BELVEDERE TRADING

PROJECT LAB FINAL PRESENTATION

Implied Volatility Term Structure Study

GENERAL GOAL

- **Identify the drivers and patterns of the change**
- **Use them to predict the SPX implied volatility term structure changes**

Summer

- Data processing
- Dimensional reduction – PCA analysis and feature selection
- Model – Regression model and Time series model
- Comparison

Autumn

- Stochastic model (Double Stochastic Dynamics)
- Define objective function
- Find optimal global initial parameter settings
- Parameters distribution analysis
- Predictor analysis

AGENDA

- Model introduction
- The objective functions
- The global initial/prior parameters setting
- The fitting results
- Result analysis
- Parameter distribution and outlier analysis
- Predictor analysis
- Improvements

MODEL INTRODUCTION

SINGLE STOCHASTIC MODEL

First consider Single Stochastic Dynamics, Heston model:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^s$$

$$dv_t = k(\theta - v_t)dt + \xi \sqrt{v_t} dW_t^v$$

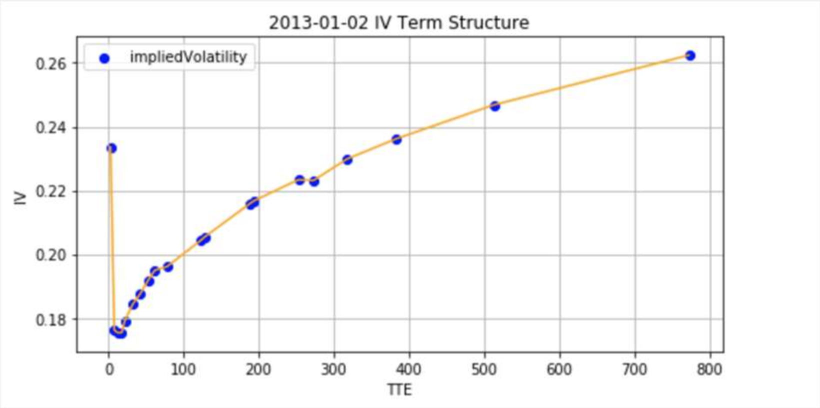
$$E[dW_t^s dW_t^v] = \rho dt$$

where θ is the long term mean, k is the mean-reverting term, ρ is the correlation between 2 brownian motions, one for underlying, one for instantaneous variance. ξ is the volatility of volatility

Assume that the fair value of implied vol should entirely be based on $E[var_t]$. The term structure will be

$$\sigma_t = \sqrt{E[annVar_t]} = \sqrt{\frac{1 - e^{-kt}}{kt}(v_0 - \theta) + \theta}$$

LIMITATION & DOUBLE STOCHASTIC MODEL



Limitation:

- Consider that the single stochastic dynamic can only fit the monotonic nonlinear model
- Assume the model has only one decay factor k

To solve the problem --> Double Stochastic Dynamics (Add another variance term)

The term structure will be

$$\sigma_t = \sqrt{E[annVar_t]} = \sqrt{\frac{1-e^{-k_{short}t}}{k_{short}t}(v_{short}-\theta_{short})+\theta_{short} + \frac{1-e^{-k_{long}t}}{k_{long}t}(v_{long}-\theta_{long})+\theta_{long}}$$

To simplified the model construction part a little, we decided to throw out the θ_{short} term.

DOUBLE STOCHASTIC MODEL

$$\sigma_t = \sqrt{E[annVar_t]} = \sqrt{\frac{1 - kappa_{short}^t}{-\ln(kappa_{short}^t)} v_{short} + \frac{1 - kappa_{long}^t}{-\ln(kappa_{long}^t)} (v_{long} - \theta) + \theta}$$

$$where \quad kappa_{short} = e^{-k_{short}}, \quad kappa_{long} = e^{-k_{long}}$$

Parameter introduction:

v_{short}: weak-persisting variance injection

v_{long}: strong-persisting instantaneous variance

θ_{long}: long term mean variance

v_{long}- θ_{long}: strong-persisting variance injection

$kappa_{short}$: weak persistence factor

$kappa_{short}$: strong persistence factor

THE OBJECTIVE FUNCTION

OBJECTIVE FUNCTION

- **Loss Function (weighted sum of squared vol and dollar error plus L2)**

$$Loss = \sum_{t=1}^{T}((1 - weight) * (\sigma_t - \widehat{\sigma_t})^2 + weight * \left((\sigma_t - \widehat{\sigma_t}) * \sqrt{t}\right)^2)$$

$$+ \lambda_{v_{short}} * \left(v_{short} - v_{short,prior}\right)^2 + \lambda_{v_{long}} * \left(v_{long} - v_{long,prior}\right)^2 + \lambda_{\theta} * \left(\theta - \theta_{prior}\right)^2$$

FITTING METHODS

Optimizer without Constraints

- Scipy.optimize.minimize(method = 'Nelder-Mead')
- Scipy.optimize.minimize(method = 'BFGS')

Optimizer with Constraints

- Scipy.optimize.minimize(method = 'SLSQP')

Optimizer with Iterative Random Jumps

- Scipy.optimize.basinhopping(method = arbitrary, w/o constraints)

PRIOR SETTING AND INITIAL VALUES

BOUNDARIES AND CONSTRAINTS

How to come up with reasonable boundaries

- Fitting without boundaries
- Intuitive adjustment
- Iteratively adjusting boundaries after examining time series plots of estimates
- Estimating from data, like θ is related to long term vol
- Convert ks, $\kappa appa_{short} = e^{-k_{short}}, \kappa appa_{long} = e^{-k_{long}}$
- $\kappa appa_{short} \in [e^{-5}, 1], \kappa appa_{long} \in [e^{-0.2}, 1]$
- $v_{short} \in [-0.5, 0.5], v_{long} \in [0, 0.2], \theta \in [0, 0.1]$

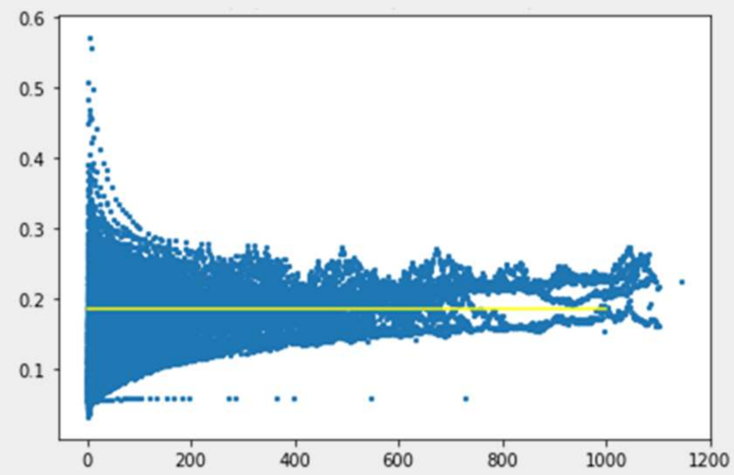
How to come up with reasonable constraints

- $v_{combined} = v_{short} + v_{long} > 0$
- $\kappa appa_{short} < \kappa appa_{long}$

PRIOR SETTING

How to find the prior setting of estimates

- Basinhopping
- Global fit of all sample data
- Set $v_{long} = \theta, v_{short} = 0$
- $\kappa appa_{short} = 0.0067, \kappa appa_{long} = 0.9347$
- $v_{long} = \theta = 0.0340$



INITIAL VALUES

Fixed initial values

- Set initial values = global fit
- Fix the initial values for each day
- Pros:
  - Easy to implement
  - Estimates are around global fit
- Cons:
  - Unable to capture continuous information from market

Combined initial values

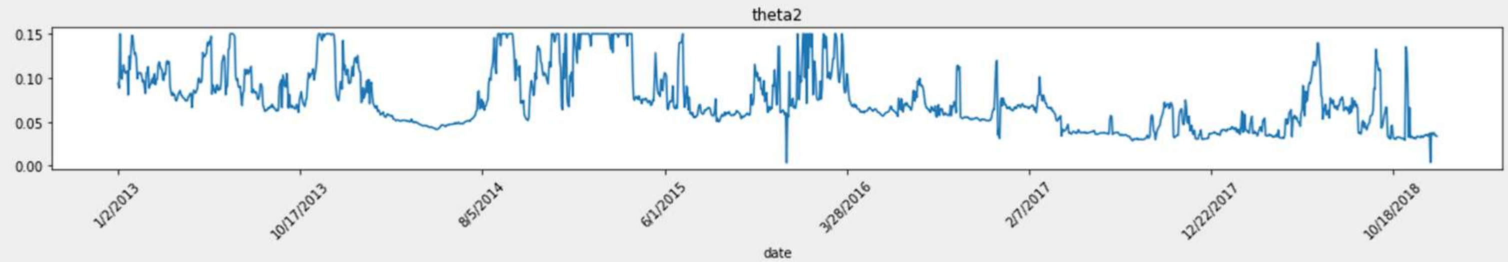
- Set initial values = global fit, generate two sets
- We compare using two sets, one is the global fit and the other is updated initials
- Optimizing rule: whichever minimizes MSE is the optimized estimates on each day
- Updating rule: last day's optimized estimates is the updated initial values set for today
- Pros:
  - Align with the goal of minimize loss function
  - Able to capture continuous information from market
- Cons:
  - Estimates may have jumps, but understandable

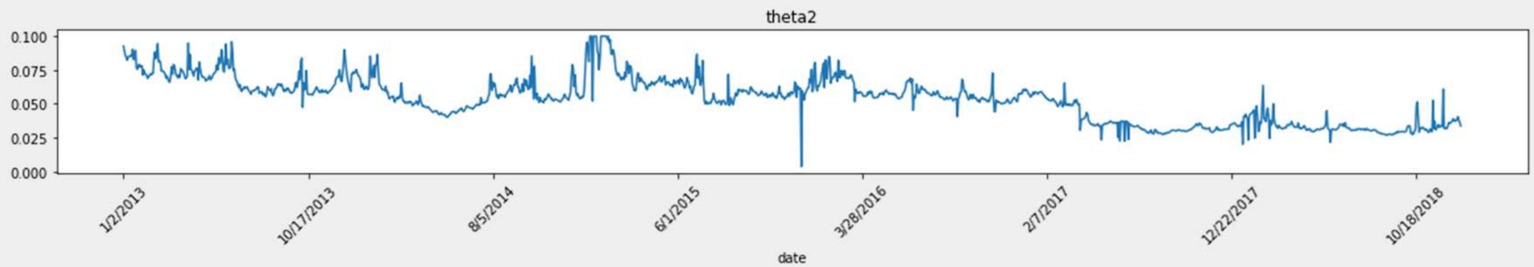
REGULARIZATION

Ridge

- Pulling estimates to global fit and stopping estimates going out of reasonable bounds
- Tradeoff: best shape of vol term structure fitting vs. smooth time series model estimates

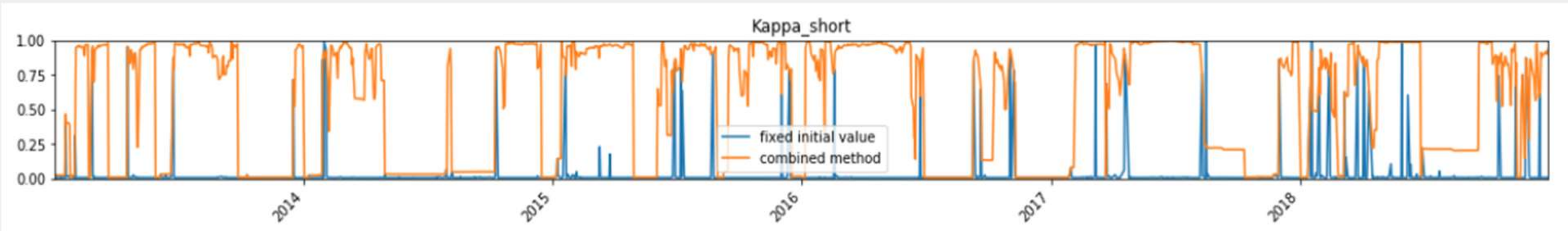
Effects

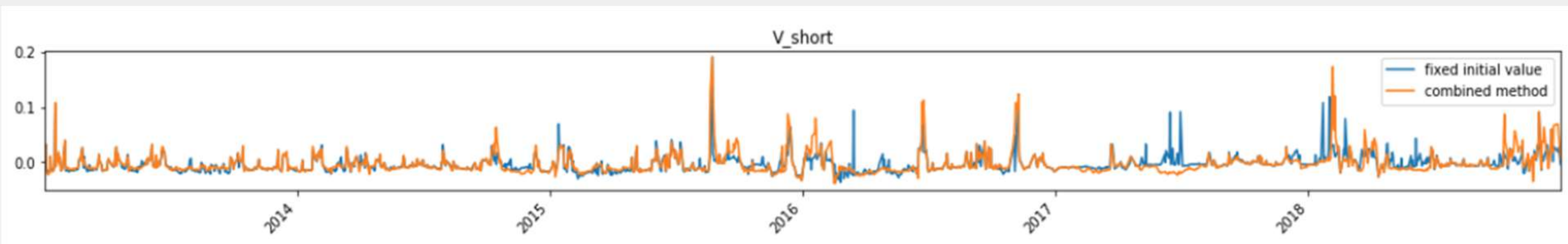


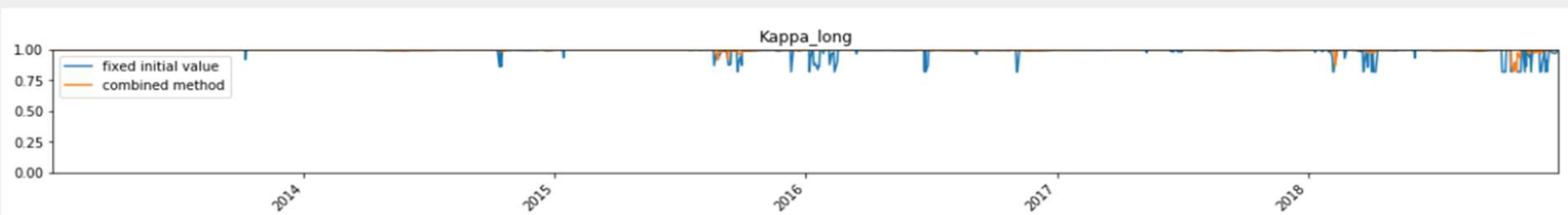


FITTING RESULTS

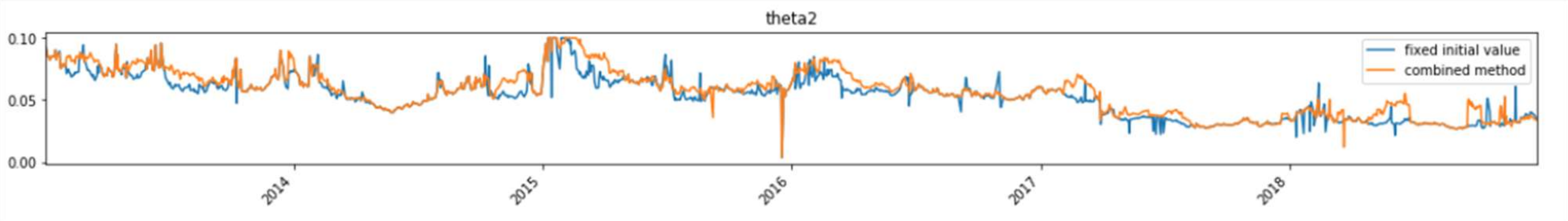
PARAMETER TIME SERIES

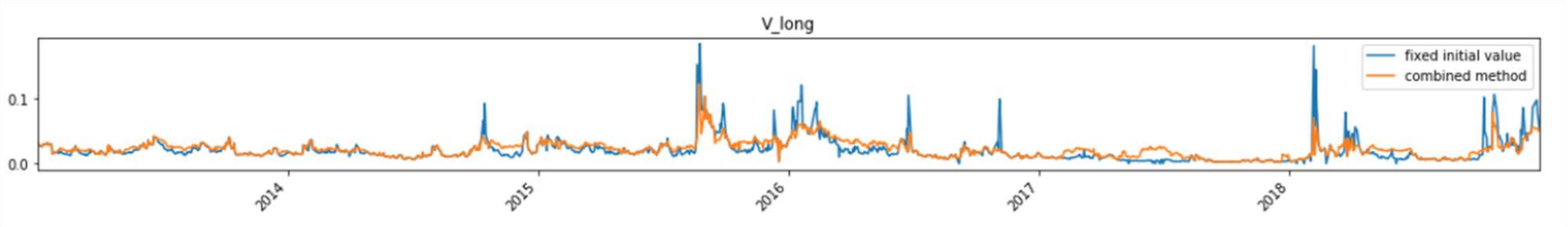


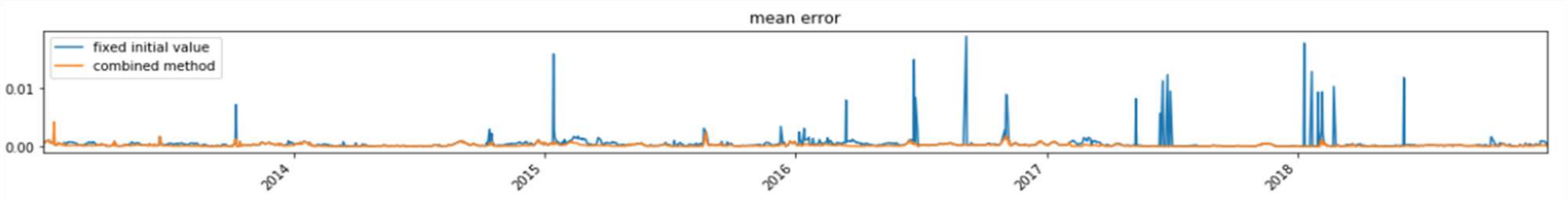




PARAMETER TIME SERIES (CONT'D)

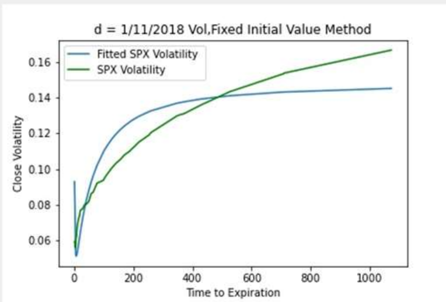


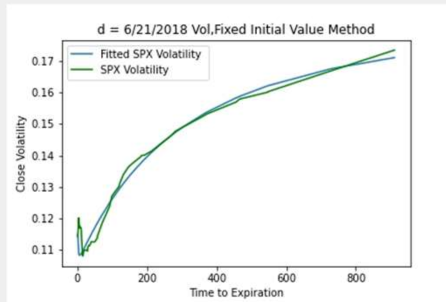


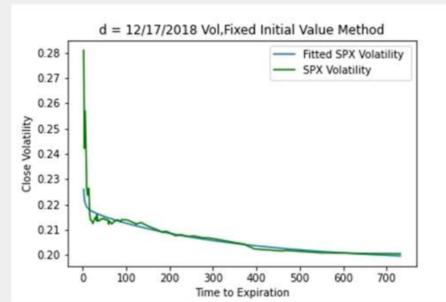


SAMPLE VOL CURVE

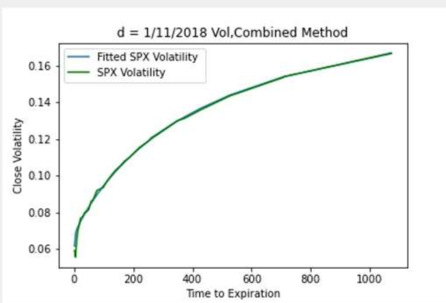
FIXED INITIAL VALUE METHOD

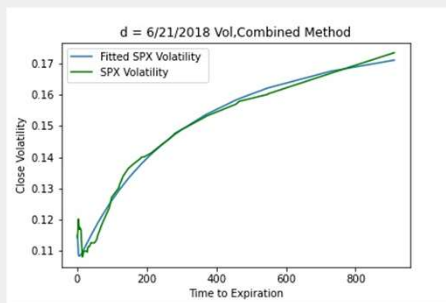


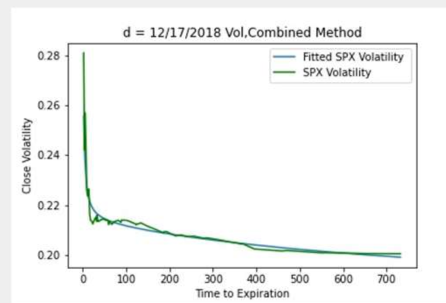




COMBINED METHOD



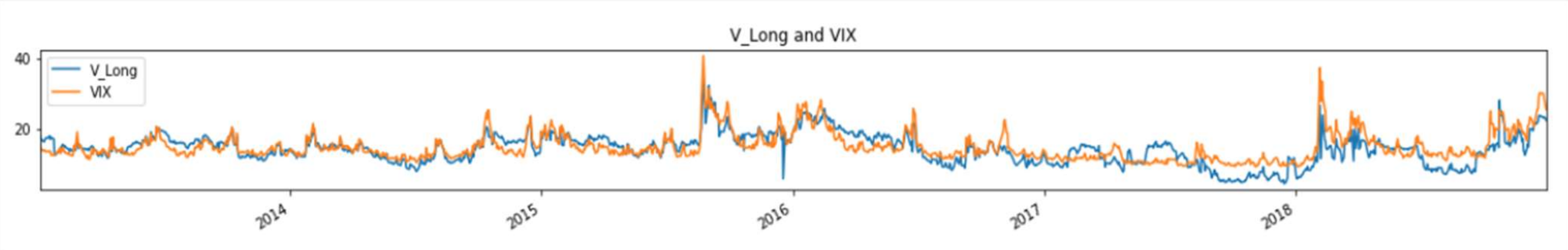


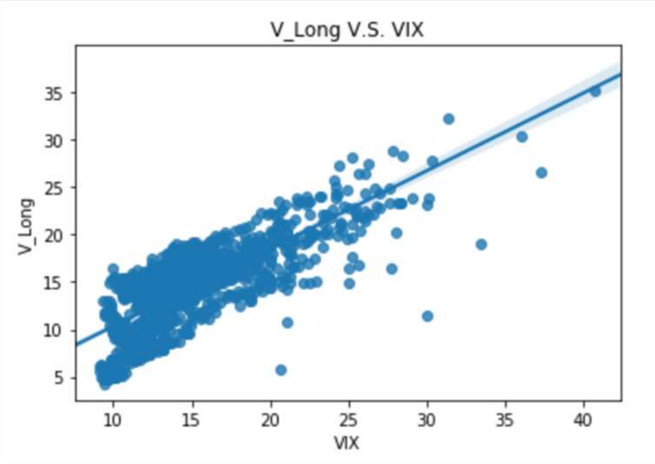


- The combined initial value method can capture the change of vol curves in different shapes, and the expiration-day effect better.

RESULT ANALYSIS

V_LONG V.S. VIX



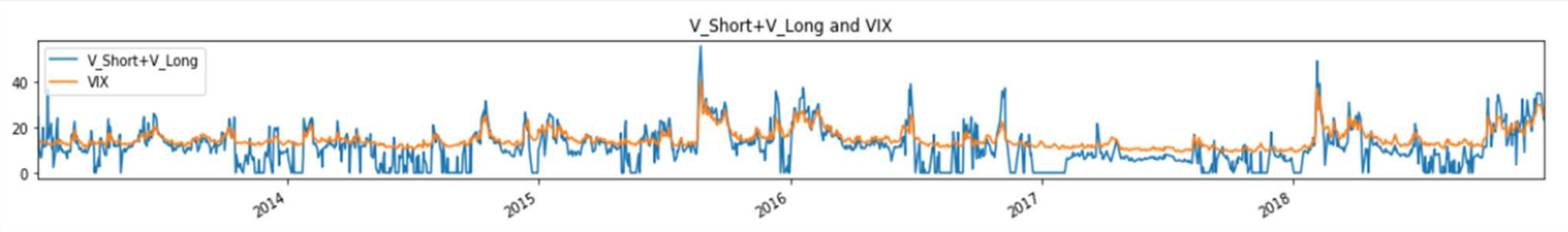


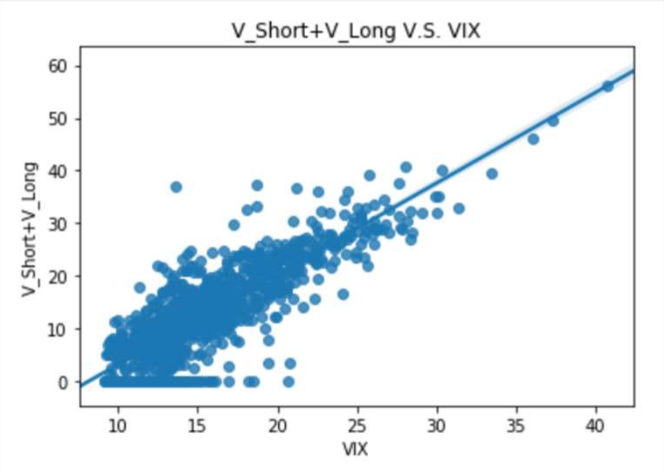
- The correlation between V_Long and VIX is 0.8106.
- The slow-reverting instantaneous vol can capture most of the VIX change.

* V_Long is transformed to vol points term (on the same basis as VIX).

RESULT ANALYSIS (CONT'D)

V_COMBINED V.S. VIX



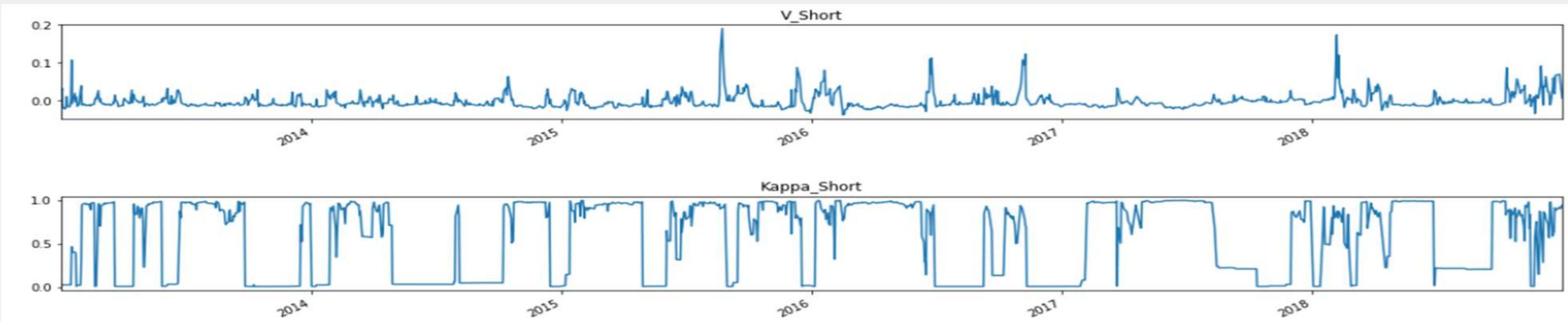


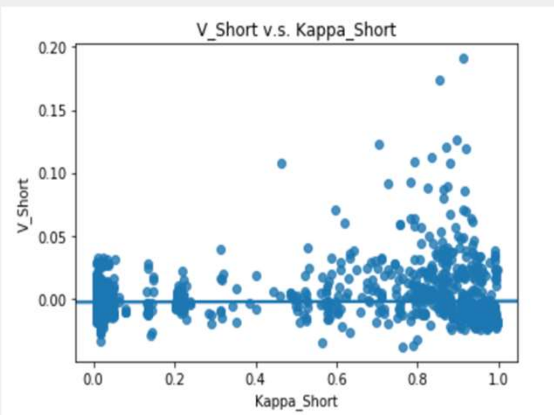
- Higher Correlation: The correlation between V_Combined and VIX is 0.8543.
- The total instantaneous vol sums up the long-term and short-term VIX change.
- Potential Problem:
- Set Theta_Short to be zero
- Scaling over time

* V_Combined (V_Long + V_Short) is transformed to vol points term.

RESULT ANALYSIS (CONT'D)

V_SHORT V.S. KAPPA_SHORT

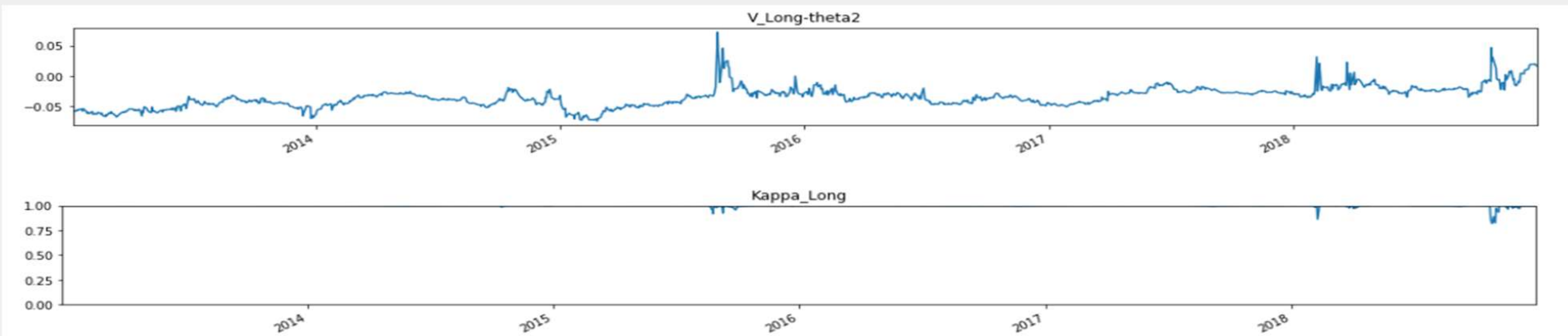


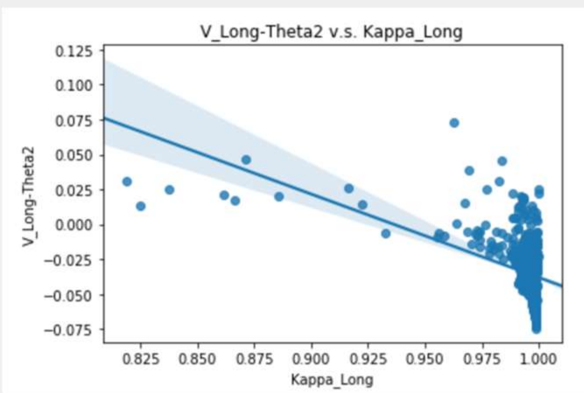


- The correlation between v_Short and Kappa_Short is 0.0157.
- The weak persistence factor and the weak-persisting variance injection have very low correlation in our model.
- The short-term change has more noise.

RESULT ANALYSIS (CONT'D)

V_LONG – THETA V.S. KAPPA_LONG



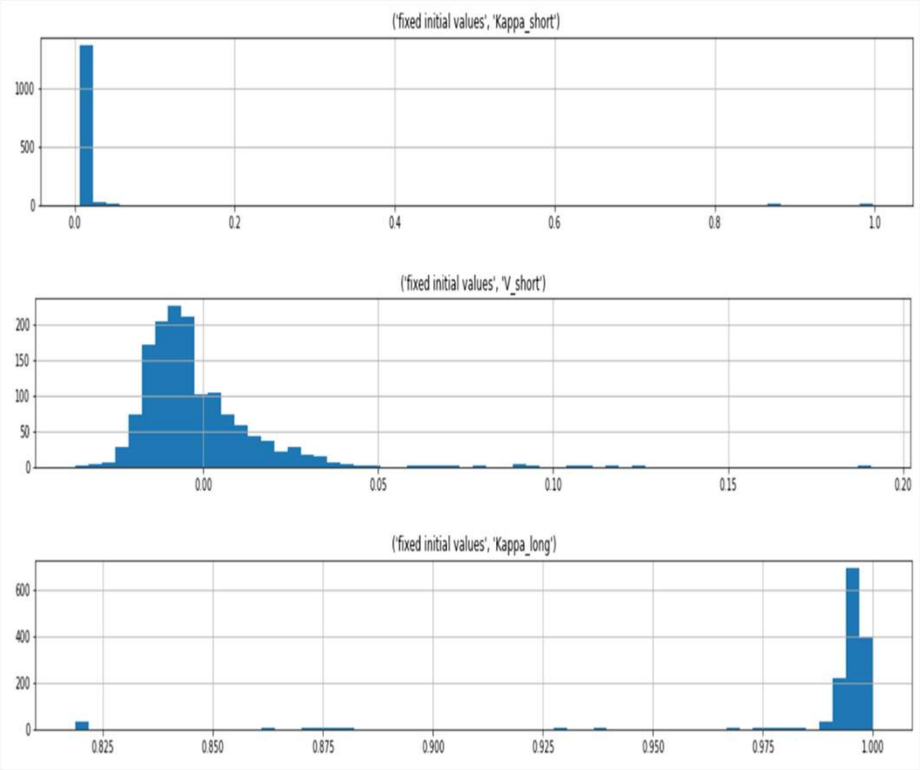


- The correlation between V_Long-Theta2 and Kappa_Long is -0.4098..
- The decay factors are concentrated to the level of 1, showing the constant volatility change pattern.
- Potential Problem: The constraint of Kappa

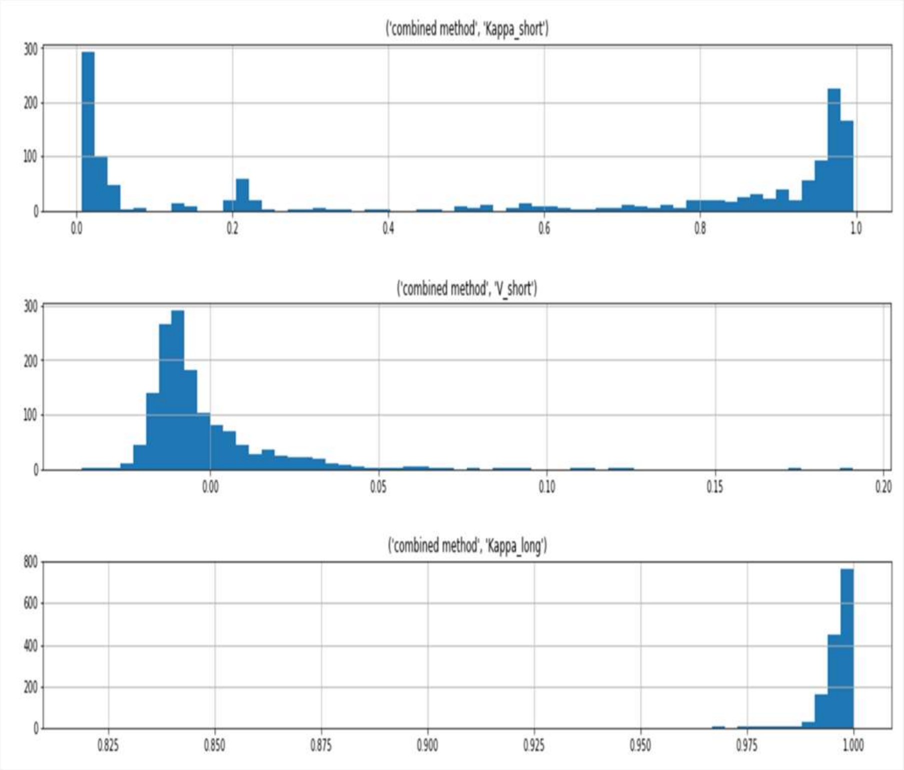
DISTRIBUTION ANALYSIS

PARAMETER DISTRIBUTION

FIXED INITIAL VALUE METHOD

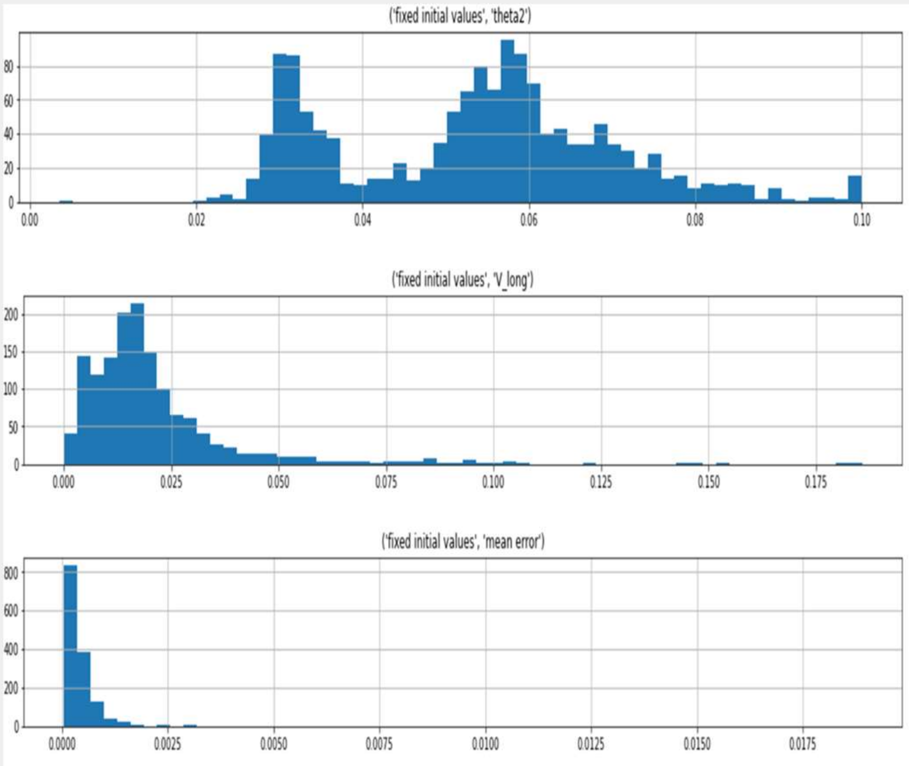


COMBINED INITIAL VALUE METHOD

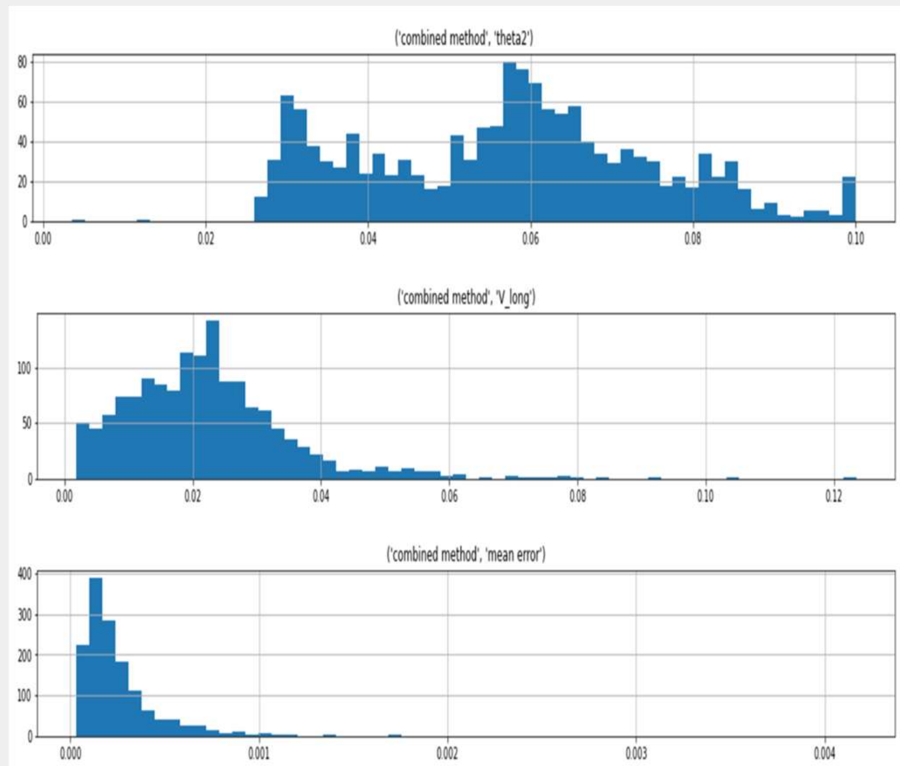


PARAMETER DISTRIBUTION (CONT'D)

FIXED INITIAL VALUE METHOD



COMBINED INITIAL VALUE METHOD



OUTLIER ANALYSIS

OUTLIER STATISTICS

Parameters	Number of Outliers (Fixed Initial Value)	Number of Outliers (Combined Initial Value)
Kappa_Short	55	0
V_Short	17	29
Kappa_Long	57	13
theta2	1	1
V_Long	39	17
Total	139	51

OUTLIER ANALYSIS - REASONS AND HOW TO IMPROVE

- Days Accounting
    - We use weekdays to calculate days to expiration. In reality, there might be bunch of holidays which we haven't accounted for.
    - Some firms in the industry also count partial days for weekend, because non-market events can still happen and will cause some jump variance to Monday open. We can, for example, count weekend as half day. This way, if you are trading a Monday expiration option on Friday open, you will regard this option as a 2.5 day option, not 2 day option.
    - Some expirations are AM and some are PM, which isn't shown in our data. Inaccurate days accounting can definitely cause data points on vol term structure to misplace, therefore throwing off our fitted parameters.

OUTLIER ANALYSIS - REASONS AND HOW TO IMPROVE

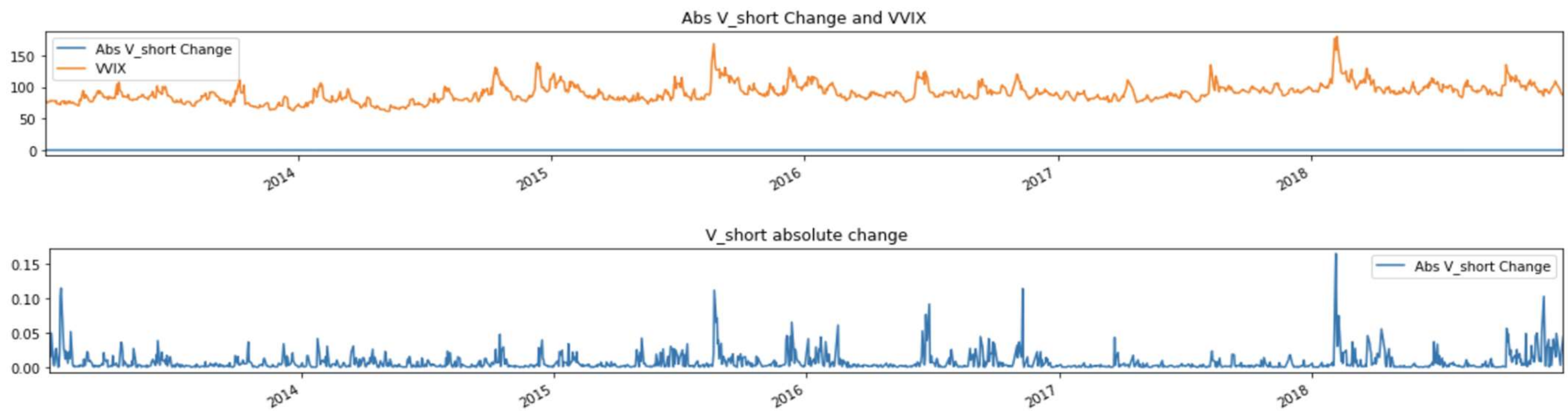
- Event Variance
  - Certain event are already put on calendar like FOMC meeting, Brexit, ect. They will injection some variance in the middle of a vol term structure.
  - Our model’s source of variance injection is only at the front.
  - In practice, if we have confident estimate of those event variances, we can simply take them out, and look at event-free vol term structure, which presents not theoretical problem for using our model.
- Maybe 2 sources of stochastic vol aren’t enough. But adding more will definitely complicate our model.

CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

V_short, V_long, V_Combined and VVIX, SPX Return, SPX Return^2

Absolute V_short Change and VVIX where V_short is weak-persisting variance injection

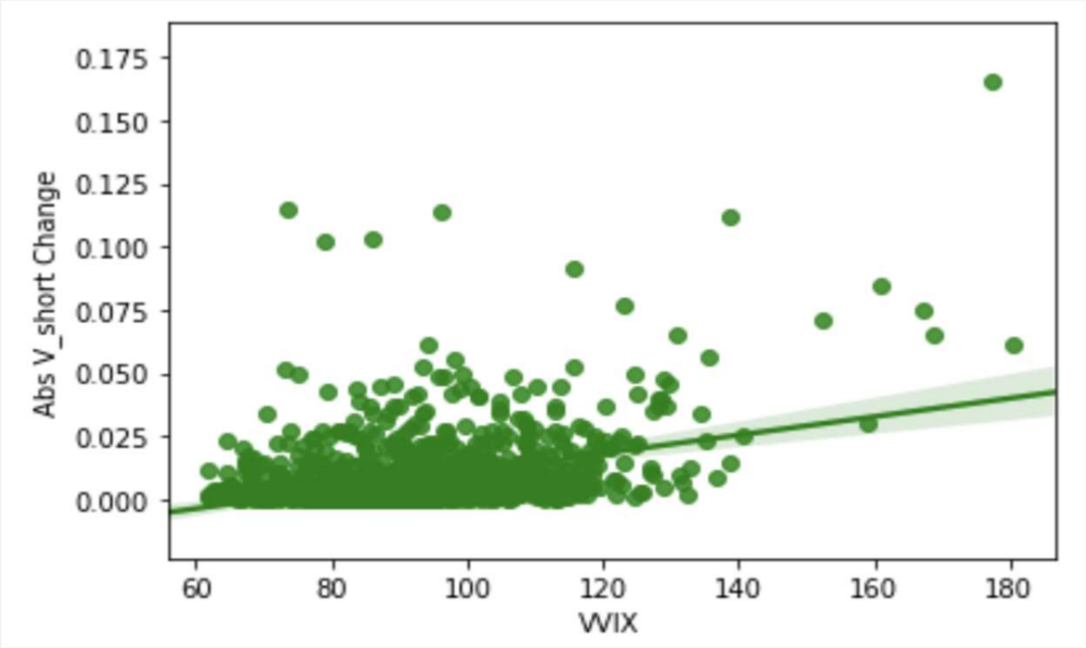
	Abs V_short Change	VVIX
Abs V_short Change	1.00000	0.40184
VVIX	0.40184	1.00000



CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

V_short, V_long, V_Combined and VVIX, SPX Return, SPX Return^2

- **Absolute V_short Change and VVIX where V_short is weak-persisting variance injection**

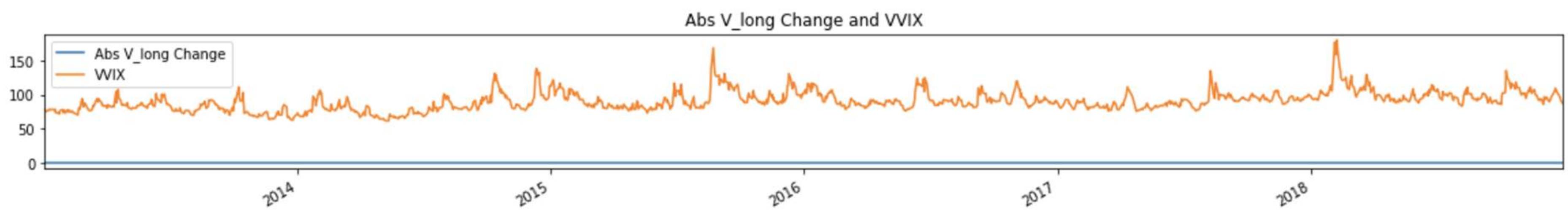


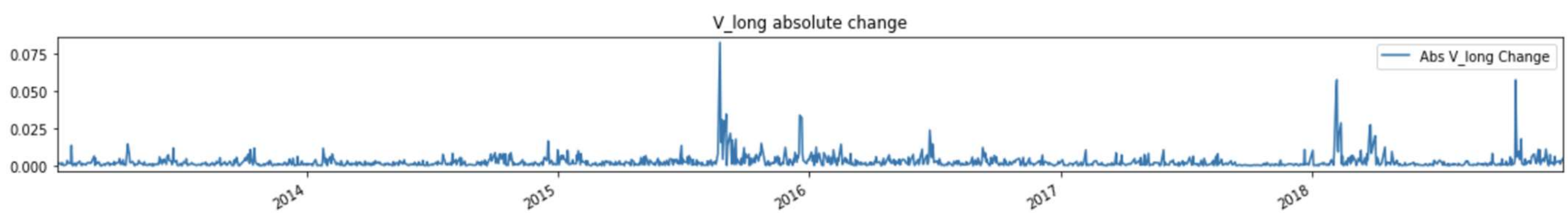
CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

V_short, V_long, V_Combined and VVIX, SPX Return, SPX_Return^2

Absolute V_long Change and VVIX where V_long is strong-persisting instantaneous variance

	Abs V_long Change	VVIX
Abs V_long Change	1.000000	0.471702
VVIX	0.471702	1.000000

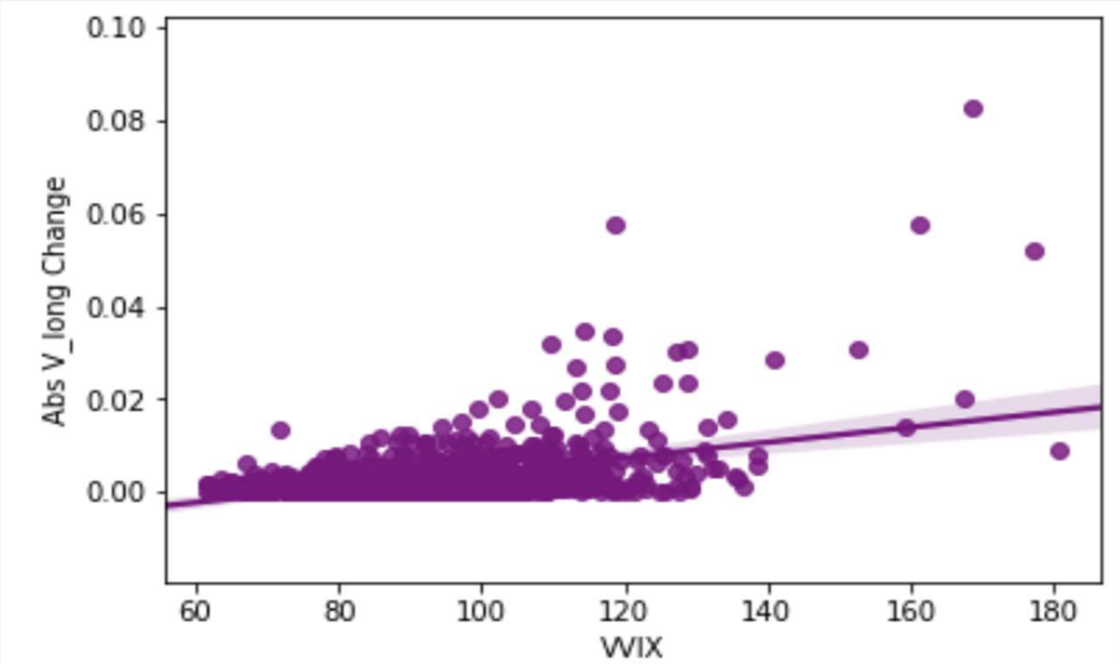




CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

V_short, V_long, V_Combined and VVIX, SPX Return, SPX_Return^2

Absolute V_long Change and VVIX where V_long is strong-persisting instantaneous variance

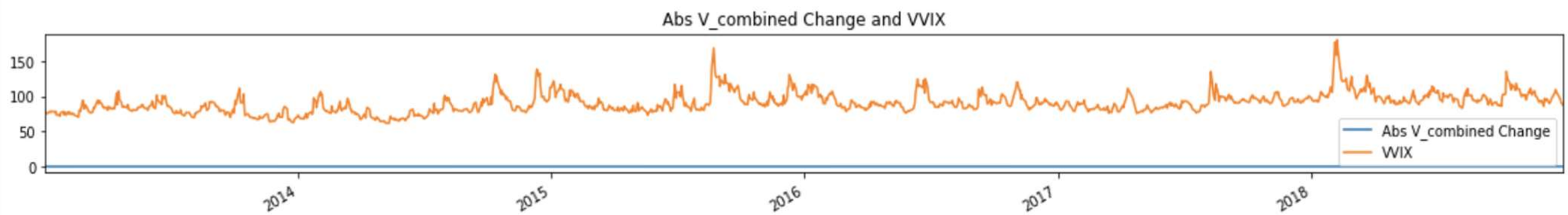


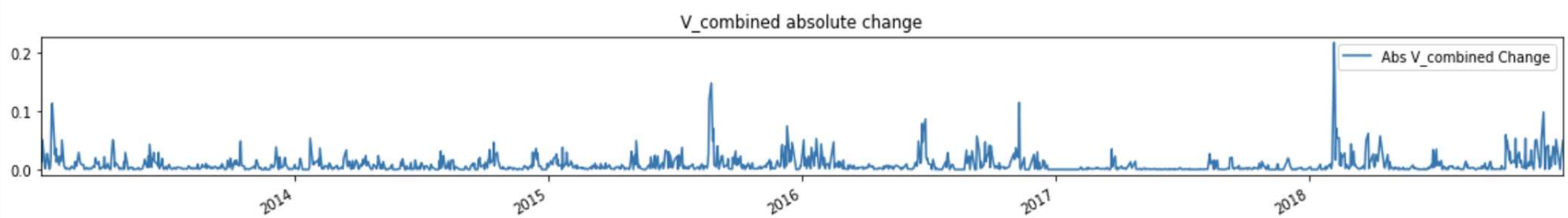
CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

V_short, V_long, V_Combined and VVIX, SPX Return, SPX_Return^2

Absolute V_combined Change and VVIX where V_combined = V_short+V_long

	Abs V_combined Change	VVIX
Abs V_combined Change	1.000000	0.444437
VVIX	0.444437	1.000000

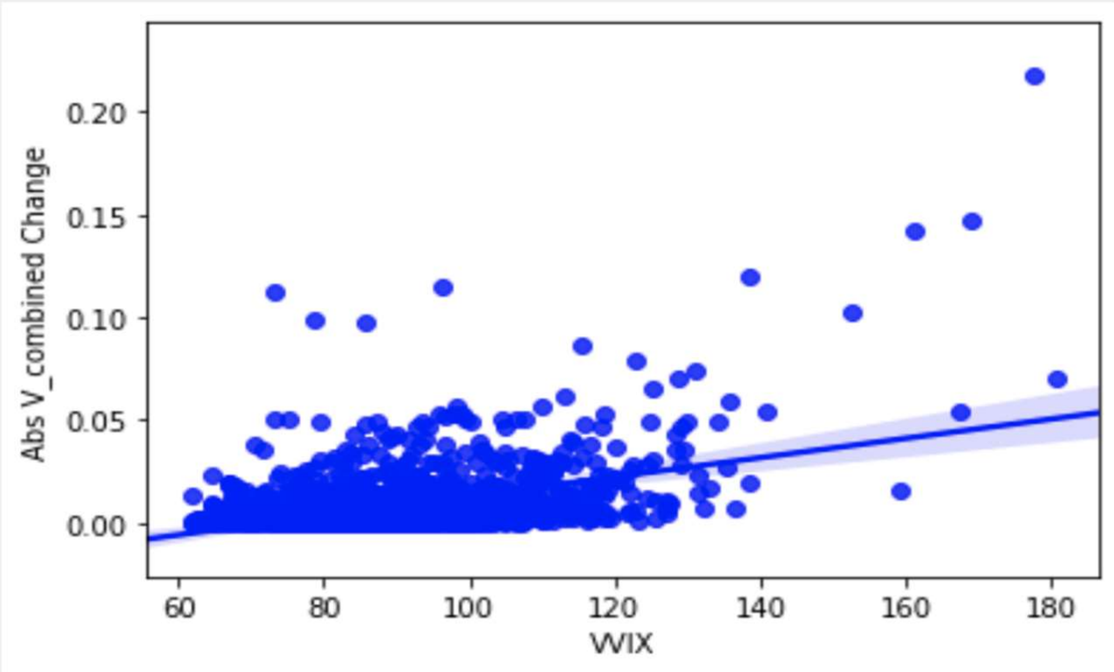




CORRELATIONS: PREDICTORS AND CHANGE IN V VALUES:

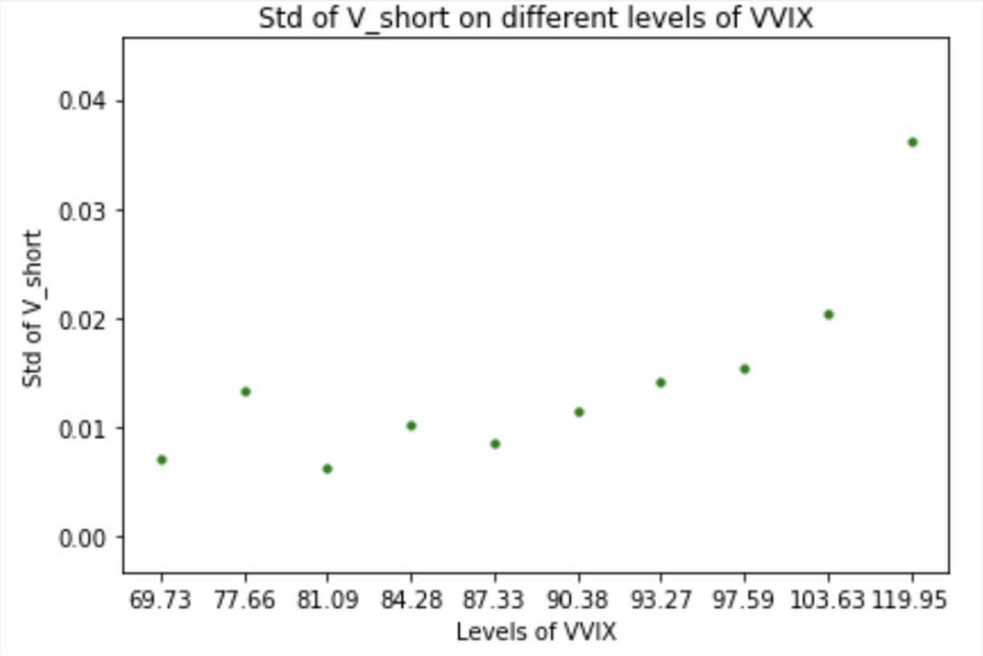
V_short, V_long, V_Combined and VVIX, SPX Return, SPX_Return^2

- **Absolute V_combined Change and VVIX where V_combined = V_short+V_long**



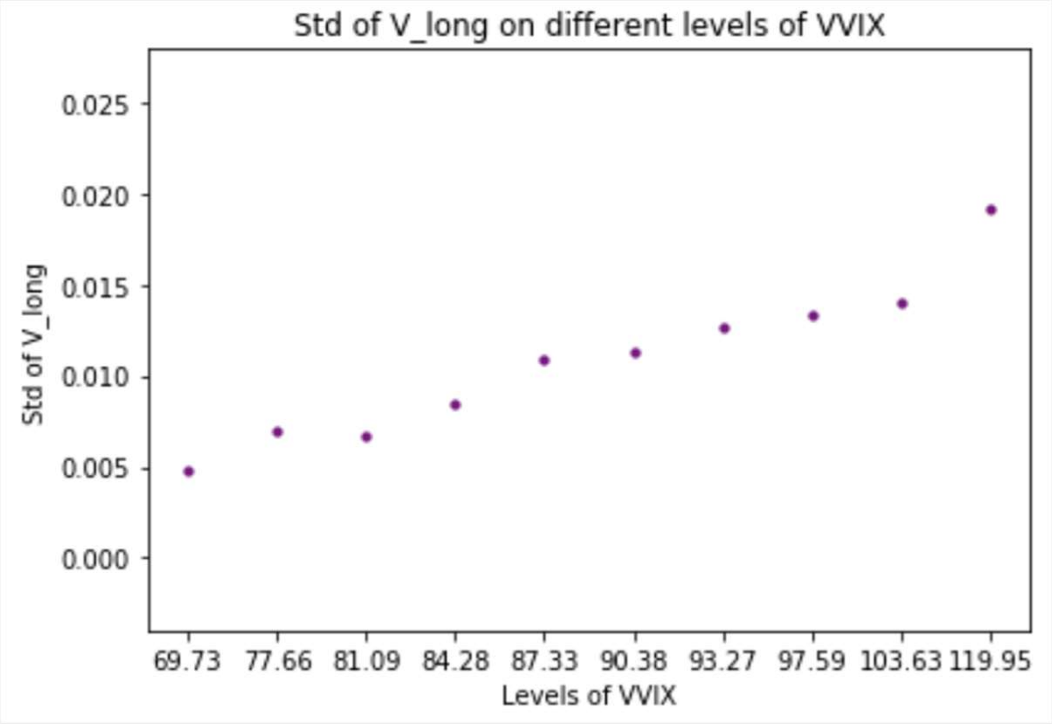
CORRELATIONS: V_SHORT, V_LONG,VIX, VVIX

- **(STD of V_short) and VVIX in 10 percentiles: [0.1,0.2,0.3,...,1.0]**



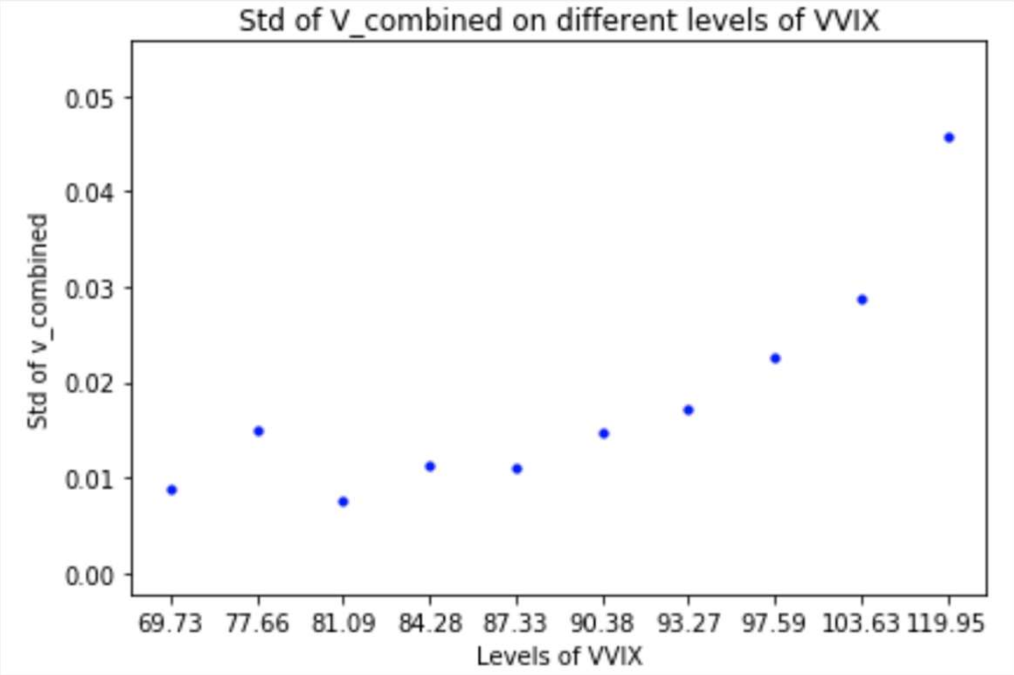
Correlations: V_short, V_long,VIX, VVIX

- **(STD of V_long) and VVIX in 10 percentiles: [0.1,0.2,0.3,...,1.0]**



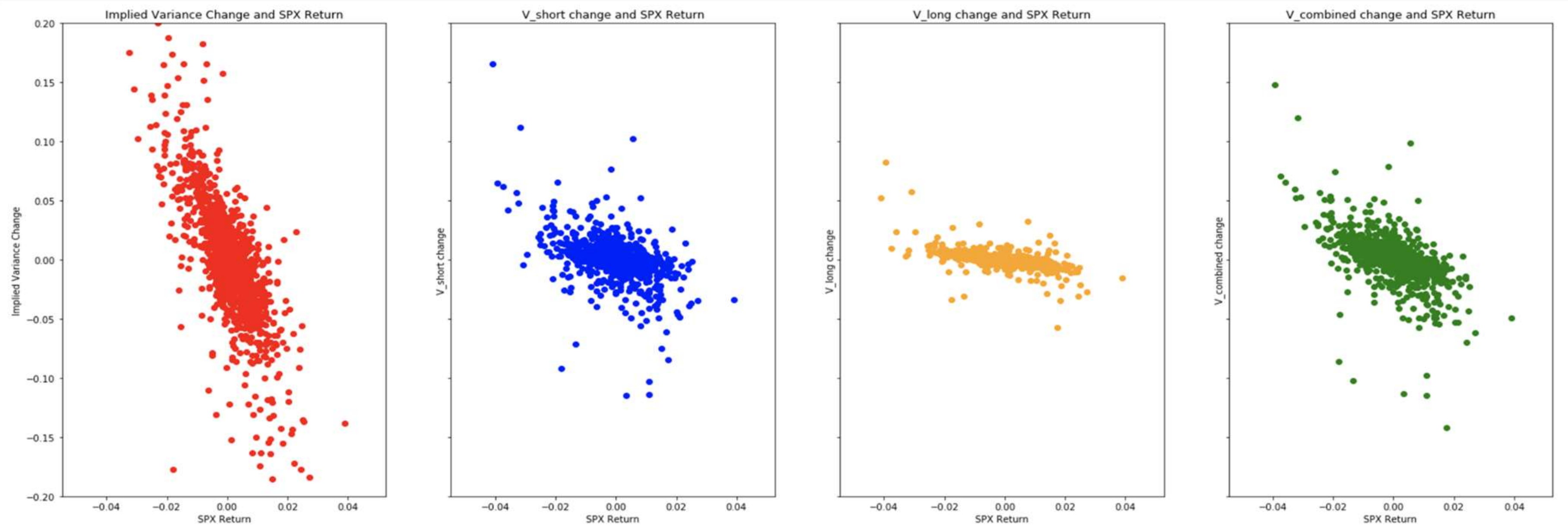
CORRELATIONS: V_SHORT, V_LONG,VIX, VVIX

- (STD of V_combined) and VVIX in 10 percentiles: [0.1,0.2,0.3,...,1.0]



CORRELATIONS: V_SHORT, V_LONG, V_O, SPX, IMPLIED_VOL

Implied Variance Change, V_short Change, V_long Change, V_combined Change and SPX Return
Note: implied variance is the interpolated from volatility curve using at 5-day TTE




```
Implied Variance Change    -0.716274
V_short_combined           -0.449977
V_long_combined            -0.566964
V_combined                 -0.568996
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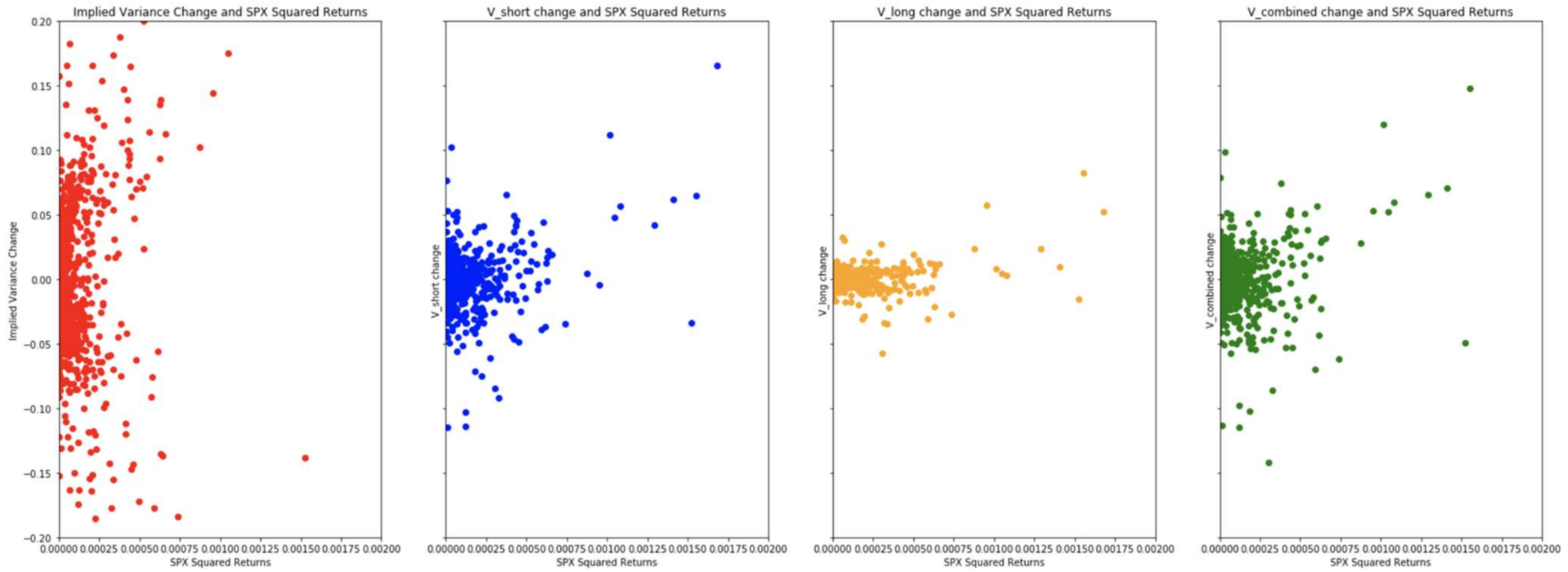
CORRELATIONS: V_SHORT, V_LONG, V_O, SPX, IMPLIED_VOL

Implied Variance Change, V_short Change, V_long Change, V_combined Change and SPX Return^2



Implied Variance Change	0.267779
V_short_combined	0.206502
V_long_combined	0.239377
V_combined	0.254418

SUMMARY AND FUTURE IMPROVEMENTS: SUMMARY

- **Double Stochastic Model replaced PCA and studied the dynamics of parameters**
- **Optimization:**
  - **Objective Function: mix_sum_of_squares, mix_sum_of_squares_ridge**
  - **Boundary & Constraints: Ks(mean_reverting term), Vs(instantaneous variance term), Theta (long-term mean variance term)**
  - **Prior Values: Basinhopping method to find prior values**
  - **Optimizer: run optimizer year by year**
  - **Estimated Parameters: the dynamics of parameters combining initial values and yesterday's fit**
  - **Outlier Analysis: check if the fit is good - check how many trading dates where outliers were shown**
- **Predictors & parameters**

SUMMARY AND IMPROVEMENTS: IMPROVEMENTS

- **Construct regression model based on relationships in predictors and parameters for predictions**
- **Out-of-sample backtesting to compare the performance of predictions**
- **Adjust constraints for Basinhopping to get better estimations**
- **Consider vol_curve_change = d(vol_curve)/d(v_short)(K_short) * [beta(v_short~spxret) +beta(v_short~spxret^2)+beta(v_short~dt)]+d(vol_curve0/d(v_long)(K_long).... And possibly other factors if we could find reliable predictors such as d(vol_curve)/d(K_long)*beta(K_long~predictor)**

ANY QUESTIONS?

THANK YOU