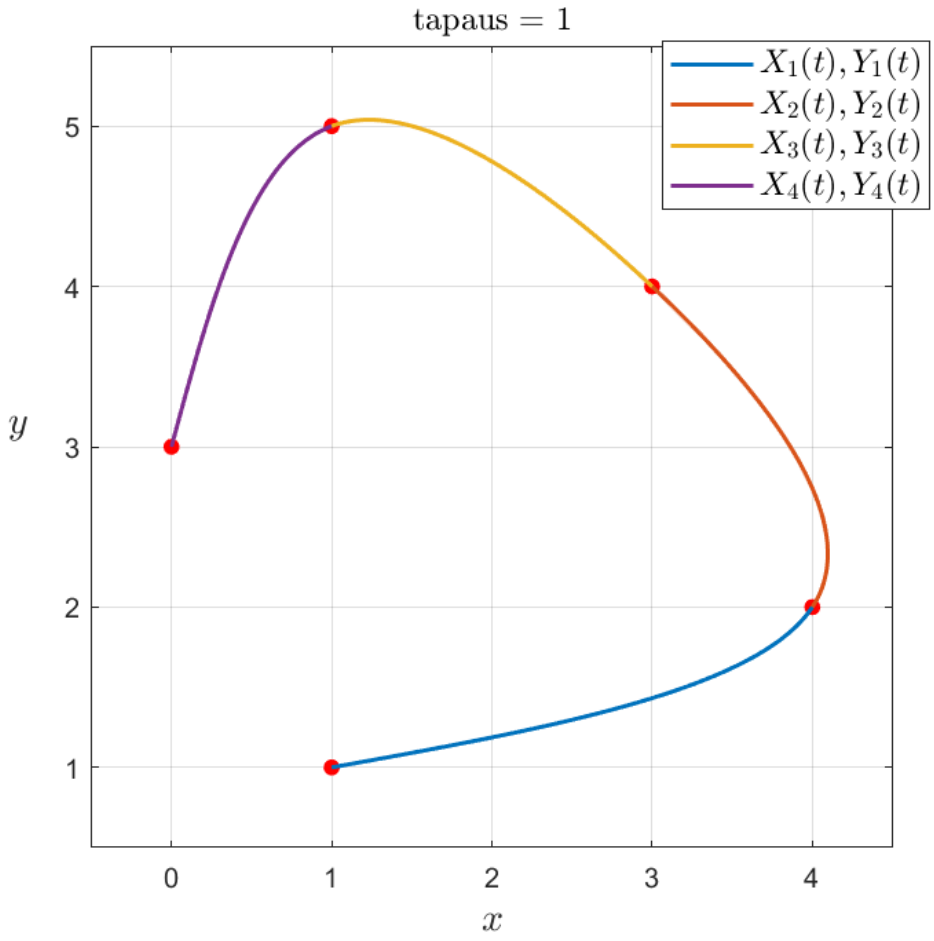
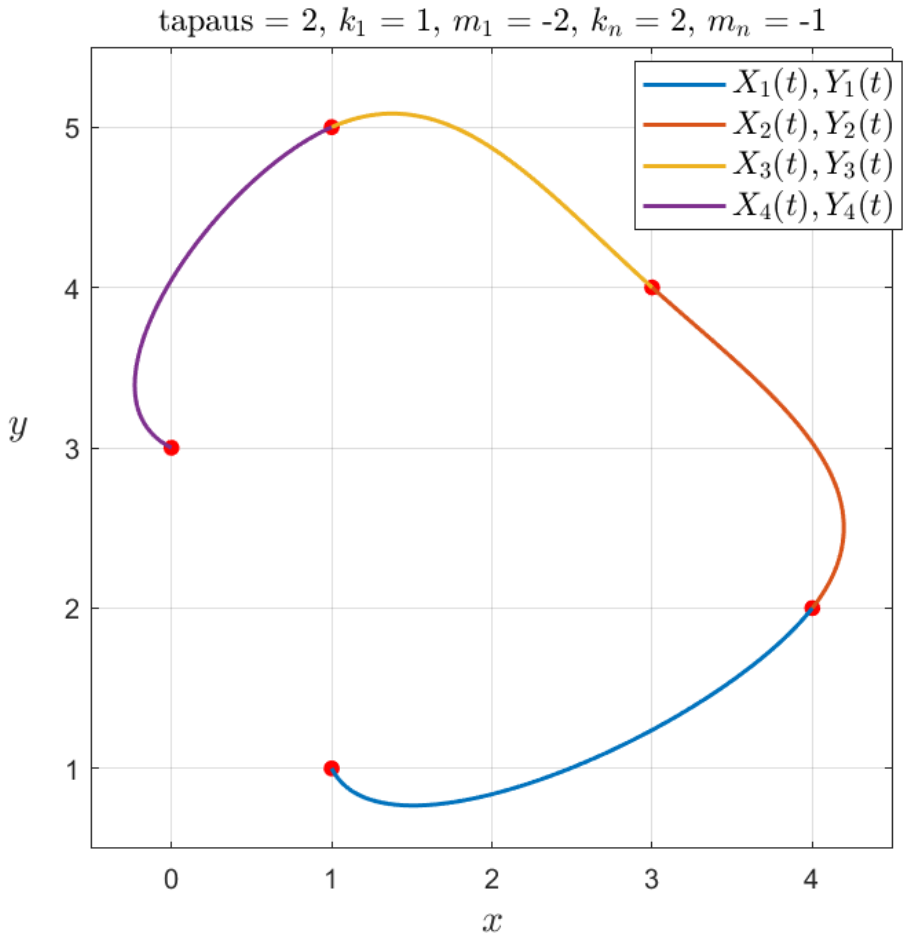
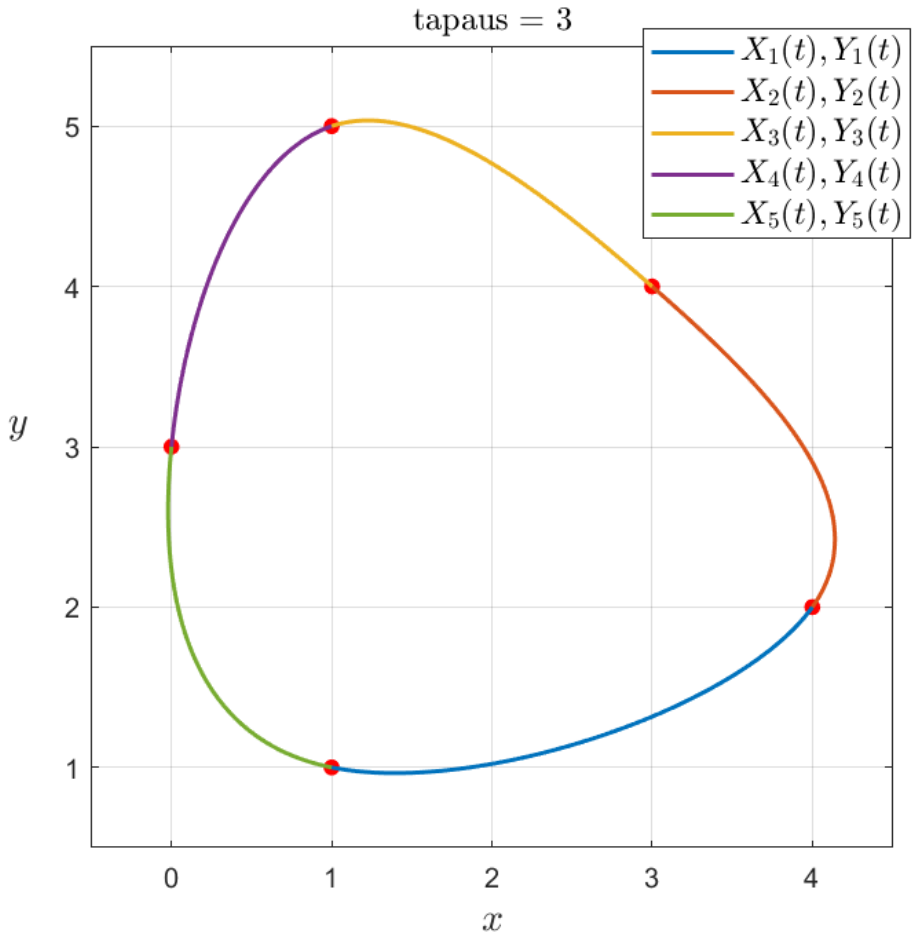
1. Given points

$$[x_1, y_1],\ [x_2, y_2], \ldots, [x_n, x_n]$$

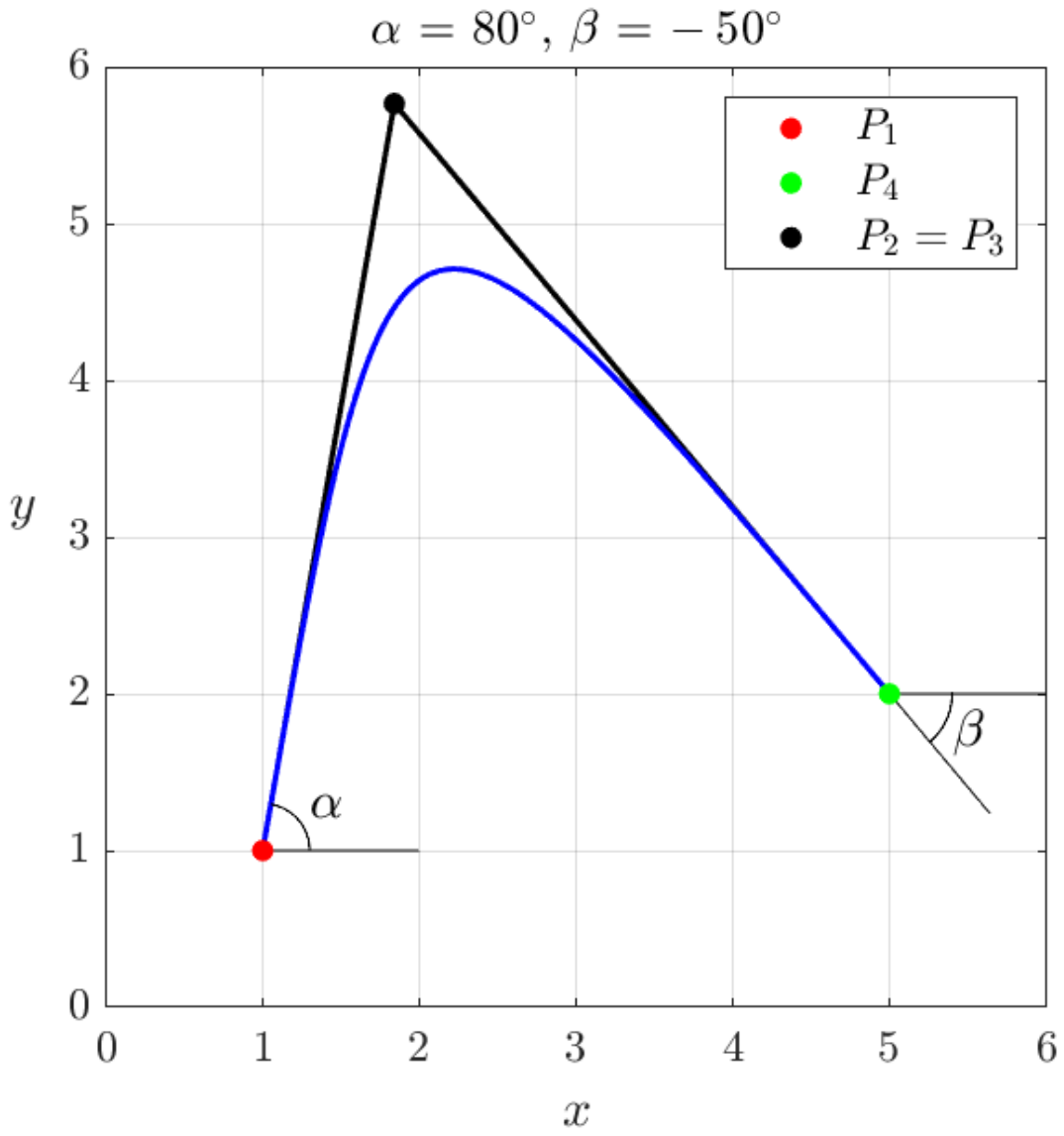
and case = 1,2 tai 3 (natural, clamped, periodic), calculate the coefficients of the corresponding parametric spline curve and draw a picture like below.
(if case = 2, values k_1, m_1, k_n, m_n are alo given, and if case = 3, then $x_1 = x_n$, $y_1 = y_n$)



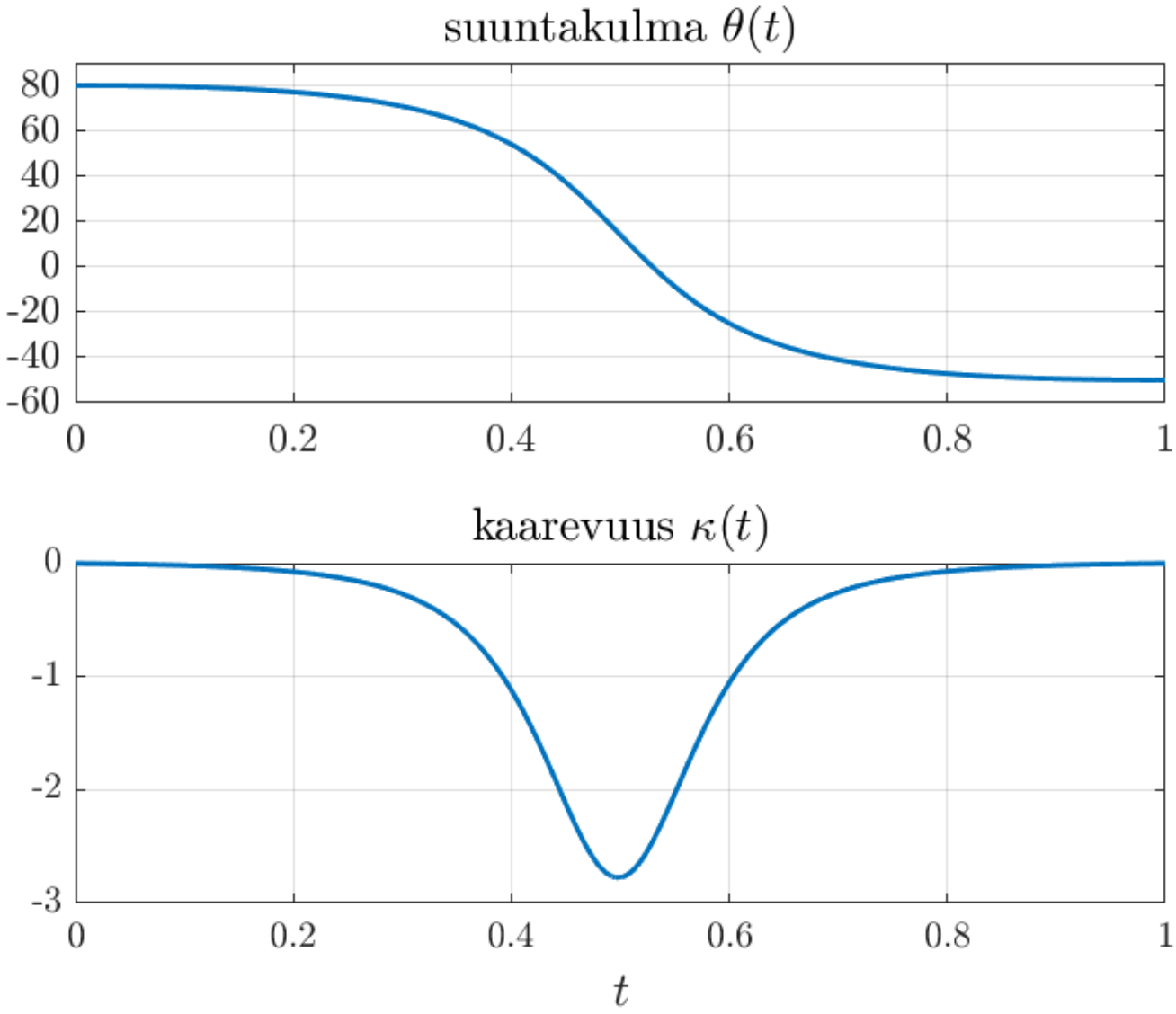

tapaus = 2, k_1 = 1, m_1 = -2, k_n = 2, m_n = -1
$X_1(t), Y_1(t)$
$X_2(t), Y_2(t)$
$X_3(t), Y_3(t)$
$X_4(t), Y_4(t)$
y
x
0
1
2
3
4
5


tapaus = 3
$X_1(t), Y_1(t)$
$X_2(t), Y_2(t)$
$X_3(t), Y_3(t)$
$X_4(t), Y_4(t)$
$X_5(t), Y_5(t)$
y
x
0
1
2
3
4
5

2. Given points P_1 and P_4 and angles α and β, find point $P_2 = P_3$ such that the direction angle of the Bezier-curve determined by points $P_1 - P_4$ is α and β at P_1 and P_4 (and curvature $= 0$). Draw a picture



Draw also the graphs of the direction angle $\theta(t)$ and curvature $\kappa(t)$ of the Bezier-curve



Hint: solve r and s from the linear system

$$\begin{cases} P_{2x} = P_{1x} + r\cos(\alpha) = P_{4x} + s\cos(\beta) \\ P_{2y} = P_{1y} + r\sin(\alpha) = P_{4y} + s\sin(\beta) \end{cases}$$

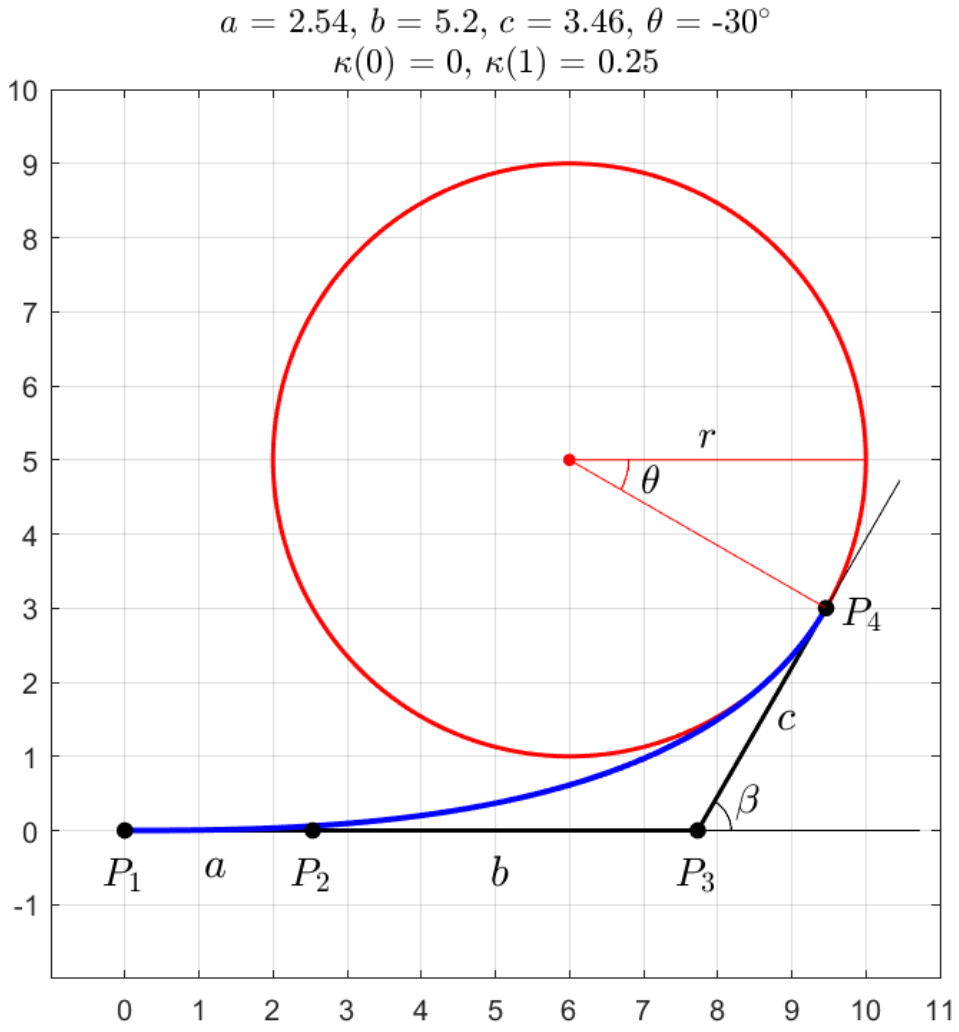
3. Given the center $[x_0, y_0]$ and radius r of a circle and angle θ, calculate points

$$P_1 = [x_1, 0], P_2 = [x_2, 0], \ P_3 = [x_3, 0]$$

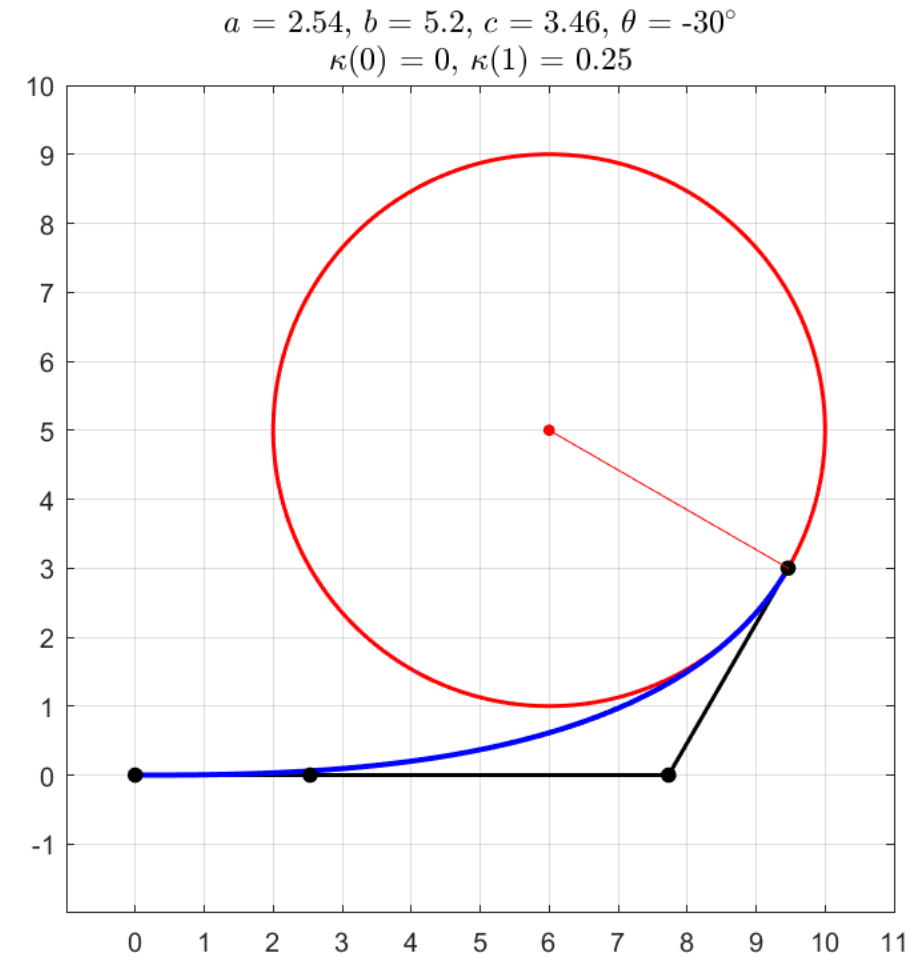
such that if

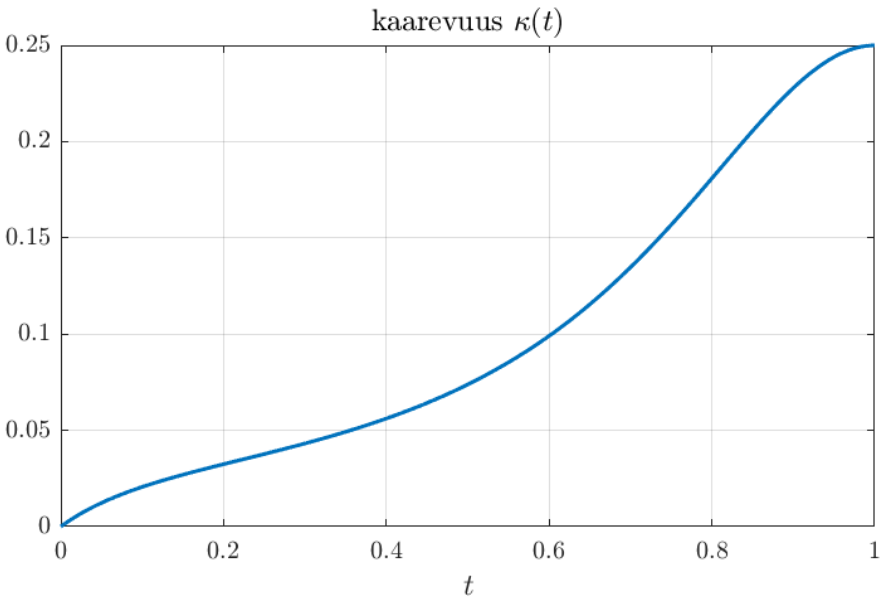
$$P_4 = [x_0 + r\cos(\theta), \ y_0 + r\sin(\theta)]$$

then the direction angle of P_3P_4 is $\beta = \theta + 90°$ and the Bezier curve determined by $P_1 - P_4$ has curvature 0 at P_1 and $1/r$ at P_4.

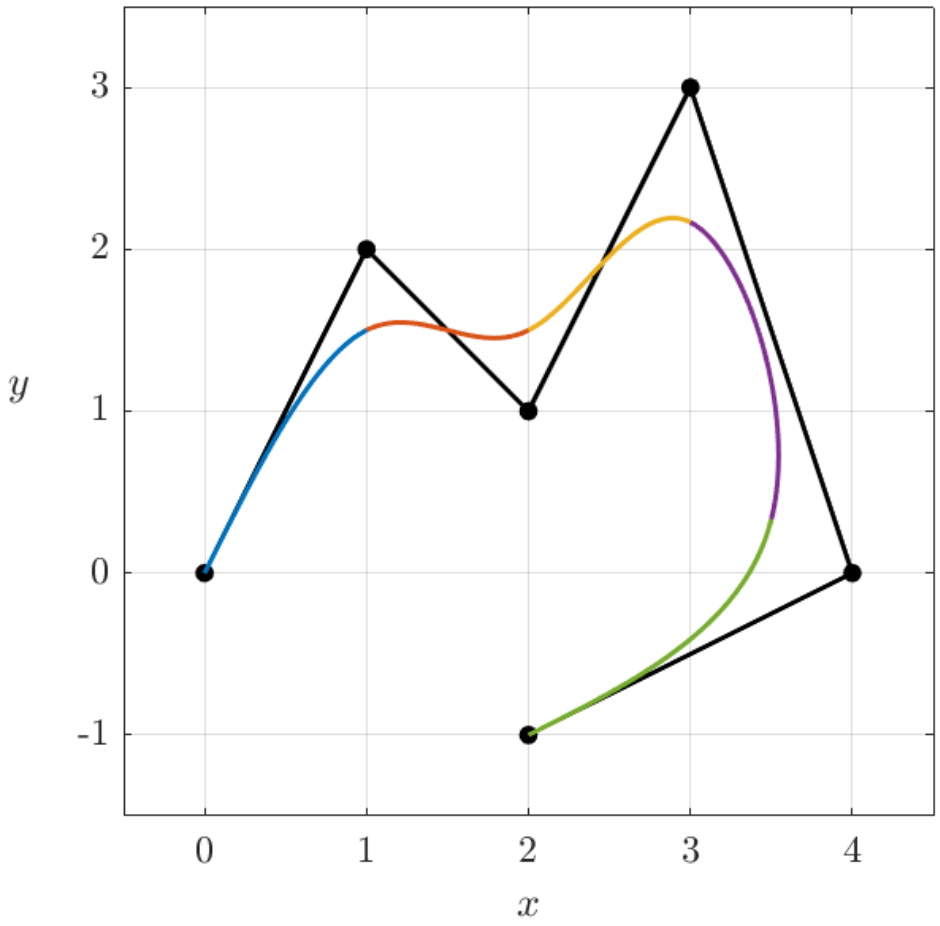


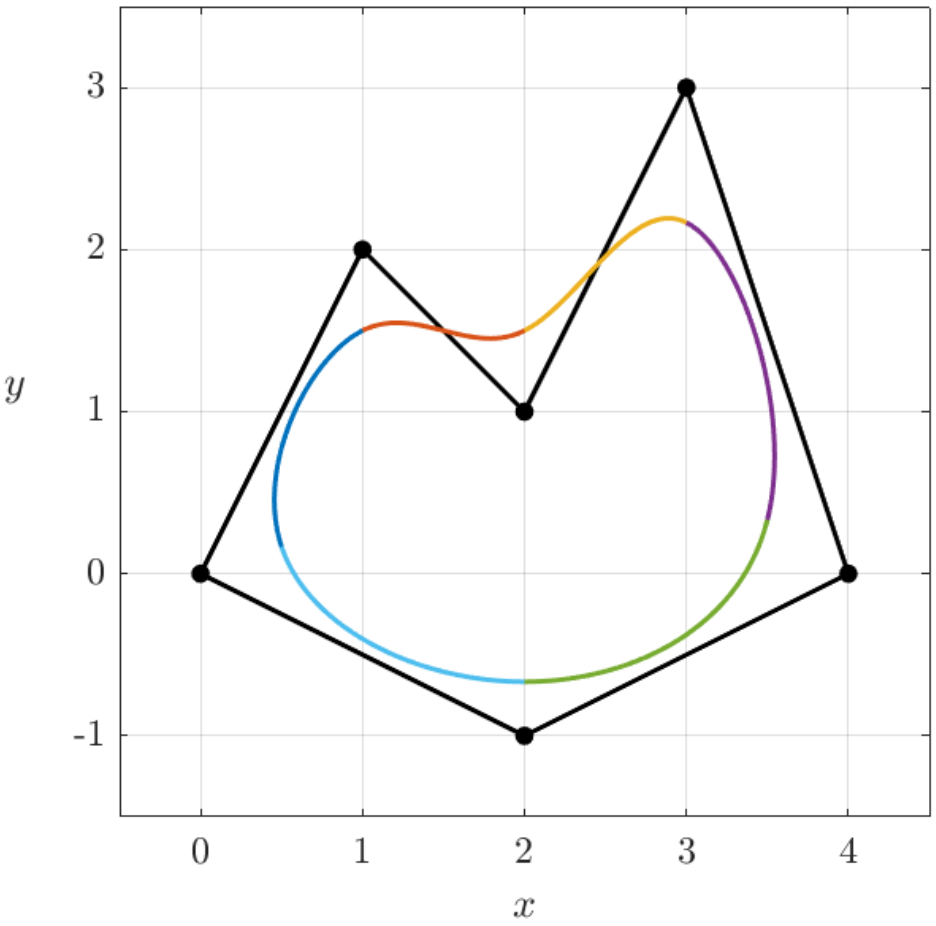
Draw pictures like below





4. Given points $B_1, B_2, \ldots, B_n$, calculate Bezier splines approximating polygons $B_1B_2 \ldots B_n$ and $B_1B_2 \ldots B_nB_1$, and draw pictures like below




3
2
1
0
-1
y
0
1
2
3
4
x