



## UNIVERSITY EXAMINATIONS

### SECOND SEMESTER, 2018/2019 ACADEMIC YEAR

### EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION AND BSC. COMPUTER SCIENCE

### MATH 314: NUMERICAL ANALYSIS

**STREAM: Y3S2**

**TIME: 2.00-4.00 PM**

**EXAMINATION SESSION: JAN-APRIL**

**DATE: 4/04/2019**

#### INSTRUCTIONS

- Time allowed : 2 Hours
- This examination paper contains five questions
- Answer QUESTION ONE and any other TWO questions
- Start each question on a fresh page
- Indicate each question number clearly at the top of each page
- Do not write on the question paper
- Scientific calculators may be used
- Observe further instructions from the booklet

#### QUESTION ONE (30 MARKS)

- a) Use Newton-Raphson method to find the root near 2 of the equation

$$x^4 - 11x + 8 = 0 \quad \text{accurate to 5dp.} \quad (5 \text{ Marks})$$

- b) The population of a town is as follows:

Year	1941	1951	1961	1971	1981	1991
Population	20	24	29	36	46	51

Estimate the population increase during the period between 1946 to 1976. (5marks)



(c) Show that  $\mu\delta = \frac{1}{2}(\Delta + \nabla)$  (3 Marks)

(d) Evaluate  $\int_0^{1.2} \frac{dx}{1+x^2}$

using Simpsons three eighth rule taking 7 ordinates points (5 Marks)

(e) Compute and interpret the condition number for the function

$f(x) = \sin x$  for  $a = 0.51\pi$  (4Marks)

(f) i)  $\sin(0.1) = 0.09983$  and  $\sin(0.2) = 0.19867$ , find an approximate value of  $\sin(0.17)$  by Lagrange interpolation (4marks)

ii) Use the secant method to determine the roots of the equation

$f(x) = x^3 - 4x + 1$  (4marks)

## QUESTION TWO (20 MARKS)

(a) Evaluate the integral

$$\int_0^{1.2} e^x$$

using Weddle's rule taking  $n = 6$  correct to 5d.p. (7 Marks)

(b) Apply Lagrange's interpolation formula to find a polynomial which passes through the points (0,-20), (1,-12), (3,-20) and (4,24) (6 Marks)

(c) Use Newton's Forward interpolation formula to find the value of  $\cos 52^\circ$  from the following set of data

x	$45^\circ$	$50^\circ$	$55^\circ$	$60^\circ$
y=f(x)	0.7071	0.6248	0.5736	0.5000

(7 Marks)

**QUESTION THREE (20 MARKS)**a) For the function  $f(x)$ 

$x$	4	5	6	7	8	9	10
$f(x)$	-10	12	56	128	264	380	572

Find  $f'(4.5)$  using Gregory Newton difference.

(10marks)

b) Find the value of  $\int_0^{0.6} e^x + 1 \, dx$  taking  $n=6$  correct to five significant figures using

i) Trapezoidal rule

(3marks)

ii) Simpson's  $\frac{1}{3}$  rule

(3marks)

iii) Simpson's  $\frac{3}{8}$  rule

(4 marks)

**QUESTION FOUR (20 MARKS)**a) A slider in a machine moves along a fixed straight rod. Its distance  $x(m)$  along the rod are given in the following table for various values of time  $t(\text{seconds})$ 

$t(s)$	1	2	3	4	5	6
$x(m)$	0.0201	0.0844	0.3444	1.0100	2.3660	4.7719

Find the velocity and acceleration of the slider at time  $t = 6s$ 

(10 Marks)

b)

$x$	0	1	2	3	4
$Y=f(x)$	1	4	17		97

Determine the missing value

(5 Marks)

c) Evaluate  $\sqrt{29}$  to 5dp using Newton –Raphson Method

(5 Marks)

**QUESTION FIVE (20 MARKS)**

- (a) Find a polynomial of degree three or less to approximate  $f(x) = \sin x$  near  $x_0 = 0$  and use the polynomial to obtain the approximate value of  $\sin(0.1)$ . (7marks)
- (b) Find a unique polynomial of degree 2 or less such that  $f(0) = 1$ ,  $f(1) = 3$  and  $f(3) = 55$  using (i) Lagrange interpolation formula
- (c) (ii) Newton divided difference interpolation formula (8marks)
- (d) Find an iterative formula to find  $\sqrt{N}$  where  $N$  is a positive integer hence find  $\sqrt{7}$  (5marks)