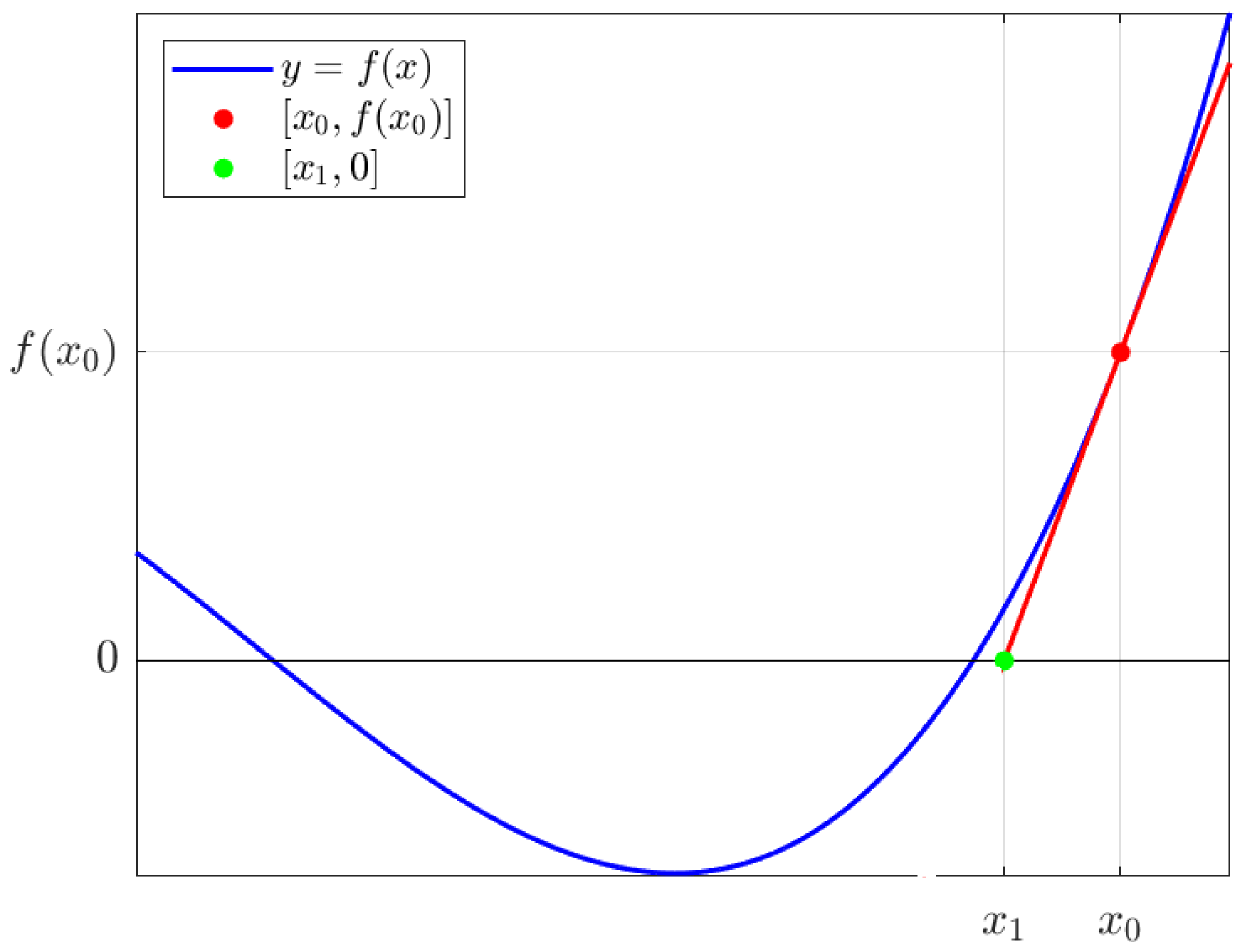
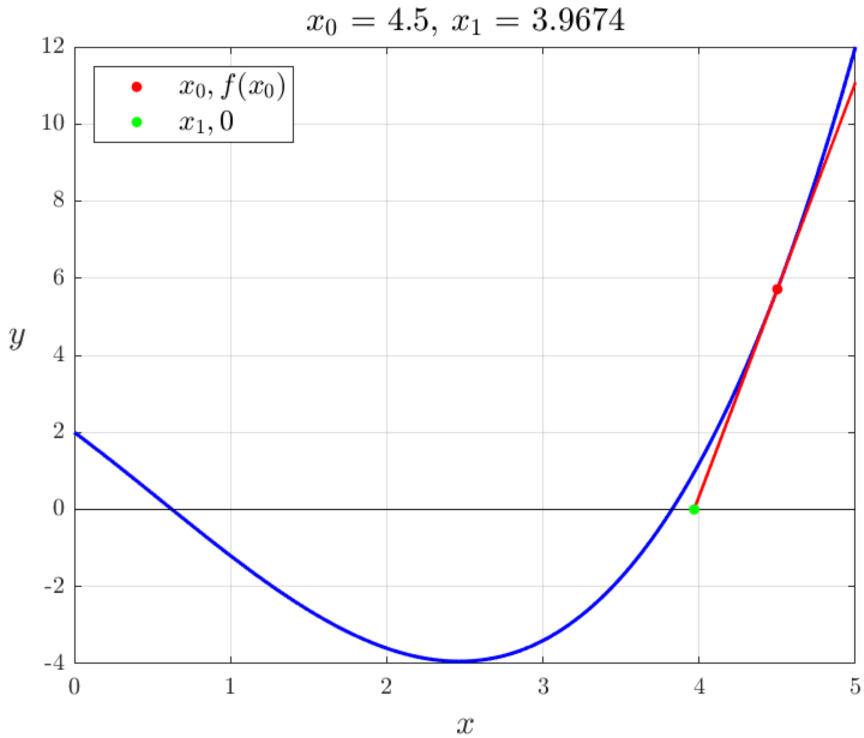
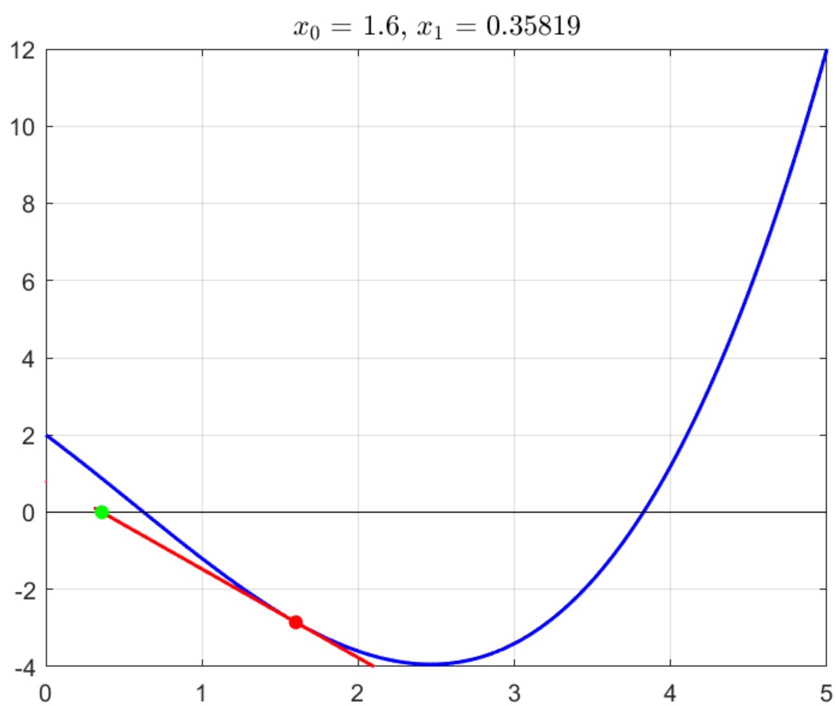
1. Given x_0, calculate the intersection x_1 of the tangent to the curve $y = f(x)$ at point $[x_0, f(x_0)]$ and x-axis, when

$$f(x) = 0.3x^3 - 0.5x^2 - 3x + 2$$



Draw a picture like below





Note: The famous Newton's method for solving an equation $f(x) = 0$ is based on this

2. The trajectory is a parabola

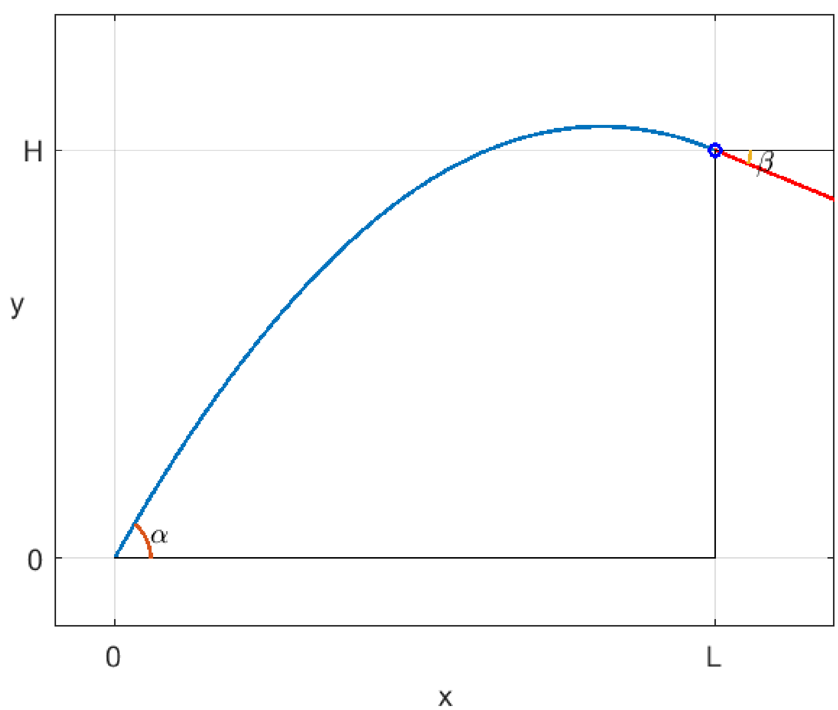
$$y(x) = ax^2 + bx$$

where

$$a = -\frac{g}{2(v_0\cos(\alpha))^2},\; g = 9.81,\; b = \tan(\alpha)$$

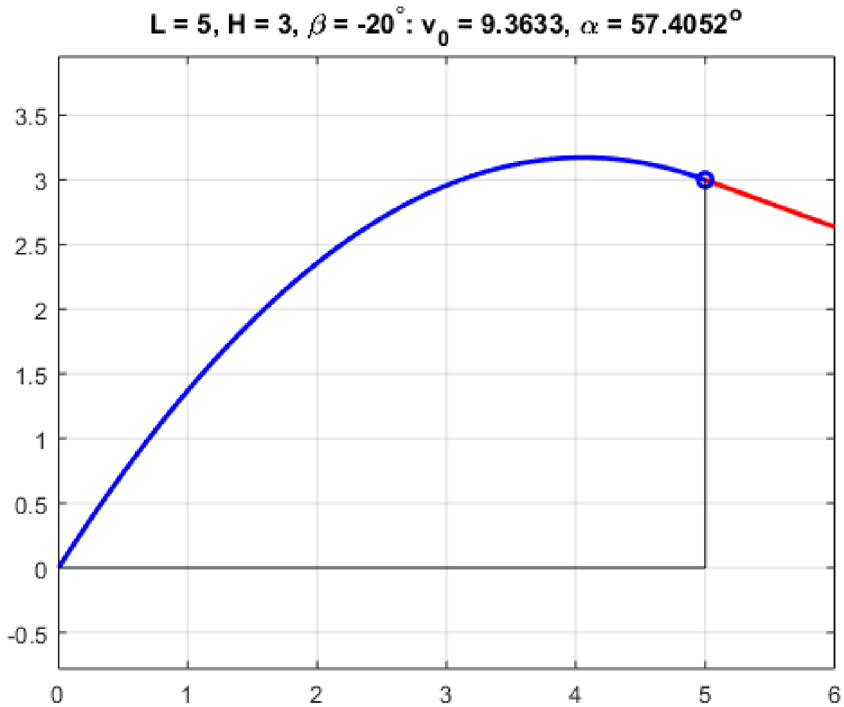
v_0 is the initial velocity and α the initial angle

Given the coordinates L and H of the target and the angle β, calculate v_0 and α such that the trajectory hits the target at angle β.



Note: β has to be on the interval $-90^\circ \ldots \tan^{-1}(H/L)$

Test by drawing the following kind of picture



hint: solve first a and b from the equations

$$y(L) = H, \quad y'(L) = \tan(\beta)$$

end then α and v_0 from the equations

$$a = -\frac{g}{2(v_0 \cos(\alpha))^2}, \quad b = \tan(\alpha)$$

3. Given

$$x_1, y_1, k_1, m_1, x_2, y_2, k_2, m_2$$

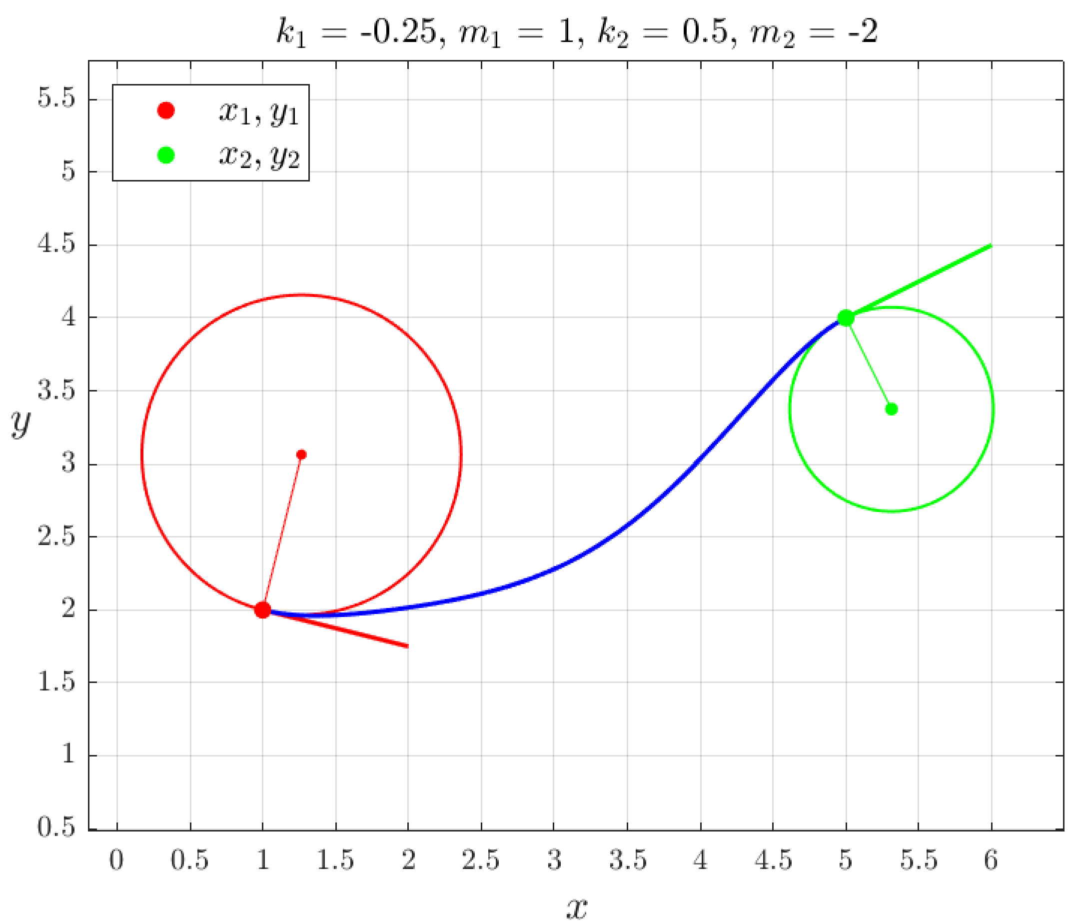
calculate coefficients $a - f$ such that the curve

$$y(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$$

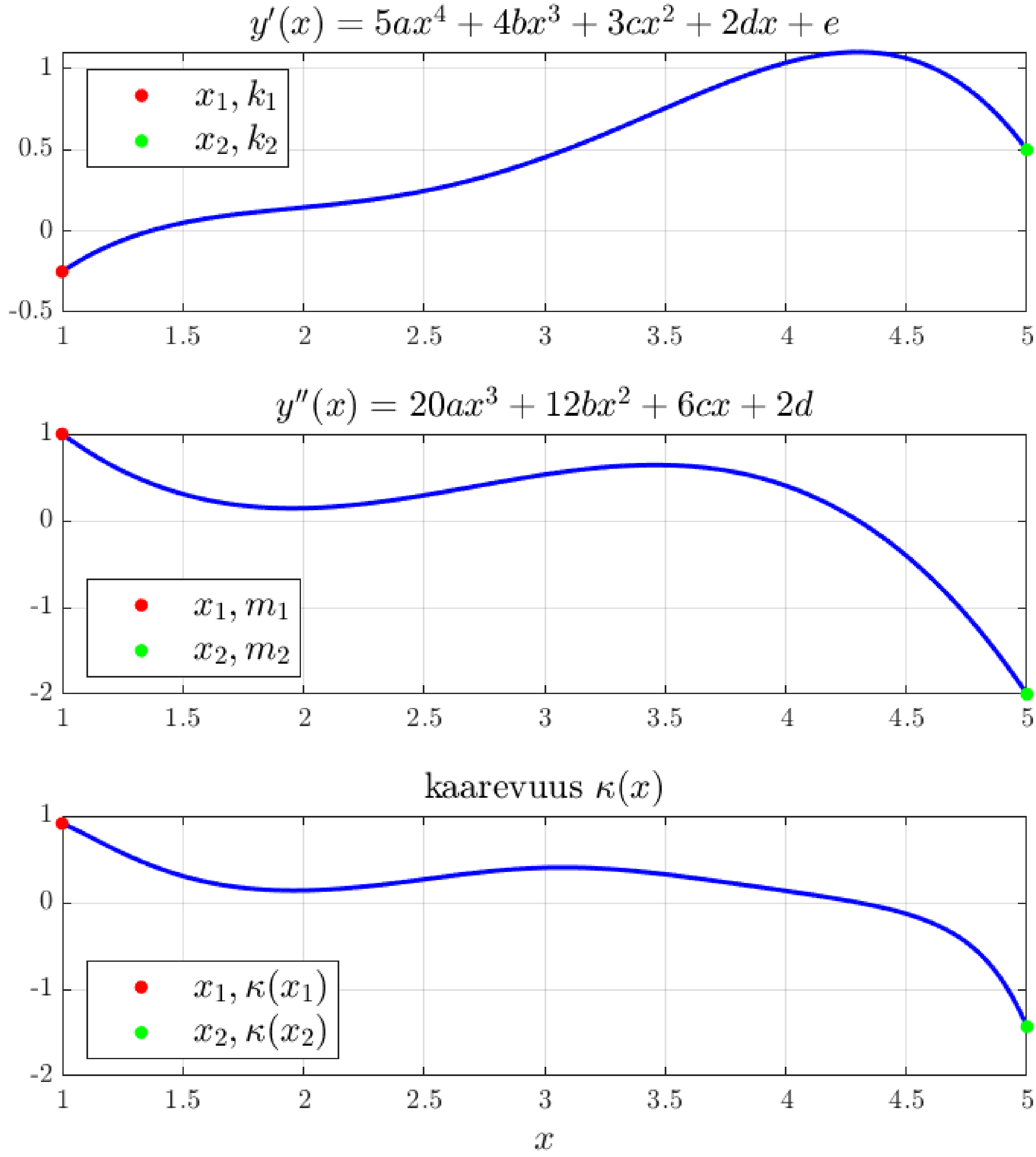
satisfies the requirements

$$y(x_1) = y_1,\ y'(x_1) = k_1, y''(x_1) = m_1$$
$$y(x_2) = y_2,\ y'(x_2) = k_2,\ y''(x_2) = m_2$$

and draw a picture like below



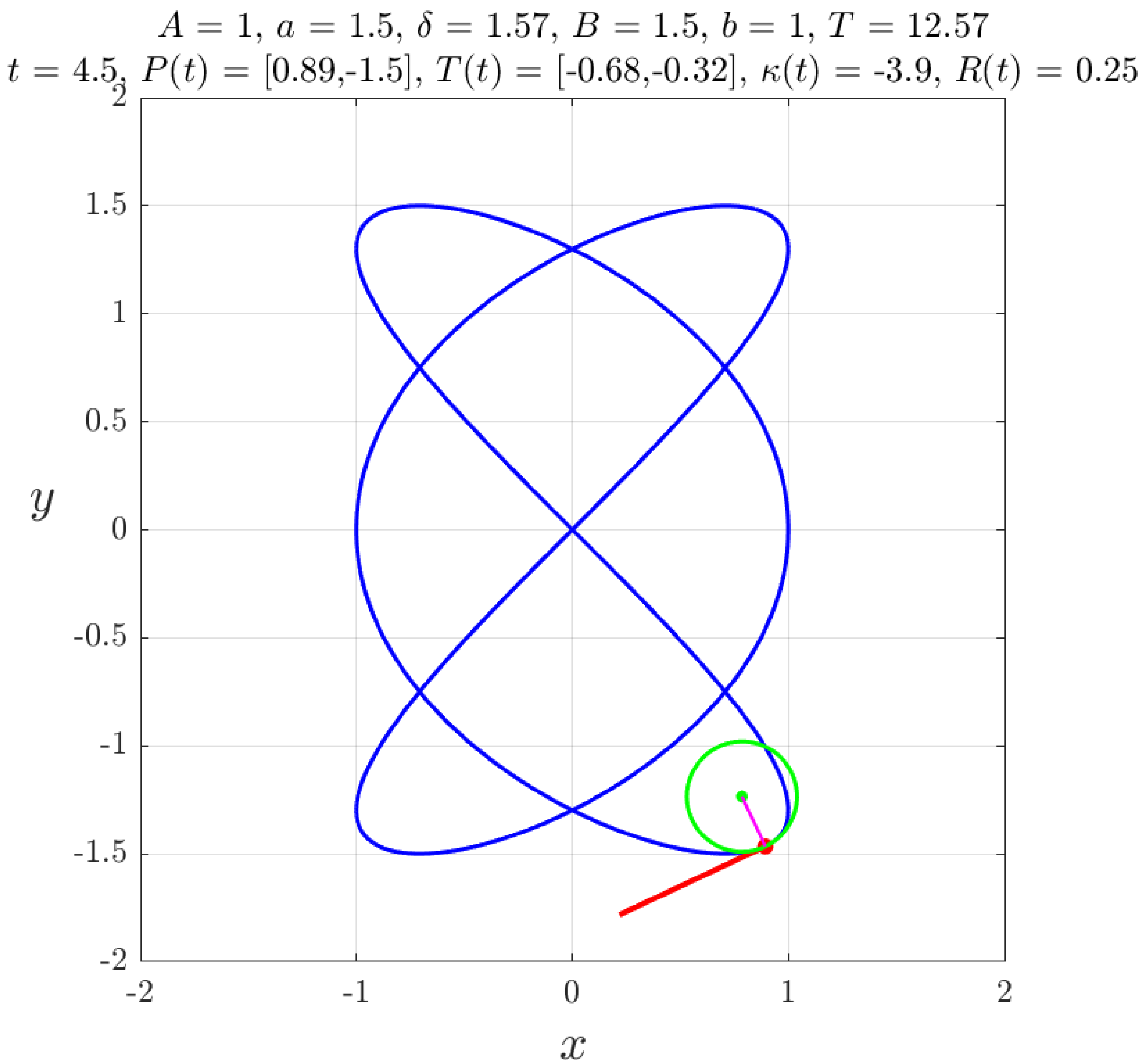
Draw also the graphs of $y'(x)$, $y''(x)$ and the curvature $\kappa(x)$, $x = x_1 \dots x_2$



4. Given A, a, δ, B, b and T, draw the Lissajous curve

$$x(t) = A\sin(at+\delta),\ y(t) = B\sin(bt),\ t = 0\ldots T$$

and its tangent vector and circle of curvature at some point of the curve



Draw also the graphs of $|T(t)|$ and $\kappa(t)$, $t = 0 \ldots T$