BINARY NUMBER

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WHAT IS BINARY NUMBER?

Binary values are represented abstractly by:

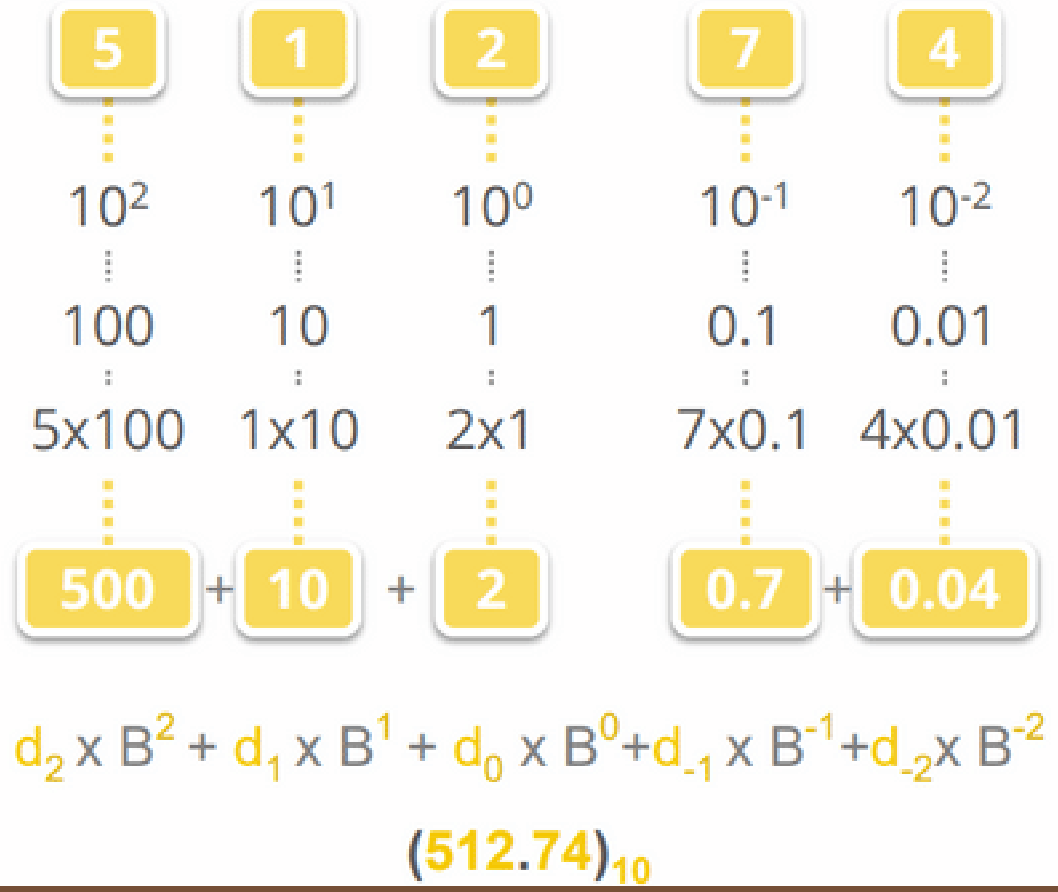
- Digits 0 and 1
- Words (symbols) False (F) and True (T)
- Words (symbols) Low (L) and High (H)
- And words On and Off

ASCII CONVERSION CHART

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Decimal Number System

- Base (also called radix) = 10
  - 10 digits
  - { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }
- Digit Position
  - Integer & fraction
- Digit Weight
  - Weight = *(Base)* ^{Position}
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation



Octal Number System

- Base = 8
  - 8 digits
  - { 0, 1, 2, 3, 4, 5, 6, 7}
- Weights
  - Weight = *(Base)* ^{Position}
- Magnitude
  - Sum of "*Digit* x *Weight*"
- Formal Notation

5	1	2	7	4
8^2	8^1	8^0	8^{-1}	8^{-2}
64	8	1	1/8	1/64
5x64	1x8	2x1	7x1/8	4x1/64
320 +	8 +	2	0.875 +	0.0625

$$o_2 \times B^2 + o_1 \times B^1 + o_0 \times B^0 + o_{-1} \times B^{-1} + o_{-2} \times B^{-2}$$

$(330.9375)_{10}$

$(512.74)_8$

Hexadecimal Number System

- Base = 16
  - 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}
- Weights
  - Weight = *(Base)* ^{Position}
- Magnitude
  - Sum of "*bit* x *Weight*"
- Formal Notation

1	E	5	7	A
16^2	16^1	16^0	16^{-1}	16^{-2}
256	16	1	1/16	1/256
1x256	14x16	5x1	7x1/16	10x1/256

256 + 224 + 5 + 0.4375 + 0.039

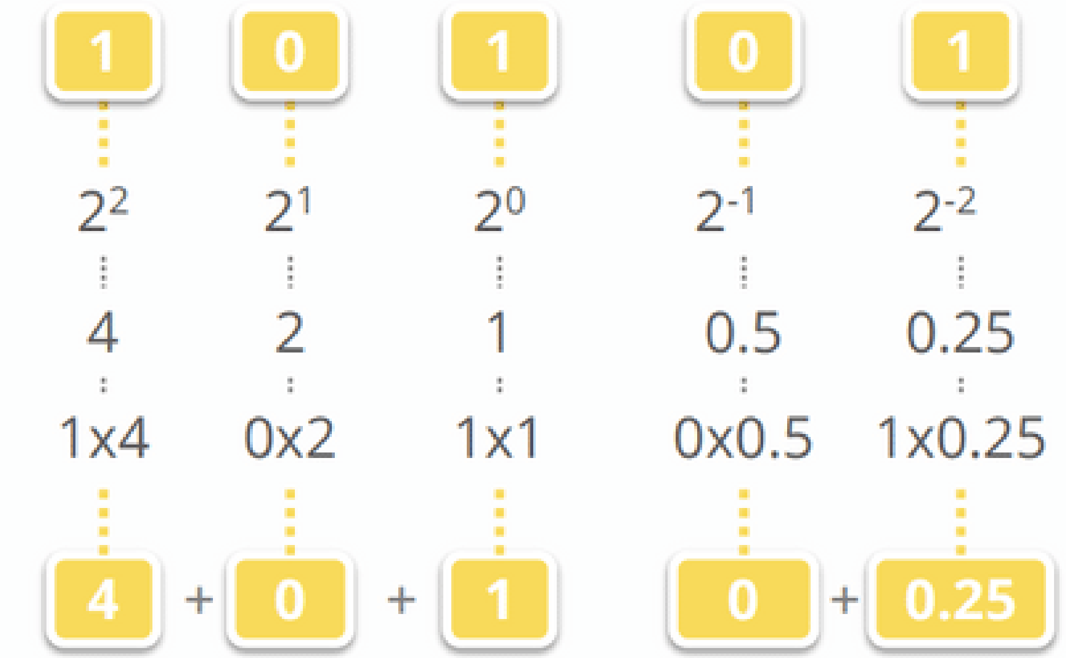
$H_2 \times B^2 + H_1 \times B^1 + H_0 \times B^0 + H_{-1} \times B^{-1} + H_{-2} \times B^{-2}$

$(458.4765625)_{10}$

$(1E5.7A)_{16}$

Binary Number System

- Base = 2
  - 2 digits { 0, 1 }, called *b*inary dig*its* or "*bits*"
- Weights
  - Weight = *(Base)* ^{Position}
- Magnitude
  - Sum of "*bit* x *Weight*"
- Formal Notation
- Groups of bits
  - 4 bits = *Nibble*, 8 bits = *Byte*



$b_2 \times B^2 + b_1 \times B^1 + b_0 \times B^0 + b_{-1} \times B^{-1} + b_{-2} \times B^{-2}$

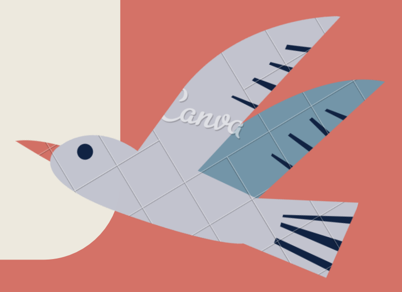
$(5.25)_{10}$

$(101.01)_2$

the power of 2

n	2^n
0	$2^0=1$
1	$2^1=2$
2	$2^2=4$
3	$2^3=8$
4	$2^4=16$
5	$2^5=32$
6	$2^6=64$
7	$2^7=128$

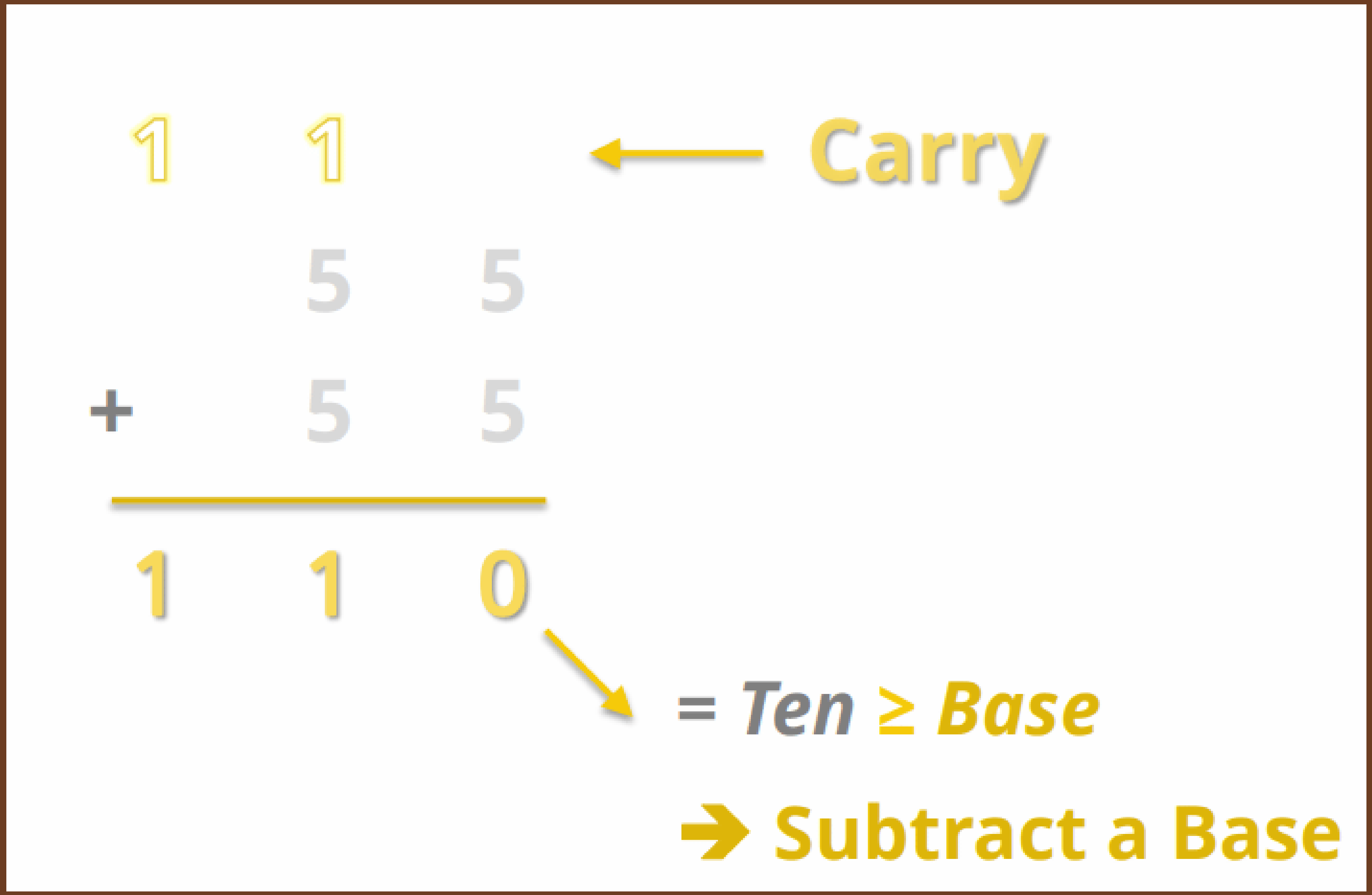
n	2^n
8	$2^8=256$
9	$2^9=512$
10	$2^{10}=1024$
11	$2^{11}=2048$
12	$2^{12}=4096$
20	$2^{20}=1M$
30	$2^{30}=1G$
40	$2^{40}=1T$



BINARY ARITHMETIC

ADDITION

Decimal addition



BINARY ARITHMETIC

ADDITION

Binary addition - Column addition

$$
\begin{array}{cccccccc|l}
 & 1 & 1 & 1 & 1 & 1 & 1 & & \\
 & & 1 & 1 & 1 & 1 & 0 & 1 & = 61 \\
+ & & & 1 & 0 & 1 & 1 & 1 & = 23 \\
\hline
 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & = 84
\end{array}
$$

$\geq (2)_{10}$

DECIMAL NUMBER SYSTEM

The decimal number system comprises digits from 0-9 that are 0, 1, 2, 3, 4, 5, 6, 7, 8 & 9. The base or radix of the decimal number system is 10 because the total number of digits available in the decimal number system is 10. All the other digits can be expressed with the help of these 10 digit numbers.

Decimal (integer) to binary

- Divide the number by the "base '(2)"
- Take the remainder (0 or 1)
- take the quotient and repeat the division

Decimal (integer) to binary

$(13)_{10}$

	Quotient	Remainder	Coefficient
13 / 2 =	6	1	$a_0 = 1$
6 / 2 =	3	0	$a_1 = 0$
3 / 2 =	1	1	$a_2 = 1$
1 / 2 =	0	1	$a_3 = 1$

Answer: $(13)_{10} = (\underset{\text{MSB}}{a_3}\, a_2\, a_1\, \underset{\text{LSB}}{a_0})_2 = (1101)_2$

Decimal (fraction) to binary

- Multiply the number by the base '(2)'
- Take the integer (0 or 1) as a coefficient
- Take the resultant fraction and repeat the division

Decimal (fraction) to binary

$(0.625)_{10}$

		Integer		Fraction	Coefficient
0.625	* 2 =	1	.	25	$a_{-1} = 1$
0.25	* 2 =	0	.	5	$a_{-2} = 0$
0.5	* 2 =	1	.	0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\, a_{-2}\, a_{-3})_2 = (0.101)_2$

MSB   LSB

Decimal (integer) to octal

$(175)_{10}$

	Quotient	Remainder	Coefficient
$175 / 8 =$	21	7	$a_0 = 7$
$21 / 8 =$	2	5	$a_1 = 5$
$2 / 8 =$	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_3\ a_2\ a_1\ a_0)_2 = (257)_8$

MSB LSB

Decimal (fraction) to octal

$(0.3125)_{10}$

	Integer		Fraction	Coefficient
$0.3125 * 8 =$	2	.	5	$a_{-1} = 2$
$0.5 * 8 =$	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_2 = (0.24)_8$

MSB LSB

Example 1:

Convert $(27.35)_8$ to the base of 10.

Answer:

$$(27.35)_8 = 7*8^0 + 2*8^1 + 3*8^{-1} + 5*8^{-2} = 7 + 16 + .375 + .078125$$
$$= (23.45)_{10}$$

Example 2 :

Convert 1101111 to decimal.

Answer:

$$(1101111)_2 = 1*2^0 + 1*2^1 + 1*2^2 + 1*2^3 + 0*2^4 + 1*2^5 + 1*2^6$$
$$= 1 + 2 + 4 + 8 + 32 + 64 = (111)_{10}$$

Converting binary to decimal

$$(a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3})_2$$

where a, is a binary digit or bit (either 0 or 1)

Can be converted to decimal number using :

$$(\underbrace{a_5a_4a_3a_2a_1a_0}_{\text{Integer}}\underbrace{.a_{-1}a_{-2}a_{-3}}_{\text{Fraction}})_2 = a_0 \times 2^0 + a_1 \times 2^1 + a_2 \times 2^2 + a_3 \times 2^3 + \ldots$$

$$+a_{-1} \times 2^{-1} + a_{-2} \times 2^{-2} + \ldots$$

$$(a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3})_2 = a_0 + 2\,a_1 + 4\,a_2 + 8\,a_3 + 16\,a_4 + 32\,a_5 + 64\,a_6$$

$$+\frac{1}{2} * a_{-1} + \frac{1}{4} * a_{-2} + \frac{1}{8} * a_{-3}$$

Example 1:

Convert $(110111.101)_2$ to decimal.

Answer :

$$(110111.101)_2 = 1*2^0 + 1*2^1 + 1*2^2 + 0*2^3 + 1*2^4 + 1*2^{*5} + 1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 55.625$$

Or

2^5	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}
1	1	0	1	1	1	.	1	0	1

$$32 + 16 + 0 + 4 + 2 + 1 + 1/2 + 0 + 1/8$$

If a binary value is made of n bits of ones, then its decimal value is $2^n - 1$.

Converting decimal to binary

To convert an integer number from decimal to binary, divide the decimal number by the new base (2 for binary), which will result in a quotient and a remainder (either 0 or 1). The first remainder will be the least significant bit of the binary number. Continually divide the quotient by the new base, while taking the remainders as each subsequent bit in the binary number, until the quotient becomes

Example 1 :

Convert 34 bit in decimal to binary

Answer :

	Quotient	Remainder
34/2 =	17	$0 = a_0$
17/2=	8	$1 = a_1$
8/2	4	$0 = a_2$
4/2	2	0 = a3
2/2	1	$0 = a_4$
1/2	0	$1 = a_5$
Therefore $34 = (100010)_2$		

If a binary number is made of all ones, then by using the equation $2^n - 1$, it can be converted to decimal.

Converting decimal fraction to binary

A decimal number representation of $(0.XY)_{10}$ can be converted into base of 2 and represented by $(0.a_{-1}, a_{-2}, a_{-3}, \text{etc.})_2$.

The fraction number is multiplied by 2, the result of integer part is a_{-1} and fraction part multiply by 2, and then separate integer part from fraction, the integer part represents a_{-2}; this process continues until the fraction becomes 0.

$(0.35)_{10} = (\qquad)_2$

0.35*2	=	0.7	=	0	+	0.7	$a_{-1} = 0$
0.7*2	=	1.4	=	1	+	0.4	$a_{-2} = 1$
0.4*2	=	0.8	=	0	+	0.8	$a_{-3} = 0$
0.8*2	=	1.6	=	1	+	0.6	$a_{-4} = 1$
0.6*2	=	1.2	=	1	+	0.2	$a_{-5} = 1$

Sometimes, the fraction does not reach 0 and the number of bits use for the fraction depends on the accuracy that the user defines, therefore the 0.35 = 0.010011 in binary

THANK YOU