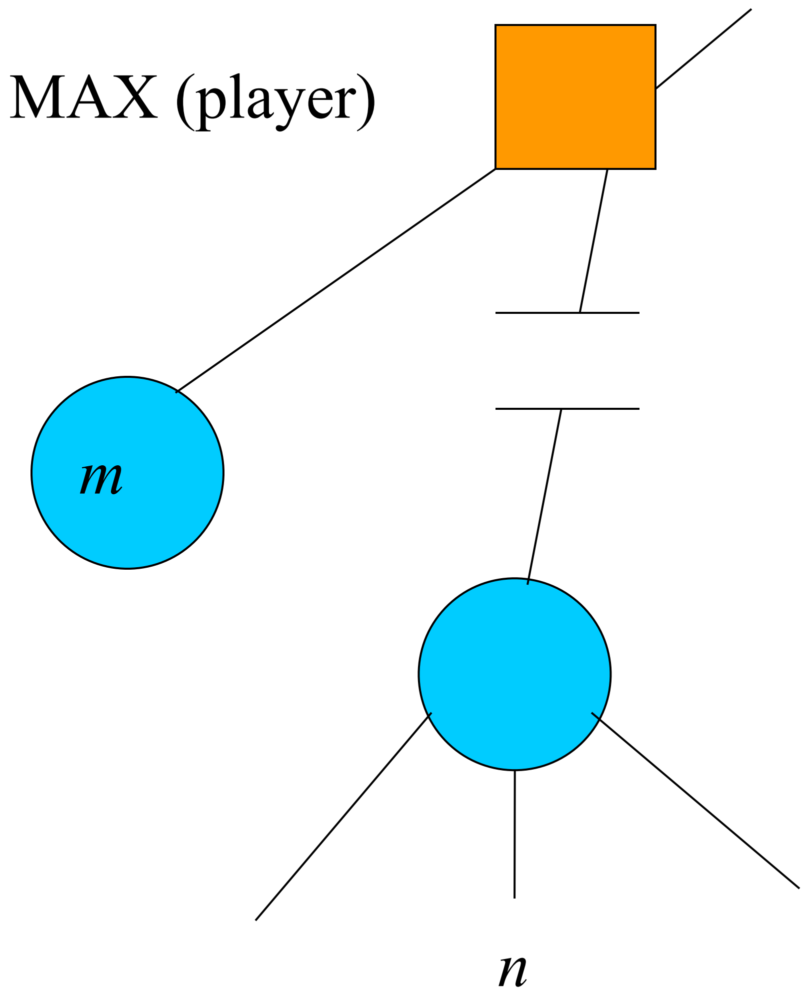


If an option is bad compared to other available options, there is no point in expending search time to find out exactly how bad.

Pruning: The Idea



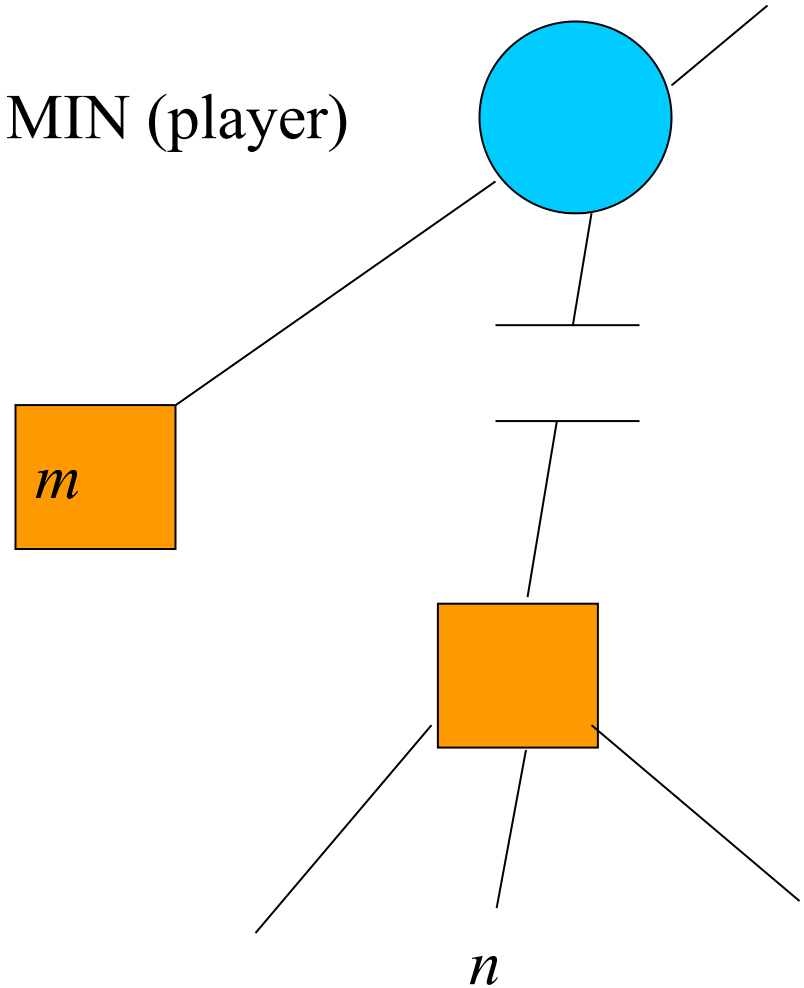
Alpha-Beta Pruning: The Concept



If *m* > *n*, **Max** would choose the *m*-node to get a guaranteed utility of at least *m.*

The Min node with utility *n* (or less) would never be reached; stop further evaluation.

Alpha-Beta Pruning: The Concept



If $m < n$, **Min** would choose the m-node to get a guaranteed utility of at most m.

The Max node with utility n (or more) would never be reached; stop further evaluation.

Alpha-Beta Pruning Algorithm

Depth first search

- only considers nodes along a single path from root at any time

α = highest-value choice found for MAX higher up in the tree

(initially, α = −infinity)

β = lowest-value choice found for MIN higher up in the tree

(initially, β = +infinity)

Pass current values of α and β down to child nodes during search.

Update values of α and β during search:

- MAX updates α at MAX nodes
- MIN updates β at MIN nodes

Prune remaining branches at a node when $\alpha \geq \beta$

Alpha-Beta Pruning Algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** an action
  $v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
  return the *action* in ACTIONS(*state*) with value v

Assuming the root node is Max

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    $\alpha \leftarrow$ MAX(α, v)
    if $\alpha \geq \beta$ **then return** v
  return v

---

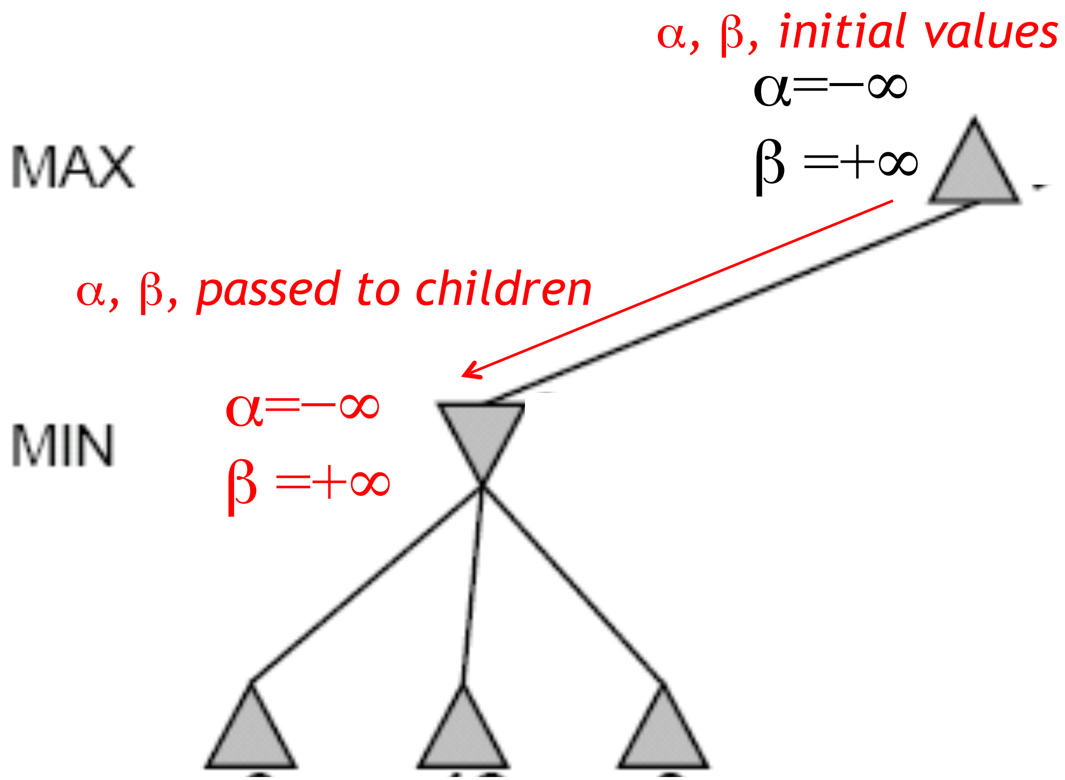
function MIN-VALUE(*state*, α, β) **returns** *a utility value*
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    $\beta \leftarrow$ MIN(β, v)
    if $\alpha \geq \beta$ **then return** v
  return v

When to Prune

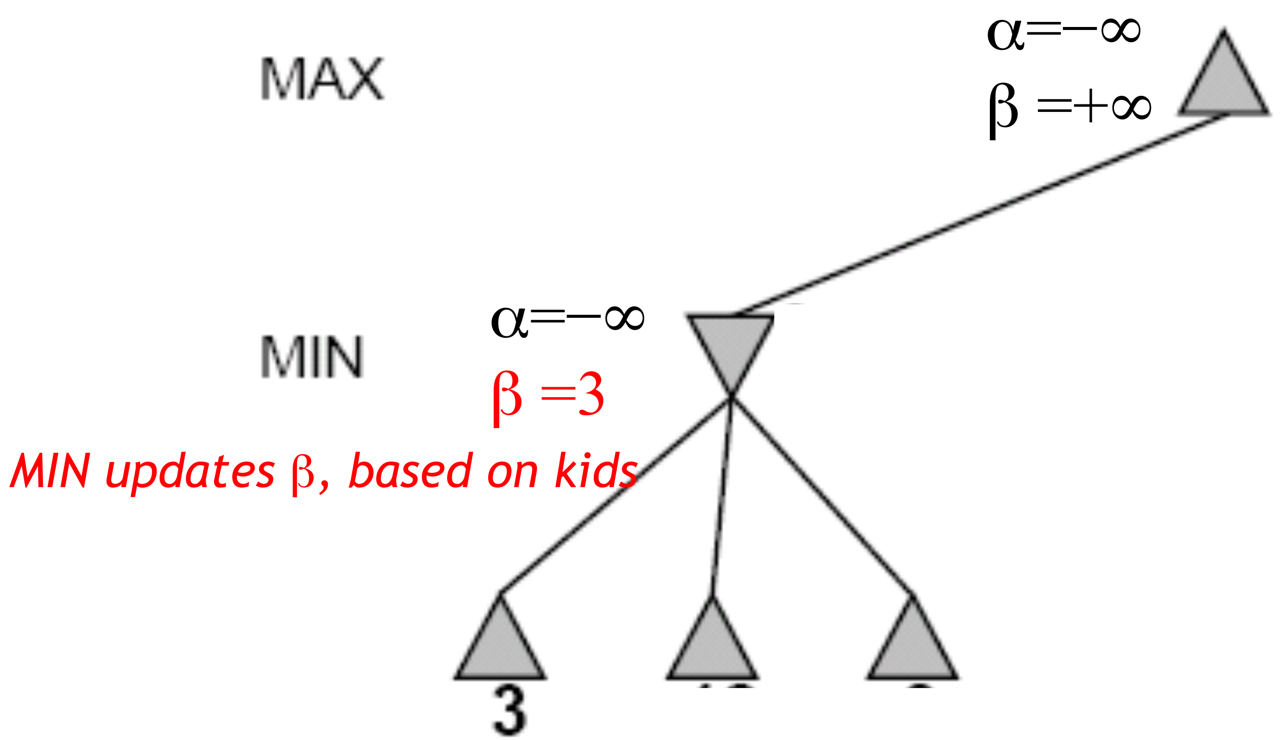
- **Prune whenever $\alpha \geq \beta$**
  - Prune below a Max node whose alpha value becomes greater than or equal to the beta value of its ancestors.
    - **Max nodes update alpha** based on children's returned values.
  - Prune below a Min node whose beta value becomes less than or equal to the alpha value of its ancestors.
    - **Min nodes update beta** based on children's returned values.

Alpha-Beta Example

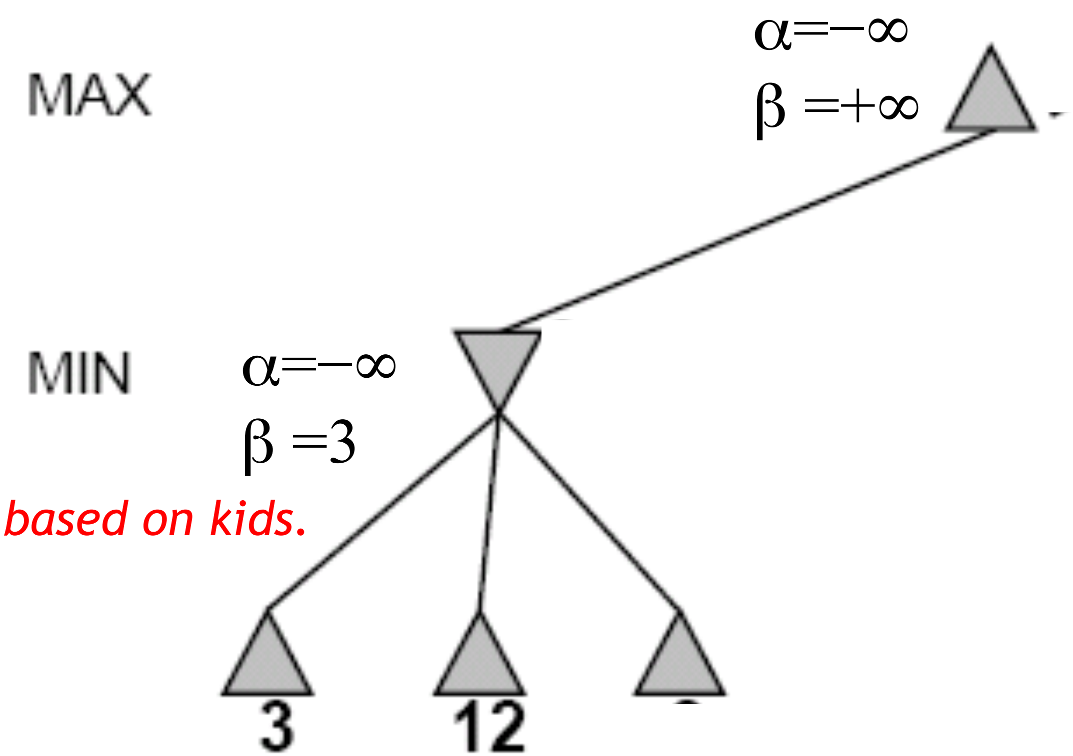
Do DF-search until first leaf



Alpha-Beta Example (continued)

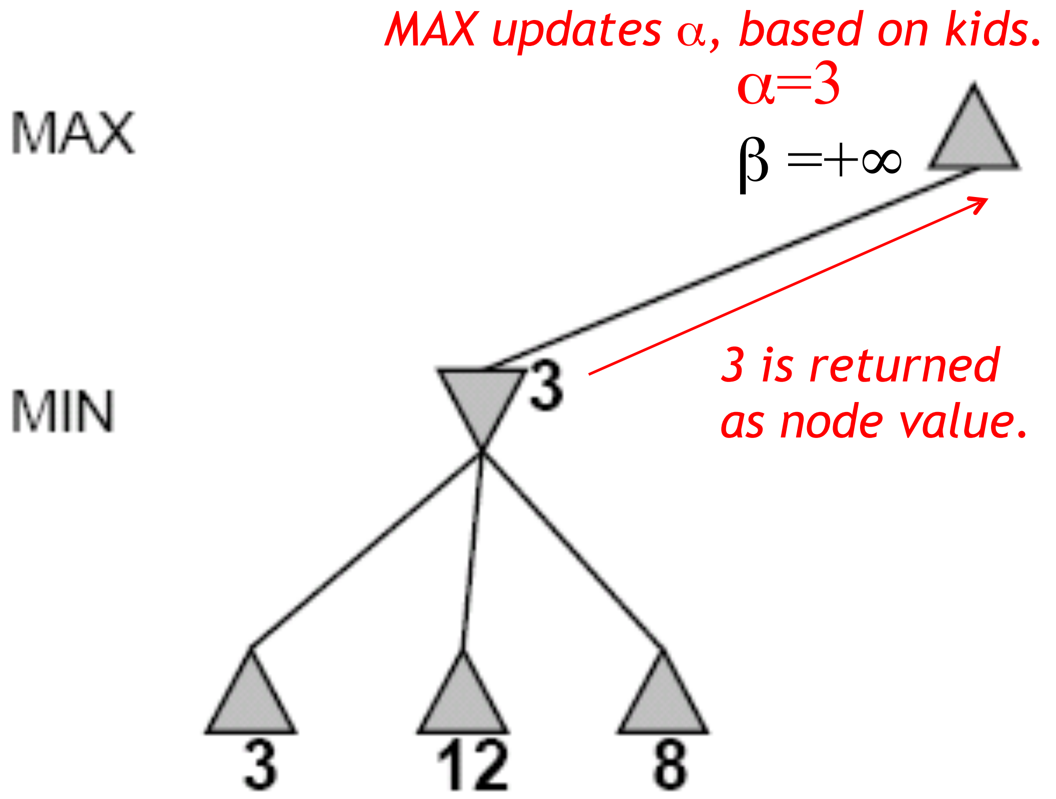


Alpha-Beta Example (continued)

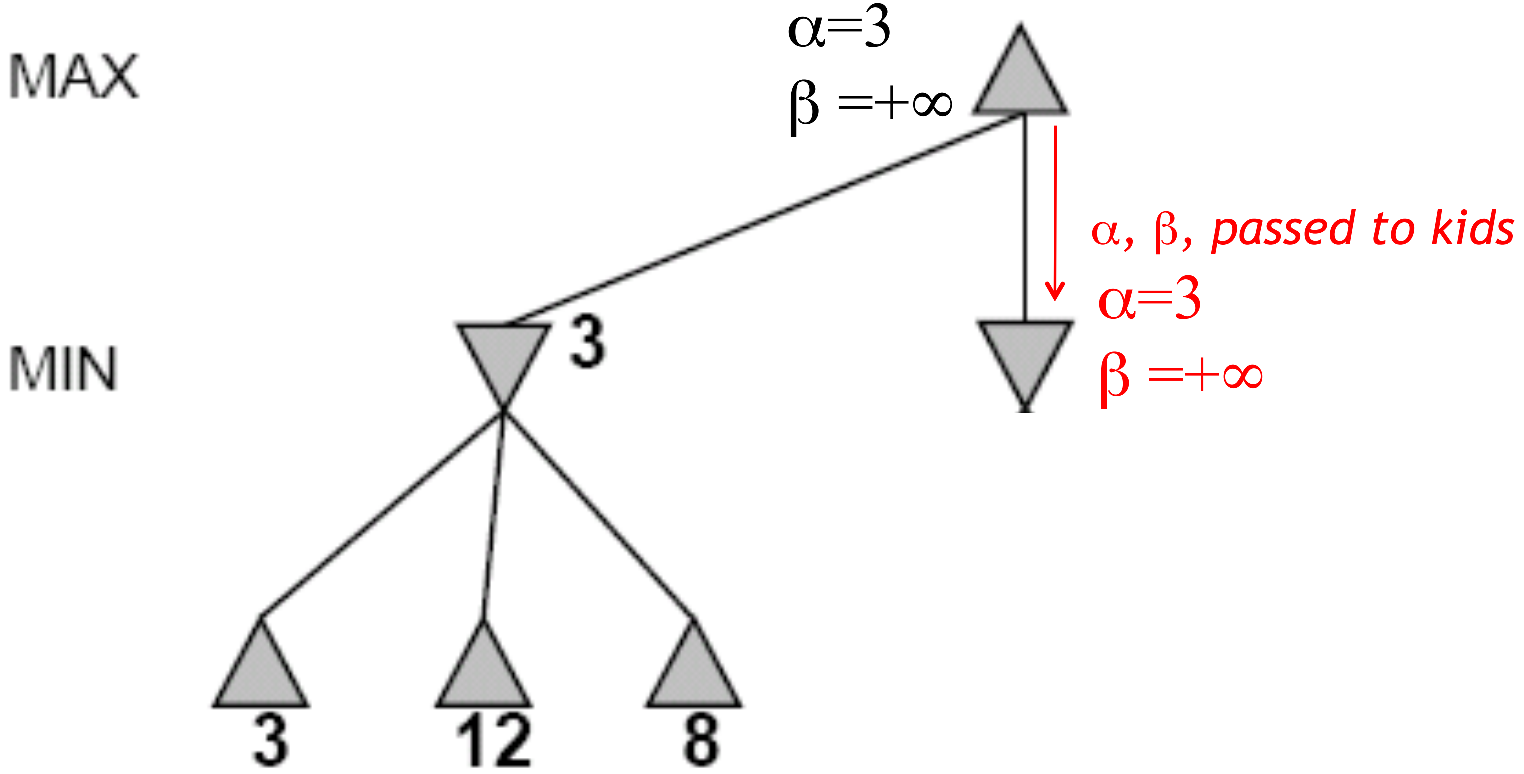


MIN updates β, based on kids.
No change.

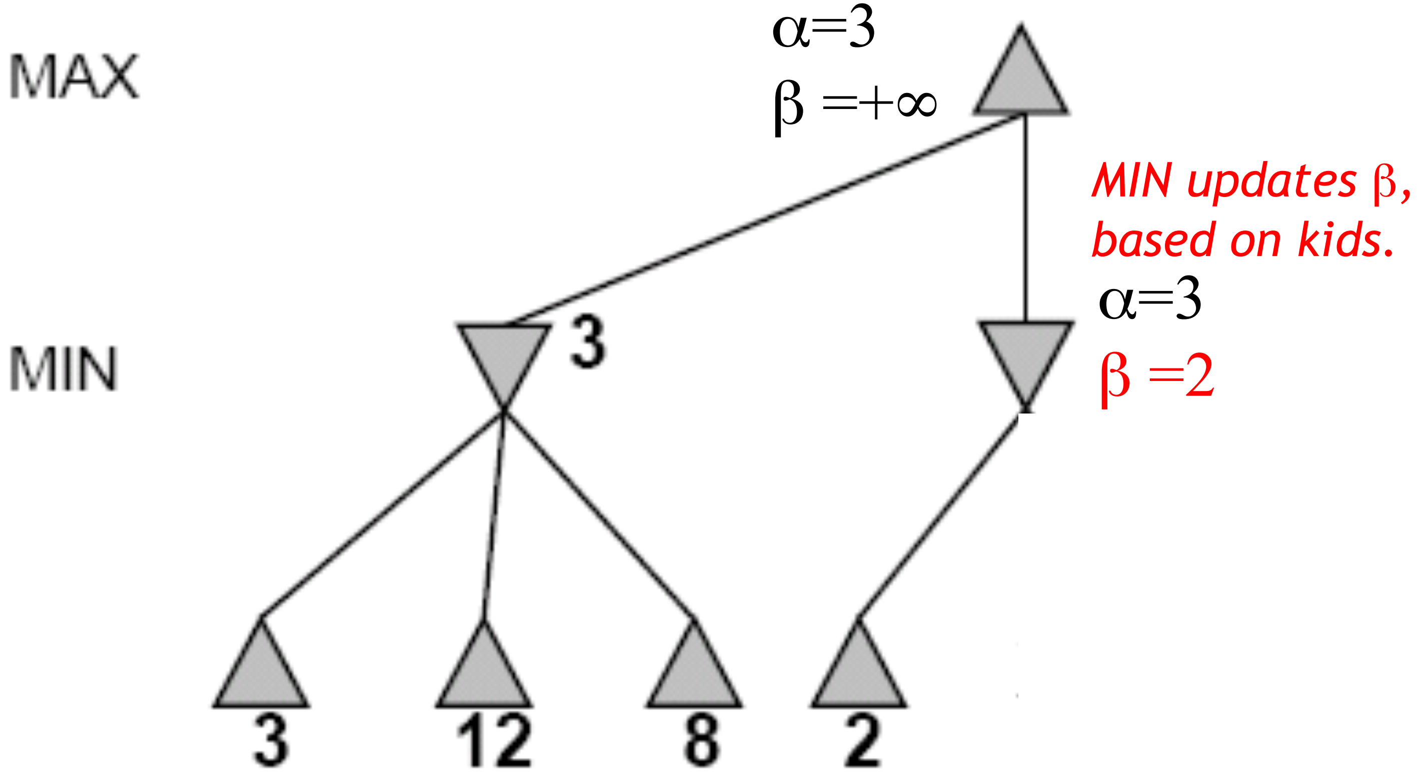
Alpha-Beta Example (continued)



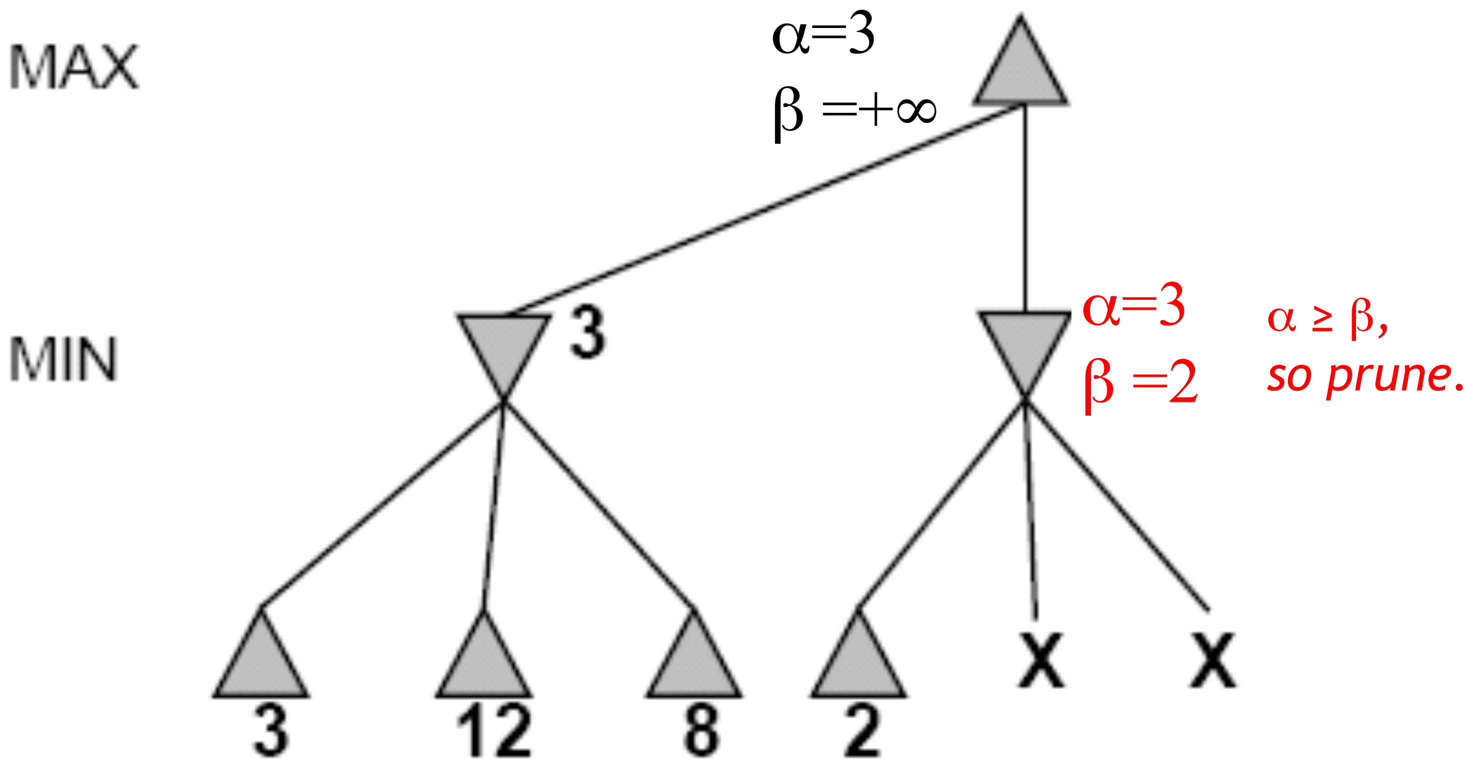
Alpha-Beta Example (continued)



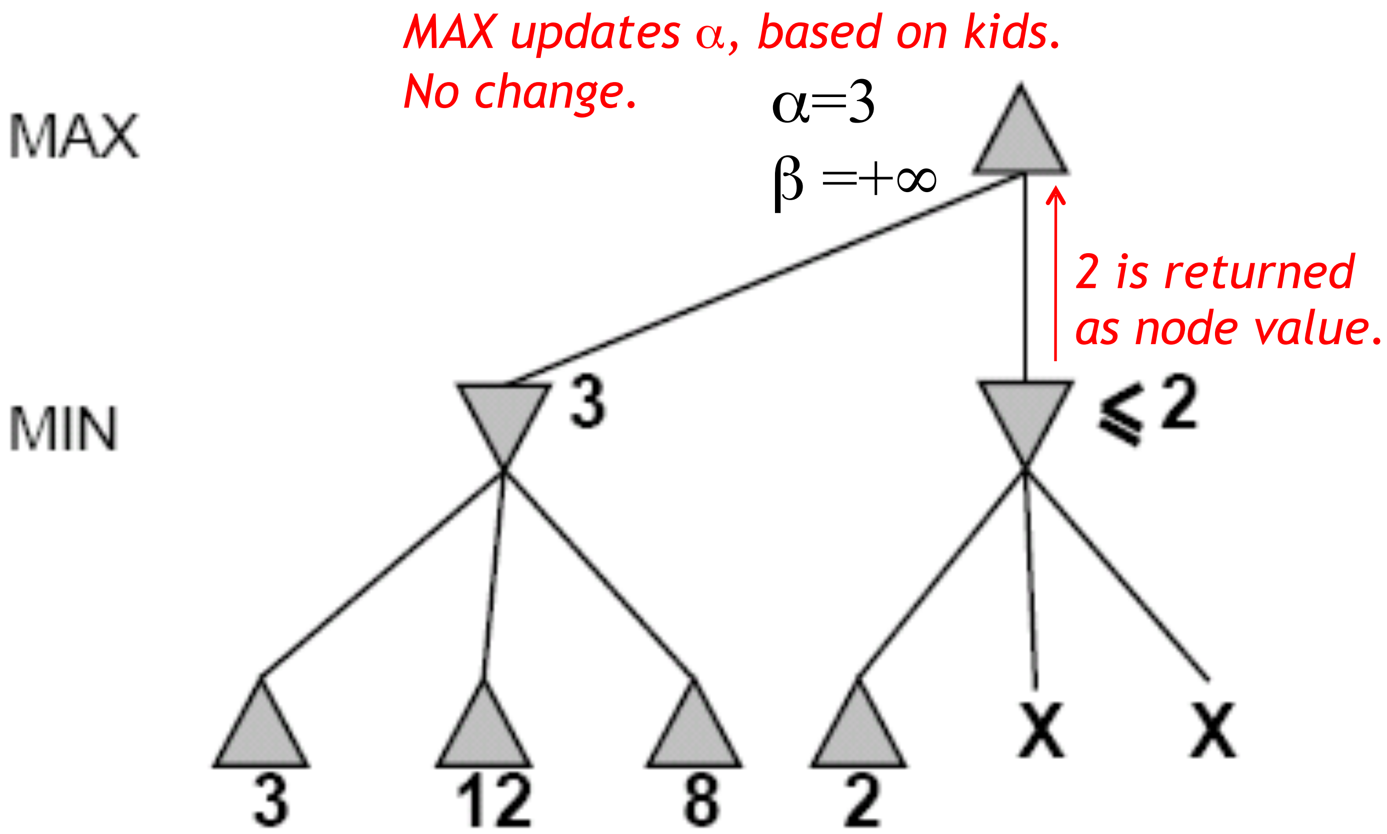
Alpha-Beta Example (continued)



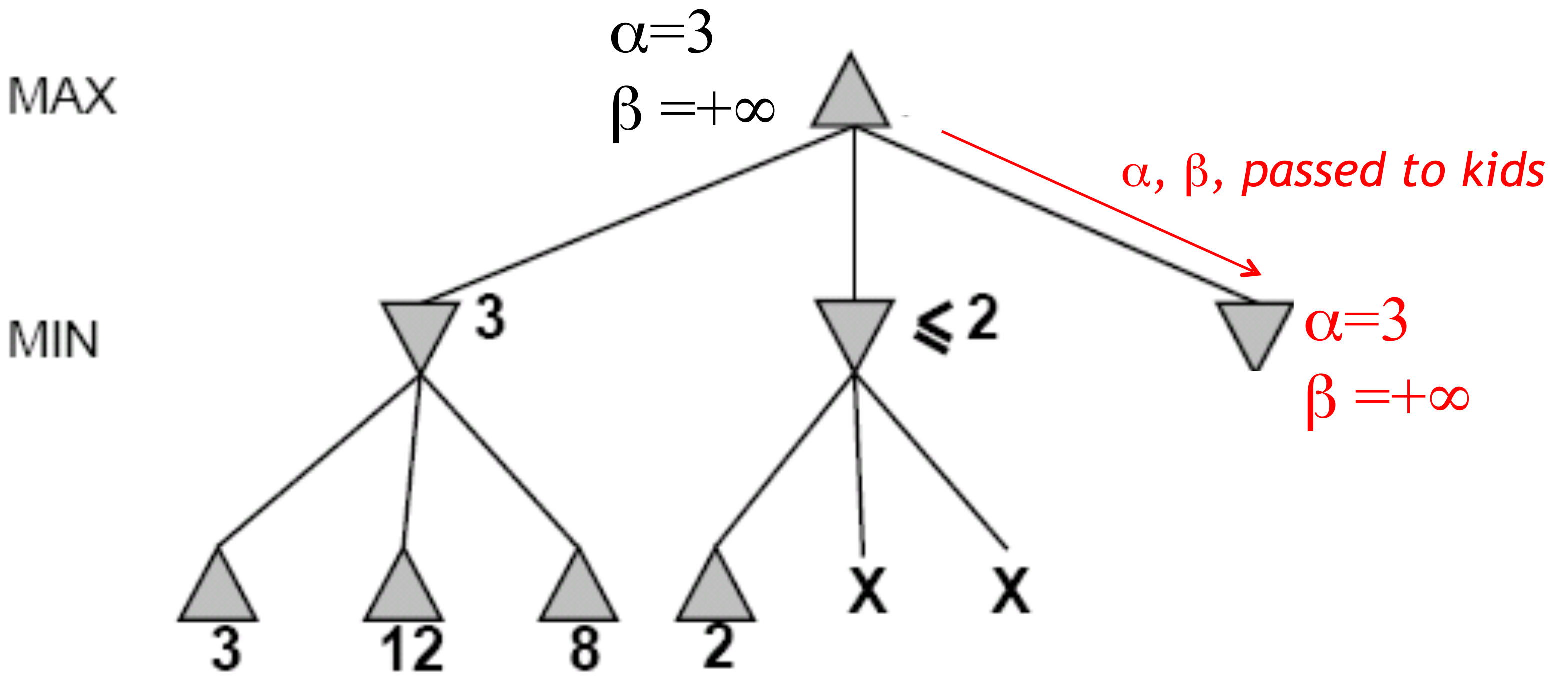
Alpha-Beta Example (continued)



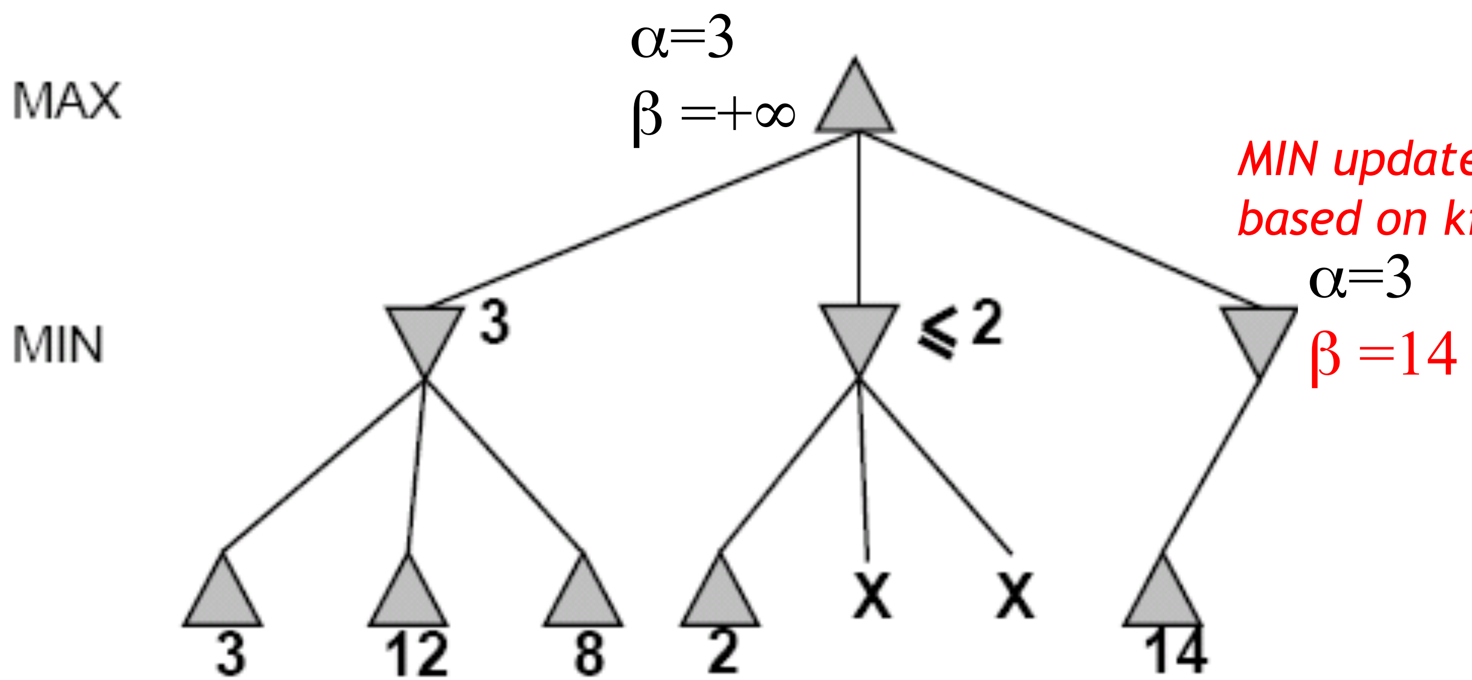
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)

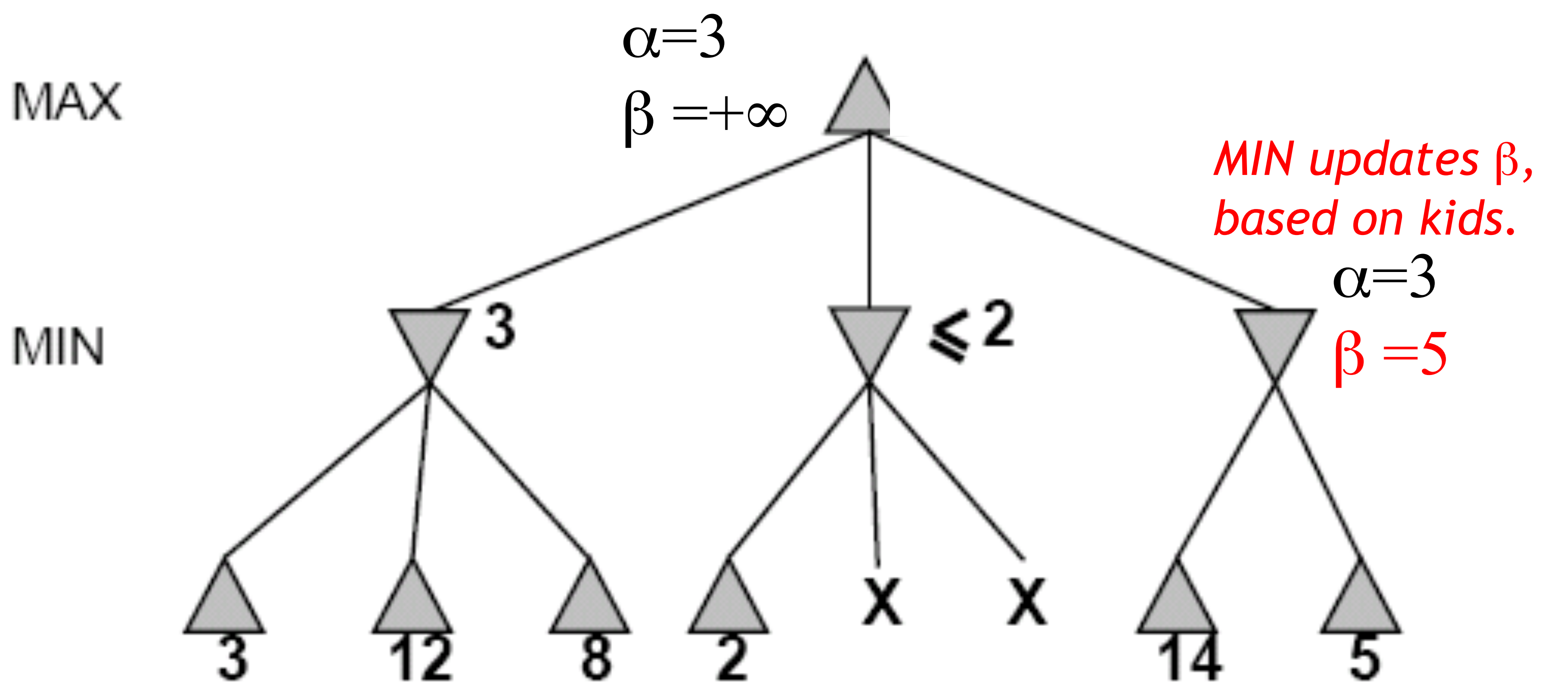


Alpha-Beta Example (continued)

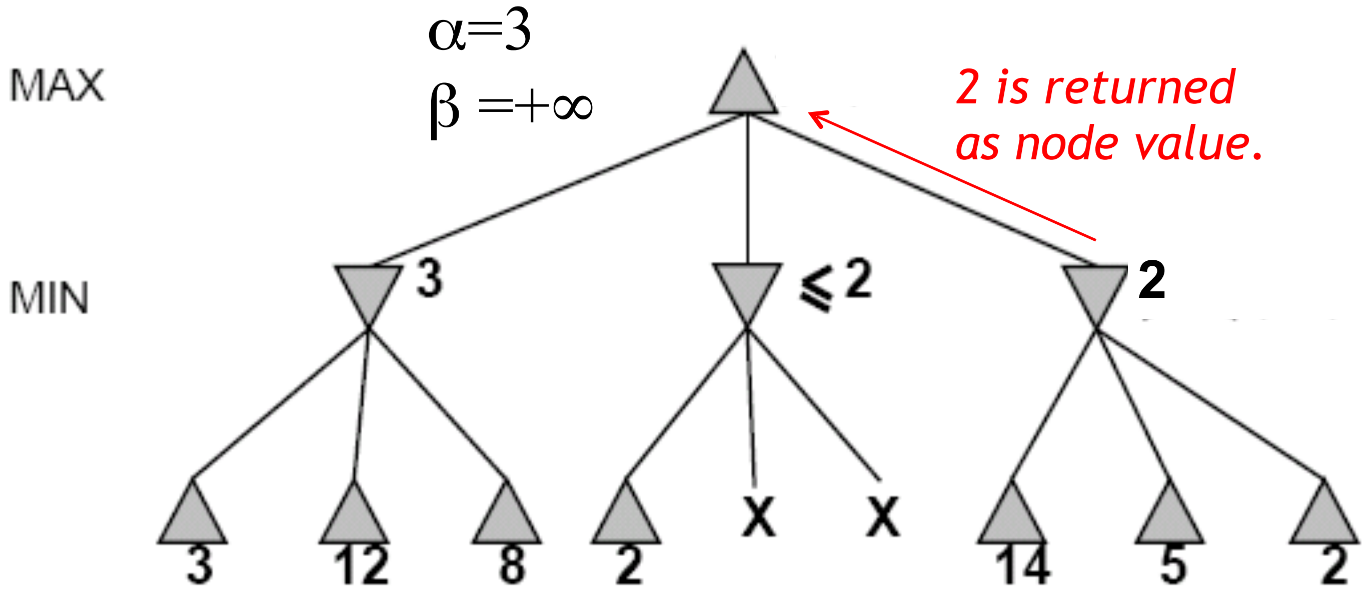


MIN updates β, based on kids.

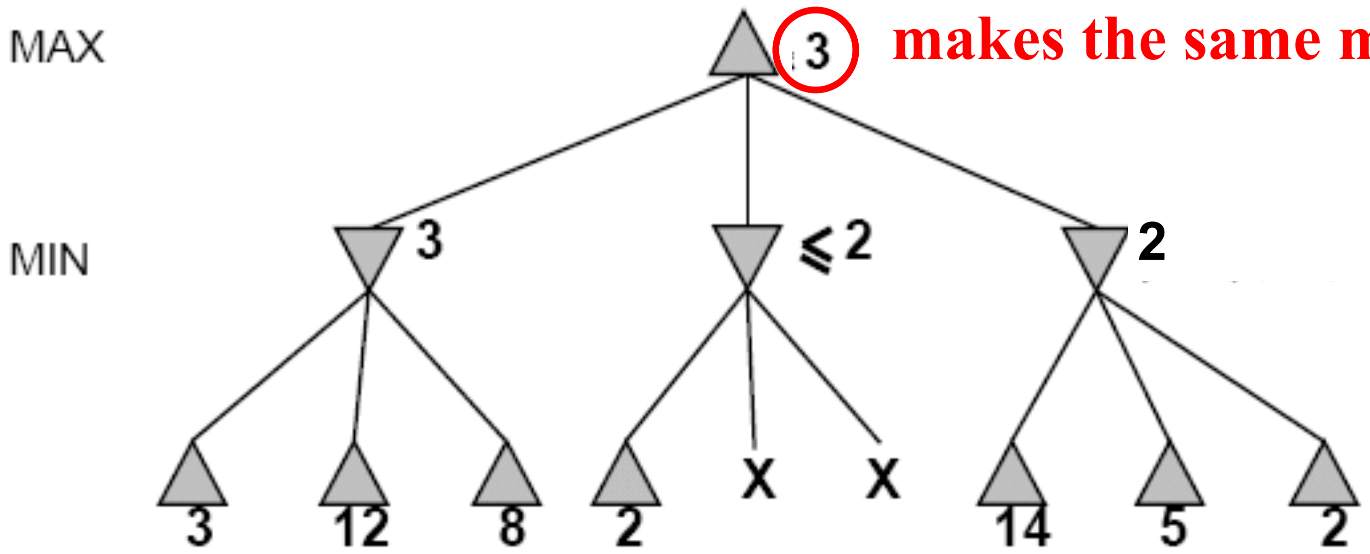
Alpha-Beta Example (continued)



Alpha-Beta Example (continued)

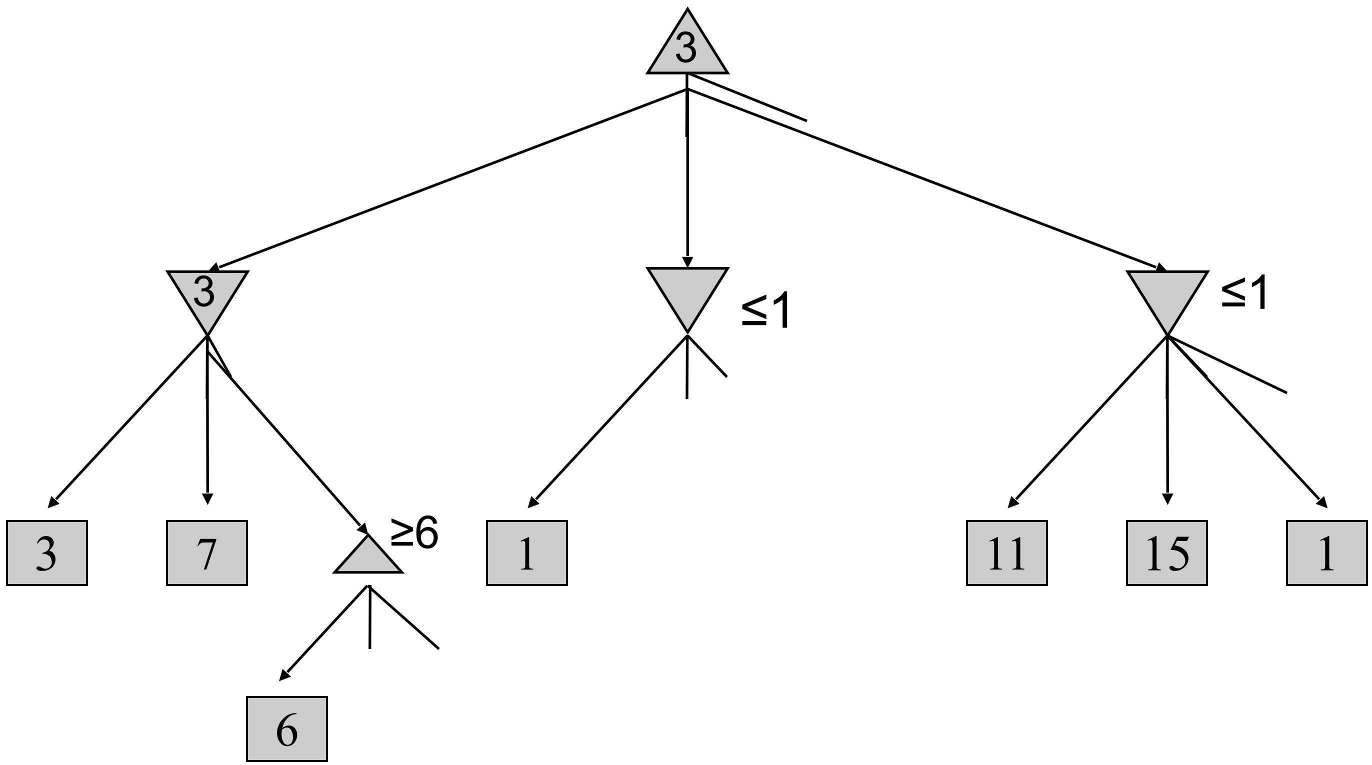


Alpha-Beta Example (continued)

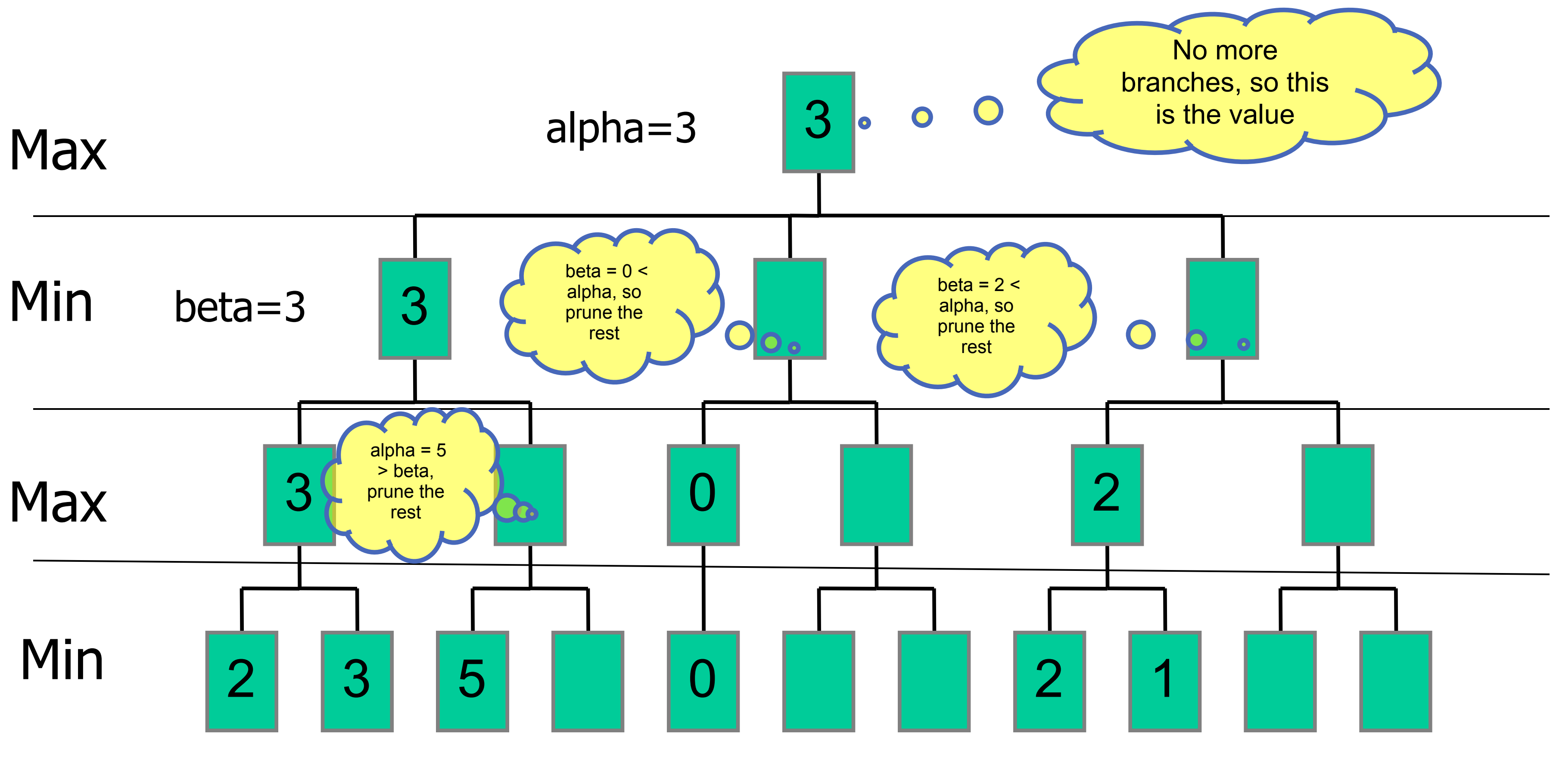


Max calculates the same node value, and makes the same move!

Alpha-Beta Pruning Example 2



Alpha-Beta Pruning Example 3



Effectiveness of Alpha-Beta Search

- Pruning does not affect the final result (optimal move).

- An entire sub-tree can be pruned.

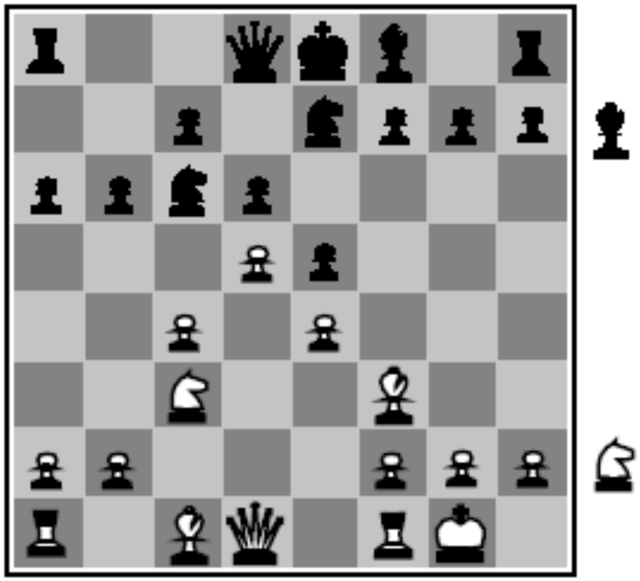
- Worst-Case
  - branches are ordered so that no pruning takes place. In this case alpha-beta gives no improvement over exhaustive search
- Best-Case
  - each player's best move is the left-most child (i.e., evaluated first)
- Good move *ordering* improves effectiveness of pruning
  - E.g., sort moves by the remembered move values found last time.
  - E.g., expand captures first, then threats, then forward moves, etc.
  - E.g., run Iterative Deepening search, sort by value last iteration.

- In practice often get $O(b^{(d/2)})$ rather than $O(b^d)$
  - this is the same as having a branching factor of sqrt(b),
    - $(\text{sqrt}(b))^d = b^{(d/2)}$,i.e., we effectively go from b to square root of b
  - e.g., in chess go from b ~ 35 to b ~ 6
    - this permits much deeper search in the same amount of time

Static (Heuristic) Evaluation Functions

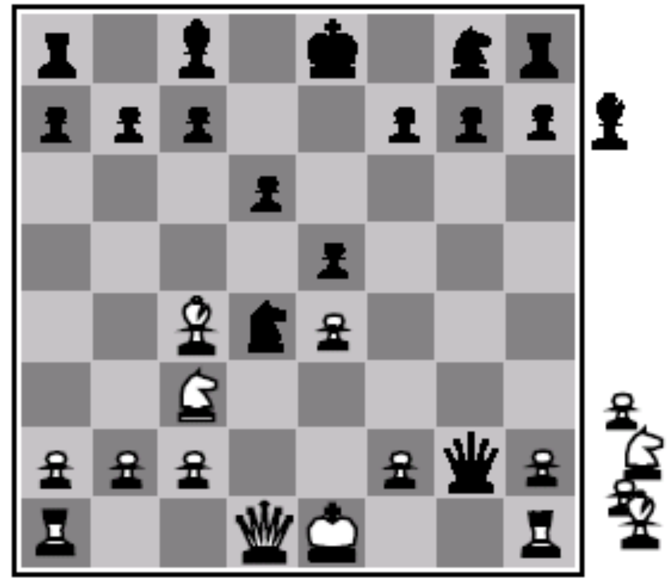
- Estimates how good the current board configuration is for a player.
- Typically, evaluate how good it is for the player, how good it is for the opponent, then subtract the opponent's score from the player's.
  - Othello: Number of white pieces - Number of black pieces
  - Chess: Value of all white pieces - Value of all black pieces
- Typical values from -infinity (loss) to +infinity (win) or [-1, +1].
- If the board evaluation is X for one player, it's -X for the other.
- This allows to perform **cut-off search**: after a maximum depth is reached, use a heuristic evaluation function instead of actual utility.

Evaluation functions



Black to move

White slightly better



White to move

Black winning

For chess, typically *linear* weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $w_1 = 9$ with
$f_1(s) =$ (number of white queens) – (number of black queens), etc.