Optimal Strategies in two player influence allocation Ising model Network

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Inspiration drew from unexpected field

Finding parallels between sometimes strikingly dissimilar branches of research can prove revolutionary in explaining real-world phenomena from a new angle.

This notion has been recognised and resulted in many statistical Physics models being used to explain a wide range of social phenomena.

One of these models, **the Ising Model**, originally used to explain ferromagnetism, can be used to model opinion dynamics and social influence.

The Ising Model

Connection between the Ising Model and a simulation of real-life opinion dynamics is maintained under the following set of analogues:

- *Social network of N individuals each holding dynamic binary opinion* → Lattice with N sites, each attributed with spin given by $\sigma_i \in \{+1, -1\}$.
- *Opinion exchange of neighbouring individuals* → Interaction term $J_{i,j}$ between adjacent lattice site pairs, where $J_{i,j}$ represents the amount of influence of agent i on agent j
- *Opinion alignment within closely related agents* → non-negativity of the interaction term $J_{i,j}$
- *Individual's opinion bias due to e.g. influence of political campaign* → external magnetic field, given by h_j , experienced by agent j∈N
- *Overall network's opinion* → lattice average magnetisation given by:

$$M(h) = \sum_i < \sigma_i > \qquad (1)$$

The total energy of this system is captured by the Hamiltonian function given by:

$$H(\sigma) = -\sum_{<i\,j>} J_{ij}\sigma_i\sigma_j - \mu\sum_j h_j\sigma_j \qquad (2)$$

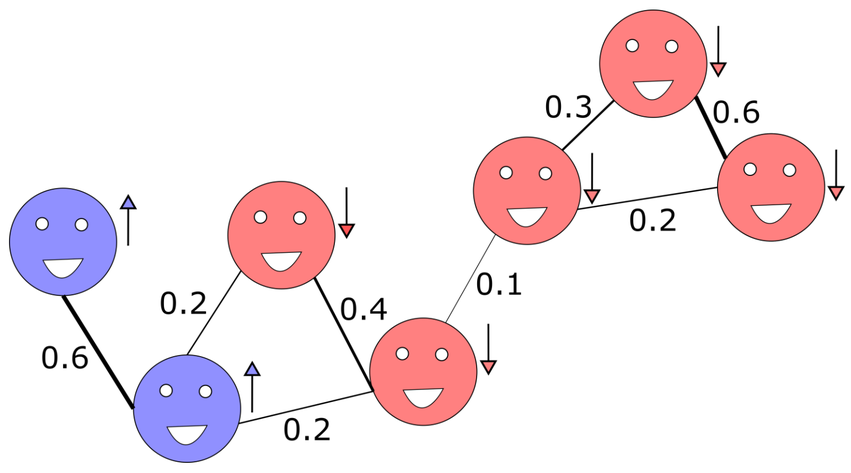

Figure 1) Example of a simple network in some ordered state without the external influence. The thickness of edges connecting nodes corresponds to interaction strengths between each pair. For simplicity of comparison, interaction strength has been also provided as a number.

Objectives of the Project

The project objective is a specific case study of a general problem *of Ising influence maximisation* problem understood as maximising the magnetisation of a network given a budget of external magnetic field.

Just as presented in [1], we will study the Ising Model under the Mean Field (MF) approximation with conditions under which MF magnetisation is concave. Whilst [1] considered a single external agent, our project will consider the setting with **two external agents e.g. two parties' political campaigns** each affecting the network for an opposite total spin alignment.

We will determine optimal strategies of agents by structuring the problem as a simultaneous game with continuous strategy. Using gradient ascent techniques we will find the best response functions for each agent and consequently Nash Equilibria (NE)[2]. We will study how these NE change for different network topologies, model's temperature and agents' budgets.

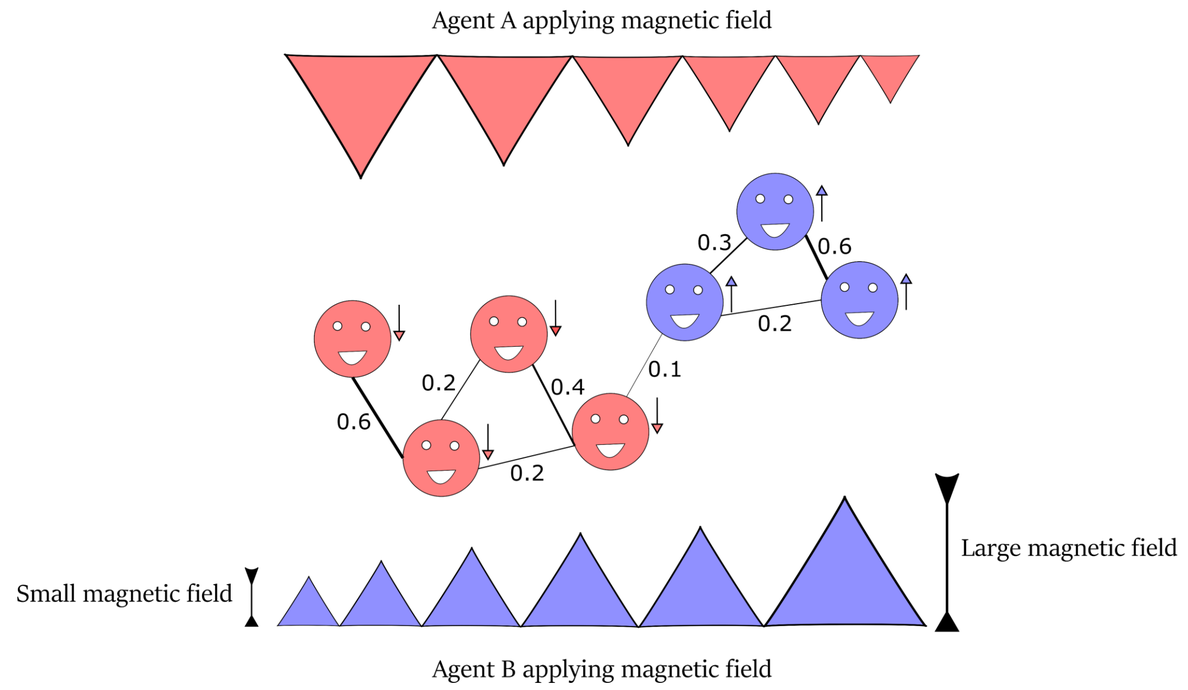

Figure 2) Simple diagram of competitive influence maximisation model. External agents distribute their magnetic field budget throughout the network. Each node's resulting alignment is due to the experienced net external magnetic field and neighbours' influence.

Relevance of the project and its potential impact

The emergence of social media made people's lives intertwined as never before. This resultant complex social network was seen in its early days as a tool to promote social awareness over global political issues.

However, in recent years we have seen that it can be as well exploited to manipulate democratic processes thus posing threat to the democratic rule. This political interference has been seen to be achieved by e.g. the escalated use of Twitterbots voicing political rumours and targeting groups of individuals in the network. The use of bot accounts has been found to be significant in swaying public opinion in events such as The US 2016 presidential elections[3] or the Brexit referendum[4].

The study of opinion spread and the extent of external agents' influence in opinion formation is crucial in understanding political processes in the digital age era. In the future, this understanding may result in the ability to predict or quickly recognise anti-democratic political interferences that may occur.

References:

[1]C. Lynn and D. Lee, "Maximizing Influence in an Ising Network: A Mean-Field Optimal Solution", in 30th Conference on Neural Information Processing Systems, Barcelona, Spain, 2016.
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[3] A. Badawy, E. Ferrara and K. Lerman, "Analyzing the Digital Traces of Political Manipulation: The 2016 Russian Interference Twitter Campaign," in 2018 IEEE/ACM International Conference on Advances in Social Networks Analysis and Mining (ASONAM), Barcelona, Spain, 2018, pp. 258-265
[4] M. T. Bastos and D. Mercea, "The Brexit Botnet and User-Generated Hyperpartisan News," Social Science Computer Review, vol. 37, no. 1, pp. 38–54, 2017.