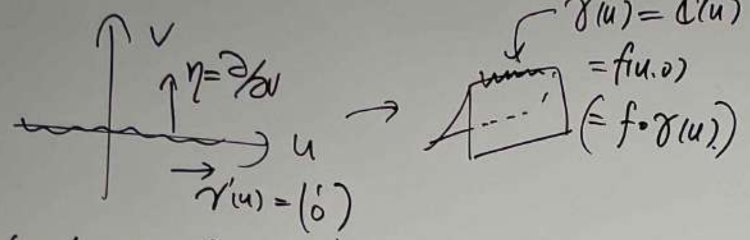
$f_v(u,0) = 0$



$\vee \ \det(\gamma'(u), \eta(u)) > 0$

$\lambda = \det(f_u, f_v, \nu)$

$\lambda_\eta^{(u,0)} = \lambda_v^{(u,0)} = \det(f_u, f_{vv}, \nu)(u,0)$

$= \det(\hat{\gamma}'(u), \mathfrak{X}(u), \sigma \nu_3(u))$ $\quad \sigma = \pm 1.$

$= \det(e(u), \cos\theta\, n - \sin\theta\, b, \sigma(\sin\theta\, n + \cos\theta\, b))$

$= \underbrace{\det(e, n, b)}_{=1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sigma\sin\theta \\ 0 & -\sin\theta & \sigma\cos\theta \end{pmatrix}$

$= \sigma$

$\varepsilon_\eta = \mathrm{sgn}\left(\underbrace{\det(\gamma', \eta)}_{正} \cdot \underbrace{\lambda_\eta(u,0)}_{\sigma} \right)$

$= \sigma$

$\mathfrak{X}(u) = f_{vv}(u,0)$

m-type edge

$\iff \exists \xi, \eta$

- $\eta^i f(u,0) = 0 \quad (\forall i = 1, \cdots, m-1)$
- $\{\xi f(u,0), \eta^m f(u,0)\}$: 1次独立.

$\xi = \frac{\partial}{\partial u} \quad \eta = \frac{\partial}{\partial v} \quad = \{f_u(u,0), f_{\underbrace{v \cdots v}_{m}}(u,0)\}$

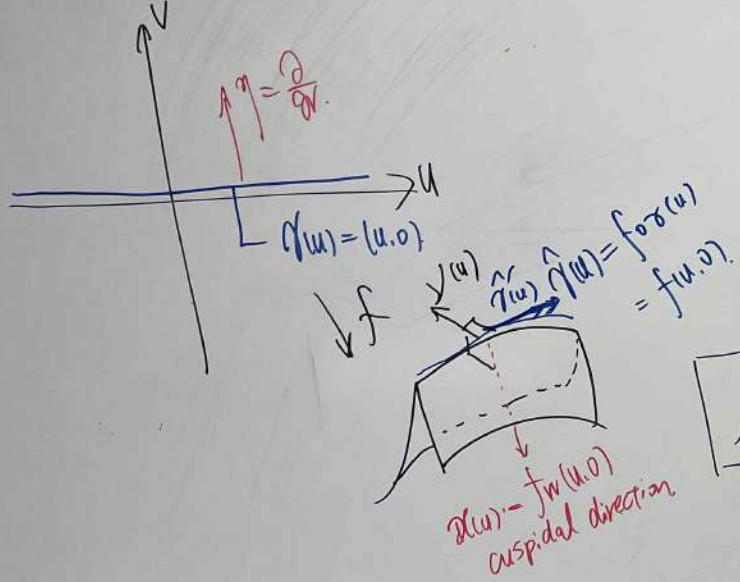
$m=2$: $f_{vv}(u,0)$.

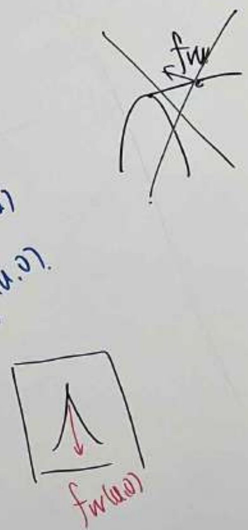
$m=2$

f: gen. cuspidal edge. at p

$\Longrightarrow$ $\exists (U; u, v)$: $\underset{(0,0)}{\overset{}{p}}$ を中心とする coord.

s.t. $\begin{cases} \cdot\ (u,0) \text{ が特異曲線.} \\ \cdot\ f_v(u,0) = 0 \\ \cdot\ \{f_u(u,0),\ f_{vv}(u,0),\ \nu(u,0)\} \text{ が ONB.} \end{cases}$





m: 偶数

f: m-type edge at p

$\Longrightarrow$ $\exists (U; u, v)$: $\underset{(0,0)}{p}$ を中心とする coord.

s.t. $\begin{cases} \circ\ (u,0) \text{ が特異曲線.} \\ \cdot\ f_v(u,0) = \cdots = f_{v^{m-1}}(u,0) = 0 \\ \cdot\ \{f_u(u,0),\ f_{v^m}(u,0),\ \nu(u,0)\} \text{ : ONB.} \end{cases}$

Def. $\eta(u) := f_{v^m}(u,0)$

を cuspidal direction という.

HW この場合も

$\kappa_s = \kappa \cos\theta$ は成立するか?

$$K_s = \varepsilon_\gamma \cdot \frac{\det(\hat{\gamma}', \hat{\gamma}'', \nu)}{\|\hat{\gamma}'\|^3}$$

$$\varepsilon_\gamma = \mathrm{sgn}\left(\det(\gamma', \eta) \cdot \hat{\lambda}_\eta\right)$$

$\boxed{\boxed{\eta^{m-1}\lambda} \text{ と変更}}$

$$\lambda = \hat{\lambda}^{m-1}$$

$$\eta\lambda = (m-1)\,\hat{\lambda}^{m-2} \cdot \eta\hat{\lambda}$$

$$\vdots$$

$$\boxed{\eta^{m-1}\lambda(u,0)} = (m-1)!\,\boxed{\eta\hat{\lambda}(u,0)}$$

同じ符号.

$\boxed{\text{Hattori Prop 17}}$

m: 偶数

f: m-type edge at p

$\Longrightarrow \exists (U; u, v)$: $\underset{(0,0)}{p}$ を中心とする coord.

s.t.
$\begin{cases} \cdot\ (u,0) \text{ が特異曲線.} \\ \cdot\ f_v(u,0) = \cdots = f_{v^{m-1}}(u,0) = 0 \\ \cdot\ \{f_u(u,0), f_{v^m}(u,0), \nu(u,0)\} : \text{ONB.} \end{cases}$

Def. $\chi(u) := f_{v^m}(u,0)$

を cuspidal direction という.

HW この場合も $K_s = \kappa\cos\theta$ は成立するか?

$$f_v = v^{m-1} \ \exists\, \psi(u,v)$$

$$f_{v^m}(u,0) = (m-1)!\ \psi(u,0).$$

$$\therefore\ \psi(u,0) = \frac{1}{(m-1)!} f_{v^m}(u,0) = \frac{1}{(m-1)!}\eta(u).$$

$$\lambda = \det(f_u, f_v, \nu)$$

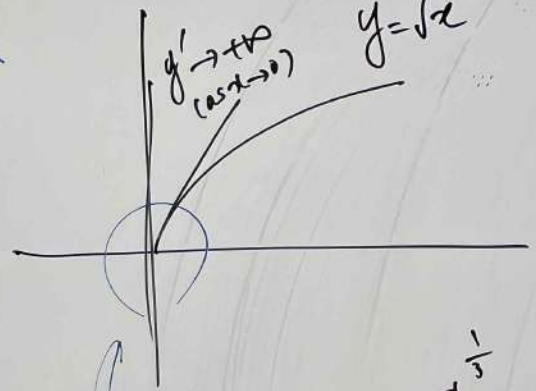
$$= v^{m-1}\cdot \det(f_u, \psi, \nu) = \hat{\lambda}^{m-1}$$

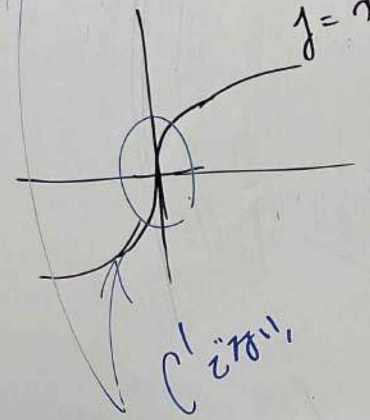
$$\hat{\lambda} := v\cdot\Big\{\underbrace{\det(f_u,\psi,\nu)}_{\neq 0}\Big\}^{\frac{1}{m-1}} : C^\infty \text{級}$$

$m-1$：奇数

$\det(f_u, \psi, \nu)$の符号によらず

$(m-1)$乗根がとれる.





$$\partial_v^{m-1}\lambda(u,0) = \lambda_{v^{m-1}}(u,0)$$

$$= (m-1)!\cdot \det(f_u, \psi, \nu)(u,0)$$

$$= \cancel{(m-1)!}\ \det\Big(\gamma'(u), \frac{1}{\cancel{(m-1)!}}\eta(u), \nu(u,0)\Big)$$

$$= \det\big(e(u), \cos\theta\, n(u) - \sin\theta\, b(u), \sigma(\sin\theta\, n + \cos\theta\, b)\big)$$

$$= \sigma.$$

$f(u,v) = (u, v^3, v^4)$.

$f_u = (1, 0, 0)$

$f_v = (0, 3v^2, 4v^3)$

$= v^2 \underbrace{(0, 3, 4v)}_{=:\psi\ (\neq 0)}$

$\nu := \dfrac{f_u \times \psi}{\|f_u \times \psi\|}$ とおく

>0 を示す必要あり!

$\left(\because f_u \times \psi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 4v \end{pmatrix} = \begin{pmatrix} 0 \\ -4v \\ 3 \end{pmatrix} \neq 0 \right)$

$$\begin{cases} \circ\ f_u \cdot (f_u \times \psi) = \det(f_u, f_u, \psi) = 0 \\ \circ\ f_v \cdot (f_u \times \psi) = \det(\underset{v^2\psi}{\underset{\shortparallel}{f_v}}, f_u, \psi) = 0 \end{cases}$$

lin. dependent.

より, ν は f の

C^∞ 級, 単位法ベクトル場.

Rem.

$\nu := \dfrac{f_u \times f_v}{\|f_u \times f_v\|}$ とすると,

$(u, 0)$ で $\|f_u \times f_v\| = 0$ となり

ν は特異点をこえて

C^∞ 級であることを示していない.

$f_\theta \times f_\varphi = \sin^3\theta\ \varphi^2 \cdot \boxed{\text{non-zero}}$

$\tilde{\nu} := \dfrac{f_\theta \times f_\varphi}{\varphi^2} = \sin^3\theta\ \boxed{\text{non-zero}} \neq 0.$

とおく.

$\nu := \dfrac{\tilde{\nu}}{\|\tilde{\nu}\|}$ は C^∞ 級単位法ベクトル場.

$$\begin{cases} f_u(u,0) \neq 0 \\ f_v(u,0) = 0 \end{cases}$$ のとき　$\mathrm{rank}(df) = 1$

f がフロント $\iff$ $\nu_v(u,0) \neq 0$.

$$g(\theta,\varphi) = \begin{pmatrix} 2\sin\theta \\ 4\cos\theta\cos\varphi \\ 4\cos\theta\sin\varphi \end{pmatrix}$$ ：楕円面のパラメータ表示.

$\left(|\theta| < \frac{\pi}{2},\ |\varphi| < \pi\right)$.

$$f(\theta,\varphi) = g(\theta,\varphi) + \frac{1}{d}\nu(\theta,\varphi)$$ ：平行曲面.

HW. $K_S = K\cos\theta$

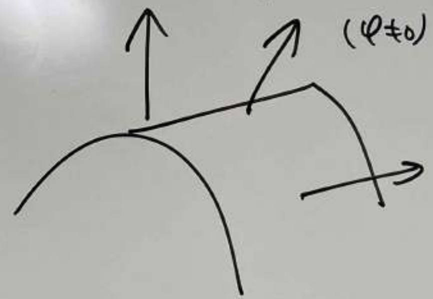
- 修論をまとめる.

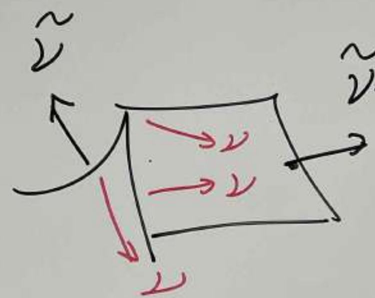
- イントロ（楕円面の平行曲面を入れる）
- 準備（m-type edge, …）
- n次 Koss 計量の等長実現定理 (Hattori, 定理6.7) ← 証明せず.
- 主定理A（HNSUY の Thm III）.
- 主定理B（HNSUY の Thm IV）

$\sigma =$

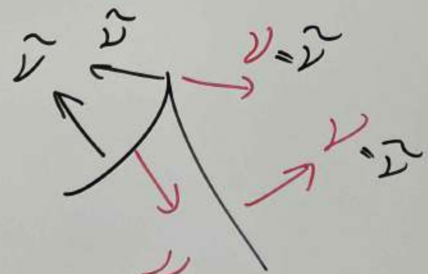
$$\nu = \frac{f_\theta \times f_\varphi}{\| f_\theta \times f_\varphi \|}$$

$(\varphi \neq 0)$





$$\tilde{\nu} = \frac{f_\theta \times f_\varphi}{\| f_\theta \times f_\varphi \|}$$



$$\tilde{\nu} = \frac{1}{|\varphi| \cdots} \begin{pmatrix} \cdot\,\varphi \\ \cdot\,\varphi \\ \cdot\,\varphi \end{pmatrix}$$

$$= \operatorname{sgn}(\varphi) \boxed{\frac{1}{\cdots} \begin{pmatrix} \vdots \end{pmatrix}} = \nu$$