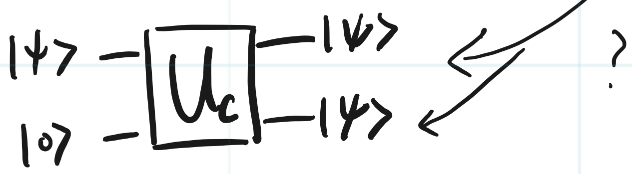
No-Cloning Theorem

{Is there any gates to clone 1st q-bit to 2nd q-bit like this below?}

$|\psi\rangle$ — arbitral state q-bit

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



Assume Operator: U_c, does following,

$$U_c(|\psi\rangle \otimes |0\rangle) = |\psi\rangle \otimes |\psi\rangle$$

→ Find out this is true?

Expand Left Side

$$U_c(|\psi\rangle \otimes |0\rangle)$$

$$= U_c((\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle)$$

$$= U_c(\alpha|00\rangle + \beta|10\rangle)$$

$$= \alpha U_c|00\rangle + \beta U_c|10\rangle$$

↳ if U_c clone st bit to 2nd bit,

$$= \alpha|00\rangle + \beta|11\rangle$$

↳ this is same as

$$\alpha|00\rangle + 0|01\rangle + 0\cdot|10\rangle + \beta|11\rangle$$

now, compare on right side

Expand Right Side

$$(\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)$$

$$= \alpha\cdot\alpha|00\rangle + \alpha\beta|01\rangle + \beta\alpha|10\rangle + \beta\beta|11\rangle$$

$$= \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

see expanded Left side

Are they true?

$$\begin{cases} \alpha^2 = \alpha \\ \beta^2 = \beta \\ \alpha\beta = 0 \end{cases}$$

⇒ NOT POSSIBLE

because since α or β has to be 0

but $\alpha^2 = \alpha$, $\beta^2 = \beta$ like 1