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Gaussian Process Regression for Analysis of Educational Data

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Overview

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- Background
  - Univariate Normal Properties and Derivation
- Methods of Theory and Code
  - The Bivariate Normal
  - Multiple Regression
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- References
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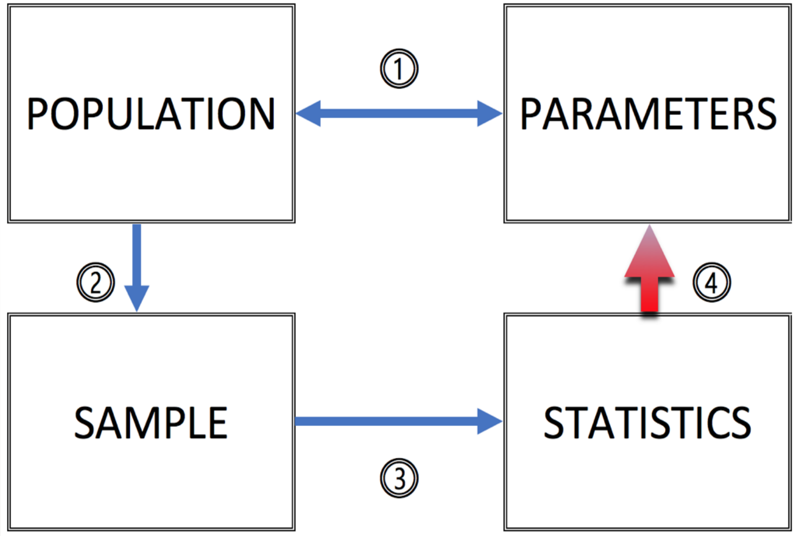
Introduction/Purpose

- My research focus: investigating patterns and trends in the educational system, and how they correlate with geography, race, and socioeconomic status. I believe once patterns and trends are identified this information can be used to promote equity in the educational system.
- In order to identify these patterns, we use statistics to model these trends, in addition other real-world data.
- At this point in my research, I am still learning methods and will discuss how I plan to apply them in future work.



Background

- The two approaches to statistical analysis are univariate and multivariate.
  - Univariate examines one variable (ex. weight of women)
  - Multivariate examines two or more variables simultaneously (ex. height and weight of women)

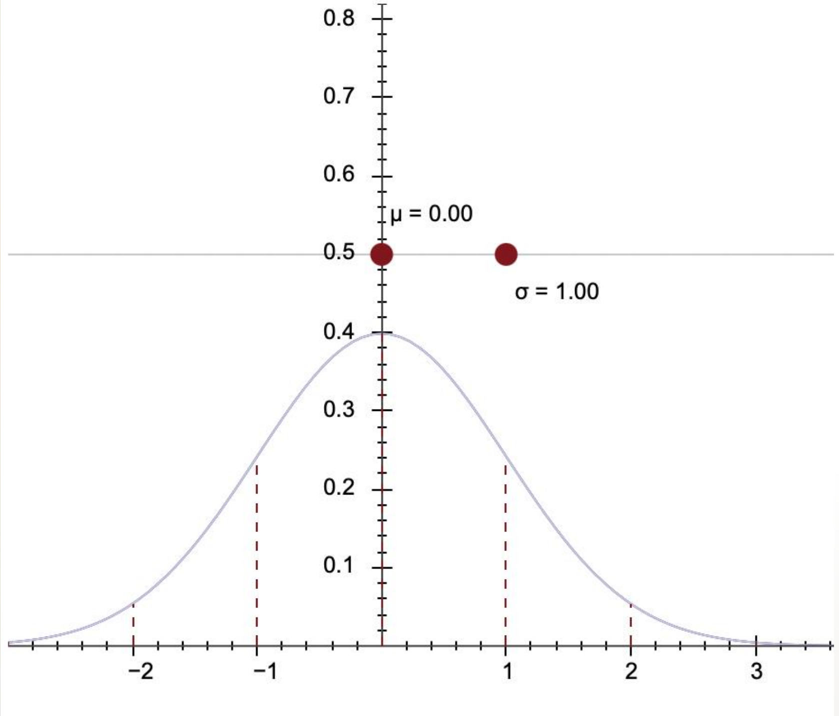
- The Univariate Gaussian Distribution, also known as Normal Distribution, is a continuous probability distribution for one random variable, x
  - The probability distribution function (PDF) is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < \infty$

- There are two parameters for the distribution
  - $\mu = mean = E(x) = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\, dx$
  - $\sigma^2 = variance = E(x-\mu)^2$

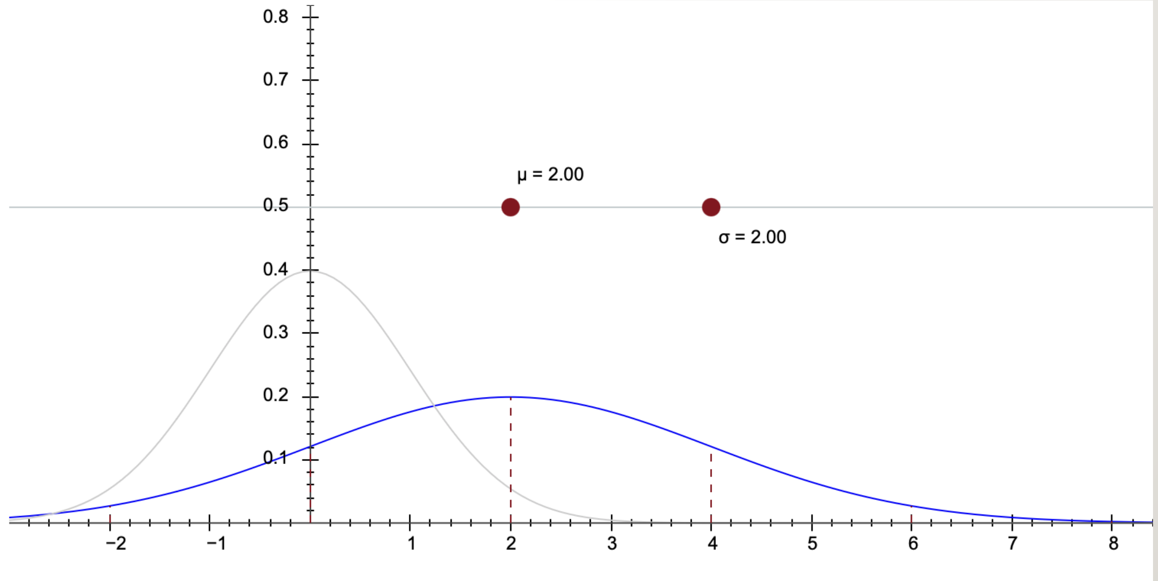
Background cont.

- The parameters μ and σ^2 affect the location and spread of the Gaussian probability density function

- A Standard Gaussian distribution function, where $\mu = 0$ and $\sigma = 1$.

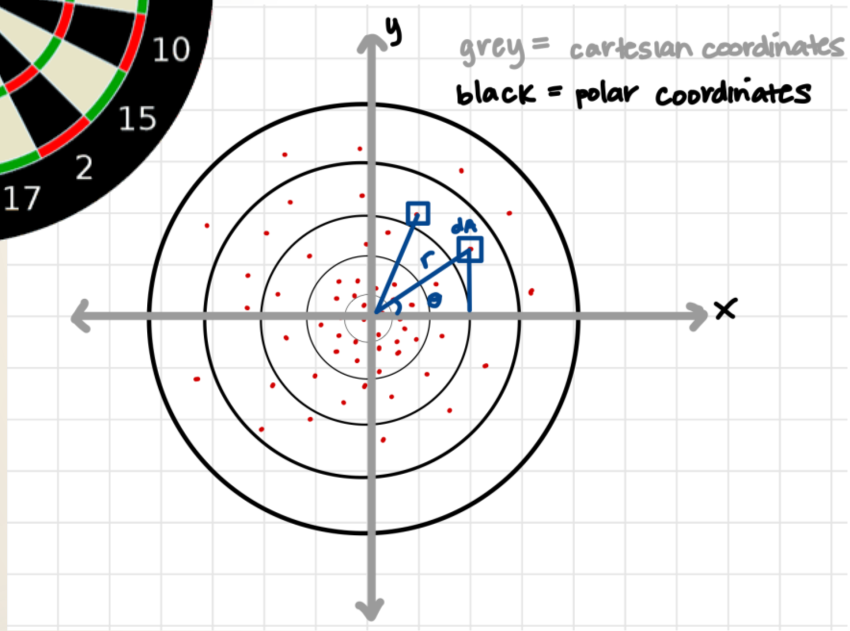


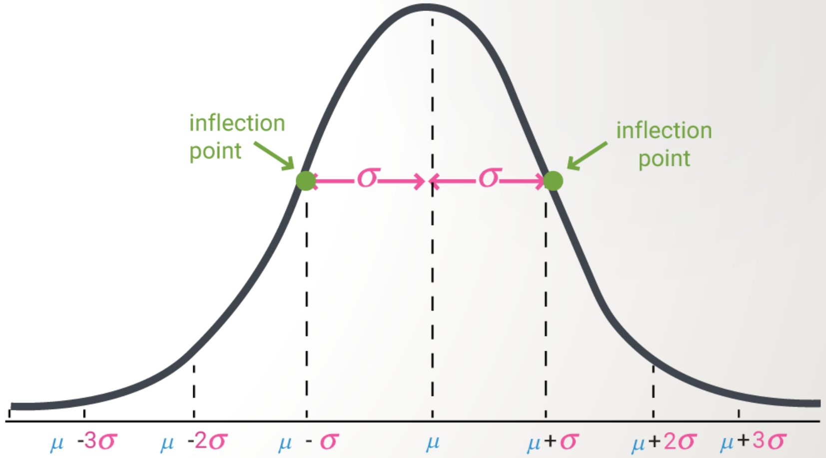
- A Gaussian distribution function where $\mu = 2$ and $\sigma = 2$. Notice how the μ $parameter$ changes the center of the curve and the increase in the σ $parameter$ spreads the curve out.



Background cont.

- How was the univariate probability distribution function derived?

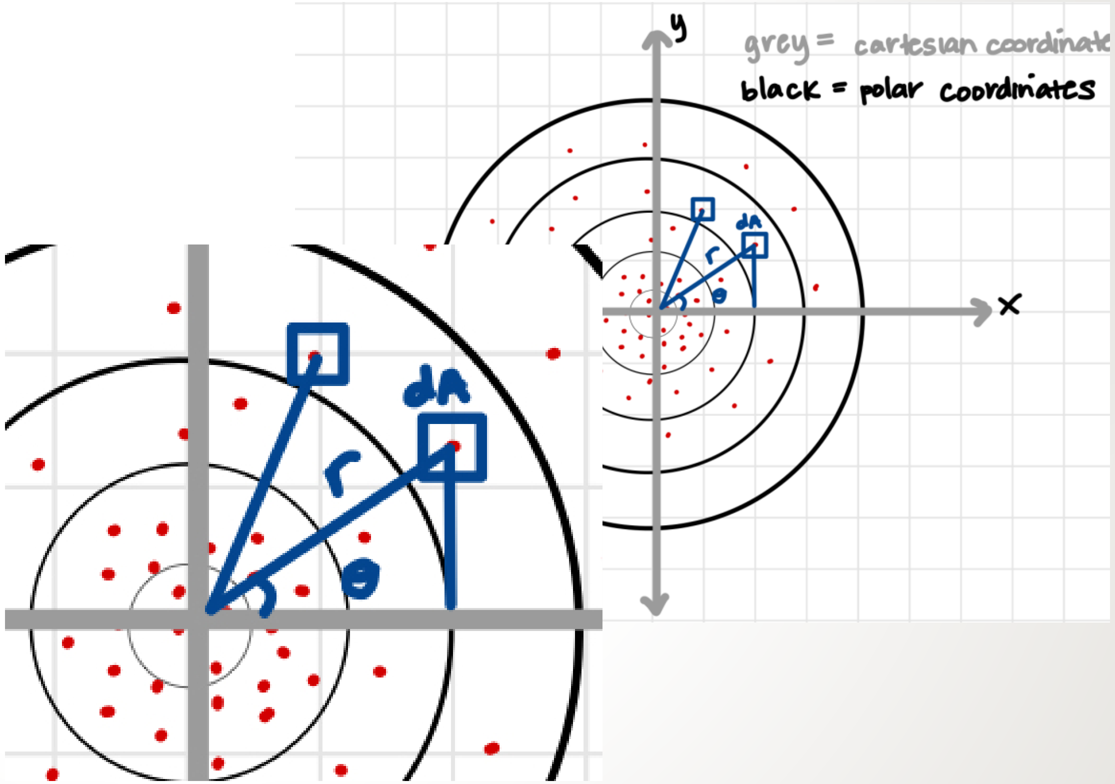




Background cont.

- φ *is* a function that describes the relative probability of finding darts at different locations

Let $\varphi = Probability\ Density\ Function$



$$\varphi dA = \varphi(r)dA$$

$$\varphi(r)dA = \varphi(x) * \varphi(y)dA$$

$$\varphi\left(\sqrt{x^2+y^2}\right) = \varphi(x) * \varphi(y)$$

$$\varphi\left(\sqrt{x^2+y^2}\right) dA = \frac{\varphi(x)\varphi(y)}{\lambda^2}$$

$$The\ most\ general\ case\ \ \varphi(x) = ae^{bx^2}$$

$$\int_{-\infty}^{\infty} \varphi(x)dx = 1 \ \therefore a\int_{-\infty}^{\infty} e^{bx^2} = 1$$

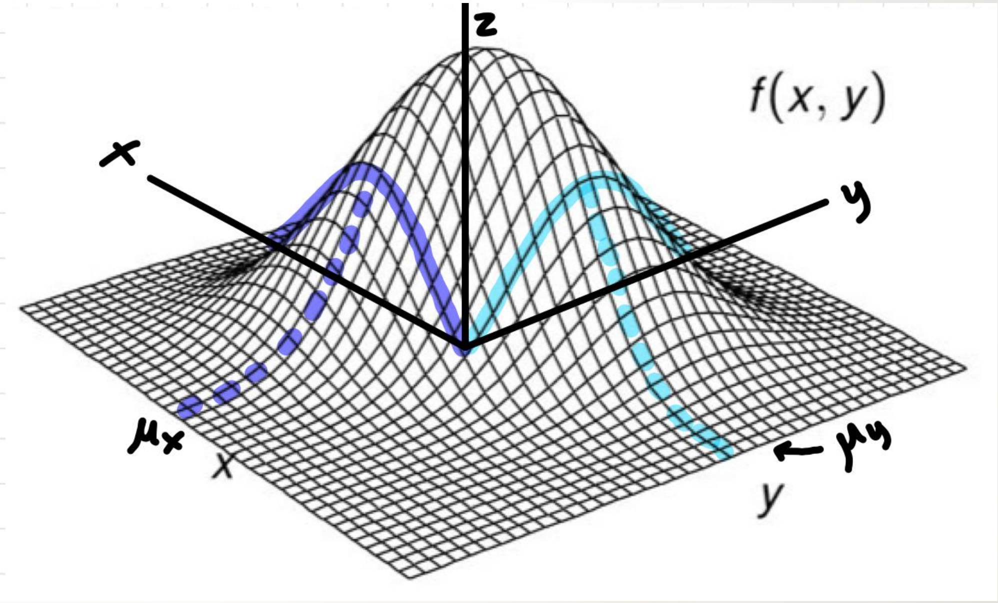
$$\varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}\ with\ mean = \ \mu\ and\ variance = \ \sigma^2$$

Methods - Theory

- The Bivariate Gaussian Distribution examines two independent random variables.

- The Marginals for random variables X and Y are both normal.
  - $X \sim N(\mu_x, \sigma_x)$
  - $Y \sim N(\mu_y, \sigma_y)$

- The Multivariate Gaussian Distribution probability Function
  - $P(X,Y) = (2\pi)^{-\frac{\kappa}{2}}|\Sigma|^{-\frac{1}{2}}e^{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)}$



Methods – Theory cont.

Bivariate Case

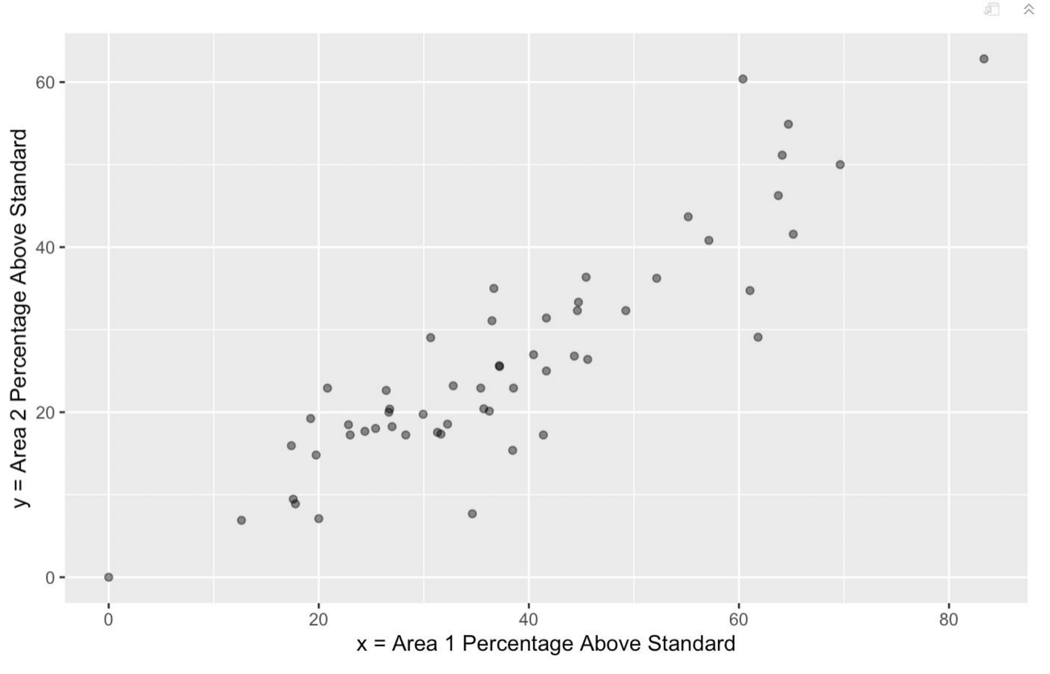
$$\vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

Multivariate Care

$$\vec{\mu} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{bmatrix}$$

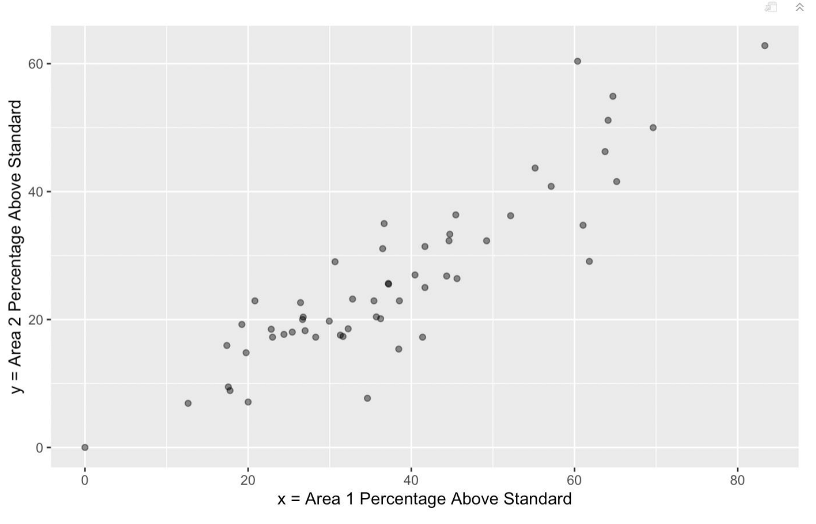


- σ_{21} *and* σ_{12} are always equal = equal to covariance.
- Covariance is proportional is correlation.
- Correlation is the scale at which two variables are positively, negatively, or not correlated.
  - $\rho = corr(x, y) = cov(x, y)/\sigma_x \sigma_y$, $-1 < \rho < 1$

Methods - Code

- R is a language and environment for statistical computing and graphics. R provides a wide variety of statistical (linear and nonlinear modeling, classical statistical tests, time-series analysis, classification, clustering) and graphical techniques.

- This scatter plot from the previous slide was produced in R.



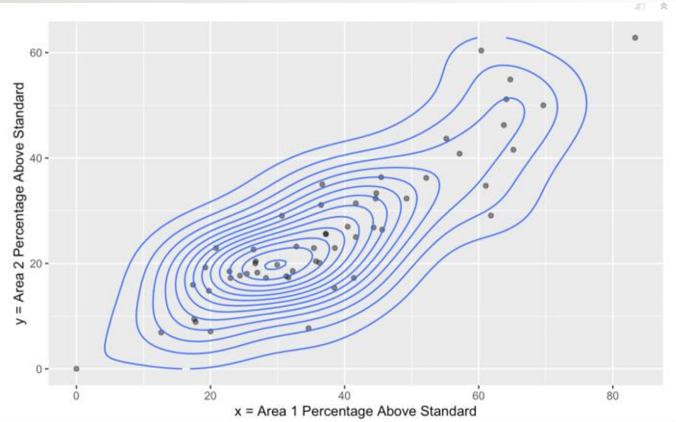
- R code to produce this scatter plot

```
```{r}
xx2<-(as.numeric(LBT2$`Area 1 Percentage Above Standard`))
xx3<-(as.numeric(LBT2$`Area 2 Percentage Above Standard`))


p2 <- ggplot(LBT2, aes(x = xx2, y = xx3)) +
  geom_point(alpha = .5) +
  #geom_density_2d() +
  labs(x="x = Area 1 Percentage Above Standard",y="y = Area 2 Percentage Above Standard")

p2
```
```

Methods - Code




````
```{r}
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  geom_point(alpha = .5) +
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  labs(x="x = Area 1 Percentage Above Standard",y="y = Area 2 Percentage Above Standard")

p2
```
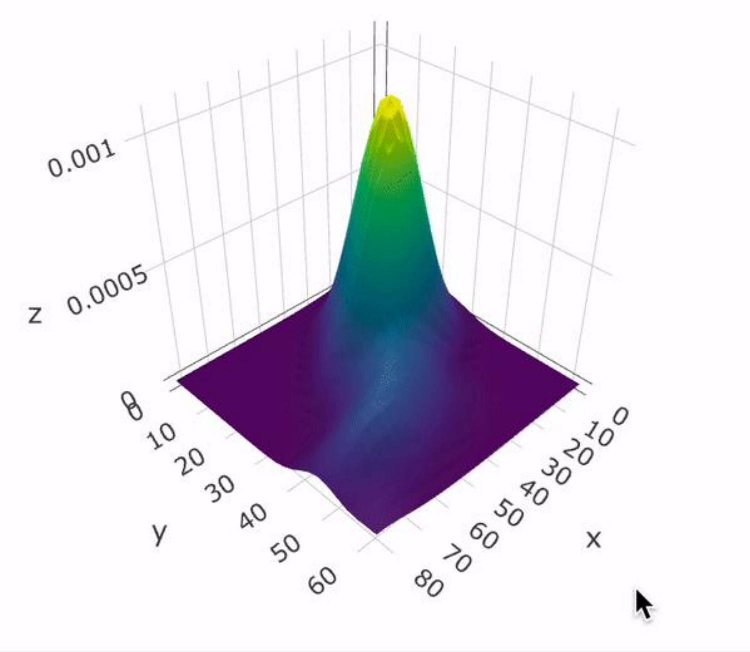
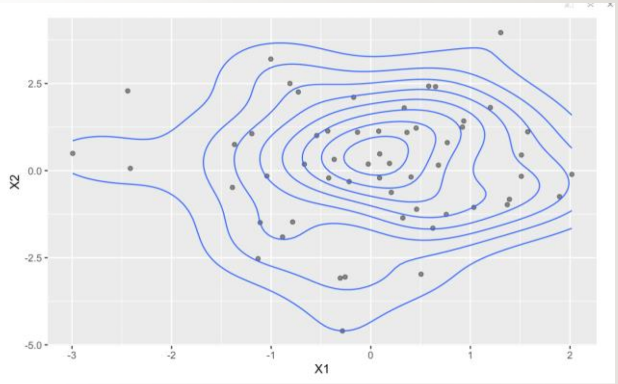
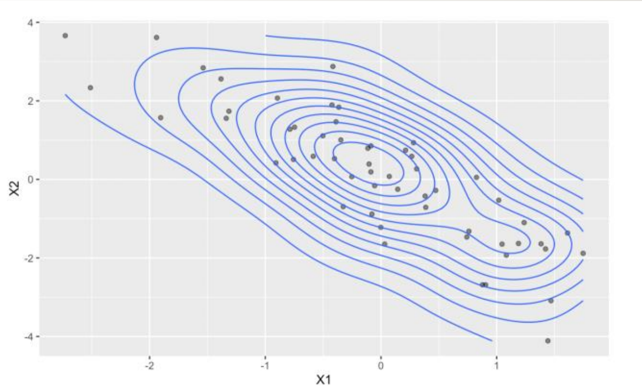
````




````
```{r}
dens <- kde2d(xx2, xx3, h=c(28,28))

plot_ly(x = dens$x,
        y = dens$y,
        z = dens$z) %>% add_surface()
```
````



- Covariance is related to the shape of the bivariate function.



Methods – Theory and Code

- Linear Regression is a model that assumes a linear relationship between two random variables. The analysis is used to predict the value of one variable based on another variable.
  - Variables X1 and X2 in previous scatter diagram
- The linear relationship between the two random variables are expressed in the form
  - $y = \beta_0 + \beta_1 x + \varepsilon$
- Use the Least Squares Method to predict β_0 *and* β_1
- The linear regression formula predicts β_0 *and* β_1 in R and provides other valuable statistical information.

```
Call:
lm(formula = LBT2$`Mean Scale Score` ~ xx2 + xx3, data = LBT2)

Residuals:
    Min      1Q  Median      3Q     Max 
-62.399  -3.832   0.409   7.982  17.777 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2414.5995     4.0251 599.885  < 2e-16 ***
xx2            1.4112     0.2184   6.461 3.32e-08 ***
xx3            0.7910     0.2691   2.940  0.00486 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 11.88 on 53 degrees of freedom
Multiple R-squared:  0.8903,	Adjusted R-squared:  0.8862 
F-statistic: 215.1 on 2 and 53 DF,  p-value: < 2.2e-16
```

Future Work

Gaussian Process Regression in Educational Data

- Gaussian process regression is the combination of looking at the Gaussian distribution as a flexible tool for modeling complex relationships that are challenging for linear regression to model.

- Use this process to study and analyze actual data sets.
  - $\begin{bmatrix} y \\ f_* \end{bmatrix} \sim N(0, \begin{bmatrix} K(X,X) + \sigma_n^2 I & K(X,X_*) \\ K(X_*,X) & K(X_*,X_*) \end{bmatrix})$
  - $\kappa(X,X') = \sigma_f^2 exp\left\{-\frac{1}{2}\sum_{d=1}^{D}\frac{1}{\ell_d^2}(x_d - x_d{}')^2\right\}$
  - (X,y) data used to build the model
  - (X^*, f_*) data used to validate the model
  - K is covariance based on our choice of covariance function κ

References

- "Multivariate Normal Distribution." *Wikipedia*, Wikimedia Foundation, 21 July 2021, en.wikipedia.org/wiki/Multivariate_normal_distribution.
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- Wackerly, Dennis D., et al. *Mathematical Statistics with Applications*. Brooks/Cole, 2012.
- Yin, Min, et al. "Key Course Selection for Academic Early Warning Based on Gaussian Processes." *Lecture Notes in Computer Science*, 2016, pp. 240–247., doi:10.1007/978-3-319-46257-8_26.
- *YouTube*, YouTube, 11 Apr. 2021, www.youtube.com/watch?v=N-bI-Dsm-rw&t=1675s.

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