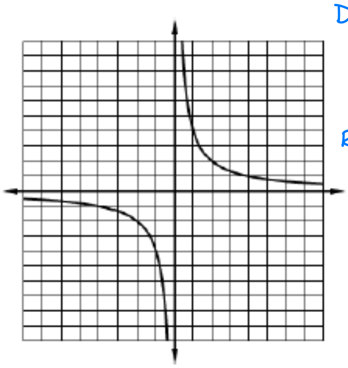
Pre-Calc Final Exam Review – Day 1

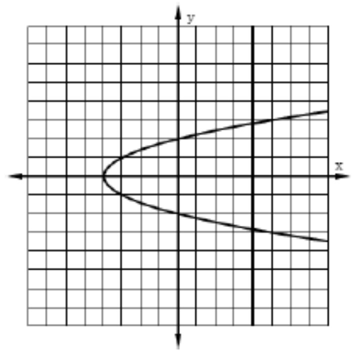
Domain and Range

State the domain and range for each of the relations.



D:
$(-\infty, 0) \cup (0, \infty)$

R:
$(-\infty, 0) \cup (0, \infty)$

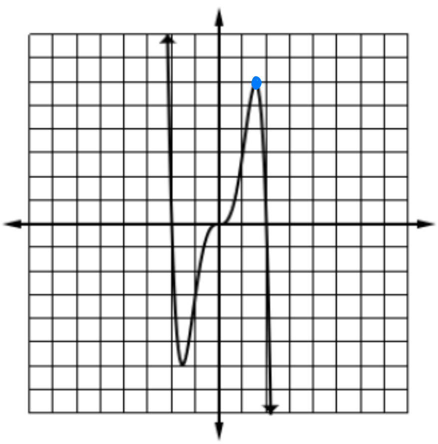


D:
$[-4, \infty)$

R:
$(-\infty, \infty)$

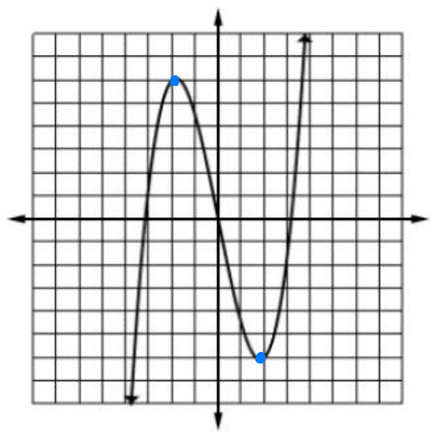
Extrema and Intervals:

Give the estimated coordinates and classify the extrema for the graph of each function.



Extrema: $(1.5, 6)$ relative max

$(-1.5, -6)$ relative min

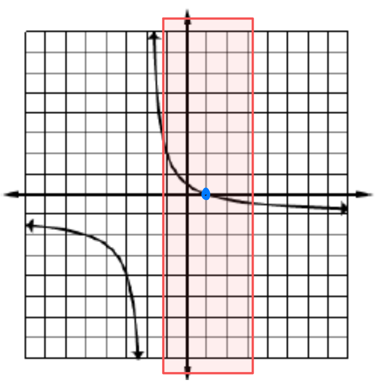


Extrema: $(-2, 6)$ rel. max

$(2, -6)$ rel. min

Closed Intervals

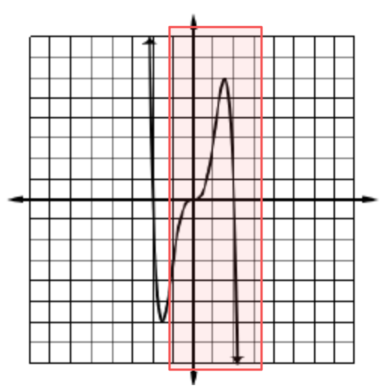
State the x-intercepts and intervals on which each function increases and decreases over the closed interval [-1,3]



x-intercept(s):
$(1, 0)$

Increasing interval(s):
None

Decreasing interval(s):
$[-1, 3]$



x-intercept(s):
$(0, 0)$, $(2, 0)$

Increasing interval(s):
$[-1, 1.5)$

Decreasing interval(s):
$(1.5, 3]$

Comparing Functions:

Consider the function below:
$f(x) = (x+5)(x+1)^2(x-3)$ Zeros = $\{-5, -1, 3\}$

a. Over the closed interval [-3, 1], what are the zeros of the function?

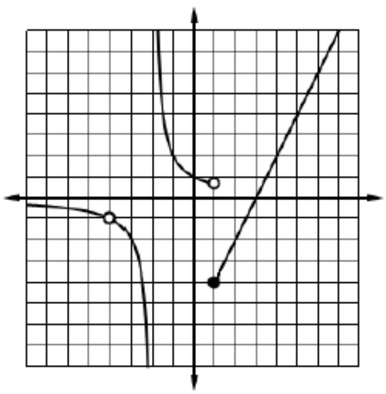
$(-1, 0)$

b. Consider the function $h(x) = \sin\frac{1}{2}(x) - 3$ between the same closed interval from [-3, 1]. Identify all the intervals for which $h(x) > f(x)$.

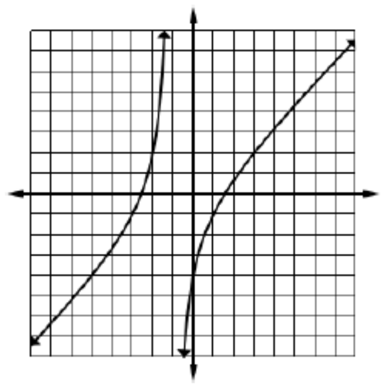
$[-3, -1.483) \cup (-0.545, 1]$

Continuity:

Determine if the function is continuous. If not, identify the type and location of discontinuity.



x = -4 (point)
x = 1 (jump)
x = -2 (infinite)



x = -1
(infinite)

Degree and End Behavior:

Use the leading coefficient and degree to describe the end behavior or each polynomial function.

$f(x) = \frac{5}{4}x^7 - x^5 + 6x^3 - 1$

Degree = 7
LC = $\frac{5}{4}$

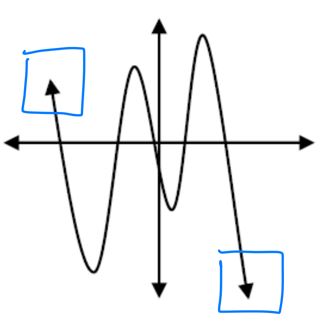
As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

$f(x) = -8x^4 + 6x^3 - 9x + 4$

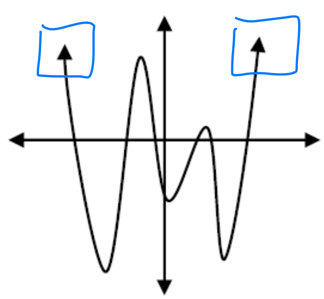
Degree = 4
LC = -8

As $x \to \infty$, $f(x) \to -\infty$
As $x \to -\infty$, $f(x) \to -\infty$

Describe both the degree and leading coefficient of the polynomial given the shape of the graph.



Odd
Negative



Even
Positive.

Zeros and Multiplicity:

Identify the zeros, the multiplicities, and the effect they have on the graph.

$$f(x) = \underbrace{x^5}_{0}\underbrace{(5x-1)^2}_{\frac{1}{5}}\underbrace{(4x+3)^3}_{-\frac{3}{4}}$$

$x = 0 \rightarrow$ mult. 5 $\rightarrow$ cubic

$x = \frac{1}{5} \rightarrow$ mult. 2 $\rightarrow$ bounce

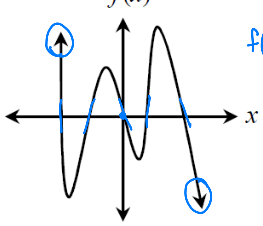
$x = \frac{-3}{4} \rightarrow$ mult. 3 $\rightarrow$ cubic

Write an equation that could represent the function with the given zeros.

$\frac{1}{2}$(mult.2), 5

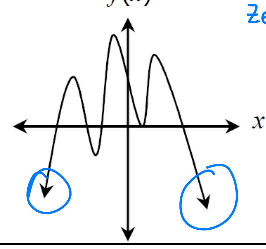
$f(x) = (2x-1)^2(x-5)$

Write an equation that could represent the graph with the given characteristics.



Zeros: $0, -4, -6, 3, 7$

$f(x) = -x(x+4)(x+6)(x-3)(x-7)$



Zeros: $-7, -4, -2, 1, 4$

$\uparrow$ mult 2

$f(x) = -(x+7)(x+4)(x+2)(x-1)^2(x-4)$

Rational Functions:

Simplify each rational expression.

$$\frac{p^2-p-72}{3p^2-28p+9} \longrightarrow p^2-28p+27$$

$$\left(p-\frac{27}{3}\right)\left(p-\frac{1}{3}\right)$$

$$\frac{\cancel{(p-9)}(p+8)}{\cancel{(p-9)}(3p-1)}$$

$$\boxed{\frac{p+8}{3p-1}}$$

$$\frac{5p^2+29p-6}{5p^2-26p+5} = \frac{p^2+29p-30}{p^2-26p+25} = \frac{\left(p+\frac{30}{5}\right)\left(p-\frac{1}{5}\right)}{\left(p-\frac{25}{5}\right)\left(p-\frac{1}{5}\right)}$$

$$= \frac{(p+6)\cancel{(5p-1)}}{(p-5)\cancel{(5p-1)}}$$

$$\boxed{\frac{p+6}{p-5}}$$

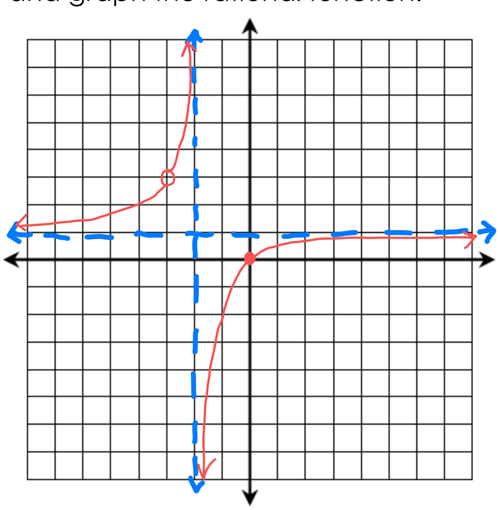
Rational Graph:

Identify all key features and graph the rational function.

$$f(x) = \frac{x^2+3x}{x^2+5x+6}$$

$$\frac{x\cancel{(x+3)}}{\cancel{(x+3)}(x+2)}$$

$x+3 \rightarrow$ Hole $x=-3$



Domain: $\{x \mid x \neq -3, -2\}$

Range: $\{y \mid y \neq 1, 3\}$

x-intercept(s): Num $= 0$ $\quad x = 0$

y-intercept: $x = 0$ $\quad y = 0$

Vertical Asymptote: Dem $= 0$ $\quad x+2=0$, $x=-2$

Horizontal Asymptote: same degree $\quad \frac{LC}{LC} = \frac{1}{1}$ $\quad y = 1$

Slant Asymptote: None

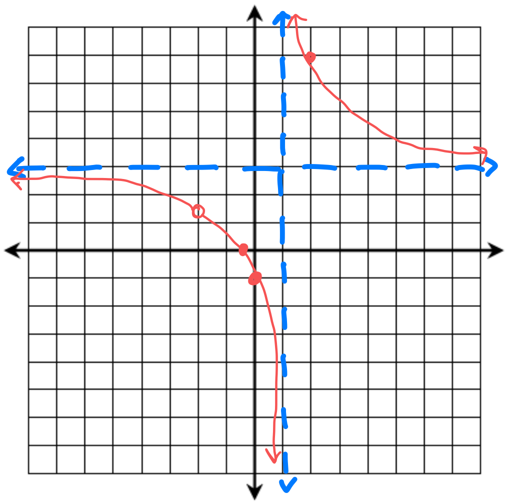
Hole(s): $(-3, 3)$

$$f(x)=\frac{3x^2+7x+2}{x^2+x-2}$$

$$\frac{\cancel{(x+2)}(3x+1)}{\cancel{(x+2)}(x-1)}$$

Test @ $x=2$

$$\frac{3(2)^2+7(2)+2}{(2)^2+2-2}=7$$



Domain: $\{x \mid x \neq -2, 1\}$

Range: $\{y \mid y \neq 5/3, 3\}$

x-intercept(s): $(-1/3, 0)$

y-intercept: $(0, -1)$

Vertical Asymptote: $x=1$

Horizontal Asymptote: $y=3$

Slant Asymptote: None

Hole(s): $(-2, 5/3)$

Logarithms: $\log_b a = x \longleftrightarrow b^x = a$

Write each equation in exponential form.

1. $\log_2 \frac{1}{8} = -3$

$$2^{-3}=\frac{1}{8}$$

3. $\ln_e 7 = x$

$$e^x = 7$$

Write each equation in logarithmic form.

4. $32^{\frac{4}{5}} = 16$ $\quad \log_{32} 16 = \frac{4}{5}$

5. $e^4 = x$

$$\ln x = 4$$

Evaluate the following, using change of base formula when necessary.

7. $\log_{\frac{1}{2}} 16$

$$\frac{\log 16}{\log \frac{1}{2}} = -4$$

9. $\ln 36$

$$3.5835$$

Solve each equation using logarithms.

$$\left(\frac{1}{3}\right)^{x-5} - 7 = 4$$

$$\left(\frac{1}{3}\right)^{x-5} = 11$$

$$\log_{\frac{1}{3}} 11 = x-5$$

$$\frac{\log 11}{\log \frac{1}{3}} + 5 = x$$

$$\boxed{x = 2.8173}$$

$$2 \cdot e^{4x-8} + 8 = 23$$

$$\frac{2 \cdot e^{4x-8}}{2} = \frac{15}{2}$$

$$e^{4x-8} = 7.5$$

$$\ln_e 7.5 = 4x-8$$

$$\frac{\ln 7.5 + 8}{4} = x$$

$$\boxed{x = 2.5037}$$

Logarithmic Applications:

1. The population P of a city is $P = 2500e^{kt}$ where t = 0 represents 2000. In 1945, the population was 1350. Find the value of k, and use this value to estimate the population in the year 2024.

1945
$(t = -55)$

$1350 = 2500e^{k(-55)}$

$\frac{27}{50} = e^{-55k}$

$\frac{\ln_e \frac{27}{50}}{-55} = \frac{-55k}{-55}$

$k = 0.0112$

2024
$(t = 24)$

$P = 2500e^{(0.0112)(24)}$

$P = 3270.98$

$\boxed{P = 3271 \text{ people}}$

2. The number of bacteria N in a culture is given by the model $N = 100e^{kt}$ where t is the time (in hours). If N = 300 when t = 5, estimate the time required for the population to double in size.

$300 = 100e^{k(5)}$

$3 = e^{5k}$

$\ln 3 = 5k$

$k = 0.2197$

$200 = 100e^{0.2197t}$

$2 = e^{0.2197t}$

$\ln 2 = 0.2197t$

$\boxed{t = 3.155 \text{ hours}}$

3. A roast turkey is taken from an oven when its temperature has reached 185 degrees Fahrenheit and placed on a table in a room where the temperature is 75 degrees Fahrenheit. The function T that describes the amount the turkey cools after t minutes can be modeled by $T(t) = T_s + (T_0 - T_s)e^{-kt}$, where

$T(t)$ represents the Temperature of the turkey, 150
t represents the number of minutes after the turkey is taken out of the oven, 30
T_s represents the surrounding temperature in degrees Fahrenheit, 75
T_0 represents the initial temperature of the turkey as it is taken out of the oven, and 185
k represents a constant

a. If the temperature of the turkey is 150 degrees Fahrenheit after half an hour, when will the turkey cool to 100 degrees Fahrenheit?

$150 = 75 + (185 - 75)e^{-k(30)}$

$75 = 110e^{-30k}$

$\frac{15}{22} = e^{-30k}$

$\frac{\ln_e \frac{15}{22}}{-30} = \frac{-30k}{-30}$ $k = 0.0128$

$100 = 75 + 110e^{-0.0128t}$

$25 = 110e^{-0.0128t}$

$\frac{5}{22} = e^{-0.0128t}$

$\frac{\ln \frac{5}{22}}{-0.0128} = \frac{-0.0128t}{-0.0128}$ $\boxed{t = 115.75 \text{ min}}$

b. Over time, to what temperature does the turkey cool? Explain.

75 as that is room temperature.