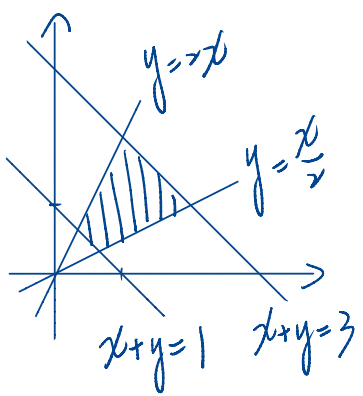
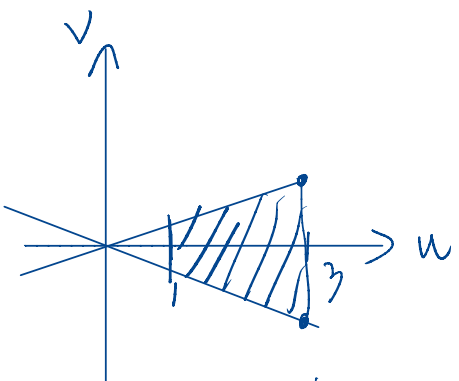
Calculus 112550015 資工一 劉家琪

Sec 15.9

30. $\iint_R \frac{y}{x}\,dA$ let $u = x+y$ $v = x-y$

$x = \frac{1}{2}(u+v)$ $y = \frac{1}{2}(u-v)$





$y = 2x \Rightarrow (u-v) = 2(u+v) \Rightarrow u = -3v$ $v = -\frac{1}{3}u$

$y = \frac{x}{2} \Rightarrow 2(u-v) = u+v \Rightarrow u = 3v$ $v = \frac{1}{3}u$

$$\iint_R \frac{y}{x}\,dA = \int_1^3 \int_{-\frac{1}{3}u}^{\frac{1}{3}u} \frac{u-v}{u+v} \cdot \frac{1}{2}\,dv\,du$$

$$= \int_1^3 \int_{\frac{1}{3}u}^{\frac{u}{3}} \frac{-w+2u}{w} \cdot \frac{1}{2}\,dw\,du = \frac{1}{2}\int_1^3 -w + 2u \cdot \ln w \Big|_{\frac{1}{3}u}^{\frac{u}{3}}\,du$$

$$= \int_1^3 -\frac{u}{3} + u\ln 2\,du = \left(-\frac{1}{3} + \ln 2\right)\cdot\frac{8}{2} = -\frac{4}{3} + 4\ln 2 \quad \#$$