Supervised classification - improving capacity learning

0. Import library

Import library

In [1]:

```
# Import libraries

# math library
import numpy as np

# visualization library
%matplotlib inline
from IPython.display import set_matplotlib_formats
set_matplotlib_formats('png2x','pdf')
import matplotlib.pyplot as plt

# machine learning library
from sklearn.linear_model import LogisticRegression

# 3d visualization
from mpl_toolkits.mplot3d import axes3d

# computational time
import time

import math
```

1. Load and plot the dataset (dataset-a.txt)

The data features for each data i are $x_i = (x_{i(1)}, x_{i(2)})$.

The data label/target, y_i, indicates two classes with value 0 or 1.

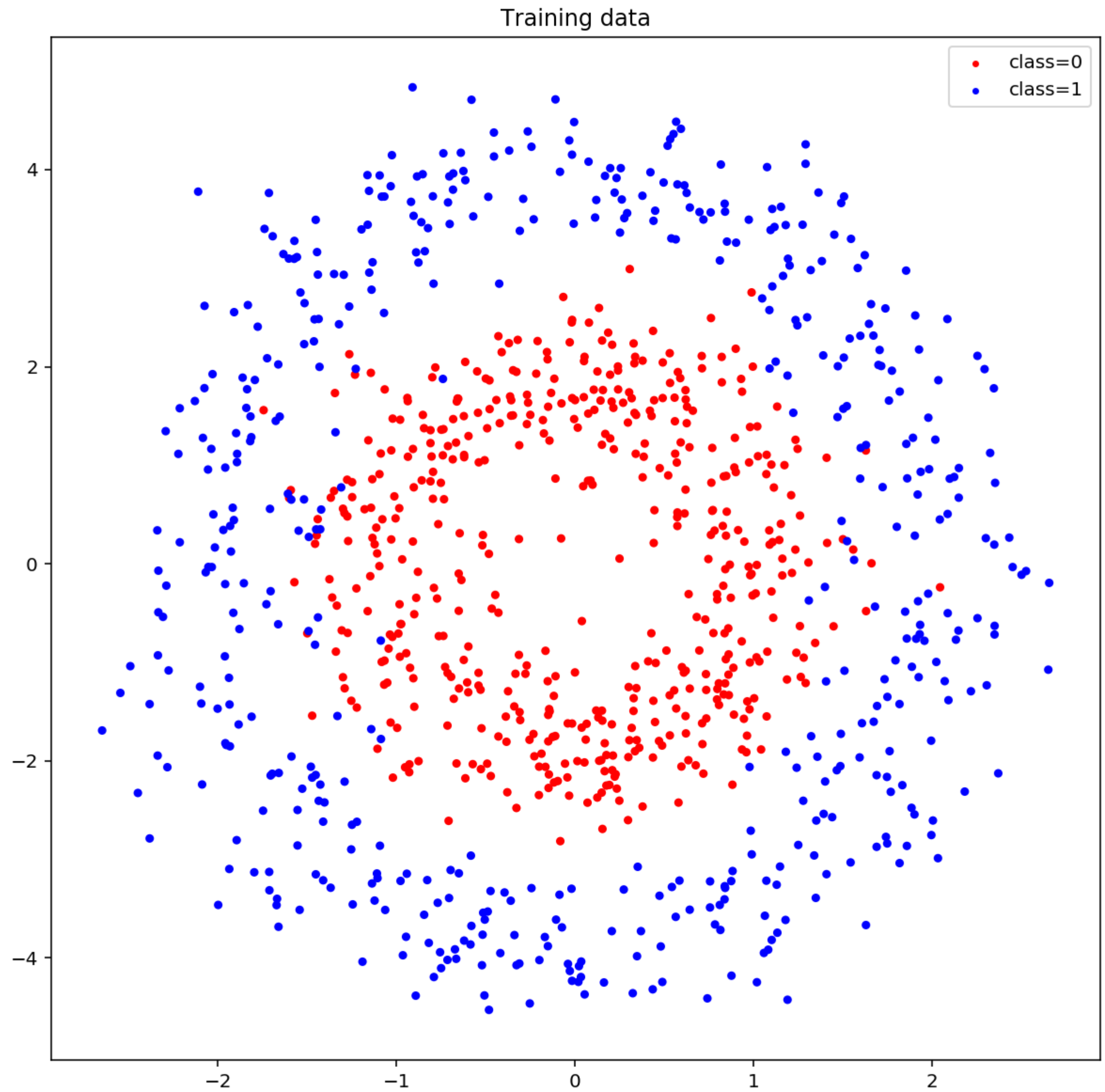
Plot the data points.

You may use matplotlib function `scatter(x,y)`.

In [14]:

```
# import data with numpy
data = np.loadtxt('dataset-a.txt', delimiter=',')

# number of training data
n = data.shape[0]
print('Number of the data = {}'.format(n))
print('Shape of the data = {}'.format(data.shape))
print('Data type of the data = {}'.format(data.dtype))

# plot
x1 = data[:,0] # feature 1
x2 = data[:,1] # feature 2
idx = data[:,2] # label

idx_class0 = (idx==0) # index of class0
idx_class1 = (idx==1) # index of class1

plt.figure(1,figsize=(10,10))
plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```

```
Number of the data = 1000
Shape of the data = (1000, 3)
Data type of the data = float64
```



2. Define a logistic regression loss function and its gradient

In [15]:

```
# sigmoid function
def sigmoid(z):
    sigmoid_f = 1 / (1 + np.exp(-z))
    return sigmoid_f


# predictive function definition
def f_pred(X,w):
    p = sigmoid(X@w)
    return p


# loss function definition
def loss_logreg(y_pred,y):
    n = len(y)
    loss = (-y.T @ np.log(y_pred) - (1-y).T @ np.log(1-y_pred)) / n
    return loss


# gradient function definition
def grad_loss(y_pred,y,X):
    n = len(y)
    grad = X.T @ (y_pred - y) * 2 / n
    return grad


# gradient descent function definition
def grad_desc(X, y, w_init, tau, max_iter):

    L_iters = np.zeros([max_iter]) # record the loss values
    w = w_init # initialization
    for i in range(max_iter): # loop over the iterations
        y_pred = f_pred(X,w) # linear predicition function
        grad_f = grad_loss(y_pred,y,X) # gradient of the loss
        w = w - tau * grad_f # update rule of gradient descent
        L_iters[i] = loss_logreg(y_pred,y) # save the current loss value

    return w, L_iters
```

3. define a prediction function and run a gradient descent algorithm

The logistic regression/classification predictive function is defined as:

$$p_w(x) = \sigma(Xw)$$

The prediction function can be defined in terms of the following feature functions f_i as follows:

$$X = \begin{bmatrix} f_0(x_1) & f_1(x_1) & f_2(x_1) & f_3(x_1) & f_4(x_1) & f_5(x_1) & f_6(x_1) & f_7(x_1) & f_8(x_1) & f_9(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2) & f_3(x_2) & f_4(x_2) & f_5(x_2) & f_6(x_2) & f_7(x_2) & f_8(x_2) & f_9(x_2) \\ \vdots & & & & & & & & & \\ f_0(x_n) & f_1(x_n) & f_2(x_n) & f_3(x_n) & f_4(x_n) & f_5(x_n) & f_6(x_n) & f_7(x_n) & f_8(x_n) & f_9(x_n) \end{bmatrix}$$

$$\text{and} \qquad w = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{bmatrix}$$

where $x_i = (x_i(1), x_i(2))$ and you can define a feature function f_i as you want.

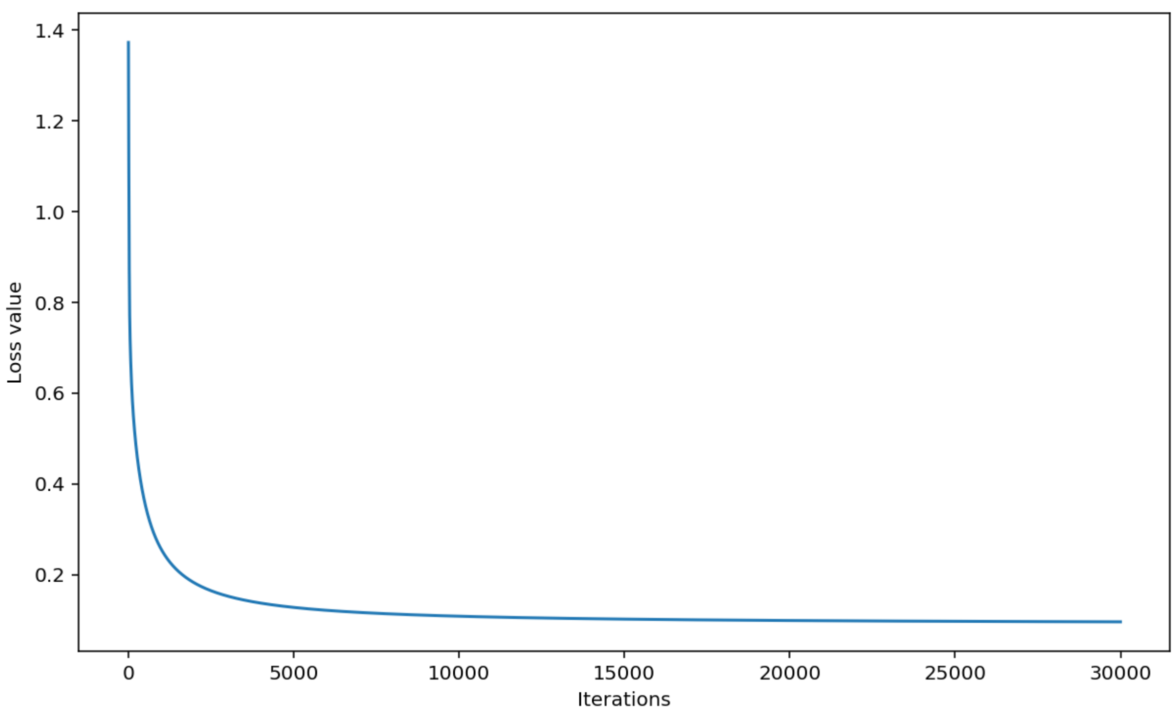
You can use at most 10 feature functions $f_i, i = 0, 1, 2, \cdots, 9$ in such a way that the classification accuracy is maximized. You are allowed to use less than 10 feature functions.

Implement the logistic regression function with gradient descent using a vectorization scheme.

In [29]:

```
import math
# construct the data matrix X, and label vector y
n = data.shape[0]
X = np.ones([n,10])

X[:,1] = x1
X[:,2] = x2
X[:,3] = x1**2
X[:,4] = x2**2
X[:,5] = x1**3
X[:,6] = x2**3
X[:,7] = np.sin(x1)**3
X[:,8] = np.sin(x2)**3
X[:,9] = np.tanh(x1)**2

y = data[:,2][:,None] # label

# run gradient descent algorithm
start = time.time()
w_init = np.array([0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5])[:,None]
tau = 1e-2; max_iter = 30000
w, L_iters = grad_desc(X,y,w_init,tau,max_iter)

# plot
plt.figure(3, figsize=(10,6))
plt.plot(np.array(range(max_iter)), L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```

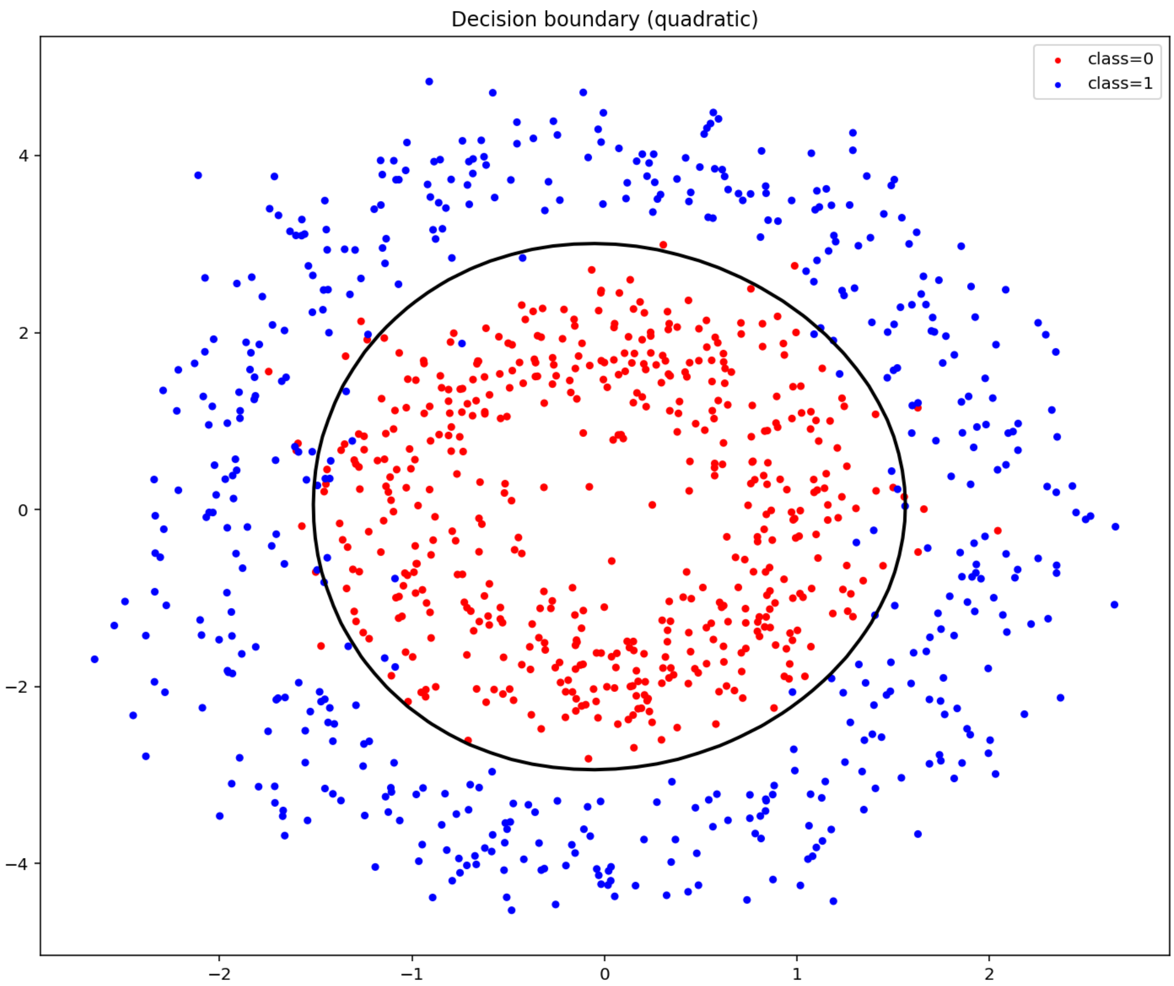
```
C:₩Users₩KSY₩Anaconda3₩lib₩site-packages₩ipykernel_launcher.py:16: RuntimeWarning: di
vide by zero encountered in log
  app.launch_new_instance()
C:₩Users₩KSY₩Anaconda3₩lib₩site-packages₩ipykernel_launcher.py:16: RuntimeWarning: in
valid value encountered in matmul
  app.launch_new_instance()
```



4. Plot the decision boundary

In [32]:

```
# compute values p(x) for multiple data points x
x1_min, x1_max = X[:,1].min(), X[:,1].max() # min and max of grade 1
x2_min, x2_max = X[:,2].min(), X[:,2].max() # min and max of grade 2
xx1, xx2 = np.meshgrid(np.linspace(x1_min, x1_max), np.linspace(x2_min, x2_max)) # create meshgrid
X2 = np.ones([np.prod(xx1.shape),10])

X2[:,1] = xx1.reshape(-1)
X2[:,2] = xx2.reshape(-1)
X2[:,3] = xx1.reshape(-1)**2
X2[:,4] = xx2.reshape(-1)**2
X2[:,5] = xx1.reshape(-1)**3
X2[:,6] = xx2.reshape(-1)**3
X2[:,7] = np.sin(xx1.reshape(-1))
X2[:,8] = np.sin(xx2.reshape(-1))
X2[:,9] = np.tanh(xx1.reshape(-1))

p = f_pred(X2,w)
p = p.reshape((len(xx1), len(xx2)))

# plot
plt.figure(4,figsize=(12,10))

plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, 1, linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (quadratic)')
plt.show()
```

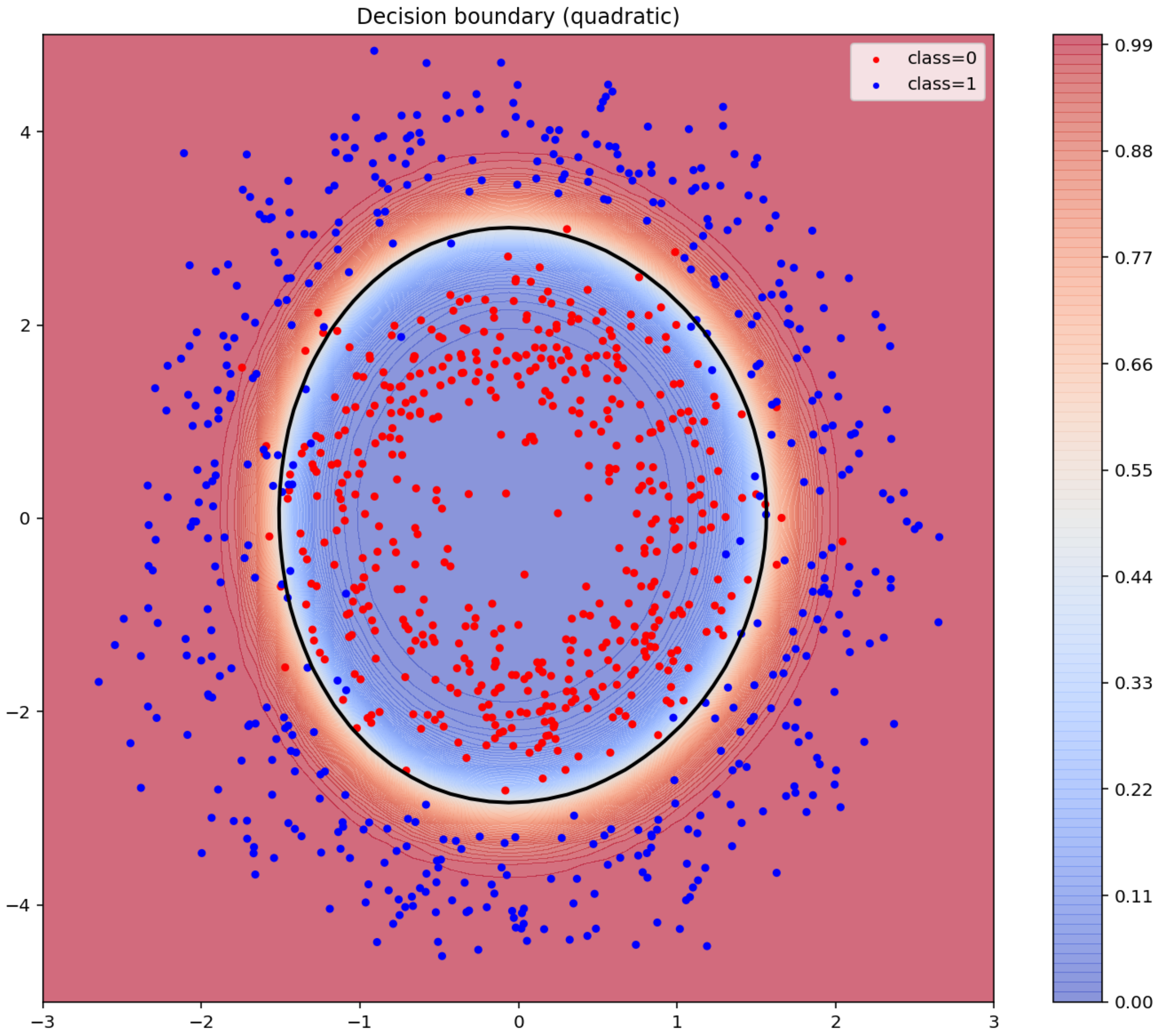


5. Plot the probability map

In [40]:

```
# compute values p(x) for multiple data points x
x1_min, x1_max = -3, 3 # min and max of grade 1
x2_min, x2_max = -5, 5 # min and max of grade 2

grid_x1 = np.linspace(x1_min, x1_max)
grid_x2 = np.linspace(x2_min, x2_max)
print(grid_x1[0])

xx1, xx2 = np.meshgrid(grid_x1, grid_x2) # create meshgrid

X2 = np.ones([np.prod(xx1.shape),10])
X2[:,1] = xx1.reshape(-1)
X2[:,2] = xx2.reshape(-1)
X2[:,3] = xx1.reshape(-1)**2
X2[:,4] = xx2.reshape(-1)**2
X2[:,5] = xx1.reshape(-1)**3
X2[:,6] = xx2.reshape(-1)**3
X2[:,7] = np.sin(xx1.reshape(-1))
X2[:,8] = np.sin(xx2.reshape(-1))
X2[:,9] = np.tanh(xx1.reshape(-1))

p = f_pred(X2,w)
p = p.reshape((len(xx1), len(xx2)))

# plot
plt.figure(4,figsize=(12,10))

ax = plt.contourf(xx1, xx2, p, 100, vmin=0, vmax=1, cmap='coolwarm', alpha=0.6)
cbar = plt.colorbar(ax)
cbar.update_ticks()

plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, 1, linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (quadratic)')
plt.show()
```

```
-3.0
```



6. Compute the classification accuracy

The accuracy is computed by:

$$\text{accuracy} = \frac{\text{number of correctly classified data}}{\text{total number of data}}$$

In [76]:

```
# compute the accuracy of the classifier
n = data.shape[0]

# plot
x1 = data[:,0] # feature 1
x2 = data[:,1] # feature 2
idx_class0 = (data[:,2]==0) # index of class0
idx_class1 = (data[:,2]==1) # index of class1

X3 = np.ones([n,10])

X3[:,1] = x1
X3[:,2] = x2
X3[:,3] = x1**2
X3[:,4] = x2**2
X3[:,5] = x1**3
X3[:,6] = x2**3
X3[:,7] = np.sin(x1)**3
X3[:,8] = np.sin(x2)**3
X3[:,9] = np.tanh(x1)**2

p = f_pred(X3,w)

#print('class1 length =', np.sum(idx_class1))
#print(p[1], idx_class1[1])
#some_p = p.reshape(-1)*idx_class1
#print(p.shape, idx_class1.shape, some_p.shape, some_p[1])
idx_class1_pred = p.reshape(-1)*idx_class1
idx_class0_pred = p.reshape(-1)*idx_class0
print(idx_class0_pred[999])

print(np.count_nonzero(idx_class1_pred >= 0.5))
print(np.count_nonzero(idx_class0_pred >= 0.5))
print(np.sum(idx_class0), np.sum(idx_class1))

correct_data = np.count_nonzero(idx_class1_pred >= 0.5) + (np.sum(idx_class0) - np.count_nonzero(id

print('total number of data =', n)
print('total number of correctly classified data = ', correct_data)
print('accuracy(%) = ', correct_data / n * 100)
```

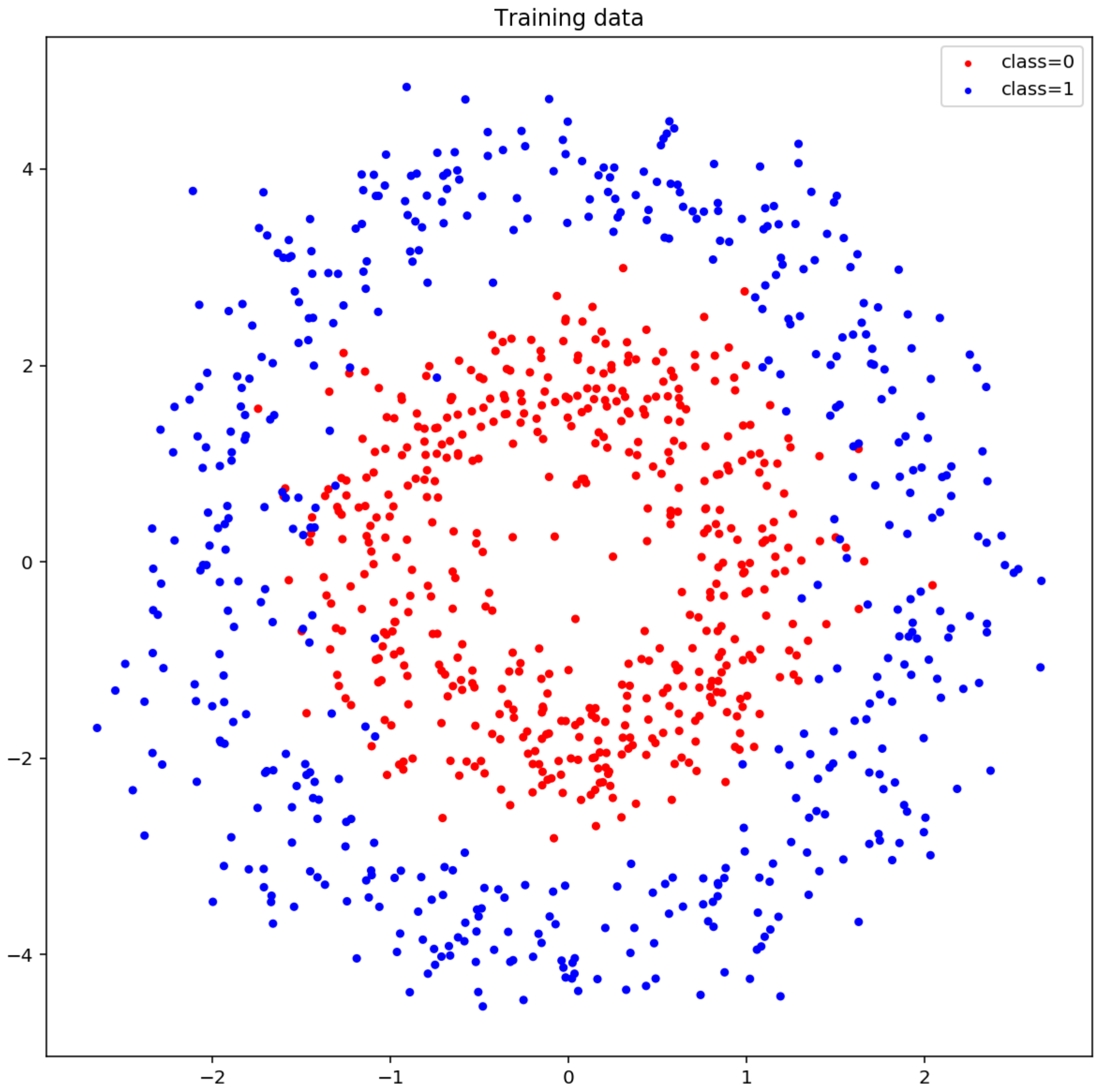
```
0.0
478
14
500 500
total number of data = 1000
total number of correctly classified data =  964
accuracy(%) =  96.39999999999999
```

Output using the dataset (dataset-a.txt)

1. Visualize the data [1pt]

In [6]:

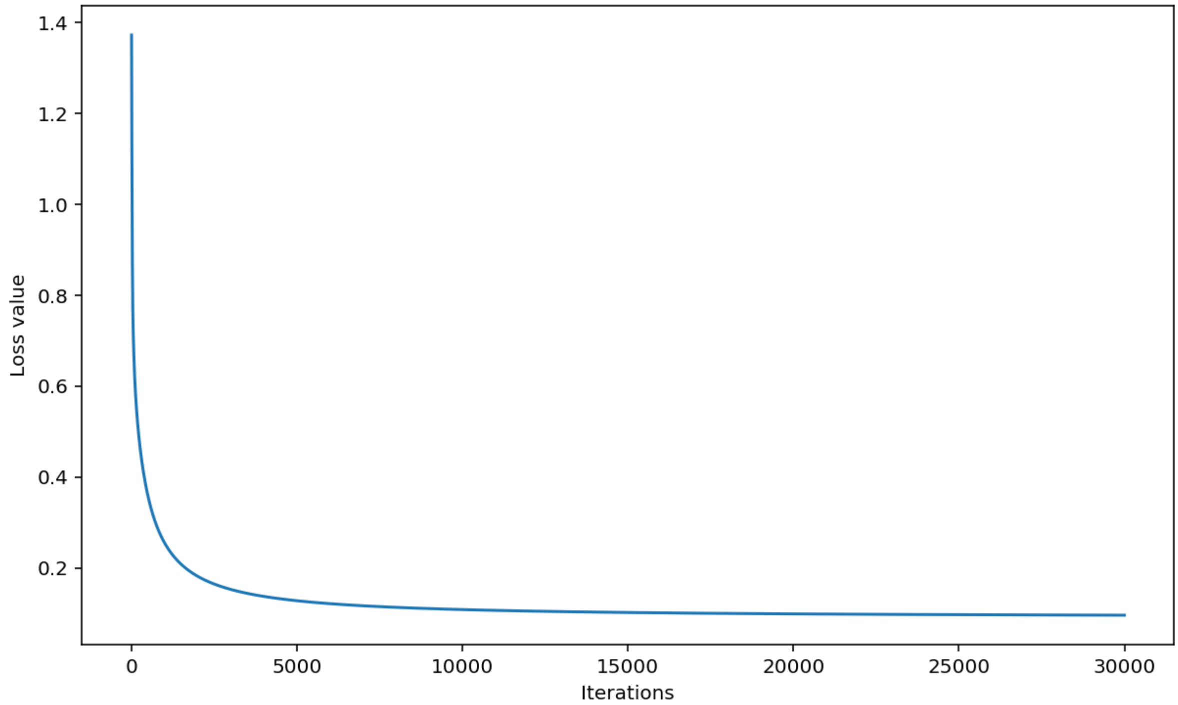
```
plt.figure(1,figsize=(10,10))
plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.title('Training data')
plt.legend()
plt.show()
```



2. Plot the loss curve obtained by the gradient descent until the convergence [2pt]

In [31]:

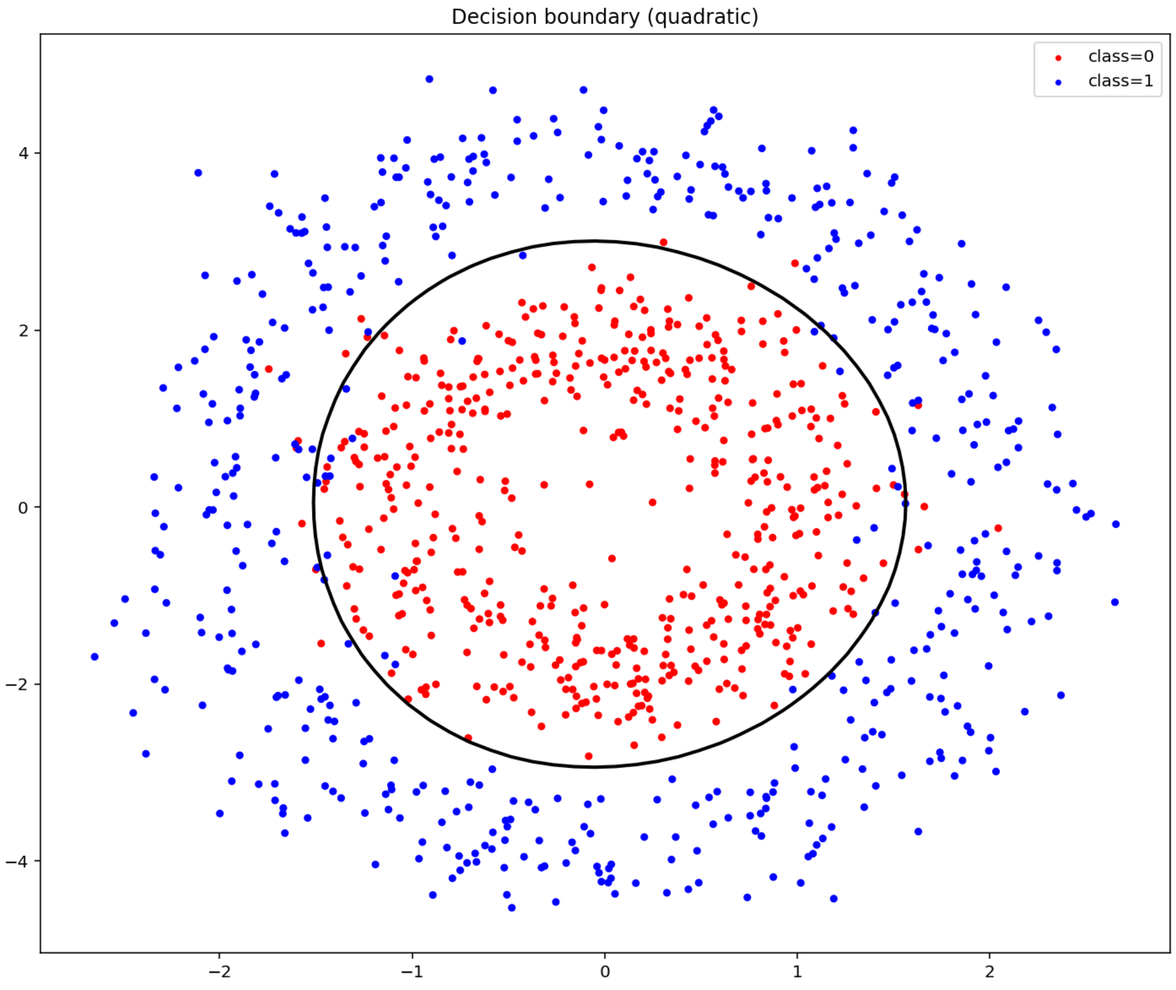
```
# plot
plt.figure(3, figsize=(10,6))
plt.plot(np.array(range(max_iter)), L_iters)
plt.xlabel('Iterations')
plt.ylabel('Loss value')
plt.show()
```



3. Plot the decision boundary of the obtained classifier [2pt]

In [33]:

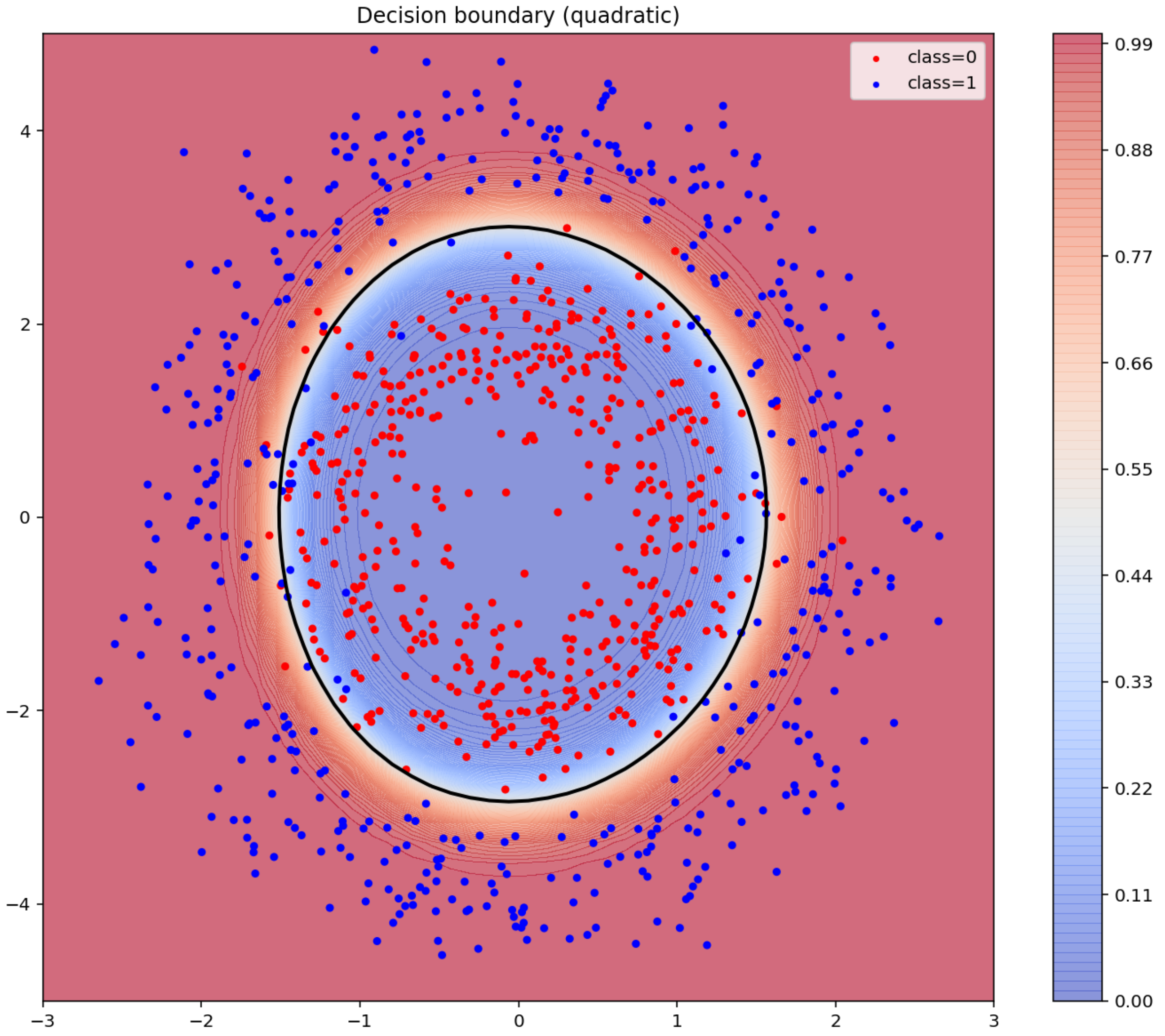
```
# plot
plt.figure(4,figsize=(12,10))

plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, 1, linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (quadratic)')
plt.show()
```



4. Plot the probability map of the obtained classifier [2pt]

In [41]:

```
# plot
plt.figure(4,figsize=(12,10))

ax = plt.contourf(xx1, xx2, p, 100, vmin=0, vmax=1, cmap='coolwarm', alpha=0.6)
cbar = plt.colorbar(ax)
cbar.update_ticks()

plt.scatter(x1, x2, s=idx_class0*50, c='r', marker='.', label='class=0')
plt.scatter(x1, x2, s=idx_class1*50, c='b', marker='.', label='class=1')
plt.contour(xx1, xx2, p, 1, linewidths=2, colors='k')
plt.legend()
plt.title('Decision boundary (quadratic)')
plt.show()
```



5. Compute the classification accuracy [1pt]

In [77]:

```
print('total number of data =', n)
print('total number of correctly classified data = ', correct_data)
print('accuracy(%) = ', correct_data / n * 100)
```

```
total number of data = 1000
total number of correctly classified data =  964
accuracy(%) =  96.39999999999999
```