Genetic Algorithm for Ballistic Missile Trajectory Optimization in a War Zone

AI Lab. Test-3

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1 Problem Statement

A ballistic missile must be launched from a fixed position to destroy an enemy weapon depot while minimizing fuel consumption and avoiding enemy air defenses. The missile must find an optimal trajectory that:

- Minimizes fuel consumption (single objective optimization)
- Avoids enemy radar and anti-aircraft zones (constraints)
- Adheres to physical flight dynamics (constraints)

The battlefield is modeled in 3D space, where the missile must navigate through waypoints while avoiding restricted zones.

2 Mathematical Formulation

2.1 Objective Function

Minimize total fuel consumption:

$$f(\mathbf{x}) = \sum_{i=1}^{N} C_i \cdot d_i + \alpha \cdot \text{Turning\_Cost}$$

where:

- $\mathbf{x}$ = trajectory (sequence of 3D waypoints)
- C_i = fuel cost per unit distance (depends on speed, altitude)
- d_i = distance between waypoints
- α = penalty for sharp turns (to maintain stability)

2.2 Constraints

- Avoid enemy defense zones (radar/surface-to-air missiles):

$$\forall (x,y,z) \in \mathbf{x}, \quad (x,y,z) \notin \text{Enemy\_Zones}$$

- Maximum acceleration limit (G-force constraint):

$$\frac{\Delta v}{\Delta t} \leq a_{\max}$$

- Minimum and maximum altitude:

$$z_{\min} \leq z \leq z_{\max}$$

- Final position must hit target within tolerance:

$$\|\mathbf{x}_{\text{final}} - \mathbf{x}_{\text{depot}}\| \leq \varepsilon$$

3 Genetic Algorithm Parameters & Ranges

Parameter	Description	Range/Values
Population Size	Number of candidate trajectories per generation	100 - 500
Generations	Maximum iterations for evolution	200 - 1000
Crossover Rate	Probability of combining two parent trajectories	0.7 - 0.95
Mutation Rate	Probability of random alterations to waypoints	0.05 - 0.2
Selection Method	Parent selection strategy	Tournament Size = 3 - 10
Encoding	Trajectory representation	3D Waypoints
Fitness Function	Combines fuel cost and penalties	$F = f(\mathbf{x}) + \lambda \cdot \text{Penalties}$
Elitism Rate	Percentage of best solutions preserved	5% - 20%

4 Additional Considerations

- **Fuel Cost Model:**
  - High altitude: Lower drag but higher fuel burn
  - Low altitude: Higher drag but stealthier
- **Enemy Zones:**
  - Spherical or cylindrical no-fly zones
- **Initialization:**
  - Random trajectories biased toward the target

5 Expected Solution

The GA should evolve trajectories that:

- Minimize fuel use
- Avoid enemy defenses
- Respect flight dynamics

The best solution will be the trajectory with the lowest fitness value after convergence.

6 Question for Solution

Design a Genetic Algorithm to optimize the missile trajectory. Discuss:

- Encoding scheme (how to represent trajectories)
- Fitness function (how to balance fuel cost and penalties)
- Selection, crossover, and mutation (what operators work best?)
- Constraint handling (penalty functions, repair methods)
- Termination criteria (when to stop evolution?)