Name: _____ **ID:** _____

Homework Problems #4 (30’**)**

The calculation process should be provided.

1. (3’) Find a closed form for the generating function for the sequence $\{a_n\}$, where
   a) $a_n = 3^n$ for all $n = 0, 1, 2, \ldots$.
   b) $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$.
   c) $a_n = \binom{8}{n}$ for all $n = 0, 1, 2, \ldots$.

2. (1’) Find the coefficient of x^{12} in the power series of $\frac{x^3}{(1+4x)^2}$.

3. (2’) Show that the coefficient $p(n)$ of x^n in the formal power series expansion of $\frac{1}{(1-x)(1-x^2)(1-x^3)\cdots}$ equals the number of partitions of n.

4. (2’) Find a formula for the probability of the union of n events in a sample space.

5. (2’) Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if
   a) the sets are pairwise disjoint.
   b) there are 50 common elements in each pair of sets and no elements in all three sets.

6. (4’) Determine whether the relation R on the set of all Web pages is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
   a) everyone who has visited Web page a has also visited Web page b.
   b) there are no common links found on both Web page a and Web page b.
   c) there is at least one common link on Web page a and Web page b.
   d) there is a Web page that includes links to both Web page a and Web page b.

7. (2’) Let R be the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of positive integers. Find:
   a) R^{-1}.
   b) $\bar{R}$.

8. (3’) Let R be the relation represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Find the matrix representing:
   a) R^{-1}.
   b) R^2.
   c) $\bar{R}$.

9. (2’) Let R be the relation on the set of ordered pairs of positive integers such that $\big((a, b), (c, d)\big) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation.

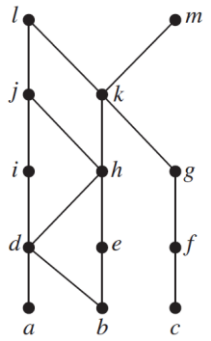
10. (2’) Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if

This homework assignment is due on E3 by **Sunday, 02 June 2024** at 23:59.

a) a is taller than b?
b) $a = b$ or a is an ancestor of b?

11. (2') Which of these are posets? Why?
    a) (Z,≠).
    b) (Z,≥).
    c) (R,≠).

12. (2') What are the maximal, minimal, greatest and least elements, according to the partial order represented by this Hasse diagram.



13. (3') In how many ways can 25 identical donuts be distributed to four police officers so that each officer gets at least three but no more than seven donuts?

This homework assignment is due on E3 by **Sunday, 02 June 2024** at 23:59.