Kalman Filtering

samedi 19 septembre 2020
17:14

Kalman Filtering (KF)

1) Model

$$x_{k+1} = A x_k + q_k \qquad q_k \sim N(0, Q)$$

$$y_k = C x_k + r_k \qquad r_k \sim N(0, R)$$

A : matrice de transition
C : ——— mesure

2) KF equations

1. INIT $x_0 \sim N(m_0, P_0)$ $\qquad p(x_k) \sim N(m_k, P_k)$

2. PREDICT

$$m_k = A . m_{k-1}$$

$$P_k = A . P_{k-1} A^T + Q \qquad P \nearrow$$

3. UPDATE

$$K_k = P_k C^T (C P_k C^T + R)^{-1}$$

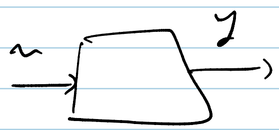
$$P_k = P_k - K_k (C P_k C^T + R) . K_k^T \qquad P \searrow$$

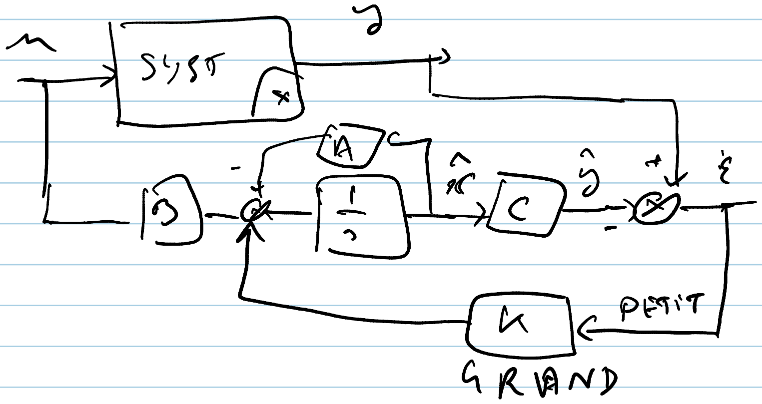
$$m_k = m_k + K_k (\underbrace{y_k - C m_k}_{\text{error innovation}})$$

3) KF as an (optimal) observer

$$x_{k+1} = A x_k + B u_k + q_k$$

$$y_k = C x_k + r_k$$





dynamique de ε

$$\varepsilon = (A - KC) . \varepsilon$$

4) EKF

Model

$$x_k = f(x_{k-1}) + q_{k-1}$$

$$y_k = h(x_k) + r_k$$

$$F = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$H = \frac{\partial h}{\partial x} = [\quad]$$

Equations

INIT $\quad m, P$

PREDICT $\quad m = f(m)$

compute $F = \left(\frac{\partial f}{\partial x}\right)\Big|_m$

$P = F.P.F^T + Q$

UPDATE $\quad$ compute $H = \left(\frac{\partial h}{\partial x}\right)\Big|_m$

$S = H.P H^T + R$

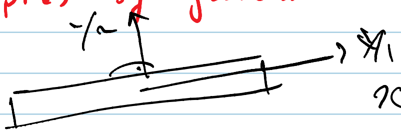
$K = P. H^T S^{-1}$

$P = P - K.S K^T$

$m = m + K(y - h(m))$

Examples of jacobian

ex



$x = \theta$ (angle)

$$\left.\begin{aligned} y_1 &= 9{,}81 . \cos x \\ y_2 &= 9{,}81 \sin x \end{aligned}\right\} \quad y = h(x)$$

$$H = \begin{pmatrix} \partial h_1(x) \\ \partial h_2(x) \end{pmatrix} = \underbrace{\begin{bmatrix} -9{,}81 . \sin x \\ 9{,}81 . \cos x \end{bmatrix}}_{1 \text{ state}} \Bigg\} 2 \text{ meas}$$

5) (E)KF IRL

$$\dot{x} = g(x)$$

5) (E)KF like

$$\dot{x} = g(x)$$

$$x_{k+1} = x_k + \Delta T \cdot g(x_k) \quad \text{EULER 1}$$

$$= f(x_k)$$

Coding

Equations

INIT m, P

PREDICT $m = f(m)$

compute $F = \left(\frac{\partial f}{\partial x}\right)_m$

$P = F \cdot P \cdot F^T + Q$

UPDATE compute $H = \left(\frac{\partial h}{\partial x}\right)_m$

$S = H \cdot P H^T + R$

$K = P \cdot H^T S^{-1}$

$P = P - K \cdot S K^T$

$m = m + K(y - h(m))$

Independent sensors

$$y_1 = h_1(x) + v_1$$

$$y_2 = h_2(x) + v_2$$

UPDATE 1

compute $H_1 = \left(\frac{\partial h_1}{\partial x}\right)_m$

$S_1 = H_1 P H_1^T + R_1$

$K_1 = P \cdot H_1^T S^{-1}$

$P = P - K \cdot S_1 K^T$

$m = m + K_1(y_1 - h_1(m))$

UPDATE 2

compute $H_2 = \left(\frac{\partial h_2}{\partial x}\right)_m$

$S_2 = H_2 P H_2^T + R_2$

$K_2 = P \cdot H_2^T S_2^{-1}$

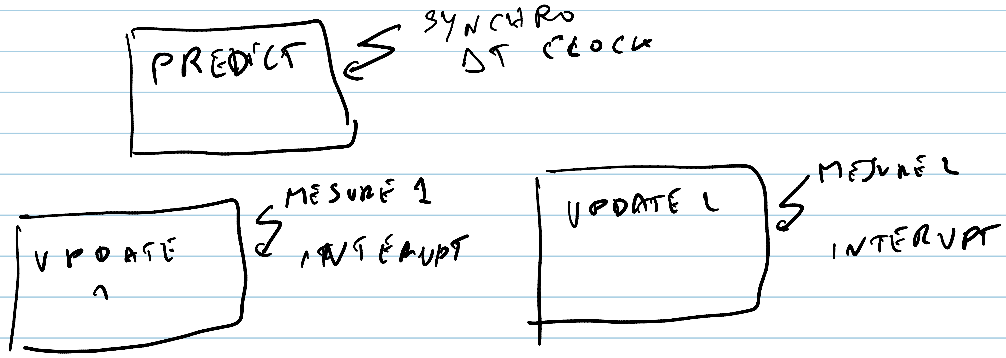
$P = P - K \cdot S_2 K^T$

$m = m + K_2(y_2 - h_2(m))$

Asynchronous reading

SYNCHRO ... CLOCK

(Asynchronous [illegible])



VRAC

* INIT x_0, P_0
  → CARAC
  → RANDOM (ex: 0)

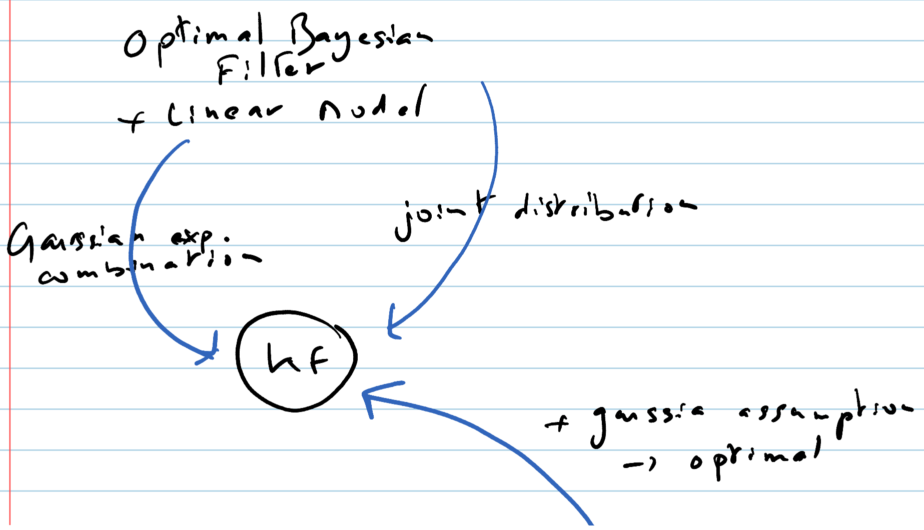
* Asynchronicity
* Prediction $P \nearrow$
  Update $P \searrow P$
* observability
* Algorithmic Issues
* Tuning

$$\cancel{Q}^{\nearrow} \quad \cancel{R}^{\nearrow} \Rightarrow K$$

$$K = \text{place}(A^T, C^T, \text{poles})$$

$$K = \text{kalman}(A, C, \cancel{Q}, \cancel{R})$$

6) Demonstration



+ gaussia assumption
-> optimal

Linear Model + Best Linear Unbiased estimator