BINARY NUMBER SYSTEM

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Bit patterns

- Bits can be used to represent patterns
- Specifically, any system or set of symbols can be translated into bit patterns
    - patterns of ones and zeros
    - 10100001101
- Example: characters from any language alphabet
- Require enough bits so that all symbols have a unique bit pattern to represent them
    - How many bits are needed to represent the English alphabet?
- Require set of symbols is *finite*

How many bits?

A bit pattern consisting of a single bit can represent at most two symbols

-possible patterns are 0 and 1

A bit pattern consisting of two bits can represent at most four symbols

-possible patterns are 00, 01, 10 and 11

In general, a bit pattern consisting of n bits can represent at most 2^n symbols

How many bits are needed to represent the English alphabet?

-we can represent 26 symbols using 5 bits ($2^5=32$)

-4 bits is not enough ($2^4=16$)

Decimal (base 10) representation

We commonly represent numbers in decimal (base 10)
Numbers are represented using patterns of the digits

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Position of each digit represents a power of ten

Example: Consider the decimal representation 2307

	2	3	0	7
	↑	↑	↑	↑
position:	3	2	1	0

$$2307 = 2\times10^3 + 3\times10^2 + 0\times10^1 + 7\times10^0$$

Base n representation

- A base n system contains n distinct symbols, the digits 0 through n - 1
- Numeric values greater than n -1 are represented by a pattern of the n symbols
- The value of any symbol in the string is found by multiplying that symbol by n^p, where p is the distance from the rightmost symbol in the pattern
- Computers represent information using bit patterns, or binary (base 2) representation
- Numbers represented in base 2 are usually called binary numbers

Binary (base 2) representation

- The binary representation contains two symbols: { 0, 1 }
- Position of each symbol represents a power of two
- What is the value of the binary representation **111**?

	1	1	1
	↑	↑	↑
position:	2	1	0

$$111 = 1\times2^2 + 1\times2^1 + 1\times2^0$$

$$= 1\times4 + 1\times2 + 1\times1$$

$$= 4 + 2 + 1 = 7$$

Binary representation

- What is the value of the binary representation **1010**?

	1	0	1	0
	↑	↑	↑	↑
position:	**3**	**2**	**1**	**0**

$$\begin{aligned} 1010 &= 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 \\ &= 1\times 8 + 0\times 4 + 1\times 2 + 0\times 1 \\ &= 8 + 0 + 2 + 0 = 10 \end{aligned}$$

Binary addition

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 1 \end{array}$$

Represent sum of binary numbers as a binary number

decimal addition	binary addition
$1+1 = 2$	$1+1 = 10$
$1+1+1 = 3$	$1+1+1 = 10+1 = 11$

Adding binary numbers

```
   101
+   10
----------
   111
```

```
   1 1     ← carry
   101
+   11
------------
  1000
```

```
  1 1      ← carry
   111
+  110
------------
  1101
```

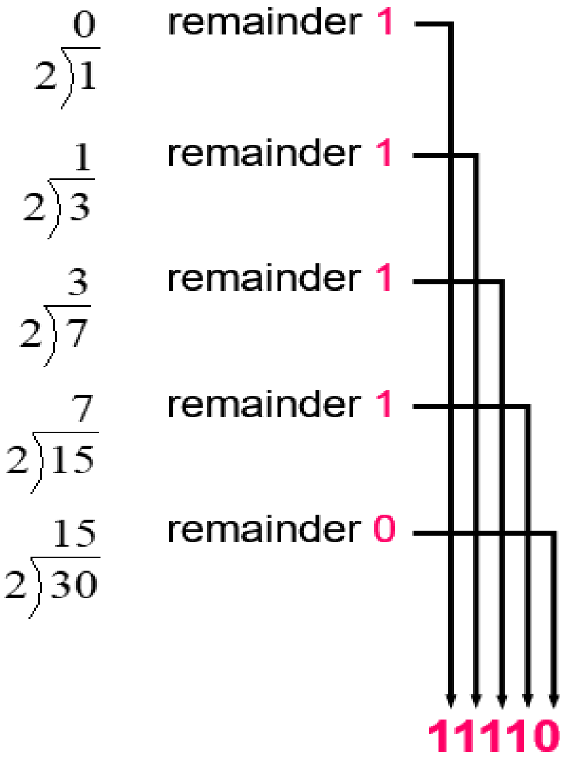
```
     1     1 1     ← carry
   10101010111
+    110000110
--------------------------
   11011011101
```

Converting decimal to binary

Decimal → → conversion → →		Binary
0 =	0×2^0	= 0
1 =	1×2^0	= 1
2 =	$1\times2^1 + 0\times2^0$	= 10
3 =	$1\times2^1 + 1\times2^0$	= 11
4 =	$1\times2^2 + 0\times2^1 + 0\times2^0$	= 100
5 =	$1\times2^2 + 0\times2^1 + 1\times2^0$	= 101
6 =	$1\times2^2 + 1\times2^1 + 0\times2^0$	= 110
7 =	$1\times2^2 + 1\times2^1 + 1\times2^0$	= 111
8 =	$1\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0$	= 1000

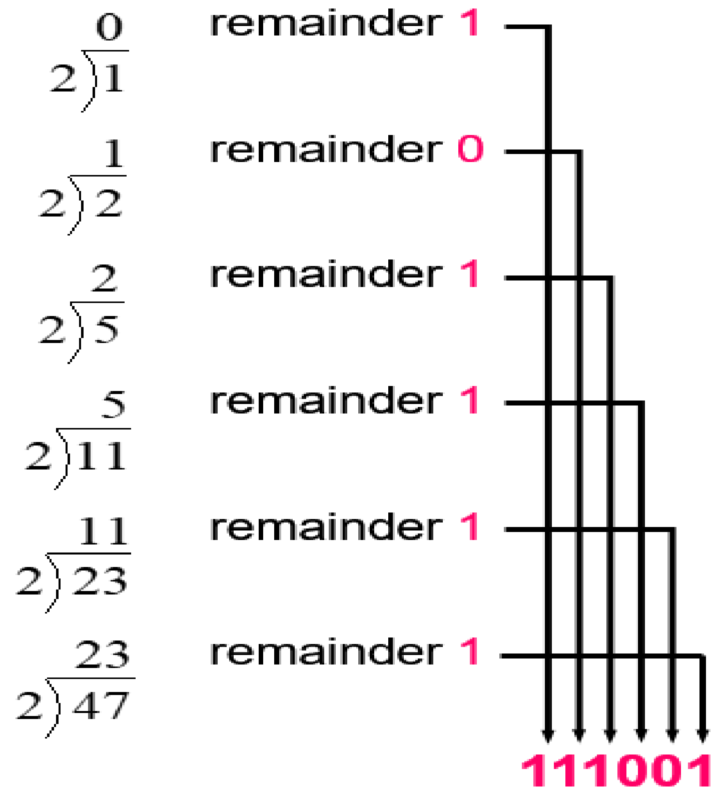
Converting decimal to binary

- Repeated division by two until the quotient is zero
- What is the binary representation of **30**?



Converting decimal to binary

- Repeated division by two until the quotient is zero
- What is the binary representation of **47**?



Problems

- Convert 1011000 to decimal representation
- Add the binary numbers 1011001 and 10101 and express their sum in binary representation
- Convert 77 to binary representation

Solutions

- Convert 1011000 to decimal representation

$$1011000 = 1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 0\times2^1 + 0\times2^0$$
$$= 64 + 16 + 8 = 88$$

- Add the binary numbers 1011001 and 10101 and express their sum in binary representation

```
   1011001
+    10101
----------------
   1101110
```

- Convert 77 to binary representation: 1001101

The End