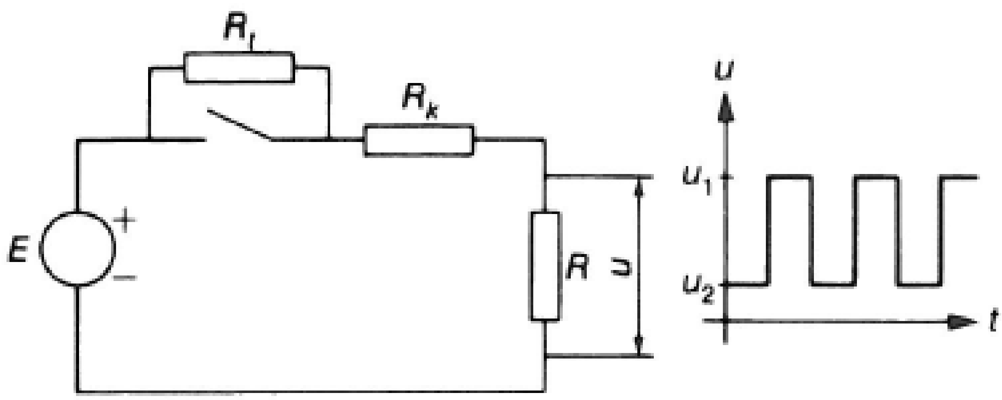
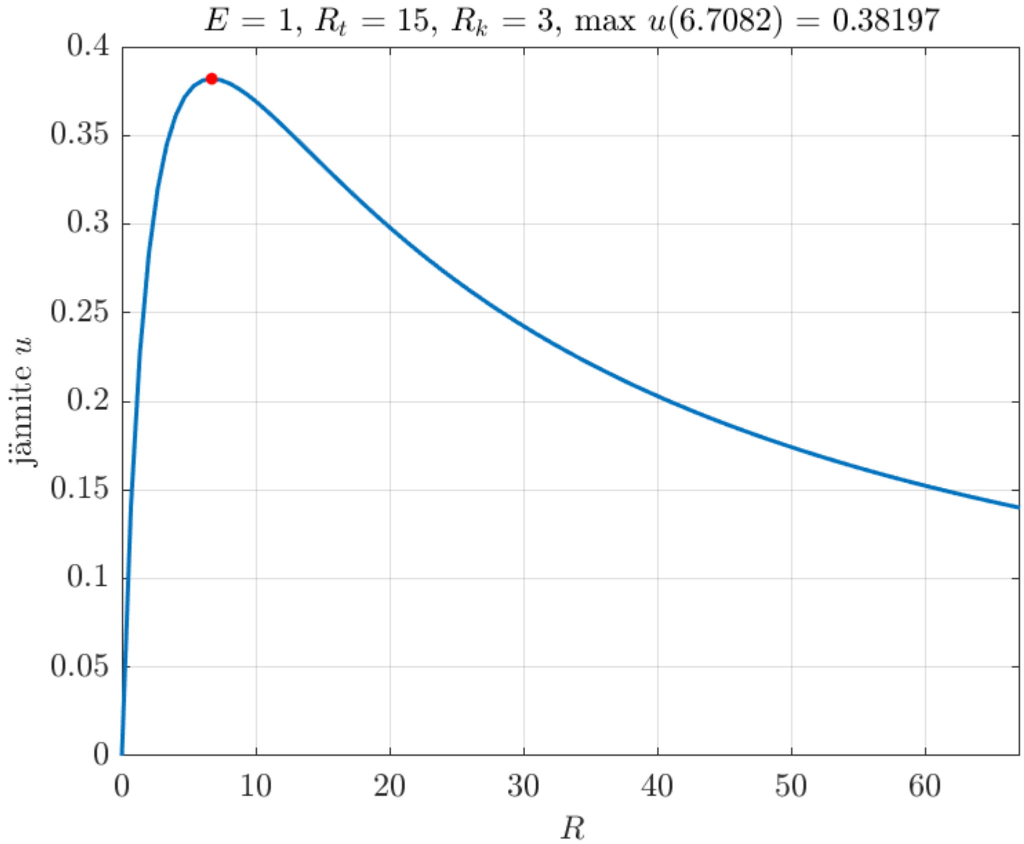
1. Given $E, R_k, R_t > 0,\ R_t > R_k$, find $R > 0$ such that the voltage

$$u(R) = \frac{E\,R}{R + R_k} - \frac{E\,R}{R + R_t}$$

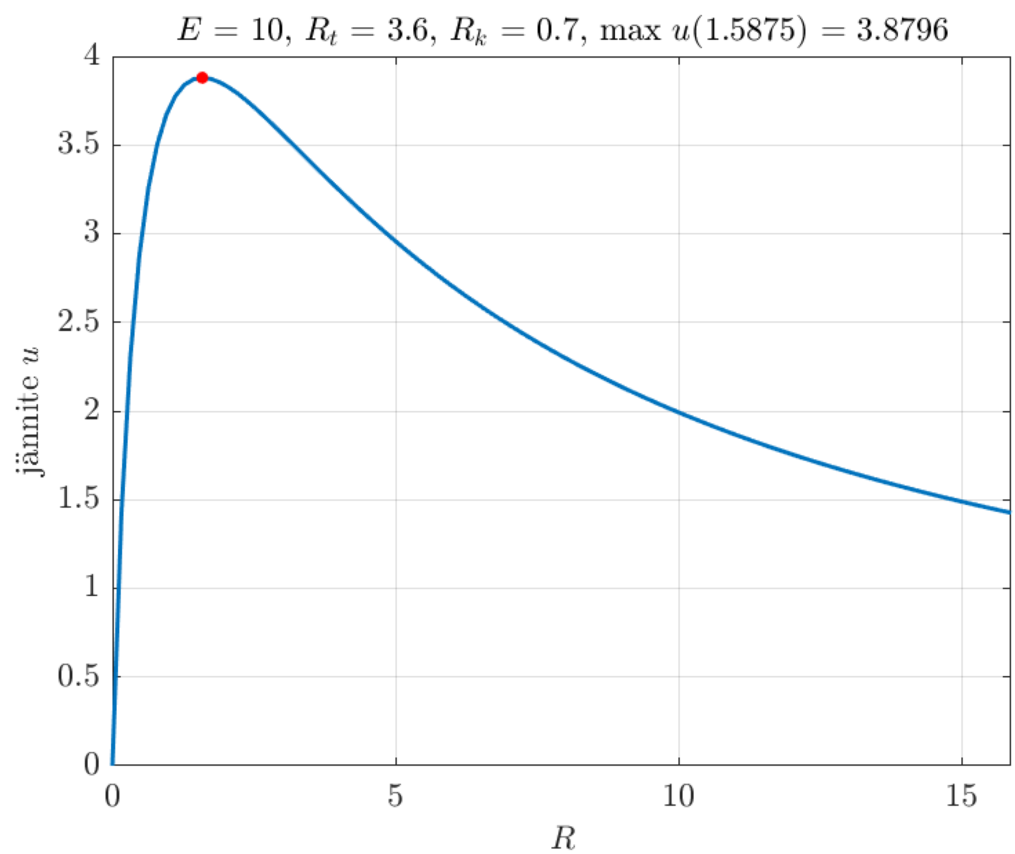
is largest and calculate the maximum value.



Check by drawing graphs of $u(R)$



$E = 1,\ R_t = 15,\ R_k = 3,\ \max\ u(6.7082) = 0.38197$
jännite u
R
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
10
20
30
40
50
60



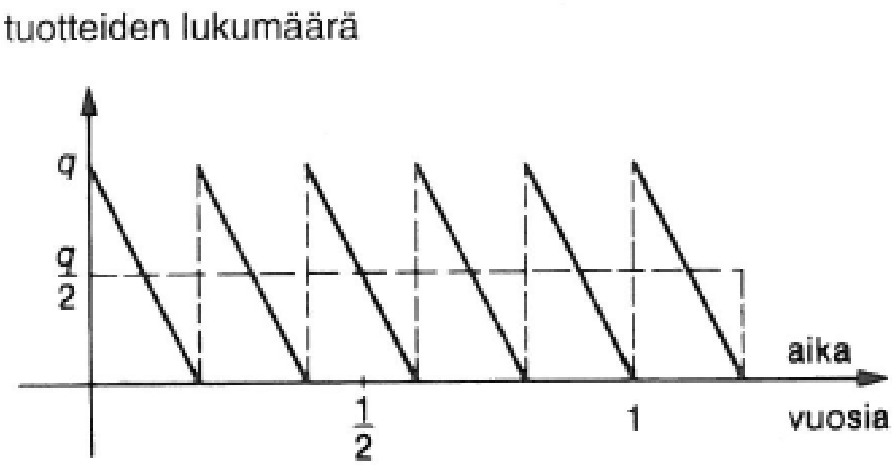
$E = 10,\ R_t = 3.6,\ R_k = 0.7,\ \max\ u(1.5875) = 3.8796$
jännite u
R
0
0.5
1
1.5
2
2.5
3
3.5
4
5
10
15

2. Given $k_v, k_e, r > 0$, find $q > 0$ such that annual cost (=vuotuiset kustannukset)

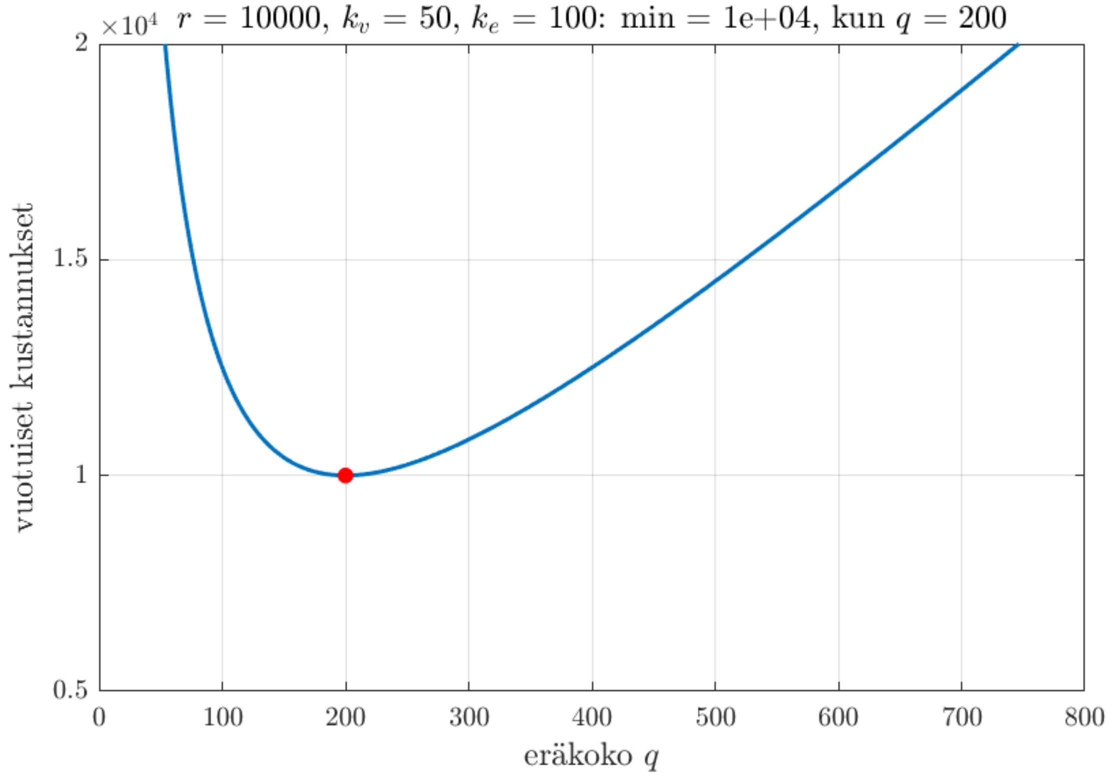
$$c(q) = k_v \cdot \frac{q}{2} + k_e \cdot \frac{r}{q}, \quad q > 0$$

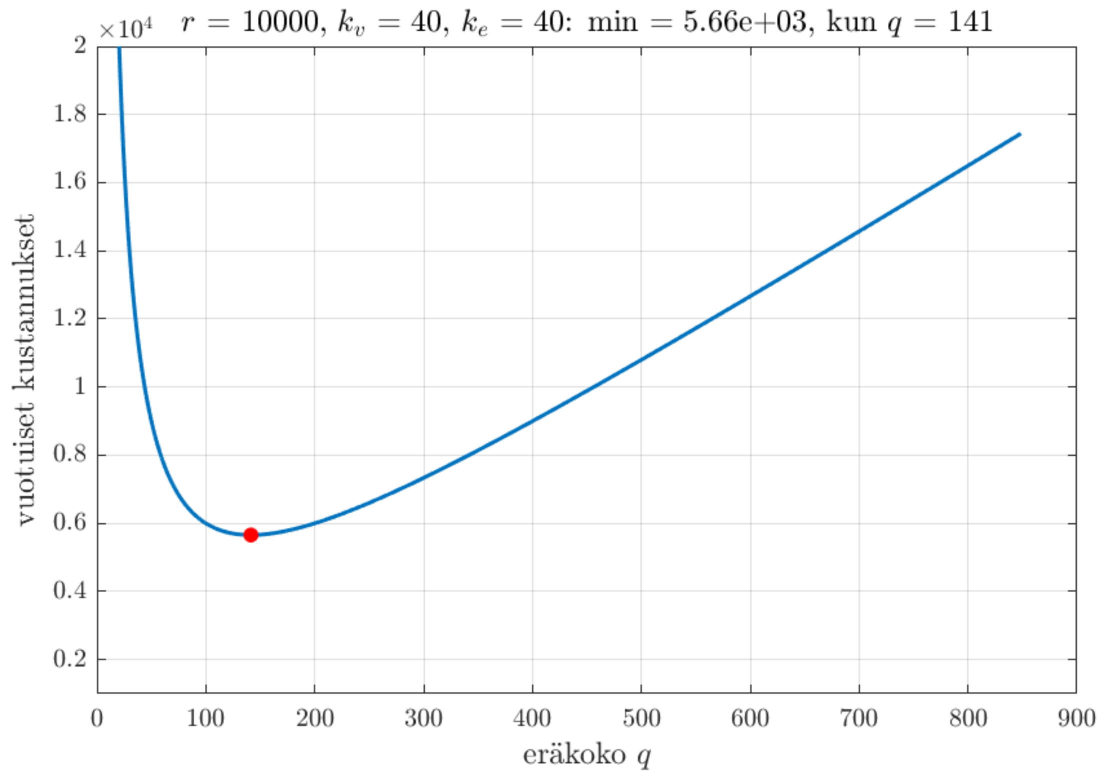
is smallest. Calculate also the smallest value
Check your formulas by drawing graphs of $c(q)$

Tuotetta myydään tasaisesti siten, että myyjän varastossa olevien tuotteiden määrä ajan funktiona on kuvion mukainen.

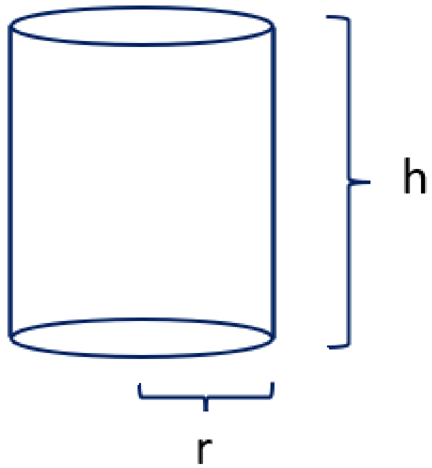


Myyjä siis tilaa tavaraa erissä, joiden suuruus on q, ja varaston koko on keskimäärin $q/2$. Oletetaan, että tuotetta myydään r kpl vuodessa, vuotuiset varastossapitokustannukset ovat keskimäärin k_v € /kpl ja jokaisen erän toimituskustannukset (kuljetus, toimistotyö yms.) ovat eräkoosta riippumatta k_e € /erä. Määritä niin sanottu optimaalinen eräkoko q_0, joka tekee vuotuiset kustannukset mahdollisimman pieniksi. Kuinka suuret vuotuiset kustannukset tällöin ovat?

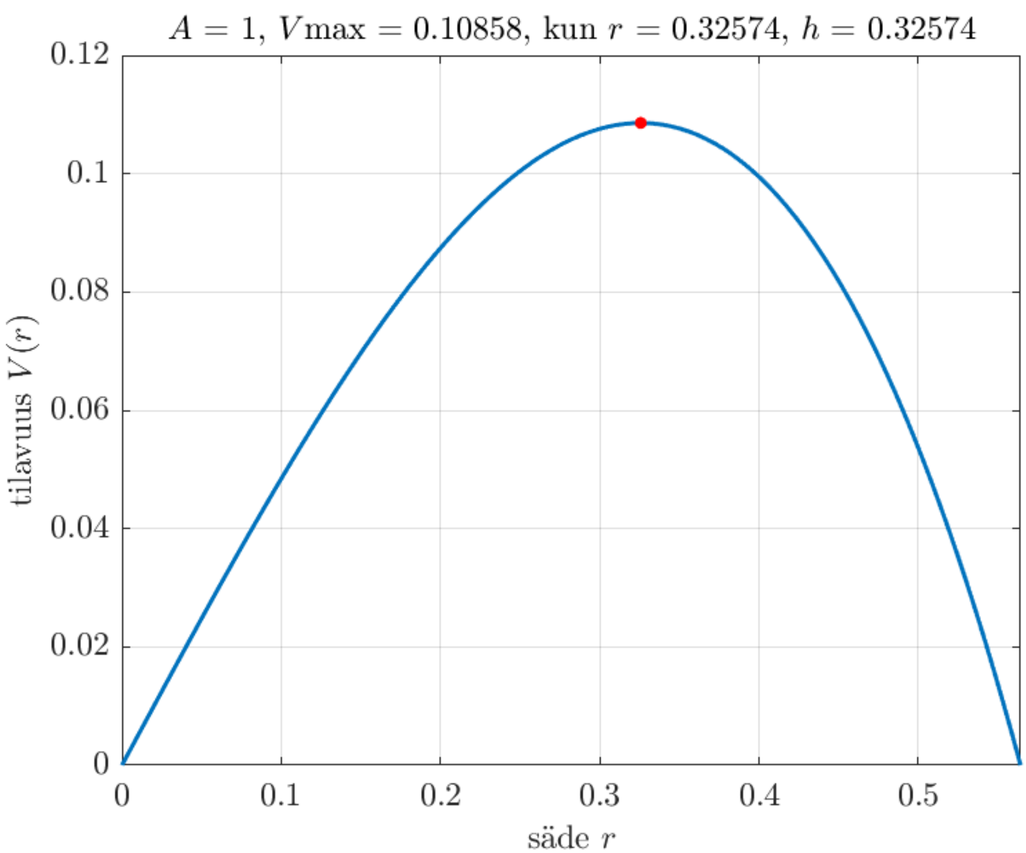

$r = 10000$, $k_v = 50$, $k_e = 100$: min = 1e+04, kun $q = 200$
$\times 10^4$
2
1.5
1
0.5
0
100
200
300
400
500
600
700
800
vuotuiset kustannukset
eräkoko q


$r = 10000$, $k_v = 40$, $k_e = 40$: min = 5.66e+03, kun $q = 141$
$\times 10^4$
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
100
200
300
400
500
600
700
800
900
vuotuiset kustannukset
eräkoko q

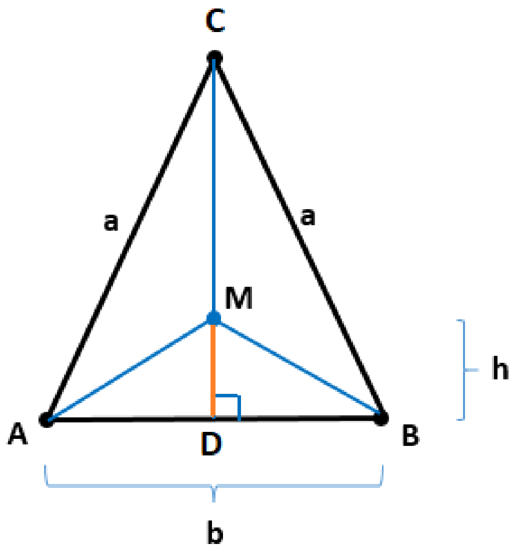
3. A cylindrical container, radius r, height h.
Volume $V = \pi r^2 h$, area $A = \pi r^2 + 2\pi r h$



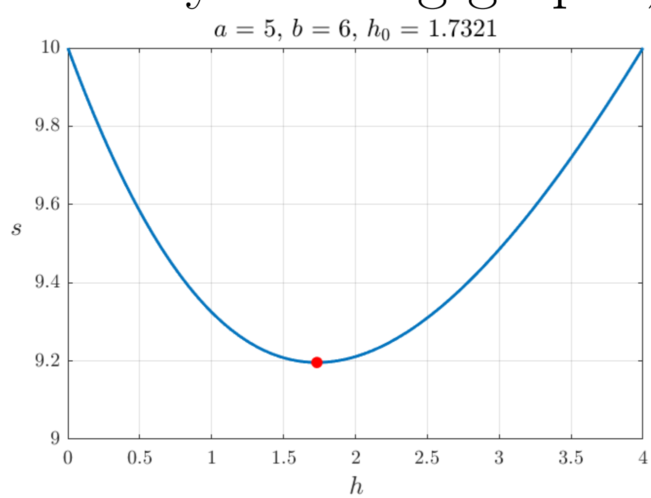
For fixed area A, find r and h such that volume V is largest. Calculate also the maximum value.
Check by drawing graphs.

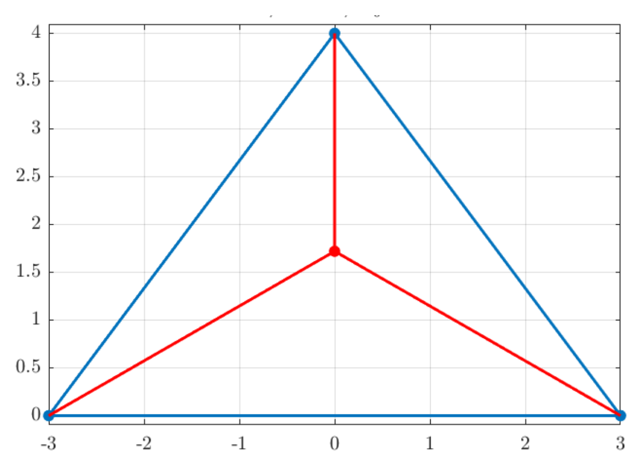


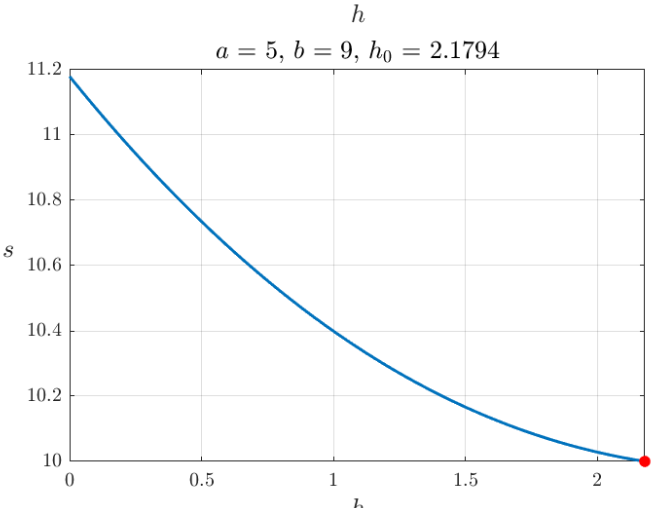
4. Given a and b, find h such that the sum of distances $s = MA + MB + MC$ is smallest and calculate the smallest value

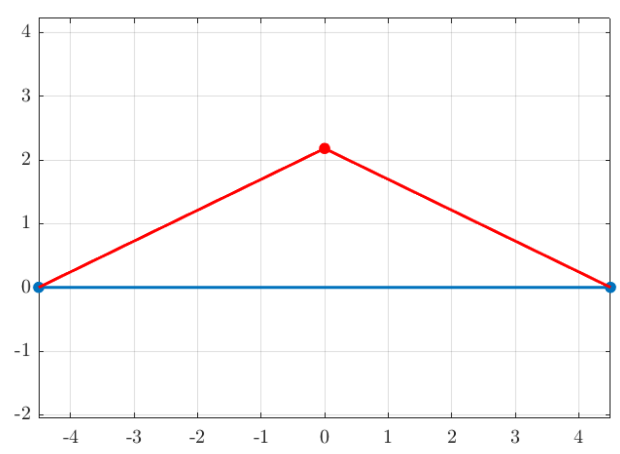


Check by drawing graphs, notice two possibilities

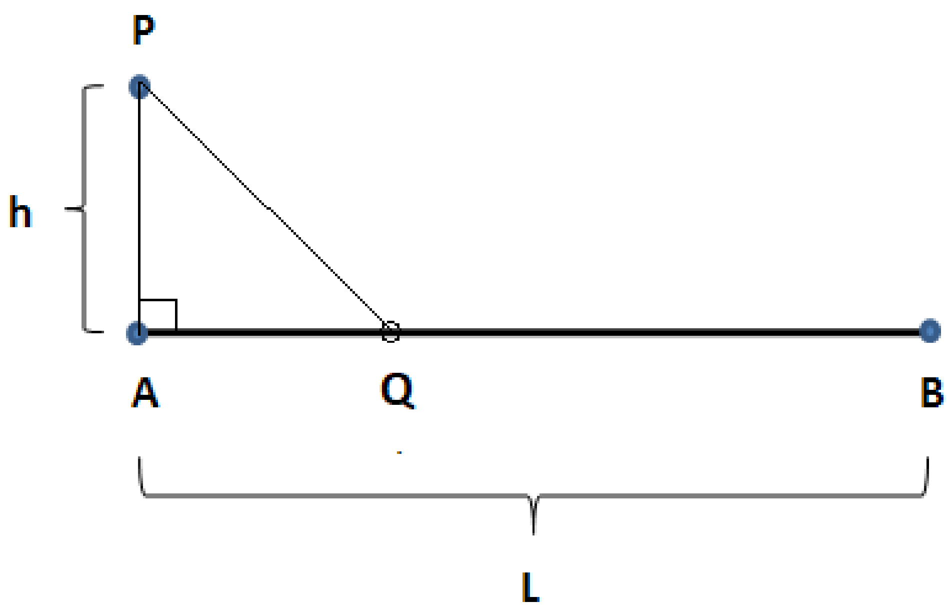




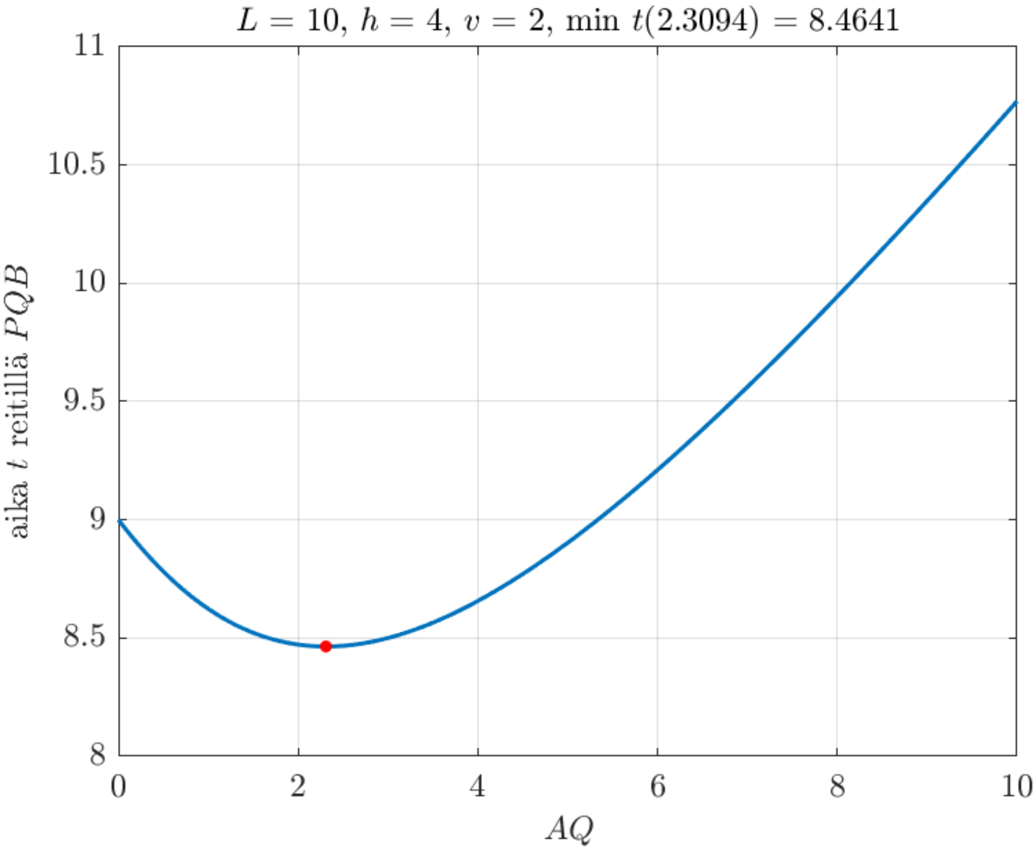




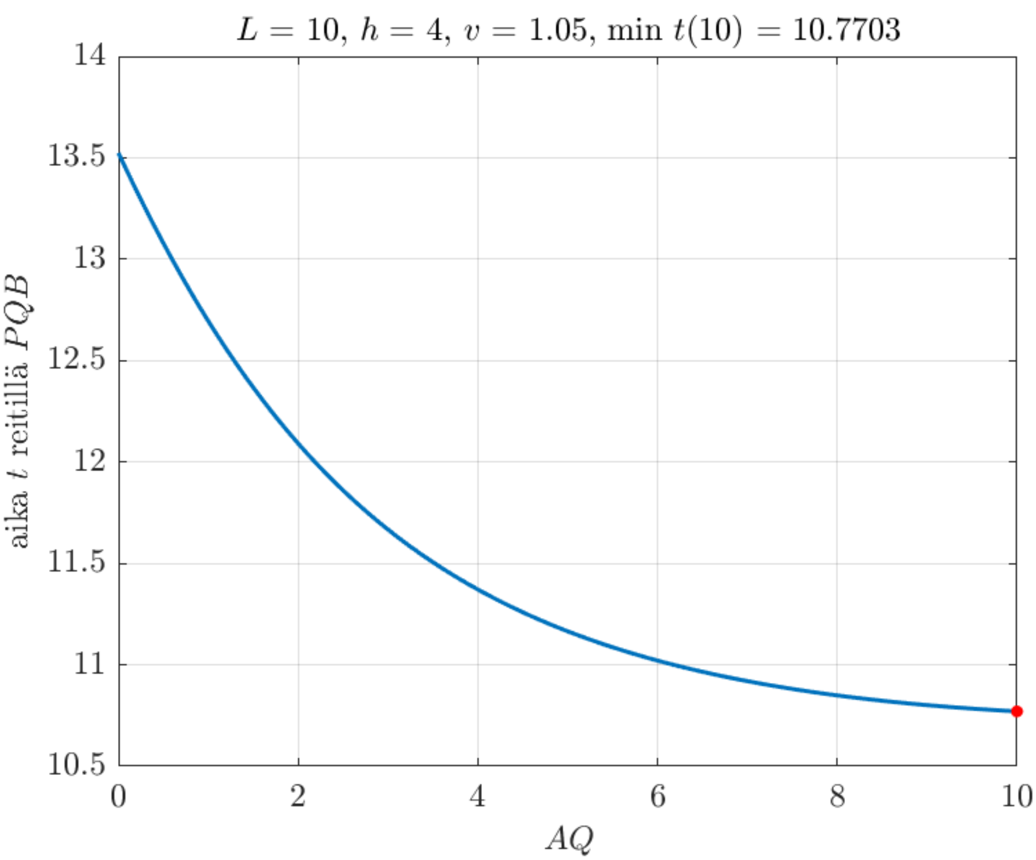
5. Given h, L and $v > 1$, find $x = AQ$ such that the traveling time PQB is smallest, when PQ is traveled at speed 1 and QB at speed v. Calculate also the smallest traveling time.



Check by drawing graphs, notice two possibilities.



$L = 10,\ h = 4,\ v = 2,\ \min\ t(2.3094) = 8.4641$
aika t reitillä PQB
AQ
11
10.5
10
9.5
9
8.5
8
0
2
4
6
8
10



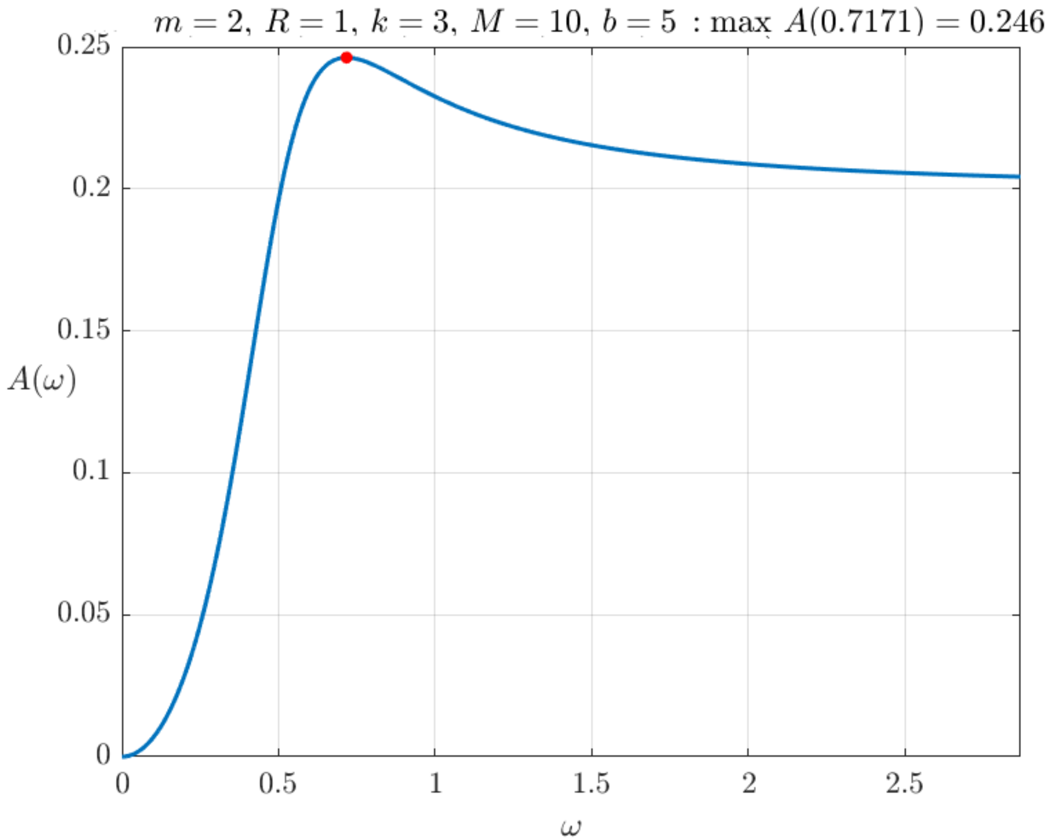
$L = 10,\ h = 4,\ v = 1.05,\ \min\ t(10) = 10.7703$
aika t reitillä PQB
AQ
14
13.5
13
12.5
12
11.5
11
10.5
0
2
4
6
8
10

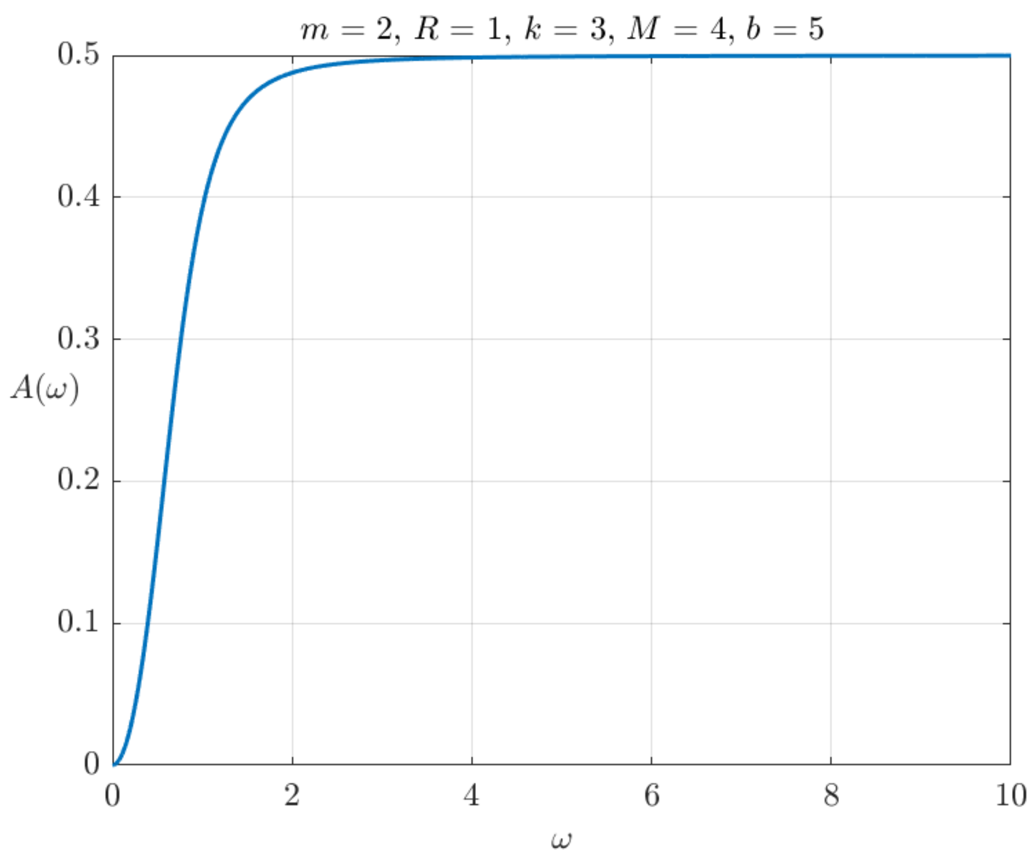
6. Given $m, R, k, M, b > 0$, find $\omega > 0$ such that the amplitude

$$A(\omega) = \frac{m\,R\omega^2}{\sqrt{(k - M\omega^2)^2 + (b\omega)^2}}$$

is largest

Check by drawing graphs, notice two possibilities


$m = 2,\ R = 1,\ k = 3,\ M = 10,\ b = 5\ : \max\ A(0.7171) = 0.246$
$A(\omega)$
ω
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5


$m = 2,\ R = 1,\ k = 3,\ M = 4,\ b = 5$
$A(\omega)$
ω
0.5
0.4
0.3
0.2
0.1
0
2
4
6
8
10