

# Class Project

## Designing a Counterweight Trebuchet

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## 1. Project Definition and Aim

This project allows us to design and build the trebuchet which can throw the object like ping-pong ball over 5 meters using at least 2 500g PASCO weight. We divide the trebuchet into 2 big parts (base tower and arms) to easily to design and build. Since the requirement ask us to create the parts which fit with limited dimension, we cut each big part into pieces that can meet the condition.

- **Base Tower**: Act like the foundation to hold firm for any movements of Arms. It includes 3 small parts which are upper supporters, supporters and base.

- Upper Supporters: 2 pieces, they locate at the top of the base tower to hold the main axis for the trebuchet. They are constructed by Laser Cutter.
- Supporters: Cut into 2 sides, which are same dimension and suitable with the box, they connected with the upper supporters as well as base by joints, then fixed by bolts and nuts. It locates underneath the supporters to maintain the balance for the trebuchet. They are built by 3D print.
- Base: created by Laser Cutter and place at the bottom of the trebuchet and keep the important task that make sure the trebuchet upright and does not lose the equilibrium when it is operated.
- Runway: the place for the projectile to stand before the trebuchet activate, also the road for it to run when the object starts working. It is produced by 3D Printer.

⇒ ***Accounting for 7 out of 12 parts.***

- **Arms**: the important part of the project which decide the throwing distance of the projectile. It combines by finger, long arm, middle arm and short arm. The Arms are created by Laser Cutter excepted Finger which is constructed by 3D Printer.

- Finger: to hold the sling of the projectile. The tool to control the release angle when the trebuchet operated.
- Long arm: connector between the finger and the middle arm.
- Middle arm: to link the long arm with the main axis. Specially, the length of the longarm can be adjusted by the joints which use to connect with the middle arm.
- Short arm: there are 2 short arms places at the end of the Arms, which are keeping the middle arm at centre. They are big and thick enough for hooking at least 1kg of mass.

⇒ ***Accounting for 5 out of 12 parts.***

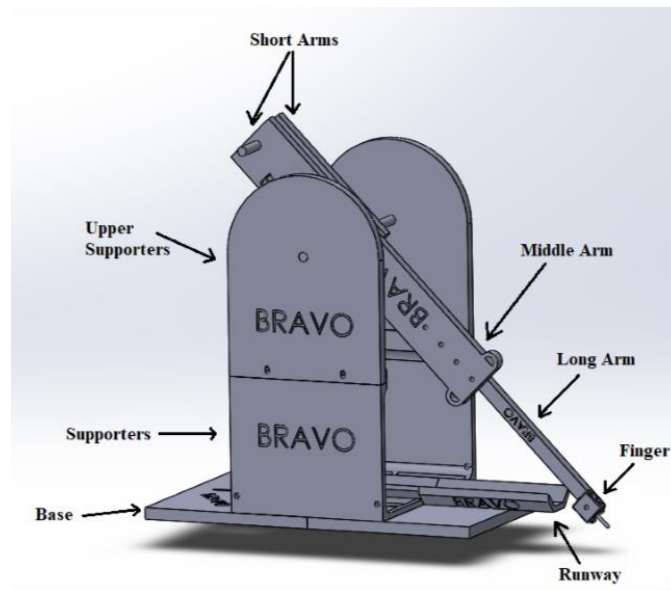


Figure 1. An overview of the trebuchet

## 2. Design Theory and Methodology

- **Conservation Energy:**

Basically, a trebuchet takes advantage of potential energy from a falling counterweight hanged at a predefined height and transfer it into a moving projectile which is laid initially on the ground. The transferred energy provides work to lift the projectile up against gravity force as well as initial velocity for it. Thus, we can somehow determine the initial velocity of the projectile right at the time it is about to launch.

On one side of the equation, we have the potential energy of the counterweight. Given its potential energy at ground level is zero, gravitational potential energy of the counterweight is achieved by:

$$E_{CW} = m_{CW} \times g \times \Delta h_{cw}$$

Where  $m_{CW}$  is the mass of the counterweight (kg),  $g$  is gravitational acceleration,  $\Delta h_{cw}$  is change is in height of the counterweight when it is dropped at the lowest point and the projectile is about to fire.

There are also additional operands, the increase in gravitational potential energy when the projectile is being lifted and rotational kinetic energy when the arm is rotating. Our energy conservation formula is eventually:  $m_{cw} \times g \times \Delta h_{cw} = \frac{1}{2} m_p \times v^2 + m_p \times g \times \Delta h_p$

Re-arrange the above formula, we have  $v = \sqrt{\frac{2 \times [(m_{cw} \times g \times \Delta h_{cw}) - (m_p \times g \times \Delta h_p)]}{m_p}}$  (1)

From the formula (1), we can achieve a much greater initial velocity for the projectile by increasing the mass and the distance dropped of the counterweight while decreasing the mass of the projectile. The greater initial velocity will eventually ensure a larger firing range, based on projectile motion analysis. [1]

- **Project motion:**

Since the projectile in this case is the Ping-Pong ball suffer Rotational Kinetic Energy when it starts to leave the trebuchet and Gravitational Potential Energy when it falls down because of gravity. The ball will move in the vertical and horizontal axis. The velocities of the projectile are  $v_x = v \times \cos \theta$  and  $v_y = v \sin \theta$  ( $v$  is the magnitude of velocity,  $\theta$  is its direction relative to the horizontal). [2]

In horizontal motion: [3]

$$v_{0x} = v_x, \quad x = x_0 + v_x \times t$$

In vertical motion:

$$y = y_0 + \frac{1}{2}(v_x + v_0)t$$

$$v_y = v_{y,0} \times g \times t$$

$$y = (y_0 + v_{y,0} \times t) - \left(\frac{1}{2} \times g \times t^2\right)$$

$$v_y^2 = v_{y,0}^2 - 2 \times g \times (y - y_0)$$

Time of flight:

$$t = \frac{2 \times v_0 \times \sin \theta}{g}$$

Maximum height reached:

$$H = \frac{2 \times v_0^2 \times \sin^2 \theta}{2 \times g}$$

Maximum Range:

$$R = \frac{v_0^2 \times \sin 2\theta}{g}$$

Where:  $v_0$  is the initial velocity (m/s).

$V_x$  is the velocity (along the x-axis) (m/s).

$V_y$  is the velocity (along the y-axis) (m/s).

$\sin\theta$  is the component along the y-axis.

$\cos\theta$  is the component along the x-axis.

$g$  is the acceleration due to gravity ( $g = 9.8 \text{ m/s}^2$ ).

$t$  is the time taken (s).

- **Rotational Dynamic**

Because the trebuchet works by using the potential energy from a falling counterweight hanged at a predefined height to launch the projectile (Ping-Pong ball), it bears a certain amount of energy which is called Rotational kinetic energy. The formula is

$$K_r = \frac{1}{2} \times I \times \omega^2 \quad [4]$$

Where:

$\omega \left( \frac{\text{radians}}{\text{sec}} \right)$  is angular speed =  $\frac{\Delta\theta}{\Delta t}$  ( $\Delta\theta$  is total covered.  $\Delta t$  is total taken) [5]

$I = \sum_I m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + \dots$  ( $m$  is the mass of the particle (kg),  $r$  is the distance from the particle to the centre of rotation (m))

Because the arm of our trebuchet has a certain mass, the Moment of Inertia has additional equation:

$I = \frac{1}{3} mL^2$  (where  $m$  is the mass of arm (kg),  $L$  is the length of the arm(m)).

- **Moments or Torques:** [6]

From the trebuchet: taking the clockwise direction as being positive, the total moment acting on the trebuchet:

$$\tau_{\text{total}} = \sum_{i=1}^n \tau_i = \tau_{\text{CW}} - \tau_{\text{projectile}}$$

In general, if the angle between the direction of the force and the line towards the pivot point is theta,  $\theta$ , the equation for the moment of the force about the pivot point is:

$$\tau = Fd\sin\theta$$

Where  $F$  is the magnitude of the force acting on the object,  $d$  is the distance from the pivot point to where the force is acting,  $\theta$  is the angle between the direction of the force and the line towards the pivot point.

It has a massless beam as well. During the movement of the trebuchet, it will experience a torque due to the unbalanced weights, which has a magnitude

$$\begin{aligned} \tau &= m_{\text{CW}} \cdot g \cdot l_{\text{SA}} \cdot \sin\theta - m_{\text{projectile}} \cdot g \cdot l_{\text{LA}} \sin\theta \\ &= g \cdot \sin\theta \cdot (m_{\text{CW}} \cdot l_{\text{SA}} - m_{\text{projectile}} \cdot l_{\text{LA}}). \end{aligned}$$

Where:

$m_{CW}$ : mass of the counterweight (kg).

$l_{SA}$ : length of the short arm (m).

$m_{projectile}$ : mass of the projectile (kg).

$l_{LA}$ : length of the long arm (m).

$g$ : gravitational acceleration ( $m/s^2$ ).

$\theta$ : the angle between the direction of the force and the line towards the pivot point.

- **Affecting Factors:** Several parameters might make a major contribution to the result and effectiveness of the trebuchet:
  - **Mass Ratio between the Counterweight and the Projectile** from law of conservation of energy, it can be concluded that heavier counterweight results in greater gravitational potential energy, which then leads to much kinetic energy converted into the projectile. In addition, heavier counterweight leads to more torque generated about the fulcrum.
  - **Arm length ratio:** Because of the mass ratio between counterweight and projectile, in order to ensure the torque applied on the long arm is large enough for the entire arm to rotate and angular acceleration of the projectile to be also large, arm ratio should be taken into accounts as well.
  - **Friction in fulcrum:** Obviously, friction in fulcrum can decrease the angular acceleration of the projectile and thus affects its launching velocity. Therefore, we decide to use a ball bearing for the rotational movement of the arm.
- **Action Plan:**
  - **Step 1:** Determine all basic parameters related to arm lengths, pivot height, mass ratio between counterweight and projectile to fulfill the requirement: firing a spherical projectile whose diameter is up to 45 mm and weight 2.7 grams over at least 5 meters.
  - **Step 2:** Determine appropriate specific dimensions for all parts of the trebuchet so that not only do they accommodate the deliverable from *step 1* but also each of which is fit into a 14 x 14 x 14 cm box.

- **Step 3:** From all the parts defined in step 2, we will decide how each part is manufactured: which part will be lasers cut, which one will be 3D printed. The parts in 3D will be carefully selected as only 4 parts are permitted.

### 3. Design Mechanism and Calculations

- **Design Mechanism** [\[7\]](#)

When the trigger holding the arm for trebuchet is released, the counterweight will fall down. During that process, the Ping-Pong ball (projectile) will be dragged and slide on the runway by the sling. The time when the Ping-Pong ball leaves the runway, it swings freely through an arc. At some point, the Ping-Pong ball will be released out of the sling because the one end of the sling will be slipped out of the finger (based on the angle between finger and long arm).

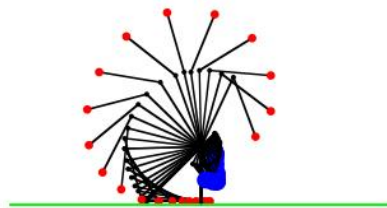


Figure 2. How trebuchet works

To be easier to understand how Ping-Pong ball slipped out of the string, it can be explained that the sling just only exerts a tension along itself. Within the direction indicated, the rotating Ping-Pong ball will exert a force on the one end of the sling, and it tends to pull the one end of the sling along a finger until that one end of the sling is slipped out of the finger and at that time, the ping-pong ball will fly a range.

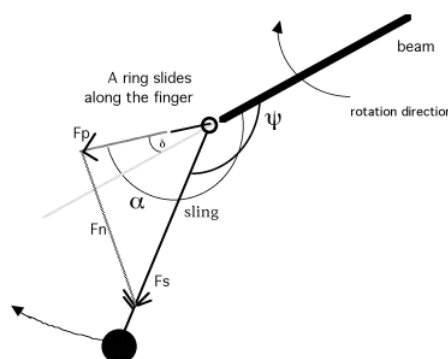


Figure 3. How Ping-Pong ball can be slipped out the sling

- **Design Calculation**

Following the action plan above, we first decided some parameters as constant for the sake of ease of calculations since the trebuchet is a complex machine. In terms of mass ratio between the counterweight and projectile, we ensure the mass of counterweight is as heavy as possible in relation to that of the projectile. Thus, we choose 4 PASCO 500 grams weights whose total mass is 2000 grams. To hang 4 PASCO weights on the short arm, we have to hang them in parallel, which makes short arm no shorter than 8 cm for the sake of durability. The length ratio between long and short arm is 4:1. The height of pivot is 23.5 cm as the height of each PASCO weight is 9cm and the short arm is 8cm, and we do not want them to hit the ground or the projectile runway when dropping. To leverage the arm ratio even more, we choose the sling whose length is close to that of the long arm, 30cm. With all these constants and based on the conservation energy formula discussed above, the initial velocity of the projectile is (assuming the long arm is nearly vertical and releasing angle is 45 degree):

$$V = \sqrt{\frac{2(m_{cw} \times g \times \Delta h_{cw} - m_p \times g \times h_p)}{m_p}}$$

$$= \sqrt{\frac{2 \times (2 \times 9.8 \times 0.1465 - 0.0027 \times 9.8 \times 0.7813)}{0.0027}} = 45.95 \text{ (m/s)}$$

The range then can be partly determined by applying the projectile motion formula, assuming the projectile released at 45 degree:

$$R = \frac{v_0^2 \sin 2\theta}{g} = 215.45 \text{ (m)}$$

Obviously, the actual result is far smaller than the calculated one as there are several factors such as air resistance, friction in fulcrum that might affects the range of firing projectile. In addition, the range is calculated given that all of potential energy generated from the counterweight is ideally transferred into kinetic energy of the projectile completely while in real life, this situation is hard to achieved. To test our assumption, due to difficulty in real life manufacturing at the first stage of this project, we rely on a virtual trebuchet simulation to examine our parameters:



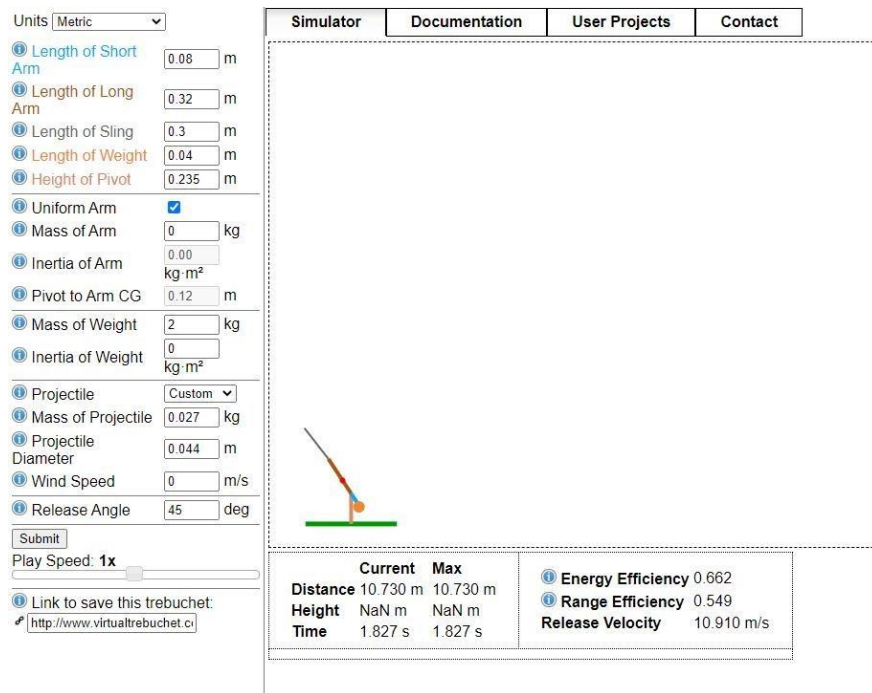


Figure 4. Projectile range in trebuchet virtual simulation.

## 4. Device Description

For each component, a picture of its 3D model should be provided together with a discussion on its primary features, its manufacturing process (3D printed or Laser Cut) and justifications for its dimensions as well as manufacturing methods.

### Bill of Material

ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Base	Acrylic Laser Cut Manufacturing	2
2	Support	3D Printing Manufacturing	2
3	UpperSupport	Acrylic Laser Cut Manufacturing	2
4	Runway	3D Printing Manufacturing	1
5	Short arm 1	Acrylic Laser Cut Manufacturing	2
6	Middle-Arm	Acrylic Laser Cut Manufacturing	1
7	Long-Arm	Acrylic Laser Cut Manufacturing	1
8	Finger	3D Printing Manufacturing	1

- **Arm**

The arm is divided into 4 parts: finger, long arm, middle arm and short arm.

- **Short Arm**

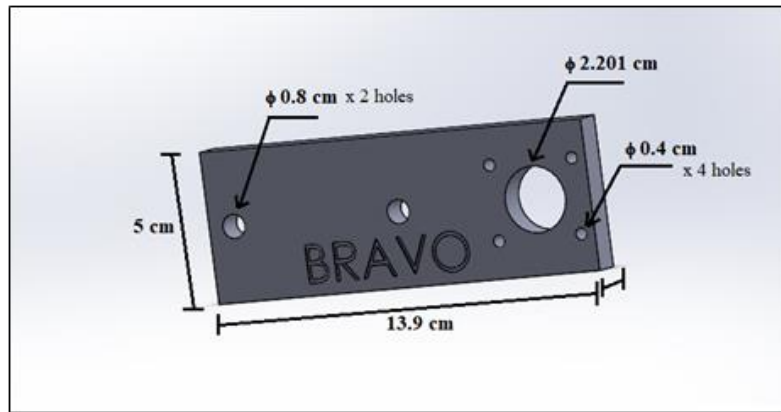


Figure 5. Short arm of the trebuchet

The short arm is connected to the middle arm at one end and the counterweights, which will be the PASCO 500g weight at the other. The short arm also has a hole with the diameter of 22 millimeter to fit the ball bearing. At the end, which is used to connect the counterweights, there are two holes. These holes are there to put a metal beam through and hang the counterweights on that beam. There are two holes to support multiple blocks. The short arm will be manufactured by the laser cutter. It has a bounding box dimension of 13.9cm x 5cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section below.

- *Middle Arm*

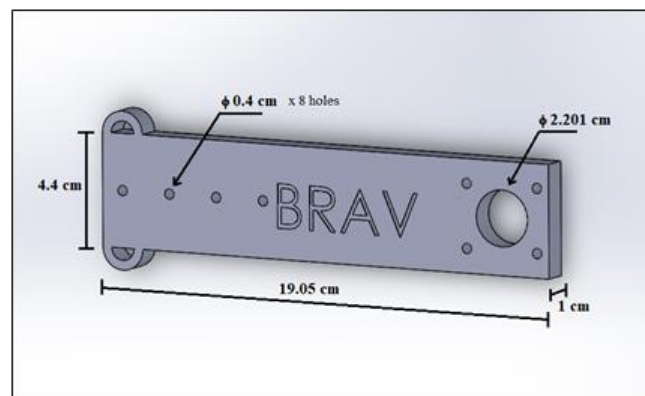
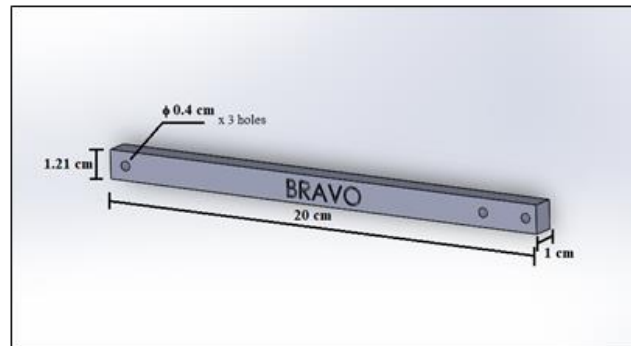


Figure 6. Middle arm of the trebuchet

The middle arm is connected to the long arm at one end and the short arm at the other. They are connected by using nuts and bolts, which are put through holes with the diameter of 4 millimeter. At the end, which is connected to the short arm, there is a big hole with the diameter of 2.2 centimeters, which will be used to put a ball bearing in. The ball bearing is there to eliminate most of the friction. At the end, which is connected to the long arm, there are 4 holes, which will be used to control the total length of the arm. At the other end, there are two holes in the shape of an arc, which is used to connect the middle arm to the trigger. The middle arm will be manufactured using the laser cutter. It

has a bounding box dimension of 19.05cm x 6.1cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section below.

- *Long Arm*

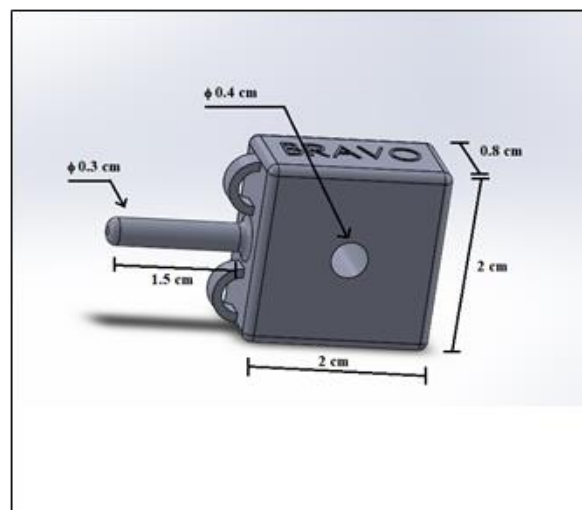


*Figure 7. Long arm of the trebuchet*

The long arm is connected to the finger at one end, and the middle arm at the other. They are connected with nuts and bolts, which will be put through the holes on the body of the long arm. These holes have the same diameter of 4 millimeter.

The long arm will be manufactured by the laser cutter. It has a bounding box dimension of 20cm x 1.21cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section below.

- *Finger*



*Figure 8. Finger of the trebuchet*

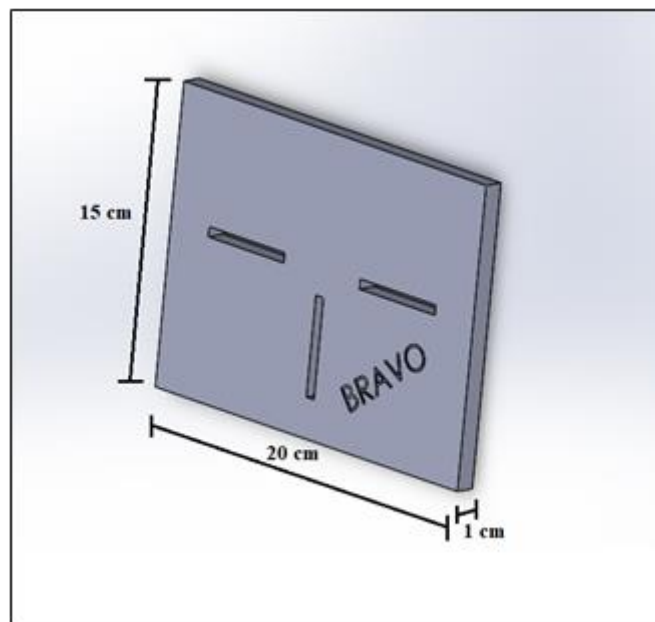
The finger is the most important part of the release mechanism of the trebuchet. It is connected to the end of the long arm, via a nut and bolt with a diameter of 4 millimetres. The finger will be 3D printed with tough resin. It has a bounding box dimension of 3.5cm x 2 cm x 1cm, which fits the

requirements for 3D printed parts. The specific dimensions will be shown in the appendix section below.

- **Tower**

The tower is divided into 4 parts: base (front - rear), support (left - right), upper support (left - right) and runway.

- *Base*



*Figure 9. Base of the trebuchet*

The base is the bottom foundation that is used to connect the support and later the whole trebuchet. There are two pieces of the base, which are connected at the longer edge (the 20cm edge) of each other. On the top of the base, there are 3 rectangular slots that cut through the base. These slots are there so that the support and the runway, which will be discussed later in the report, can be connected to via the tabs on those parts. The base will be manufactured by the laser cutter. It has a bounding box dimension of 20cm x 15cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section below.

- *Lower Support*

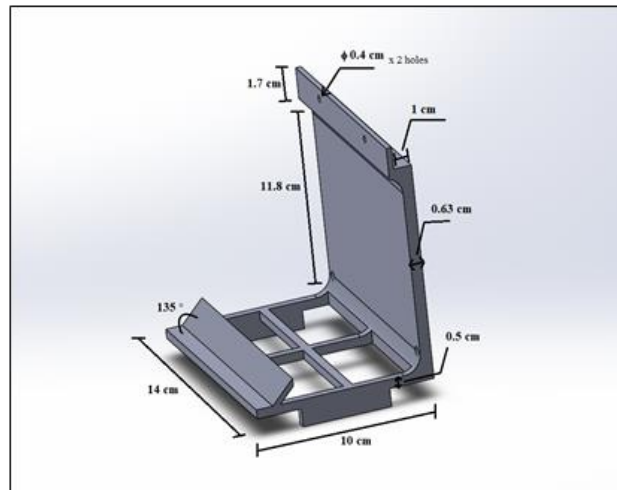


Figure 10. Lower support of the trebuchet

The support is the part that is connected to the base at the bottom, and the upper support on top. It has a distinctive "L" shape. At the bottom, there are two tabs, which are used to connect to the slots on the base. Along with that, the support has one half of the primary runway. There are two holes, with a diameter of 4 millimeters each to connect the support to the upper support via a nut and bolt. The top of the support also has a "L" shape to let the bottom of the top support settle on. The bottom of the support must have rectangular holes to reduce its volume and comply with the volume restriction for 3D printed parts. There will be another support, which is mirrored along the longer edge (the 14cm edge) of the part to complete the entire support. The illustration of this will be shown in the assembly section later in the report. The support will be manufactured by the 3D printer. It has a bounding box dimension of 14cm x 10cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section below.

#### ○ Upper Support

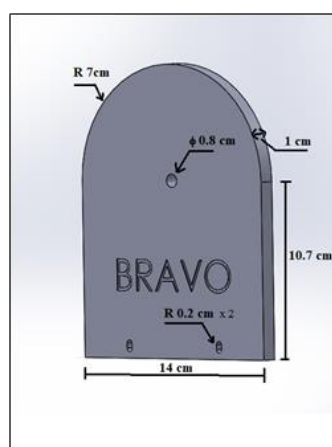
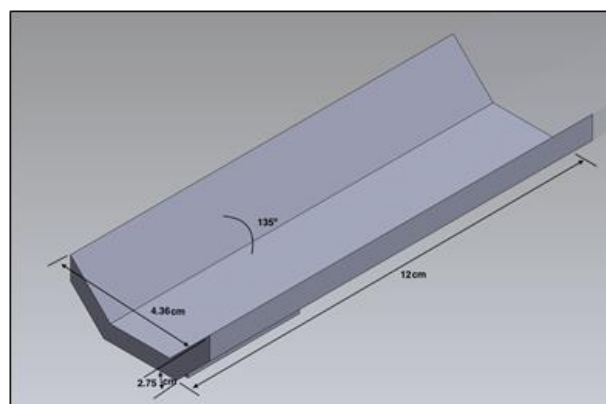


Figure 11. Upper support of the trebuchet

The upper support is the part that is connected to the support and the connector beams, which is connected to the arm. The lower half of the top support has a rectangular shape, while the upper half of it is filleted with an arc which has a radius of 7 centimeters. At the center of the filleted arc, there is a hole to put the connector beam through, which has a diameter of 0.8 centimeters. At the bottom of the upper support there are two holes, with a diameter of 0.4 centimeters each, to connect the upper support to the support using nuts and bolts.

The upper support will be manufactured by the laser cutter. It has a bounding box dimension of 14cm x 17.7cm x 1cm, which fits the dimension requirement for laser cut parts. The specific dimensions will be shown in the appendix section bellow.

- *Runway*



*Figure 12. Runway for the trebuchet*

This can be called the secondary runway, as the primary runway is integrated into the support. The primary runway extends the total distance of the runway. The purpose of the runway is to control the movement of the projectile. Initially, the projectile will be located at the end of the runway. When the arm starts swinging, the projectile will follow the runway in a straight direction, minimizing any disturbance to maximize the efficiency.

The runway has a "V" shape which was extruded to be longer. At the bottom of this runway, there is a tab, which will be used to connect to the slot on the base. It was designed in a way that will fit the projectile and will not be hit by the counterweights at the bottom most position.

The upper support will be manufactured by the 3D printer. It has a bounding box dimension of 12cm x 4.36cm x 2.75cm, which fits the dimension requirement for 3D printed parts. The specific dimensions will be shown in the appendix section bellow.

- **How to assemble all components**

- *Assemble all components for the arm*

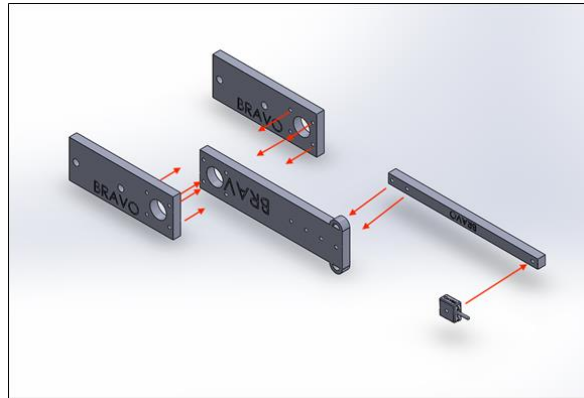


Figure 13. How to assemble the arm

As mentioned above, the parts will be connected using nuts and bolts. Firstly, two short arms are connected to the two sides of the middle arm. The four 4mm holes on the short arms need to be aligned with those on the middle arm. Four long nuts are put through the holes and secured by four bolts. Then, based on the chosen length of the entire arm, two holes of the long arm will be aligned to the corresponding holes on the middle arm. And two nuts will be put through the two holes and be secured with two nuts. Finally, the finger will be connected to the other end of the long arm. In a way so that the finger is straightly aligned with the middle arm. The hole on the finger must be aligned with that on the long arm. A nut will be put through the hole and be secured by a nut.

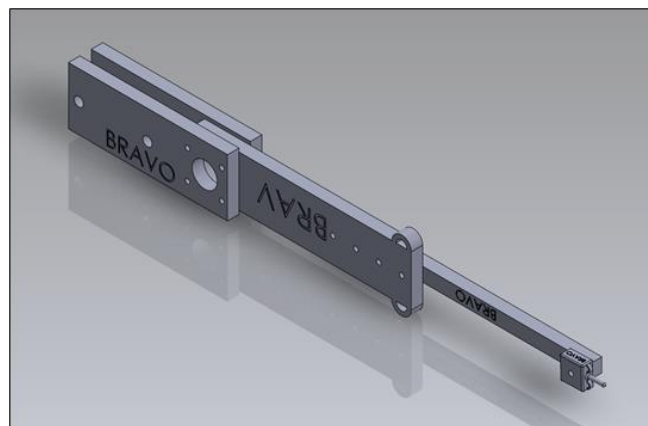


Figure 14. Completely assemble the arm

- *Assemble all components for the base-support tower*

First, the two bases and supports need to be aligned as shown in the figure. The tab at the bottom of the runway is connected to the vertical slot on the rear base. Two upper supports will be connected and aligned to the supports as shown in the figure. Four nuts will be put through the holes and be secured with four bolts. Four tabs of the two supports will now be connected to the remaining four horizontal slots on the base. Completely assemble the base-tower support

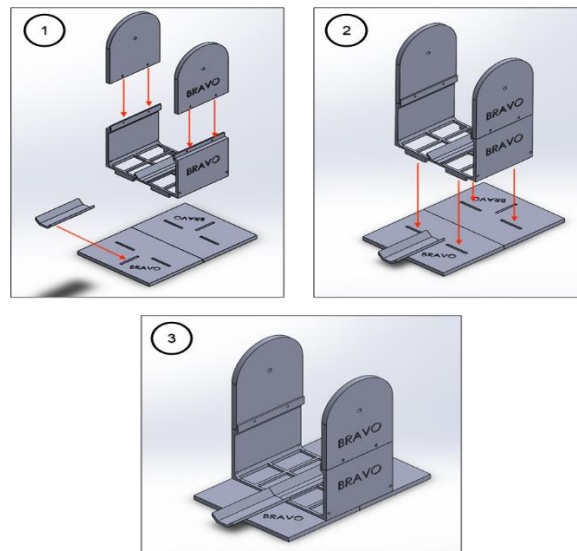


Figure 15. How to assemble base and support towers

- Completely assemble trebuchet by assembling Arm and Base-Support Towers

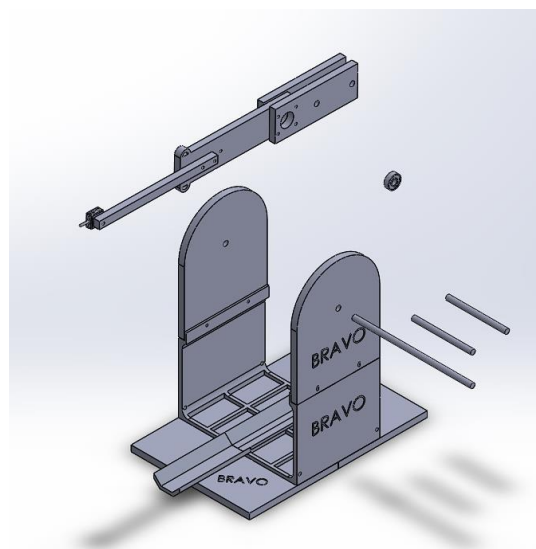


Figure 16. Assembling Arm and Base-Support Towers

For assembling the arm and the base-support towers, our group used several connecting parts including 1 Ball Bearing (with the parameter 8mm), 1 Connector Beams (with the diameter 8mm with length 200mm), 2 Mass Beams (with the diameter 8mm and the length 100mm). Our group inserts the Ball Bearing into the connection point between Short-Arm, Long-Arm and Middle-Arm of the Arm. The main reason to use Ball Bearing to reduce friction to maximize efficiency of the system. After that, we use the Connector Beam to insert through the Ball Bearing that already inserted into the connection point of the arm. Finally, we connect 2 Mass Beams at 2 connection points at the end of the Short-Arm.



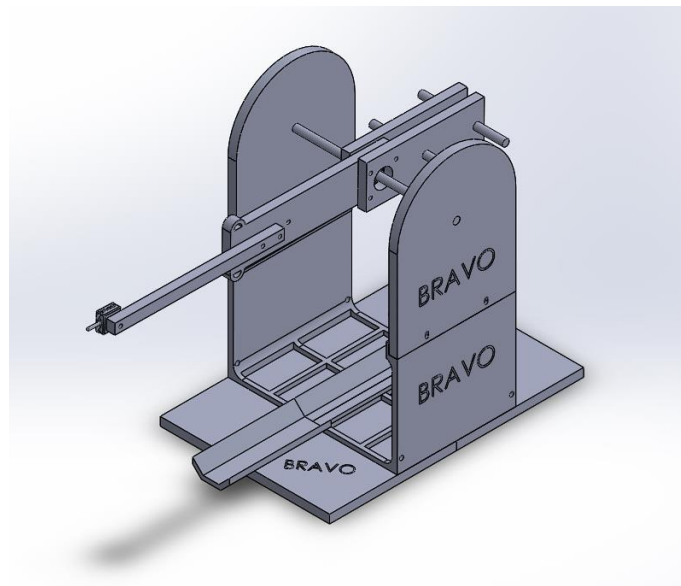


Figure 17. Completely assemble trebuchet

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## 6. Appendixes for 2D-Drawings

