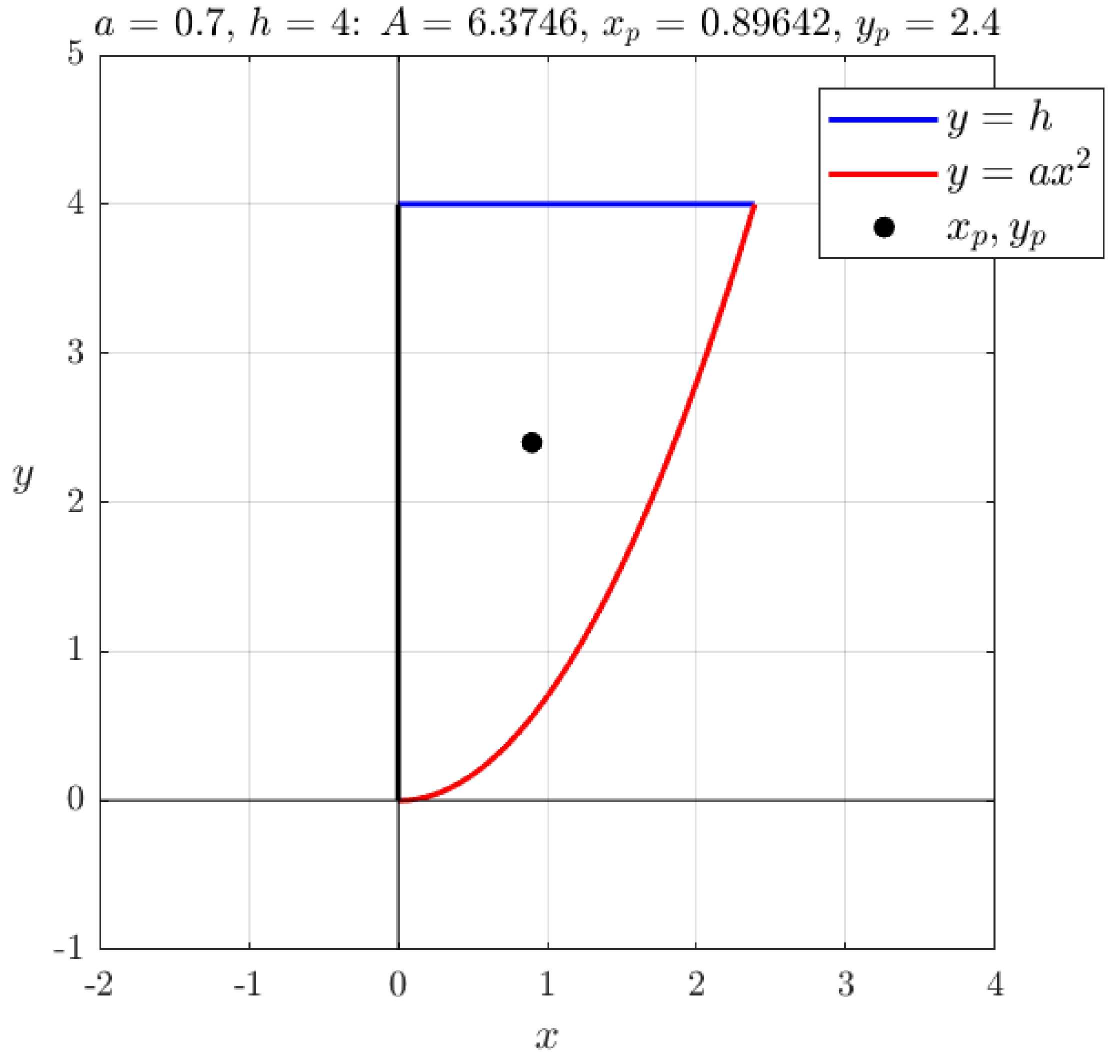
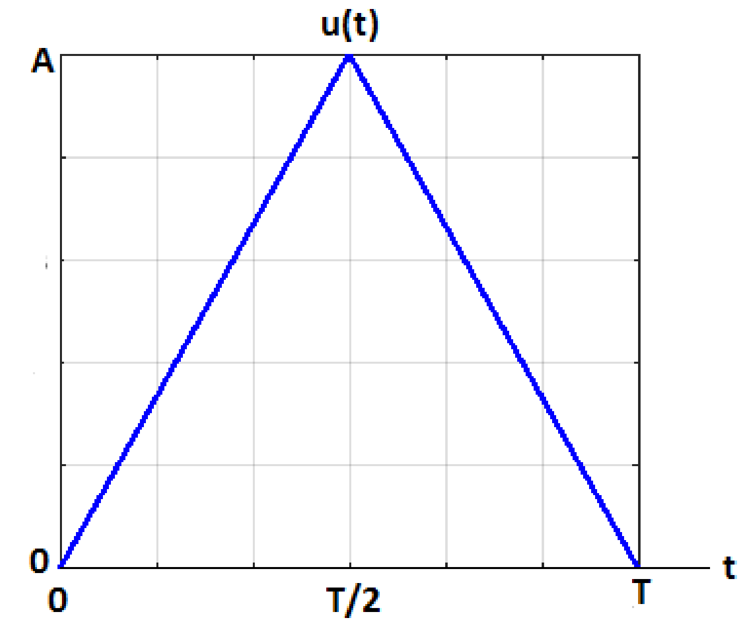
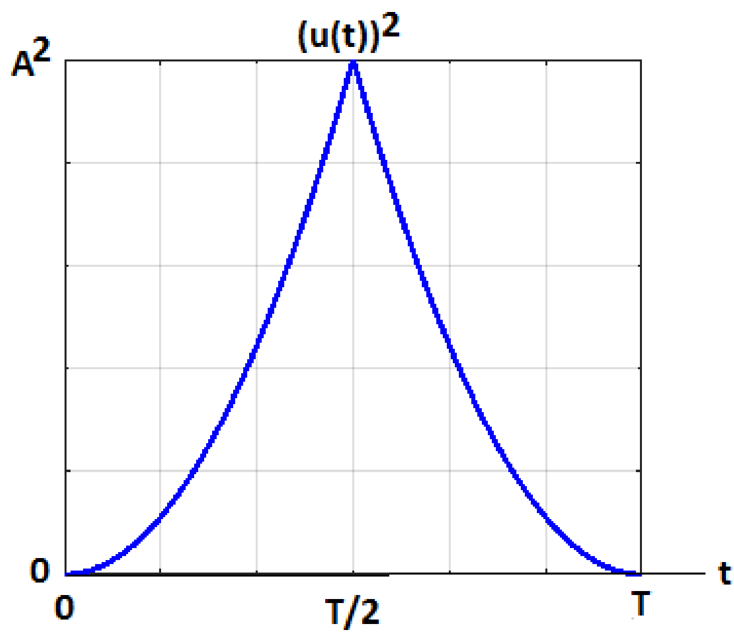
1. Given a and h, calculate the area A and the centroid $[x_p, y_p]$ of the region bounded by the line $f(x) = h$, the parabola $g(x) = ax^2$ and y-axis, and draw a picture like below



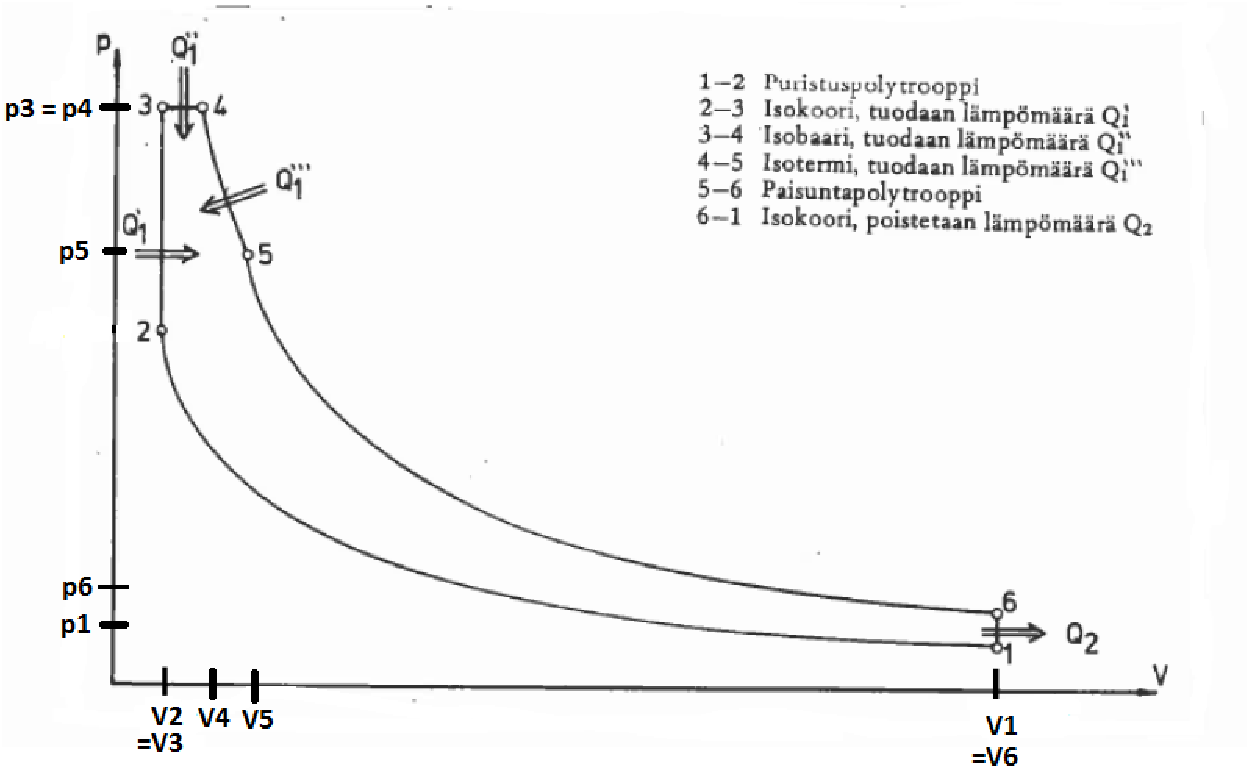
2. Given A and T, calculate the mean u_{avg} and the root mean square u_{rms} of the function $u(t)$ below.





ans. $A = 5, T = 4 \rightarrow u_{avg} = 2.5,\ u_{rms} = 2.89$

3. Given $V_1, p_1, V_2, p_3, V_4, V_5, k_1$ and k_2, calculate the area inside the pV-curve below (= work done during the cycle 1-6)



1-2: $V = V_1 \ldots V_2$, $p = p_1 \cdot \left(\dfrac{V_1}{V}\right)^{k_1}$

2-3: vertical line

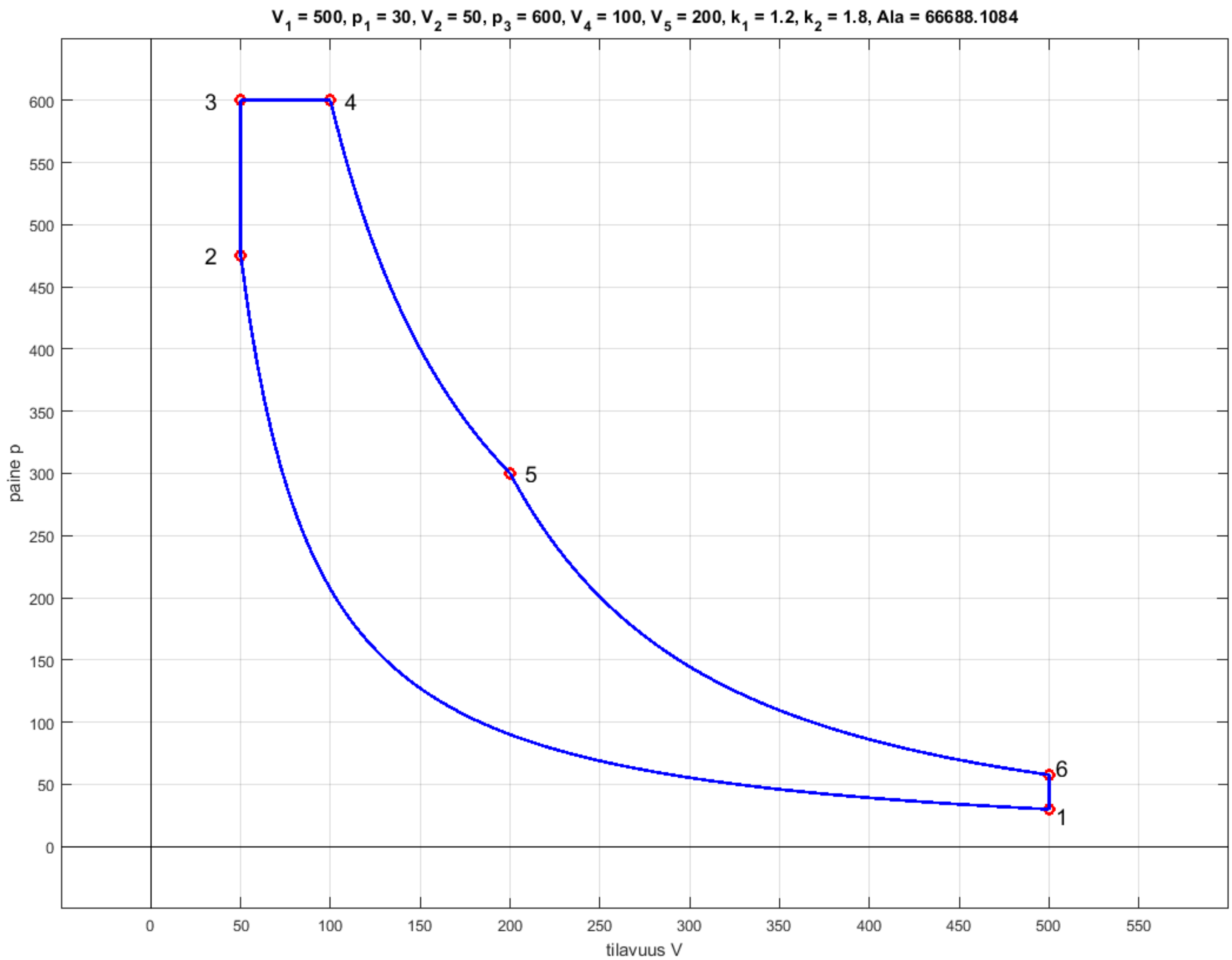
3-4: horizontal line

4-5: $V = V_4 \ldots V_5$, $p = p_4 \cdot \dfrac{V_4}{V}$

5-6: $V = V_5 \ldots V_6$, $p = p_5 \cdot \left(\dfrac{V_5}{V}\right)^{k_2}$

6-1: vertical line

Draw also a picture like below:



hint:

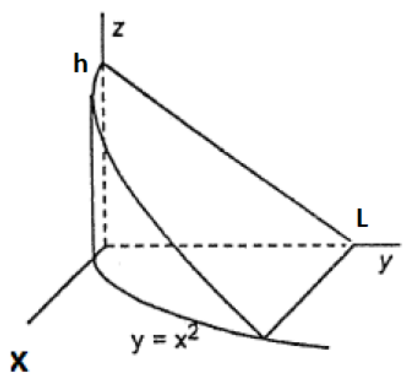
$$p_2 = p_1 \cdot \left(\frac{V_1}{V_2}\right)^{k_1},\ V_3 = V_2,\ p_4 = p_3$$

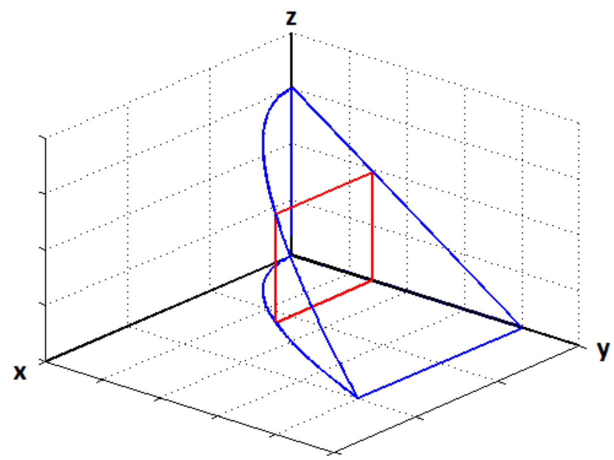
$$p_5 = p_4 \cdot \frac{V_4}{V_5},\ V_6 = V_1,\ p_6 = p_5 \cdot \left(\frac{V_5}{V_6}\right)^{k_2}$$

4. Calculate the volume

$$V = \int_0^L A(y)\, dy$$

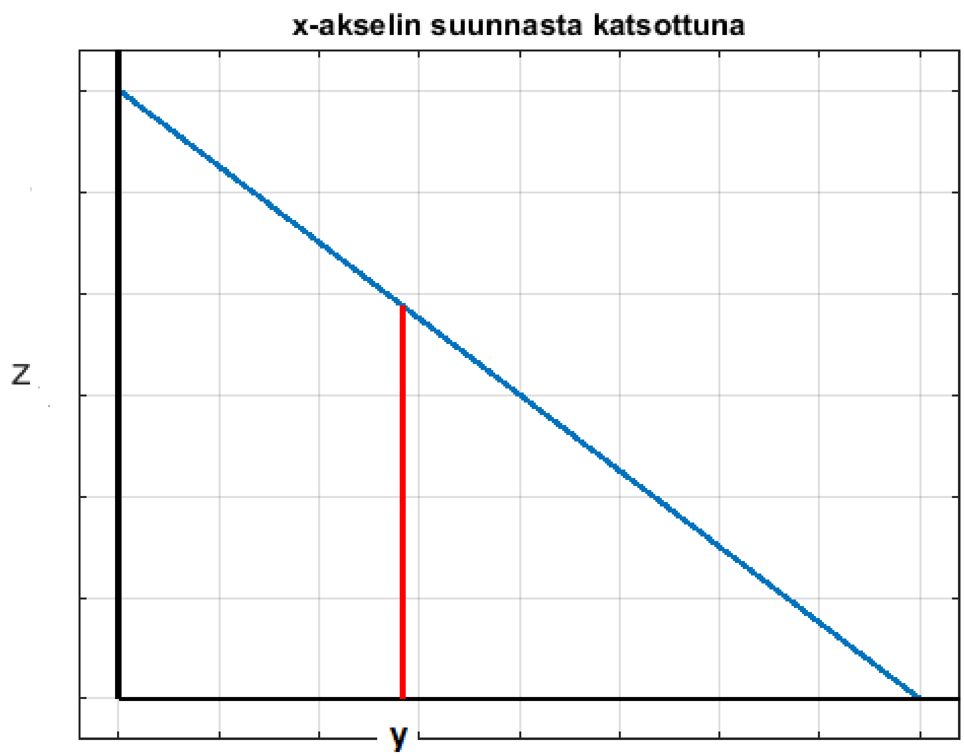
of the solid below, where $A(y)$ is the area of the cross-section at y (which is a rectangle).





ans: $h = 3, L = 4 \rightarrow V = 6.4$

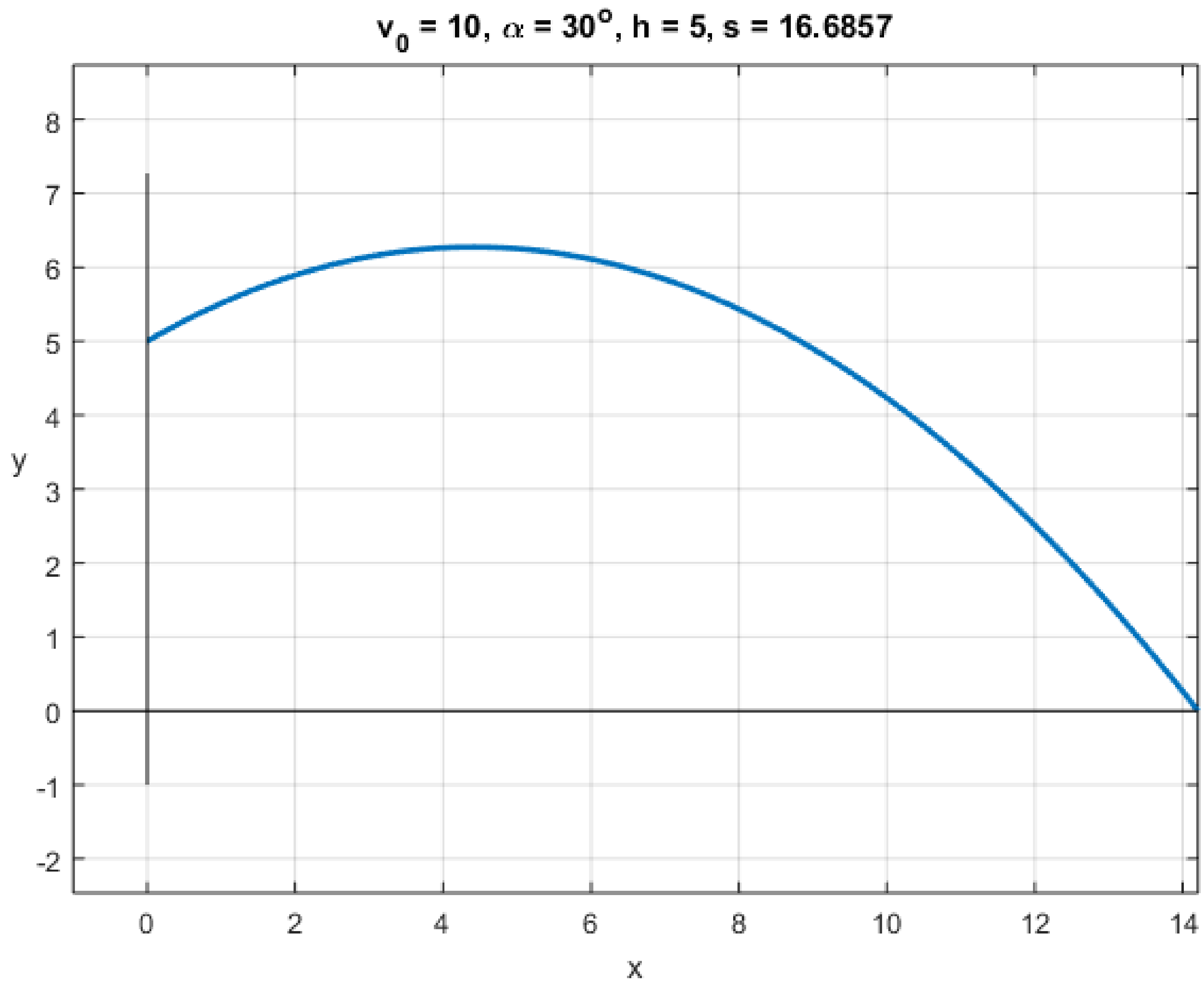

z-akselin suunnasta katsottuna
y
x


x-akselin suunnasta katsottuna
z
y

5. Given initial velocity v_0, angle α and height h, calculate (numerically) the length s of the projectile

$$y = -\frac{g}{2(v_0\cos(\alpha))^2} \cdot x^2 + \tan(\alpha) \cdot x + h, \quad g = 9.81$$

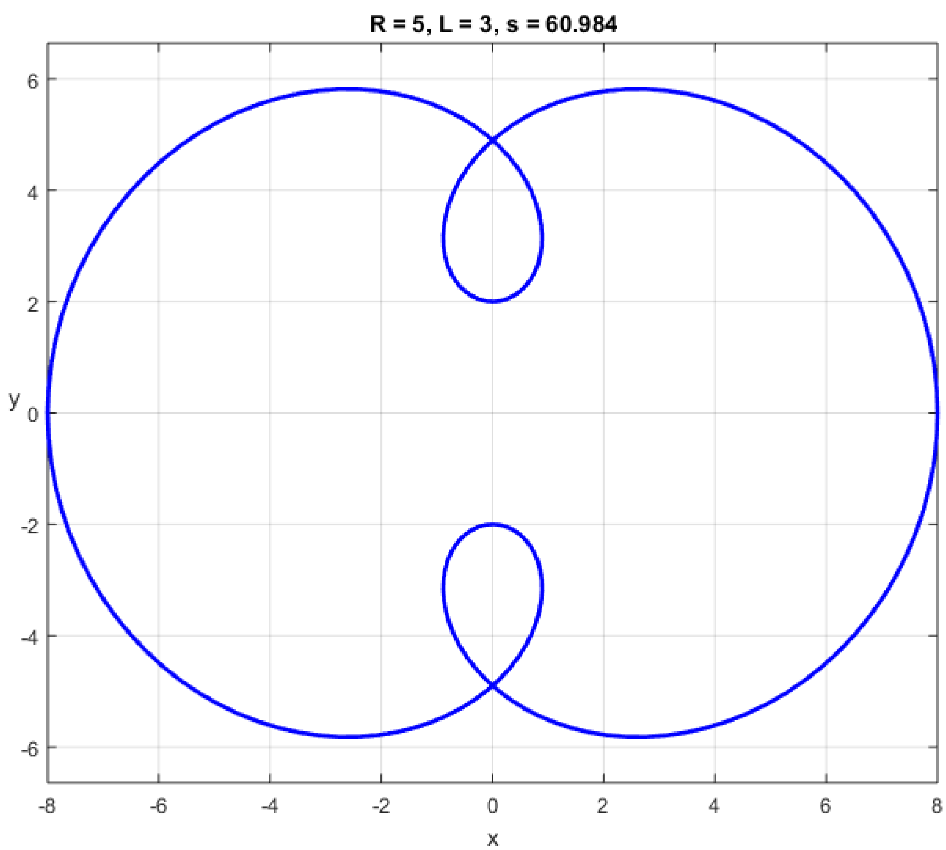
and draw a picture like below



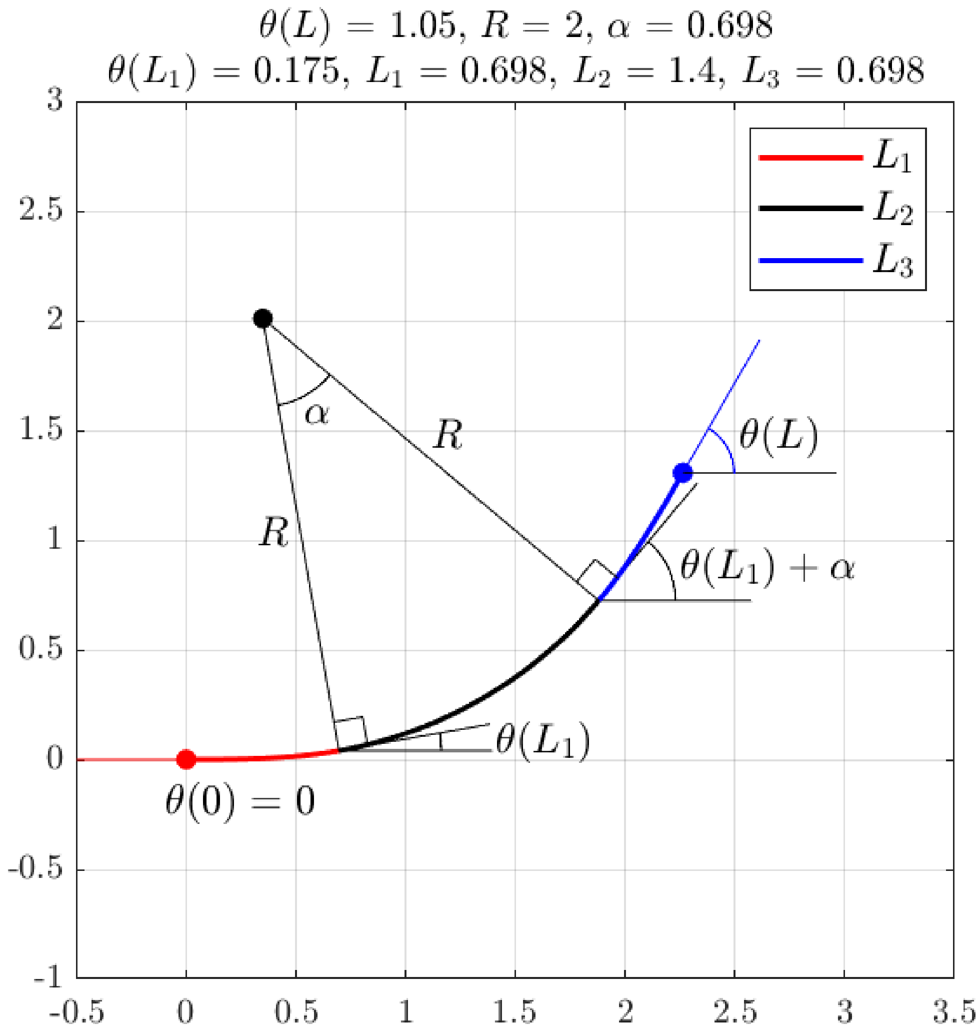
6. Given R and L, calculate (numerically) the length s of the parametric curve

$$\begin{cases} x(t) = R\cos(t) + L\cos(3t) \\ y(t) = R\sin(t) + L\sin(3t) \end{cases}, \ t = 0 \ldots 2\pi$$

and draw a picture like below



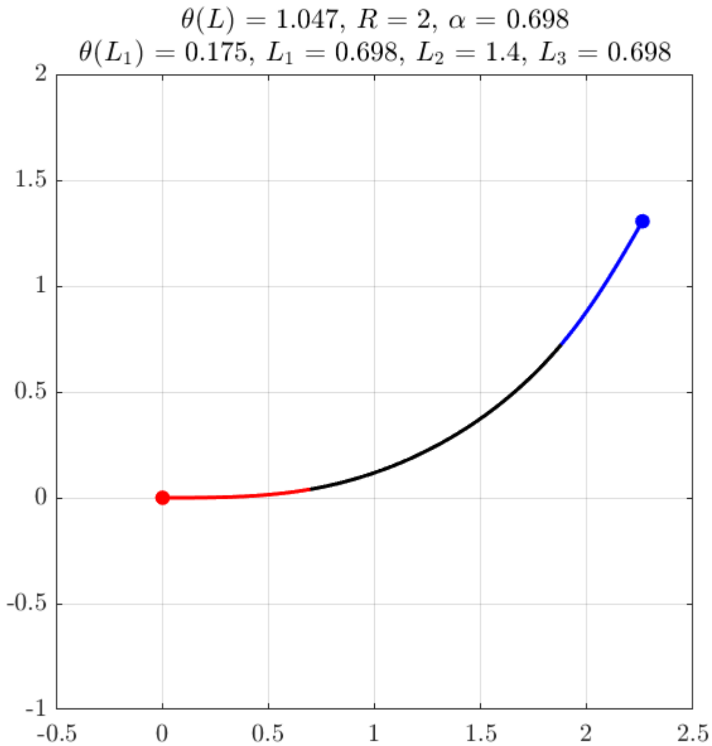
7. Given initial point $x(0), y(0)$, end angle $\theta(L)$, radius R and angle α from the interval $0 \ldots \theta(L)$, calculate points on the curve below, consisting of two Euler spirals and circular arc.



$$\kappa(0) = \kappa(L) = 0,\ \kappa(L_1) = \kappa(L_1 + L_2) = \frac{1}{R}$$

$$\theta(L_1) = \frac{\theta(L) - \alpha}{2},\ \theta(L_1 + L_2) = \theta(L_1) + \alpha$$

Draw a picture of the curve



and the graphs of curvature $\kappa(s)$ and direction angle $\theta(s)$, $s = 0 \ldots L$