

# Calculus

112550015 資工-劉家祺

Sec 15.3

50.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dA$$

$$= \lim_{a \rightarrow \infty} \iint_{D_a} e^{-(x^2+y^2)} dA$$

$$= \lim_{a \rightarrow \infty} \int_0^{2\pi} \int_0^a e^{-r^2} \cdot r dr d\theta$$

$$= \lim_{a \rightarrow \infty} \int_0^{2\pi} \int_0^a \frac{r}{e^{r^2}} dr d\theta$$

$$= \lim_{a \rightarrow \infty} \int_0^{2\pi} d\theta \cdot \frac{1}{2} \int_0^{a^2} e^u du$$

$$= \lim_{a \rightarrow \infty} \pi \cdot -1 \left[ e^{-a^2} - 1 \right]$$

$$= \lim_{a \rightarrow \infty} \pi \left[ 1 - \frac{1}{e^{a^2}} \right]$$

$$= \pi$$