Constraints on Low Energy Effective Theory: Swampland Problem and Positivity Bounds

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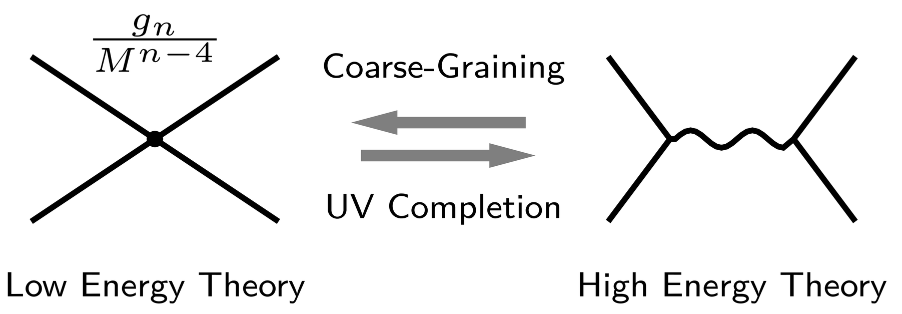
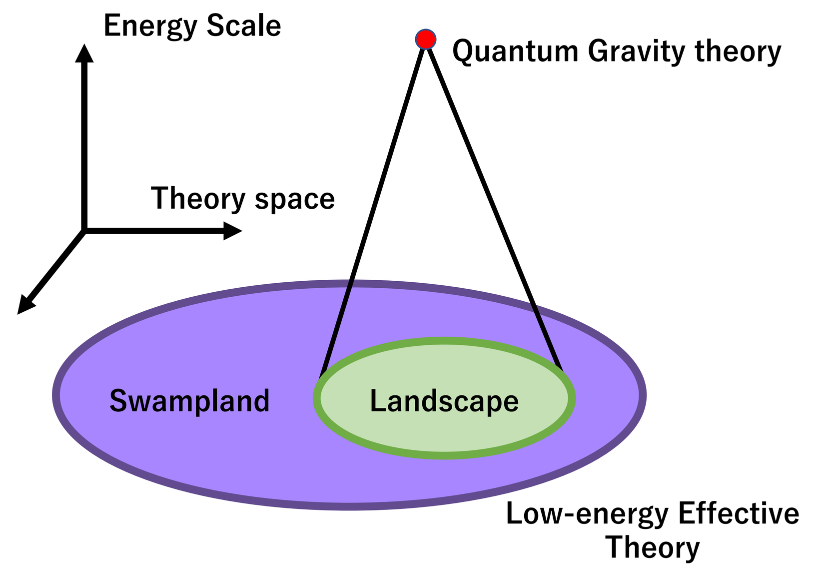
Introduction: Positivity Bounds

Positivity bounds provides the necessary conditions for the theory to be UV-complete as a set of constraints on the coupling constants of **low-energy effective theories (EFT)**.

$$\mathcal{L}_{\text{low}} = \mathcal{L}_{\text{ren}} + \sum_{n=5}^{\infty} \frac{g_n \mathcal{O}_n}{M^{n-4}}$$

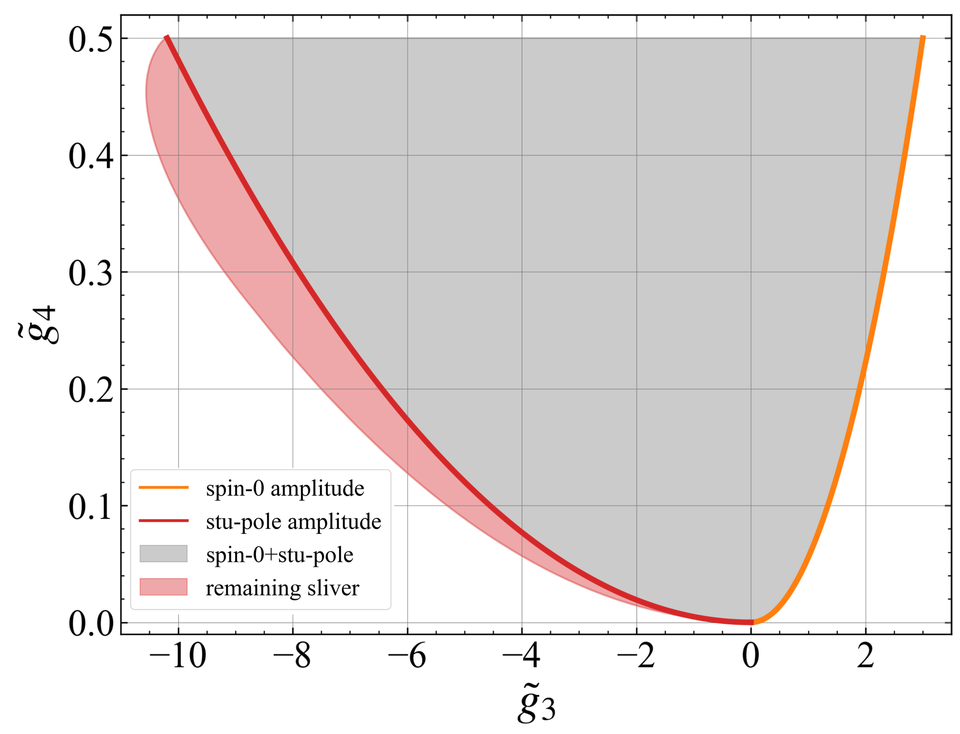




AIM: To investigate whether the **allowable region of effective couplings** derived from the positivity bounds can be entirely filled by analytical examples.

Allowable Region and Remaining Sliver

For most part of the allowed region, corresponding analytic example is already known. However, for regions near the boundary of allowed region, called **remaining sliver,** the corresponding theory has not discovered yet.



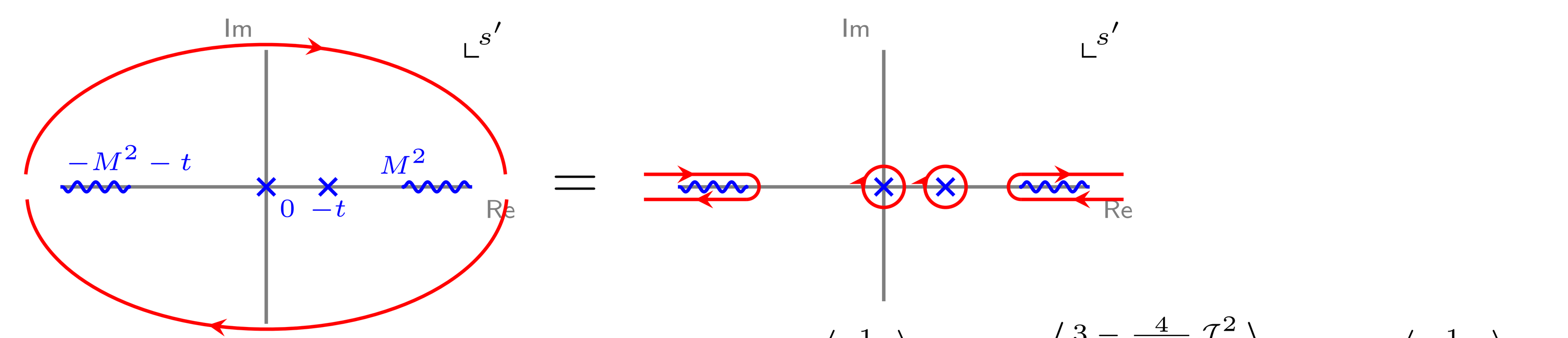
g_3 has negative value in the remaining sliver, which means the corresponding theory should contain many **higher spin particles** on their spectrum.

Setup: Massless Scalar EFT

We consider scattering amplitudes of massless scalar EFT as a starting point:

$$\mathcal{L}_{\text{low}} = -\frac{1}{2}(\partial_\mu \phi)^2 - \frac{g}{3!}\phi^3 - \frac{\lambda}{4!}\phi^4 + \frac{g_2}{2}[(\partial_\mu\phi)^2]^2 + \frac{g_3}{3}(\partial_\mu\partial_\nu\phi)^2(\partial_\sigma\phi)^2 + 4g_4[(\partial_\mu\partial_\nu\phi)^2]^2 + \dots$$

Using unitarity and analyticity of high-energy theory, we can derive **dispersive sum rules** for low-energy theory's higher-order couplings g3 and g4.



$$\mathcal{O}(s^0): \quad g_2 = \left\langle \frac{1}{m^4} \right\rangle, \quad g_3 = \left\langle \frac{3 - \frac{4}{d-2}\mathcal{J}^2}{m^6} \right\rangle, \quad g_4 = \left\langle \frac{1}{2m^8} \right\rangle,$$

$$\mathcal{O}(s^2): \quad g_4 = \left\langle \frac{1 + \frac{4-5d}{2d(d-2)}\mathcal{J}^2 + \frac{1}{d(d-2)}\mathcal{J}^4}{2m^8} \right\rangle, \dots$$

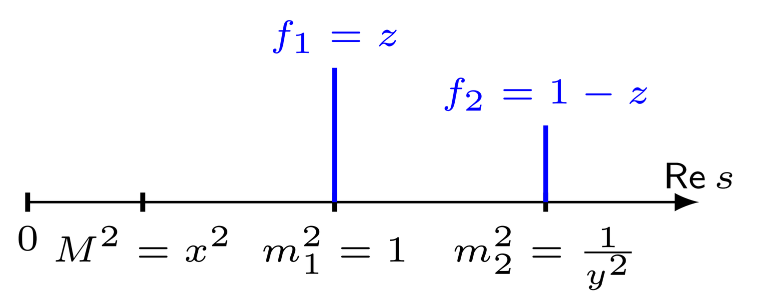
Triple Product Amplitude

A new class of tree-level scattering amplitudes that can describe UV-complete graviton amplitudes has recently been proposed, called **triple product amplitude**.

$$\mathcal{M}^{(2)}_{\text{triple}} = m_1^4 \mathcal{A}^{(2)}(s)\mathcal{A}^{(2)}(t)\mathcal{A}^{(2)}(u),$$

$$\mathcal{A}^{(2)}(s) = \frac{f_1}{-s+m_1^2} + \frac{f_2}{-s+m_2^2}$$

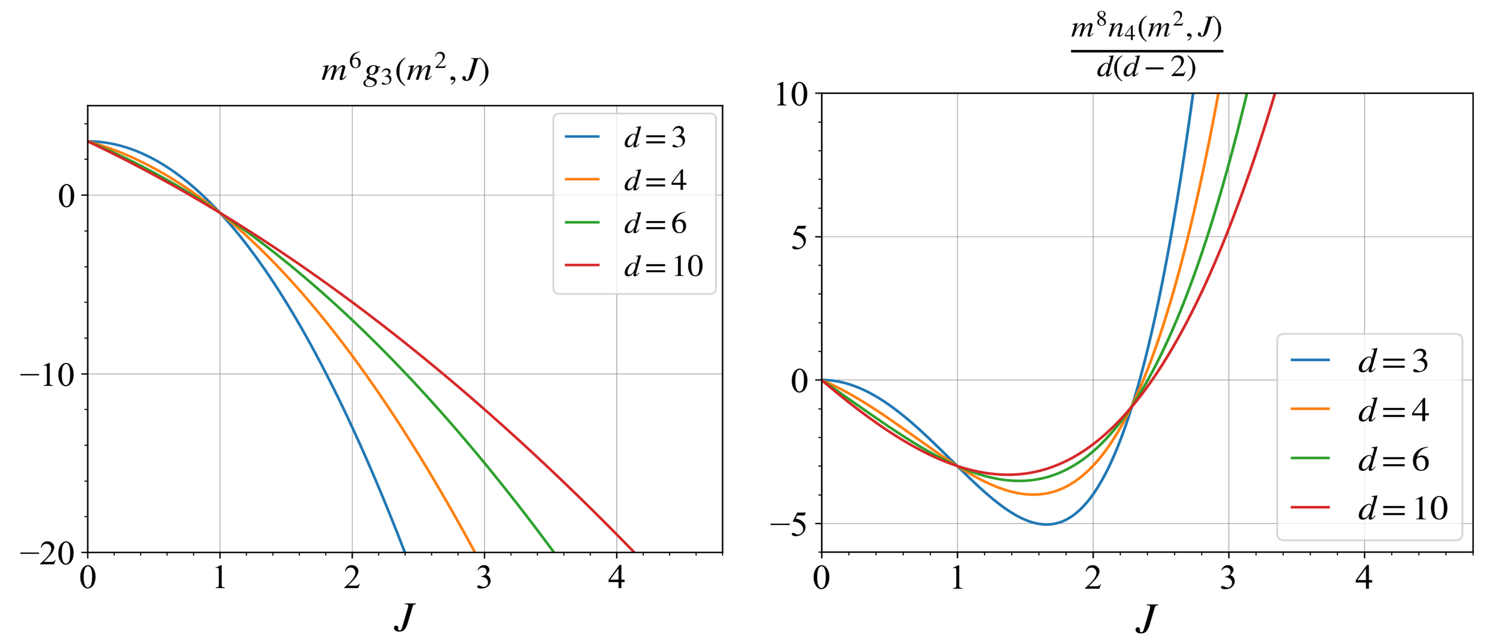
This amplitude contains **infinitely many higher spin states**. For single pole case, this amplitude saturates the known boundary of allowed region. As a next attempt, we investigated triple product amplitude **for double pole case**.



Method: Semidefinite Program

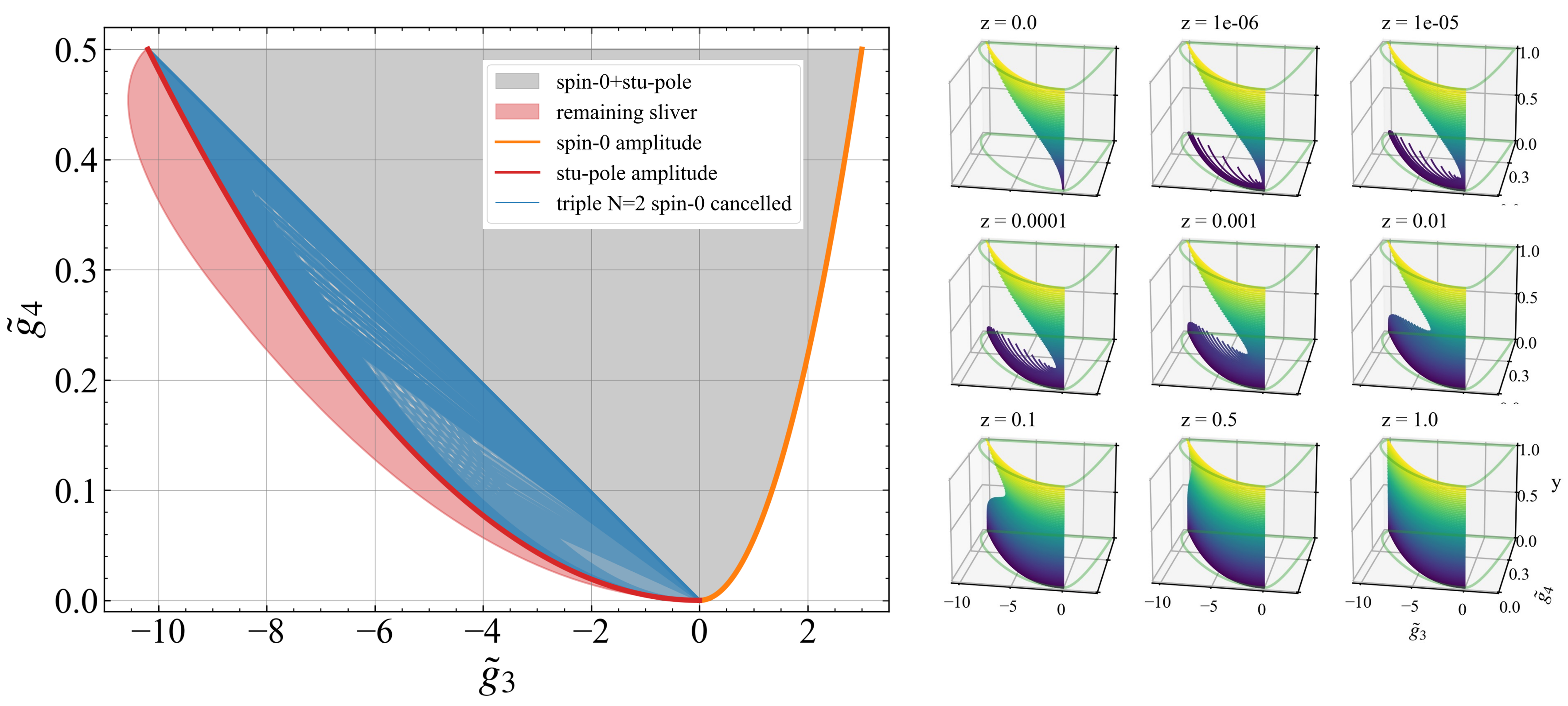
The allowable region of parameter space of couplings from multiple positivity bounds is determined by optimization problem. This can be solved numerically using **semidefinite program**.

$$\begin{cases} \text{maximize} : h := A_1 + A_2 M^2 \tilde{g}_3^{(0)} \\ \text{subject to} : 0 \leq (-A_1, -A_2, 1, c_4, c_5, \dots) \cdot v(m^2, J) \end{cases}$$



Results and Outlook

The results show that the addition of **the second pole increases the value of g_3 in every cases**, so the sufficiency of the positivity bounds is not implied by this specific case.



As a future work, we aim to further narrow the allowable region by obtaining conditions that reflecting more information from unitarity and other conditions.