KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

SECOND SEMESTER, 2018/2019 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BSC IN COMPUTER SCIENCE

MATH 111: VECTOR GEOMETRY

STREAM: [Y1S1] **TIME: 9.00-11.00 AM**

EXAMINATION SESSION: JAN-APRIL **DATE: 3/04/2019**

INSTRUCTIONS:

1.Question ONE is compulsory.

2. Attempt question ONE and any other TWO

Question One [30 Marks]

(a) Determine the angle between the following pairs

(i) $a = (9,-2)\ \ b = (4,1,8)$

(ii) $u = (3,-1,6) V = (4,2,0)$ [6mks]

a) Find the magnitude and direction of the displacement vector $\overrightarrow{AB}$ where *A* and *B* are points $(-9,7)$ and $(12,-5)$ respectively. [3 Marks]

b) Points L, M, N are the mid-points of the sides AB, BC, CA of the triangle ABC.

Show that $2\overrightarrow{AB} + 3\overrightarrow{BC} + \overrightarrow{CA} = 2\overrightarrow{LC}$ [4 Marks]

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c) Find the area of a triangle whose vertices are $P(1,3,2)\ Q(2,-1,1)\ R(-1,2,3)$

[3 Marks]

d) Determine a unit vector that is perpendicular to the plane of $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. [4 marks]

e) Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the coordinate axis.

[4 Marks]

f). Given that $z = -5 + 5i$

Determine;

i). $|z|$ (ii) Arg z (iii) z in polar form [3marks]

g). Determine the value of a such that the vectors $\underline{p} = 2\hat{i} + a\hat{j} + 4\hat{k}$ and $q = 5\hat{i} + 2\hat{j} - 4\hat{k}$ are orthogonal. [3 Marks]

Question Two [20 Marks]

a) Show that addition of vectors is associative. [3 marks]

b) Evaluate $(2\hat{i} - 3\hat{j}) \bullet \left(\hat{i} + \hat{j} - \hat{k}\right) \times \left(3\hat{i} - \hat{k}\right)$. [3 Marks]

c) Find the projection of v onto u if v = $\langle -3, 4 \rangle$ and u $\langle -5, 7 \rangle$. [4 marks]

d) Find the Cartesian form of the equation of the line of intersection of the two planes $7x - 4y + 3z = -3$ and $4x + 2y + z = 4$ [3 marks]

e) At noon two boats P and Q are at points where position vectors are $4\hat{i} + 8\hat{j}$ and $4\hat{i} + 3\hat{j}$ respectively. Both boats are moving with a constant velocity; the velocity of P is $4\hat{i} + \hat{j}$ and the velocity of Q is $2\hat{i} + 5\hat{j}$ where all distances are in kilometers and time measured in hours.

i. Find the position vectors of P and Q and $\overrightarrow{PQ}$ after *t* hours. [3 Marks]

ii. Express the distance PQ between the boats in terms of *t* [2 Marks]

iii. Show that the least distance between the boats is $\sqrt{5}$ km [2 Marks]

Question Three [20 Marks]

a). Determine the value of $z = \frac{7}{3-i} - \frac{1-2i}{3+4i}$ [4mks]

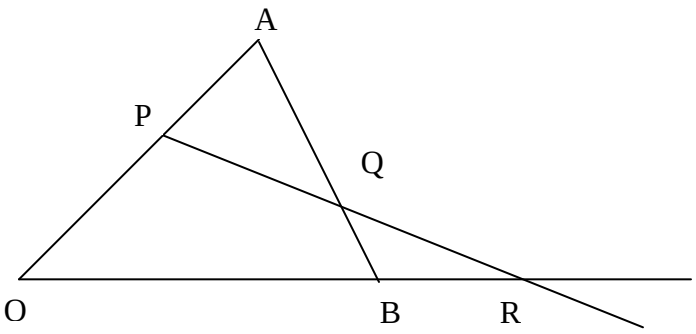
b). Find the perpendicular distance between the point $A(4,-3,10)$ and the line L whose vector equation is $\underline{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ [6 Marks]

c) Show that $Z_1 + Z_2 = Z_1 + Z_2$ [4mks]

d) Prove $|Z1 + Z_2| \leq |Z_1| + |Z_2|$ [4mks]

Question Four [20 Marks]

a) Given the triangle below prove Menelaus theorem which states that if $\overrightarrow{OP} = \alpha \overrightarrow{PA}$, $\overrightarrow{AQ} = \beta \overrightarrow{QB}$ and $\overrightarrow{BR} = \gamma \overrightarrow{RO}$ then $\alpha\beta\gamma = -1$ [6 marks]



b) Find the acute angle between the line $\frac{x+1}{4} = y-2 = \frac{z-3}{-1}$ and the plane $3x-5y+4z=5$ [3 Marks]

c) Given that *A*, *B* and *C* are the points $(1,1,1)$, $(5,0,0)$ and $(3,2,1)$ respectively find the equation which must be satisfied by the coordinates (x, y, z) of any point *P* in the plane *ABC*. [6 Marks]

d) Find the point of intersection of the lines $\frac{x-3}{-1} = \frac{y-2}{2} = \frac{z+3}{4}$ and $3x-y+2z=8$ [5 Marks]

Question Five [20 Marks]

a) Points *A, B and C* have the position vector $\hat{i}+2\hat{j}-3\hat{k}$, $\hat{i}+5\hat{j}$ and $5\hat{i}+6\hat{j}-\hat{k}$ respectively, relative to an origin

i. Show that $\vec{AB}$ is perpendicular to $\vec{BC}$ and find the area of the triangle *ABC* [4 Marks]

ii. Find the vector product $\vec{AB} \times \vec{BC}$. [3 Marks]

iii. Find the equation of the plane *ABC* in the form $\underset{\sim}{r} \bullet \underset{\sim}{n} = p$ [3 marks]

iv. The point D has position vector $4\hat{i}-\hat{j}+3\hat{k}$ find the distance of the point D from the plane *ABC* [3 marks]

v. Show that the volume of the tetrahedron *ABCD* is equal to 21 [3 Marks]

b) Find the volume of a parallelepiped whose edges are represented by $\vec{A} = 2\hat{i}-3\hat{j}+4\hat{k}$ $\vec{B} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{C} = 2\hat{i}-\hat{j}+2\hat{k}$ [4 Marks]