(9) $\nabla_X(T) = \nu S(X)$

$X = \frac{\partial}{\partial u}, \frac{\partial}{\partial v}$ を代入すると、

$$
\star
\begin{cases}
\circ \left(\frac{\alpha}{E}\right)_u + \frac{\alpha E_u + \beta E_v}{2E^2} = \frac{\nu L}{E} \\
\circ \left(\frac{\beta}{E}\right)_u + \frac{-\alpha E_v + \beta E_u}{2E^2} = \frac{\nu M}{E} \\
\circ \left(\frac{\alpha}{E}\right)_v + \frac{\alpha E_v - \beta E_u}{2E^2} = \frac{\nu M}{E} \\
\circ \left(\frac{\beta}{E}\right)_v + \frac{\alpha E_u + \beta E_v}{2E^2} = \frac{\nu N}{E}
\end{cases}
$$

$\left(\frac{\alpha}{E}\right)_u = \frac{\alpha_u E - \alpha E_u}{E^2}$ および $\star$ の分母を払うと、

$$
\star \Longleftrightarrow
\begin{cases}
\circ\ 2\alpha_u E - \alpha E_u + \beta E_v = 2E\nu L & \cdots ① \\
\circ\ 2\beta_u E - \beta E_u - \alpha E_v = 2E\nu M & \cdots ② \\
\circ\ 2\alpha_v E - \beta E_u - \alpha E_v = 2E\nu M & \cdots ③ \\
\circ\ 2\beta_v E + \alpha E_u - \beta E_v = 2E\nu N & \cdots ④
\end{cases}
$$

- ②、③ より、$\alpha_v = \beta_u$ である
- $\beta\times$①、$\alpha\times$② より、 (E_u 消去)

$$2\alpha_u\beta E - 2\alpha\beta_u E + \beta^2 E_v + \alpha^2 E_v = 2E\nu(L\beta - M\alpha)$$

$$\Longleftrightarrow -L\beta\nu + M\alpha\nu = \frac{-2\alpha_u\beta E + 2\alpha\beta_u E - \beta^2 E_v - \alpha^2 E_v}{2E}$$

$$\Longleftrightarrow -L\beta\nu + M\alpha\nu + \frac{E_v\alpha^2}{2E} + \frac{E_v\beta^2}{2E} = -\alpha_u\beta + \alpha\beta_u$$

$$= -\alpha_u\beta + 2\alpha\beta_u - \alpha\beta_u$$

$\alpha_v = \beta_u$ 代入

$$= -\alpha_u\beta + 2\alpha\alpha_v - \alpha\beta_u$$

$$\Longleftrightarrow -E_v - L\beta\nu + M\alpha\nu + \frac{E_v\alpha^2}{2E} + \frac{E_v\beta^2}{2E} = -E_v - \alpha_u\beta + 2\alpha\alpha_v - \alpha\beta_u.$$

$\therefore$ ①, ② $\Longleftrightarrow$ (1.2)

$$\frac{\alpha}{E} f_u + \frac{\beta}{E} f_v + \nu n \overset{?}{=} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

• $\beta\times$②、$\alpha\times$④ より (${E_v}$消去)

$$2\beta\beta_u E - 2\alpha\beta_v E - \beta^2 E_u - \alpha^2 E_u = 2E\nu(\beta M - \alpha N)$$

$$\Longleftrightarrow -M\beta\nu + N\alpha\nu = -\frac{2\beta\beta_u E - 2\alpha\beta_v E - \beta^2 E_u - \alpha^2 E_u}{2E}$$

$$\Longleftrightarrow -M\beta\nu + N\alpha\nu - \frac{E_u\alpha^2}{2E} - \frac{E_u\beta^2}{2E} = -\beta\beta_u + \alpha\beta_v$$

$$= \alpha\beta_v + \beta\beta_u - 2\beta\beta_u$$ $\alpha_v = \beta_u$ 代入

$$= \alpha\beta_v + \alpha_v\beta - 2\beta\beta_u$$

$$\Longleftrightarrow E_u - M\beta\nu + N\alpha\nu - \frac{E_u\alpha^2}{2E} - \frac{E_u\beta^2}{2E} = E_u + \alpha\beta_v + \alpha_v\beta - 2\beta\beta_u$$

$\therefore$ ②, ④ $\Longleftrightarrow$ (1.3)

• (1.2) $\Longleftrightarrow$ (2.1) であることを確認する

$$(1.2): -E_v - L\beta\nu + M\alpha\nu + \frac{E_v\alpha^2}{2E} + \frac{E_v\beta^2}{2E} = -E_v - \alpha_u\beta + 2\alpha\alpha_v - \alpha\beta_u.$$

$$\Longleftrightarrow \quad -\alpha_u\beta + 2\alpha\alpha_v - \alpha\beta_u + \frac{E_u}{E}\alpha\beta = -L\beta\nu + M\alpha\nu + \frac{E_v\alpha^2}{2E} + \frac{E_v\beta^2}{2E} + \frac{E_u}{E}\alpha\beta$$

両辺に $\frac{E_u}{E}\alpha\beta$ 足す

E_v 足す

$$\Longleftrightarrow \quad \alpha_u\beta - 2\alpha\alpha_v + \alpha\beta_u - \frac{E_u}{E}\alpha\beta = L\beta\nu - M\alpha\nu - \frac{E_v\alpha^2}{2E} - \frac{E_v\beta^2}{2E} - \frac{E_u}{E}\alpha\beta$$

両辺を -1倍する

$$\Longleftrightarrow \quad \frac{\alpha_u\beta}{E} - \frac{2\alpha\alpha_v}{E} + \frac{\alpha\beta_u}{E} - \frac{E_u\alpha\beta}{E^2} = \frac{L\beta\nu}{E} - \frac{M\alpha\nu}{E} - \frac{E_v\alpha^2}{2E^2} - \frac{E_v\beta^2}{2E^2} - \frac{E_u\alpha\beta}{E^2}$$

両辺を $\frac{1}{E}$倍する

$$\Longleftrightarrow \quad \frac{-E_u\alpha\beta + (\alpha_u\beta - 2\alpha\alpha_v + \alpha\beta_u)E}{E^2} = \frac{EL\beta\nu - EM\alpha\nu - E_u\alpha\beta - \frac{E_v\alpha^2}{2} - \frac{E_v\beta^2}{2}}{E^2}$$

分母を E^2に統一する

$$\Longleftrightarrow \quad \frac{-E_u\alpha\beta + E_v\alpha^2 + (\alpha_u\beta - 2\alpha\alpha_v + \alpha\beta_u)E}{E^2} = \frac{EL\beta\nu - EM\alpha\nu - E_u\alpha\beta + \frac{E_v\alpha^2}{2} - \frac{E_v\beta^2}{2}}{E^2}$$

両辺に $\frac{E_v\alpha^2}{E^2}$ 足す

これは(2.1)の等式部分と等しい。 $\therefore$ (1.2) $\Longleftrightarrow$ (2.1)

• (同様の計算で、(1.3) $\Longleftrightarrow$ (3.1) である)

(10) $d\nu(X) = -\langle S(X), T\rangle$

$$\begin{cases} \nu_u = \dfrac{-1}{E}(L\alpha + M\beta) \cdots ① \\ \nu_v = \dfrac{-1}{E}(M\alpha + N\beta) \cdots ② \end{cases}$$

② × α − ① × β より、

$$\nu_v\alpha - \nu_u\beta = \frac{-\alpha}{E}(M\alpha + N\beta) + \frac{\beta}{E}(L\alpha + M\beta)$$

$$= \frac{-M\alpha^2 - N\alpha\beta + L\alpha\beta + M\beta^2}{E}. \cdots ③$$

式(9)より、$\alpha_v\nu - \beta_u\nu = \alpha_v\nu - \alpha_v\nu = 0$ である. ③と足し合わせると

$$\nu_v\alpha - \nu_u\beta + \alpha_v\nu - \beta_u\nu = \frac{-M\alpha^2 - N\alpha\beta + L\alpha\beta + M\beta^2}{E} + 0$$

であるため、式(9)
式(10) $\Longleftrightarrow$ (1.4) ($\Longleftrightarrow$ (4.1)) である.

- ガウス方程式 $\Longleftrightarrow$ (3.2) ($\Longleftrightarrow$ (2.3))
- コダッチ方程式 $\Longleftrightarrow$ $\begin{cases} (2.4) \ (\Longleftrightarrow (4.2)) \\ (3.4) \ (\Longleftrightarrow (4.3)) \end{cases}$
  - $\frac{\partial}{\partial u}$ の係数比較
  - $\frac{\partial}{\partial v}$ の係数比較

- 式(9) $\Longleftrightarrow$ $\begin{cases} \alpha_v = \beta_u, \ (\alpha_u + \beta_v = \nu(L+N)) \\ (1.2) \ (\Longleftrightarrow (2.1)) \\ (1.3) \ (\Longleftrightarrow (3.1)) \end{cases}$

- $\alpha_v = \beta_u$、式(10) $\Longleftrightarrow$ (1.4) ($\Longleftrightarrow$ (4.1))

$$\begin{pmatrix} 0 & \cancel{(1.2)} & \cancel{(1.3)} & \cancel{(1.4)} \\ \cancel{(2.1)} & 0 & \cancel{(2.3)} & \cancel{(2.4)} \\ \cancel{(3.1)} & \cancel{(3.2)} & 0 & \cancel{(3.4)} \\ \cancel{(4.1)} & \cancel{(4.2)} & \cancel{(4.3)} & 0 \end{pmatrix}$$