

Problem 1. (18 points total). This problem has three parts. Consider the following augmented matrix

$$A = \begin{bmatrix} \underline{1} & \overset{t}{1} & 0 & \overset{s}{0} & 0 & | & 1 \\ 0 & 0 & \underline{1} & 2 & 0 & | & 2 \\ 0 & 0 & 0 & 0 & \underline{1} & | & 3 \end{bmatrix}$$

- (i) Is the matrix A above in reduced row echelon form? Why or why not?

Yes, it is.

- Because:
- ① Each pivot is to the right of the pivot above its row
 - ② The all zero rows are on the last row of the matrix
 - ③ In all the pivots position, there is a leading one
 - ④ On each pivot column, the pivot position has the only non zero one.
 - ⑤ Before all the pivots are zero.

- (ii) Write down the linear system of equations in the variables x_1, \dots, x_5 for which A is the associated augmented matrix.

$$\therefore A = \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array} \right)$$

$$\therefore \begin{cases} x_1 + x_2 = 1 \\ x_3 + 2x_4 = 2 \\ x_5 = 3 \end{cases}$$



(iii) Which of the following are solutions to the system of equations for which A is the associated augmented matrix. **Circle your answer(s).** There may be more than one correct answer, and options are listed in the form (a, b, c, d, e) which means $x_1 = a, x_2 = b, x_3 = c, x_4 = d, x_5 = e$. **Each incorrect circled answer will result in deduction of 2 points for this problem.**

☒ $(1, 0, 2, 0, 3)$

☐ $(1, 1, 2, 0, 3)$

☒ $(0, 1, 2, 0, 3)$

☐ $(1, 2, 3)$

☐ $(3, 0, 2, 0, 1)$

☐ $(1, 1, 2, 1, 3)$

☒ $(1, 0, 0, 1, 3)$

☒ $(1, 0, 4, -1, 3)$

☐ $(1, 4, -1, 3)$

☒ $(0, 1, 0, 1, 3)$

☐ $(5, 4, 3, 2, 1)$

☐ None of the above



Problem 2. (17 points total). Suppose A is a 5×4 matrix with 3 pivot positions.

(i) (2 points). How many solutions are there to $Ax = 0$? Justify your answer.

$$\therefore Ax = 0$$

\therefore it is consistent

$\therefore A$ has 3 pivot positions

$\therefore A$ has 1 free variables

$\therefore Ax = 0$ has infinitely many solutions

(ii) (5 points). Does $Ax = b$ always have a solution? Justify your answer.

No.

When $b=0$, it is always consistent, it always have a solution.

When after row reduction to RREF, if the constant column

exists $\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$ $t_1, t_2, t_3, t_4, t_5 \in \mathbb{R}$, $t_4 \neq 0$ or $t_5 \neq 0$.

$Ax = b$ is inconsistent, it does not have a solution.

(iii) (5 points). What's the dimension of $\text{Nul}(A)$ the nullspace of A ?

$$\therefore \text{rank } A = 3$$

$$\therefore \dim(\text{Nul}(A)) + \text{rank } A = n$$

$$\therefore \dim(\text{Nul}(A)) = 1$$



- (iv) (5 points). What if A has 4 pivots instead of 3. Is $Ax = b$ always consistent? Justify your answer.

5x4
No.

When after row reduction to RREF, the constant column

exists $\begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix}$, $t_1, t_2, t_3, t_4, t_5 \in \mathbb{R}$, $t_5 \neq 0$,

$Ax = b$ is inconsistent.

If $b = 0$, it is always consistent.



Problem 3. (15 points). Let $A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 2 & 3 \\ 2 & 2 & 3 \end{bmatrix}$. Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Put your answers in the blank matrices provided.

$$P = \begin{bmatrix} -\frac{3}{2} & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$CA(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 2 & 3 \\ 2 & 2-\lambda & 3 \\ 2 & 2 & 3-\lambda \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 2-\lambda & 2 & 3 \\ 2 & 2-\lambda & 3 \\ 2 & 2 & 3-\lambda \end{vmatrix} = \det \begin{pmatrix} 2-\lambda & 2 & 3 \\ 2 & 2-\lambda & 3 \\ 0 & \lambda & -\lambda \end{pmatrix}$$

$$\text{Set } CA(\lambda) = 0$$

$$\therefore CA(\lambda) = (2-\lambda) \cdot (-1)^{1+1} \det \begin{pmatrix} 2-\lambda & 3 \\ \lambda & -\lambda \end{pmatrix} + 2 \cdot (-1)^{2+1} \det \begin{pmatrix} 2 & 3 \\ \lambda & -\lambda \end{pmatrix}$$

$$= (2-\lambda) [(2-\lambda)(-\lambda) - 3\lambda] - 2 \cdot (-\lambda - 3\lambda)$$

$$= (2-\lambda) \cdot \lambda \cdot (\lambda-5) + 10\lambda$$

$$= -\lambda^2(\lambda-7)$$

$$\therefore \lambda = 0 \text{ with } m=2 \text{ or } \lambda = 7 \text{ with } m=1$$



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(extra paper for Problem 3)

$$E_{\lambda=0} = \text{null} \begin{pmatrix} 2 & 2 & 3 \\ 2 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix} = \text{null} \begin{pmatrix} 2 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \text{span} \left\{ \begin{pmatrix} \frac{3}{2} \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

$$= \text{null}(A - 0 \cdot I)$$

$$E_{\lambda=1} = \text{null} \begin{pmatrix} -5 & 2 & 3 \\ 2 & -5 & 3 \\ 2 & 2 & -4 \end{pmatrix} = \text{null} \begin{pmatrix} -5 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} = \text{span} \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$= \text{null}(A - 1 \cdot I)$$

$$\therefore P = \begin{pmatrix} -\frac{3}{2} & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



Problem 4. (15 points total). Find the orthogonal complement in \mathbb{R}^4 of the subspace $S =$

$\text{span}\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 4 \\ 5 \end{bmatrix}\right\}$. Show all your work.

$$\text{Set } S^\perp = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \mid \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 0 \text{ and } \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 5 \end{pmatrix} = 0 \right\}$$

$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \mid \begin{array}{l} x_1 + 2x_2 + 3x_3 + 4x_4 = 0 \\ 3x_1 + 4x_2 + 4x_3 + 5x_4 = 0 \end{array} \right\}$$

$$\therefore \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 3 & 4 & 4 & 5 & 0 \end{array} \right) \text{ is equivalent to the equation of } S^\perp$$

$$\therefore \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 3 & 4 & 4 & 5 & 0 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & -2 & -5 & -7 & 0 \end{array} \right)$$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2t + 3s \\ -\frac{5}{2}t - \frac{7}{2}s \\ t \\ s \end{pmatrix} = t \begin{pmatrix} 2 \\ -\frac{5}{2} \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ -\frac{7}{2} \\ 0 \\ 1 \end{pmatrix} \quad t, s \in \mathbb{R}$$

$\therefore \begin{pmatrix} 2 \\ -\frac{5}{2} \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -\frac{7}{2} \\ 0 \\ 1 \end{pmatrix}$ are linearly independent, they are a basis for S^\perp

$$\therefore S^\perp = \left\{ \begin{pmatrix} 2 \\ -\frac{5}{2} \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -\frac{7}{2} \\ 0 \\ 1 \end{pmatrix} \right\}$$



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Problem 5. (35 points). Parts (i)–(iv) of this problem are about a matrix A which admits an LU factorization $A = LU$ with

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

and the unique solution to $Ly = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$ is $y = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$.

(i) (10 points). Solve $Ax = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$ using the LU decomposition.

$$\because A = LU$$

$$\therefore LUX = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$$\therefore L(LX) = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$$\therefore LY = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$$\therefore UX = Y = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$\therefore \left(\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\therefore X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$



(ii) (5 points). Calculate $\text{rank}(A)$. Justify your answer.

$$\because AX = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix} \text{ has unique solution } \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore A$ does not have free variables.

$$\because Ly = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix} \quad \therefore \text{size}(L) = 3 \times 3 \quad \therefore \text{rank } A = 3$$

\downarrow
 3×1

$$\therefore \text{size}(U) = 3 \times 3, A = LU$$

$$\therefore \text{size}(A) = 3 \times 3$$

(iii) (5 points). What is the dimension of $\text{Nul}(A^T)$? Justify your answer.

$$\because \text{rank}(A^T) = \text{rank}(A) = 3$$

$$\text{size}(A) = 3 \times 3$$

$$\therefore \dim(\text{Nul}(A^T)) + \text{rank}(A^T) = n = 3$$

$$\therefore \dim(\text{Nul}(A^T)) = 0$$

(iv) (5 points). Give a basis for $\text{row}(A)$. Justify your answer.

$$\because \text{rank } A = 3 = n$$

$$\text{size}(A) = 3 \times 3$$

$\therefore A$ is invertible

$$\therefore \text{basis for } \text{row}(A) = \{(1 \ 0 \ 0), (0 \ 1 \ 0), (0 \ 0 \ 1)\}$$

$(1 \ 0 \ 0), (0 \ 1 \ 0), (0 \ 0 \ 1)$ are linearly independent.



(v) (5 points). Give a basis for $\text{col}(A)$. Justify your answer.

According to UV, A is invertible.

$$\therefore \text{col}(A) = \text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is basis for $\text{col}(A)$ and
it is linearly independent.

(vi) (5 points). Does the matrix $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ have an LU decomposition? If yes, find L and U . If no, explain why not.

No. Because LU factorization cannot interchange rows. B has a zero on its pivot position.



Problem 6. (18 points total). Consider the set S , defined by

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \in \mathbb{R}^5 \mid \det \begin{pmatrix} 1 & 5 & x_1 & 6 & 2 \\ 2 & 4 & x_2 & 7 & 2 \\ 3 & 3 & x_3 & 8 & 3 \\ 4 & 2 & x_4 & 7 & 2 \\ 5 & 1 & x_5 & 6 & 2 \end{pmatrix} = 0 \right\}$$

Is S a subspace of \mathbb{R}^5 ? If so, prove it. If not, explain why not.

When $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $\det \begin{pmatrix} 1 & 5 & 0 & 6 & 2 \\ 2 & 4 & 0 & 7 & 2 \\ 3 & 3 & 0 & 8 & 3 \\ 4 & 2 & 0 & 7 & 2 \\ 5 & 1 & 0 & 6 & 2 \end{pmatrix} = 0$

$\therefore 0 \in S$
 Set $t = \begin{pmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{pmatrix} \in S$ and $k = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \end{pmatrix} \in S$, $C \in \mathbb{R}$.
 \therefore change all the t to k , we get equation ② = 0.
 change all the t to $(t+k)$ we get equation ③.

WTS: $Ct \in S$
 and $(t+k) \in S$

$$\therefore ③ = ① + ② = 0$$

$$\therefore (t+k) \in S$$

$\therefore S$ is a subspace

$\therefore t_1 \cdot \det \begin{pmatrix} 2 & 4 & 7 & 2 \\ 3 & 3 & 8 & 3 \\ 4 & 2 & 7 & 2 \\ 5 & 1 & 6 & 2 \end{pmatrix} - t_2 \cdot \det \begin{pmatrix} 1 & 5 & 6 & 2 \\ 3 & 3 & 8 & 3 \\ 4 & 2 & 7 & 2 \\ 5 & 1 & 6 & 2 \end{pmatrix} + t_3 \cdot \det \begin{pmatrix} 1 & 5 & 6 & 2 \\ 2 & 4 & 7 & 2 \\ 4 & 2 & 7 & 2 \\ 5 & 1 & 6 & 2 \end{pmatrix} - t_4 \cdot \det \begin{pmatrix} 1 & 5 & 6 & 2 \\ 2 & 4 & 7 & 2 \\ 3 & 3 & 8 & 3 \\ 5 & 1 & 6 & 2 \end{pmatrix} +$
 $t_5 \cdot \det \begin{pmatrix} 1 & 5 & 6 & 2 \\ 2 & 4 & 7 & 2 \\ 3 & 3 & 8 & 3 \\ 4 & 2 & 7 & 2 \end{pmatrix} = 0$
 multiply all t by c , $\det \begin{pmatrix} 1 & 5 & Ct_1 & 6 & 2 \\ 2 & 4 & Ct_2 & 7 & 2 \\ 3 & 3 & Ct_3 & 8 & 3 \\ 4 & 2 & Ct_4 & 7 & 2 \\ 5 & 1 & Ct_5 & 6 & 2 \end{pmatrix} = ① \cdot c = 0c = 0$
 $\therefore Ct \in S$



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Problem 7. (20 points total). Suppose that $S \subset \mathbb{R}^8$ with $\dim S = 5$. Is there a subspace $V \subset \mathbb{R}^8$ with $\dim V = 2$ and $V \cap S = \{0\}$? Justify your answer.

Yes, there is.

$\because S \subset \mathbb{R}^8$ with $\dim S = 5$

$$\therefore \text{Set } S = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} t_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} t_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} t_3 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} t_4 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} t_5 +$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} t_6 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} t_7 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} t_8 \quad \text{and randomly pick 3 } t_i \text{ for example}$$

equals 0.
the rest of $t_i \neq 0$

$t_1 = t_3 = t_8 = 0$, now S is a subspace of \mathbb{R}^8 with $\dim S = 5$

Similarly, Set a $V \subset \mathbb{R}^8$ with $\dim V = 2$

$$V = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} k_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} k_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} k_3 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} k_4 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} k_5 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} k_6 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} k_7 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} k_8$$

the rest of $k_i = 0$

and randomly pick 2 $k_i \neq 0$, for example $k_1 \neq 0$ and $k_3 \neq 0$.

Now $S \cap V = \{0\}$

As long as pick 3 t_i in S equals 0 (i.e. \mathbb{R}), and pick ^{one or} 2 of the same

"1"s as t_i in k , $V \cap S = \{0\}$.



Problem 8. (14 points total). This problem has two parts.

- (i) (7 points). Prove that if $Ax \in \text{Nul}(A^T)$ then $Ax = 0$. Hint: it may help to recall that $S \cap S^\perp = \{0\}$ always holds for a subspace S .

$$\begin{aligned} \because Ax &\in \text{Nul}(A^T) \\ (\text{Nul}(A^T)) &= (\text{col}(A))^\perp \end{aligned}$$

$$\therefore Ax \in (\text{col}(A))^\perp$$

$\because Ax$ is a linear combination of $\text{col}(A)$ and x ,

$$\therefore Ax \in \text{col}(A)$$

$$\because (\text{col}(A)) \cap (\text{col}(A))^\perp = \{0\}$$

$$\therefore Ax = 0$$

- (ii) (7 points). Use part (i) above to show that $\text{Nul}(A) = \text{Nul}(A^T A)$ holds for all matrices A .

WTS: $\text{Nul}(A) = \text{Nul}(A^T A)$ which is to show that
if ① $x \in \text{Nul}(A)$ then $x \in \text{Nul}(A^T A)$ and ② if $x \in \text{Nul}(A^T A)$,
then $x \in \text{Nul}(A)$

①: \because if $x \in \text{Nul}(A)$

$$Ax = 0$$

$$\therefore A^T(Ax) = A^T \cdot 0 = 0$$

$$\therefore (A^T A)x = 0$$

$$\therefore x \in \text{Nul}(A^T A)$$

②: \because if $x \in \text{Nul}(A^T A)$,

$$\therefore (A^T A)x = 0$$

$$\therefore A^T(Ax) = 0$$

$$\therefore Ax \in \text{Nul}(A^T)$$

According to (i)

$$\therefore Ax = 0$$

$$\therefore x \in \text{Nul}(A)$$

$$\therefore \text{Nul}(A) = \text{Nul}(A^T A)$$



Problem 9. (18 points total, 3 points each) For the following questions, answer using the word "True" or the word "False". A statement is "True" when it's *always true* and otherwise is false. You **don't need to justify your answer** to receive full credit. However, to discourage guessing we will deduct 2 points for each incorrect answer given.

- (a) Let A be a matrix. The pivot columns of the reduced row echelon form of A are a basis for the row space of the transpose of A .

False

- (b) If A is invertible then $\text{rank}(A) = \text{rank}(A^{-1})$.

True

- (c) The rank of a matrix must be at least as big as the dimension of its nullspace.

False

- (d) A nonzero element of $\text{Nul}(A)$, for a square matrix A , must be an eigenvector of A .

True

- (e) If every entry of a square $n \times n$ matrix A is positive then $\det(A) \neq 0$.

False

- (f) Let A be $m \times k$ and B be $k \times n$ matrices which have reduced row echelon matrices given by matrices M and N respectively. Then the reduced row echelon matrix of AB is given by the matrix MN .

False

Not graded.



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(*extra paper*)



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(extra paper)

~~cont. p. 1~~

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{pmatrix} \quad (1 \ 2)$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \\ \downarrow \\ y$$

$$Ax = b.$$

$$\text{RREF} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



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(extra paper)

$$x_1 - 5t - 7s + 3t + 4s$$

$$x_1 + -2t - 3s$$

$$x_2 = \frac{5t + 7s}{-2}$$

$$-2x_2 = 5t + 7s$$

$$-2x_2 - 5t - 7s = 0$$

$$\begin{array}{cccc} -3 & -6 & -9 & 12 \\ 1 & 2 & 3 & 4 \\ 0 & -2 & -5 & -7 \end{array}$$

$$\begin{array}{l} 2t + 3s \\ -\frac{5}{2}t - \frac{7}{2}s \\ t \\ s \end{array}$$