Pre-Calc: Final Exam Review Day 2

Trig Functions:

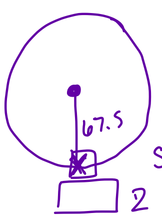
amplitude, period $\frac{2\pi}{b}$, Phase shift

$a \sin b(x-h)+k$ ← vertical shift/midline

The London Eye Ferris wheel has a diameter of 135 meters. A ride on the Eye will complete one rotation every 30 minutes. Passengers board from a platform that is 2 meters above the ground. Write an equation that models the height above ground as a function of time in minutes.

$y=-67.5\cos\frac{\pi}{15}(x\ )\ +69.5$

radius $=\frac{135}{2}=67.5$



start at min $(\cos\theta)$ *Negative

Midline : 69.5

Period : 30min

$b=\frac{2\pi}{30}=\frac{\pi}{15}$

A buoy in the Gulf of Mexico sends a signal beacon to a Coast Guard station. The behavior of the buoy can be modeled by the function $h = a\sin(bt) + 4$, where h is measured in feet above sea level. During a recent tropical storm, the height veries from 1 foot to 8 feet with a 4.5 second interval between one 8-foot height to the next. Find the equation that represents this situation.

$h=a\sin(bt)+4$

$h=\frac{7}{2}\sin\left(\frac{4\pi}{9}t\right)+4$

Amp: $\frac{\overset{max}{8}-\overset{min}{1}}{2}=\frac{7}{2}$ ft

Period $=\frac{2\pi}{b}$

$4.5=\frac{2\pi}{b}$ $\quad b=\frac{4\pi}{9}$

Trig Identities:

Prove each identity.

$$1-\frac{\cos^2 x}{1+\sin x}=\sin x$$

$1-\frac{(1-\sin^2 x)}{1+\sin x}=\sin x$

$1-\frac{\cancel{(1+\sin x)}(1-\sin x)}{\cancel{1+\sin x}}=\sin x$

$1-(1-\sin x)=\sin x$

$1-1+\sin x=\sin x$

$\sin x=\sin x$ ✓

Find the exact value of each trig function using sum and difference identities.

$$\tan\frac{5\pi}{12}$$

* $\frac{\pi}{6}+\frac{\pi}{4}=\frac{2\pi}{12}+\frac{3\pi}{12}=\frac{5\pi}{12}$

$\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)=\frac{\tan\frac{\pi}{6}+\tan\frac{\pi}{4}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{4}}=\frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}}=\frac{\frac{\sqrt{3}+3}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}}=\frac{\sqrt{3}+3}{3-\sqrt{3}}\frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$=\frac{12+6\sqrt{3}}{6}$

$=2+\sqrt{3}$

Law of Sines/Law of Cosines:

A used car lot has a large SALE balloon tied down to stakes 1500 feet apart. The angle of elevation from the stake at the west end of the lot to the balloon is 42°, and the angle of elevation from the stake at the east end of the lot to the balloon is 72°. What is the altitude of the balloon?

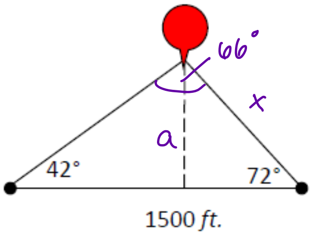
$$\frac{\sin 42}{x} = \frac{\sin 66}{1500}$$

$$x = \frac{1500 \sin 42}{\sin 66}$$

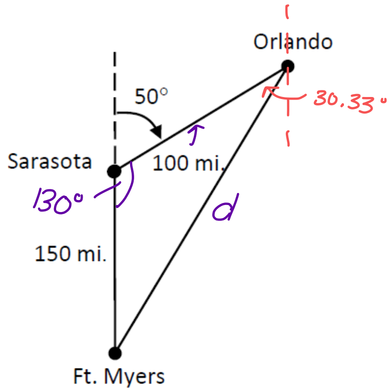
$$x = 1098.68 \text{ ft}$$

$$\sin 72 = \frac{a}{1098.68}$$

$$\boxed{a = 1044.9 \text{ ft}}$$



A jet plane flies 150 miles, from Ft. Myers, FL to Sarasota, FL, and then turns 50° towards the east heading to Orlando, FL which is a distance of 100 miles.



a. How far is the return trip from Orlando to Ft. Myers?

$$d^2 = (150)^2 + (100)^2 - 2(150)(100)\cos 130$$

$$d = \sqrt{(150)^2 + (100)^2 - 2(150)(100)\cos 130}$$

$$\boxed{d = 227.56 \text{ miles}}$$

b. Through what angle should the pilot turn at Orlando to return to Ft. Myers?

$$\frac{\sin \text{Orlando}}{150} = \frac{\sin 130}{227.56}$$

$$\sin \text{orlando} = \frac{150 \sin 130}{227.56}$$

$$\text{orlando} = \sin^{-1}\left(\frac{150 \sin 130}{227.56}\right)$$

$$= 30.33°$$

Total Turn: 130° + 30.33°

$$\boxed{160.33°}$$

Vectors:

Find the magnitude and direction angle of the vector.

m = <6, -9> Q4

$$\|m\| = \sqrt{6^2 + (-9)^2} = \sqrt{117} = \boxed{3\sqrt{13}}$$

$$\tan\theta = \frac{-9}{6} \qquad \theta = \tan^{-1}\left(\frac{-9}{6}\right) = -56.31 + 360$$

$$= \boxed{303.69°}$$

Find the following for **a = <-12, 9> and b = <-5, -1>**

$5\mathbf{b} + 2\mathbf{a}$

$5\langle -5, -1\rangle + 2\langle -12, 9\rangle$

$= \langle -25, -5\rangle + \langle -24, 18\rangle$

$= \boxed{\langle -49, 13\rangle}$

Dot Product:

Use the dot product to find the angle between the vector pair.

$\mathbf{p} = \langle -2, -9\rangle, \mathbf{q} = \langle -5, 5\rangle$

$$\cos\theta = \frac{p \cdot q}{\|p\| \cdot \|q\|}$$

$$\cos\theta = \frac{(-2)(-5) + (-9)(5)}{\sqrt{(-2)^2+(-9)^2}\sqrt{(-5)^2+(5)^2}}$$

$$\theta = \cos^{-1}\left(\frac{-35}{\sqrt{85}\sqrt{50}}\right)$$

$$\boxed{\theta = 122\ 47^\circ}$$

Polar Coordinates and Graphs:

Name three different pairs of polar coordinates that name the given point if $-2\pi \leq \theta \leq 2\pi$.

$\left(-0.5, \frac{2\pi}{3}\right)$

$\left(-0.5, -\frac{4\pi}{3}\right)$

$\left(0.5, \frac{5\pi}{3}\right)$

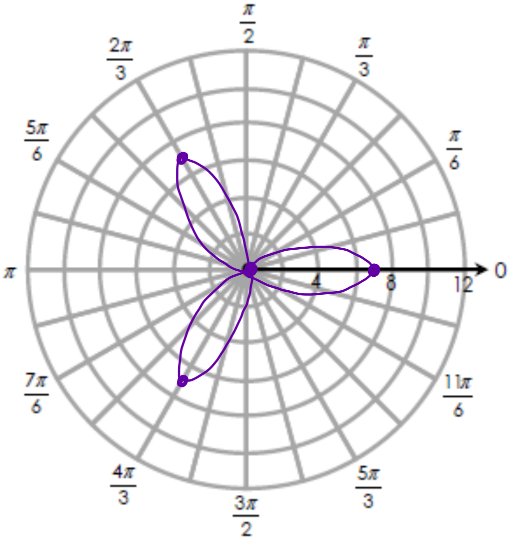
$\left(0.5, -\frac{\pi}{3}\right)$

Graph the Polar Curve.

$y = 7\cos 3\theta$

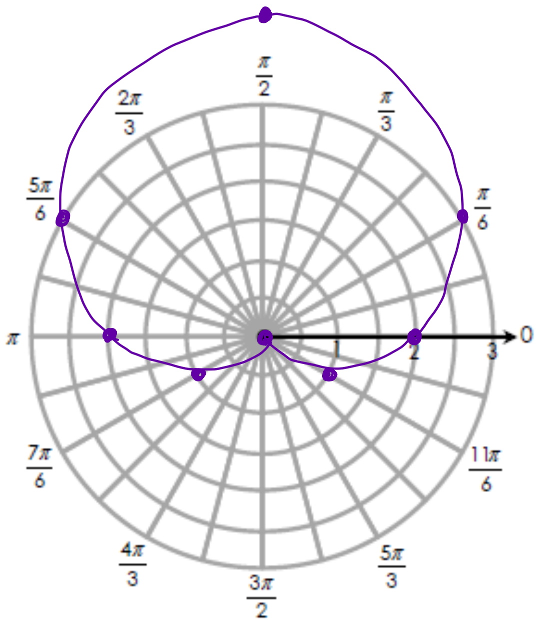
θ	Y
0	7
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-7
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	7
$\frac{5\pi}{6}$	0
π	-7

θ	Y
$\frac{7\pi}{6}$	0
$\frac{4\pi}{3}$	7
$\frac{3\pi}{2}$	0
$\frac{5\pi}{3}$	-7
$\frac{11\pi}{6}$	0
2π	7



$y = 2 + 2\sin\theta$

θ	Y
0	2
$\frac{\pi}{6}$	3
$\frac{\pi}{2}$	4
$\frac{5\pi}{6}$	3
π	2
$\frac{7\pi}{6}$	1
$\frac{3\pi}{2}$	0
$\frac{11\pi}{6}$	1
2π	2



Projectile Motion:

A golf ball is hit from the ground at 30 meters per second, traveling at an angle of elevation of 50°.

a. Write the parametric equations to describe the horizontal and vertical position of the golf ball at time *t*.

$X = t \; 30\cos 50$

$Y = -4.9t^2 + t \cdot 30\sin 50$

b. Find the horizontal distance the ball has traveled at 3 seconds.

$X = (3)(30\cos 50)$

$\boxed{X = 57.85\text{m}}$

c. Find the time at which the ball hits the ground again.

$0 = -4.9t^2 + t \cdot 30\sin 50$

$0 = t(-4.9t + 30\sin 50)$

$\boxed{t = 4.69\text{sec}}$

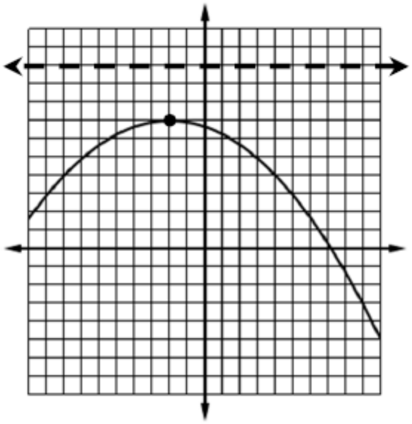
d. Find the total horizontal distance the ball traveled.

$X = (4.69)(30\cos 50)$

$\boxed{X = 90.44\text{m}}$

Conic Sections:

Write an equation for the conic section given the information below.



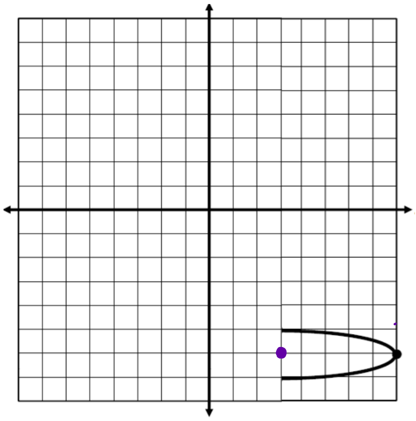
Vertex (-2,7)

Directrix $y = 10$

$p = -3$ (down)

$4p = -12$

$\boxed{(x+2)^2 = -12(y-7)}$



Co-vertices

(3,-5) and (3,-7)

$b = 1$

vertices

(8,-6)

center (3,-6)

$a = 5$

$$\boxed{\frac{(x-3)^2}{25} + \frac{(y+6)^2}{1} = 1}$$

Binomial Theorem:

Use Pascal's triangle to find the following:

1. 15th row, 9th element

5005

2. 23rd row, 17th element

100,947

Algebra Optimization:

A wire that is 40 cm long will be used to form a circle, a square, or both. The wire can be cut so that one section is used to create a circle and the other section is used to create a square.

$A_s = s^2$

Circumference $= 2\pi r$

$A_c = \pi r^2$

a. Find a function to model the total combined area of the two figures.

* Let x be the total length cut for the square

$A_s = \left(\frac{x}{4}\right)^2$

* Let $40 - x$ be the total cut for the circle (circumference)

$40 - x = 2\pi r \rightarrow r = \frac{40-x}{2\pi}$ $\quad A_c = \pi\left(\frac{40-x}{2\pi}\right)^2$

Total:

$\left(\frac{x}{4}\right)^2 + \pi\left(\frac{40-x}{2\pi}\right)^2$

b. Use Desmos to identify where the cut should be made in order to minimize the combined area of the shapes.

At $x = 22.404$ (this is the x-value at the minimum of the function)

c. What does the y-intercept of the function represent in the context of the problem?

(0, 127.324) This is if the entire length of the wire was used to make the circle, no square at all.