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***Abstract*—In this paper we discuss the aspects of kinematic design of an all-terrain 4-wheel drive rover with synchronous wheels-steer (4WDS). The system poses a hybrid-type of holonomy constrained by the locomotion design. The locomotion system is assumed to have rigid supporting devices with no dumpering effects in a first study. The proposed kinematic solution (direct/inverse) provides the capability to develop holonomic control of trajectories. This manuscript also includes a modeling analysis on a pose control law suggesting a formal resource to future intelligent navigational algorithms in practical applications. We present numerical simulation results on modeling and design of the robotic platform.**

Keywords- kinematics, rover, holonomy, steer, 4W-drive

I. Introduction

The main purpose of a mobile robot is to execute a diversity of assigned tasks in hostile environments where humans are unable to work, or are exposed to dangerous conditions. Some of such tasks are mining, surveillance, space exploration, underwater and/or volcanic sensing, and so forth. In order to increase the robot's success, it is necessary to provide higher levels of autonomy by improving its mobility and steerability capabilities (holonomy). Mobile robots are classified as holonomic and non-holonomic according to their mobility δ_m and steerability δ_s degrees. The holonomic robots does not have mechanical constraints that limits their mobility, this feature allows them to move on the surface of displacement in any direction. A system is holonomic, if the number of degrees of freedoms that can be controlled is equal to the available degrees of freedom. Likewise, non-holonomic robots have kinematic constraints in their locomotion structure, which limits their mobility on the surface of displacement [4] (e.g. cannot move laterally). In a non-holonomic system, the differential equations are not integrable in the robot's final position, which is required to know how the movement was executed as a function of time [1]. In the present study, we deal with the kinematic solution of a four-wheel drive steering robot (4WDS) depicted in figure 1. This type of robotic structure includes a synchronous steering angle for all in-wheels motor.

This manuscript is organized in the following manner. Section II describes the proposed kinematic structure. Section III discusses an analysis of the traditional nonslip kinematic condition and its unfitness. Section IV describes the proposed solution based on the kinematic rolling constraints. Section V provides numeric experimental results. Finally, summarized conclusions are given.

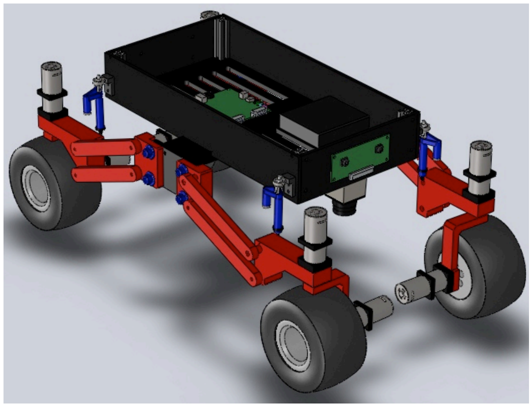

Figure 1. 4WDS laboratory prototype mechanical design.

II. 4WDS Rover Kinematic Design

In terms of holonomy, this manuscript classifies 4W (contact points) robots according to the categories of table I. The 2WD with 4 wheels are laterally synchronized; 4WD, four-wheel drive, are no steering; 4WDS, four-wheel drive all-in-one steering angle; and 4WD4S (four-wheel drive four-steer).

TABLE I. 4WDS configuration modalities (NH: NonHolonomy; H: Holonomy).

4WD configuration modes			
Holonomy	**Velocities**	**Description**	**Steer**
1: NH	φ_r, φ_l	2WD, differential drive (4W).	β=constant
2: NH	$\varphi_1,..,\varphi_4$	4WD, four independent velocities.	β=constant
3: H	$\varphi_1,..,\varphi_4$	4WDS, 4-wheel drive, synchronous 4W steering.	β(t)
4: H	$\varphi_1,..,\varphi_4$	4W4S, 4 independent	β(t)

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3: H	$\varphi_1,..,\varphi_4$	4WDS, 4-wheel drive, synchronous 4W steering.	β(t)
		velocties, 4 asynchronous steering wheels.	

As a matter of fact, the 4WDS and 4WD4S are enhancements of the 4WD structures, because extra DOFs are added. With such kinematic structures, we can also have different configuration modalities with respect to $\boldsymbol{\varphi}(t)$ and $\beta(t)$, so the robot is given with a different holonomic capability. This manuscript tackles the problem of 3:H (Table I), which is a generalization for the particular cases of 1:NH and 2:NH. For the case of 4:H, the topic is out of the scope of this technical paper and will be treated in future works. Thus, this study concerns on providing a kinematic solution of a 4WDS robotic structure by means of modeling analysis, and numerical results.

Figure 2 shows some mechanical elements related to the steering feature of the rover design. The plate (1) provides attachment between the wheels (10) suspension bars (2), (3) and the main chassis. The supporting bars (2) and (3) hold the piece (4) which is a mechanical base for the actuators (6), (11) the physical supports. There exist another mechanical device (5), which is a rigid suspension device to keep chassis and wheels fixed. A mechanic piece (8) gives rigid support to the wheel (10), the drive motor (11), steer-motor with the plate (4). In fact, the suspension support (5) is a spring-mass dumper device, which in this manuscript is assumed to be rigid to treat the problem of 4WDS as a set; contact points in a fixed position with respect to any inertial frame in use.

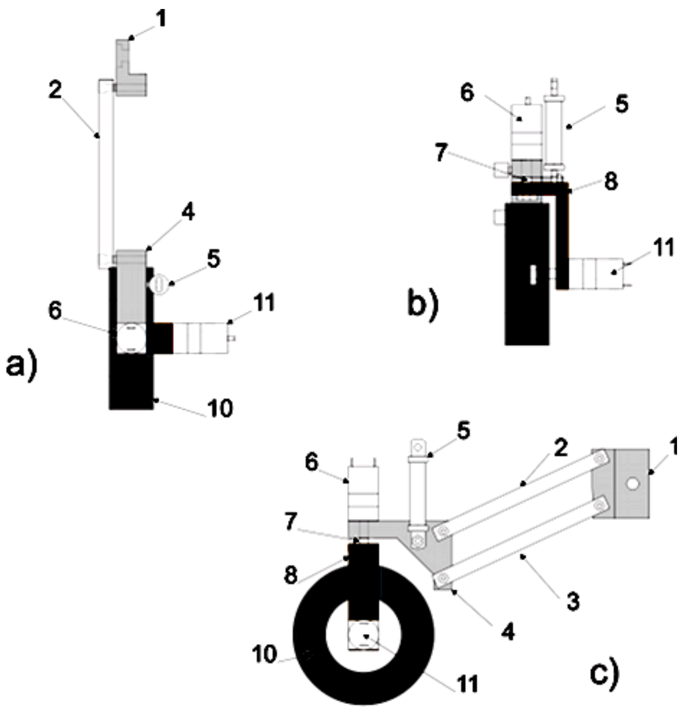

Figure 2. 4WDS mechanical devices associated with driving and steering.

III. NONSLIP KINEMATIC CONDITION

The nonslip kinematic condition refers to the orthogonal (lateral) mobility restrictions of the robot's wheels. Figure 3 depicts a top view of the 4WDS kinematics. The driven instantaneous angular velocities for each wheel are given by $\varphi_1,..,\varphi_4$. The wheels steering angles are $\beta_1,..,\beta_4$ represented in the robot's attached coordinate frame. The wheels contact points location in the polar form are given by $\alpha_1,..,\alpha_4$ and l. These parameters are the wheel angle (contact point) w.r.t. the robots geometric center.

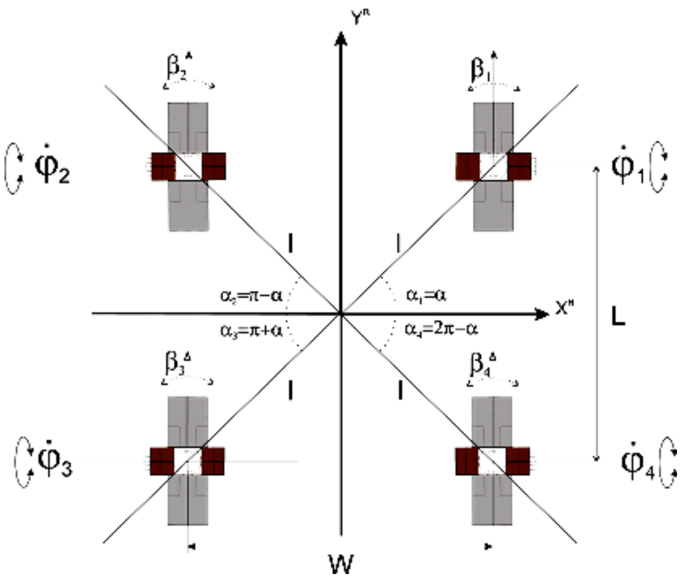

Figure 3. Rover kinematic configuration.

The orthogonal kinematic components restrictions for fixed and centered wheels is described by:

$$[\cos(\alpha+\beta)\ \ \sin(\alpha+\beta)\ \ l\sin(\beta)\ ]\mathbf{R}(\theta)d\xi/dt-r\varphi=0 \qquad (1)$$

According to the method using the nonslip kinematic conditions contributed in [3], we state from (1) that the matrix of kinematic restrictions is defined by $\mathbf{K}_1$,

$$\mathbf{K}_1=\begin{pmatrix} \cos(\alpha+\beta_1) & \sin(\alpha+\beta_1) & l\sin(\beta_1) \\ -\cos(\alpha-\beta_2) & \sin(\alpha-\beta_2) & l\sin(\beta_2) \\ -\cos(\alpha+\beta_3) & -\sin(\alpha+\beta_3) & l\sin(\beta_3) \\ \cos(\alpha-\beta_4) & -\sin(\alpha-\beta_4) & l\sin(\beta_4) \end{pmatrix} \qquad (2)$$

The fixed wheels matrix $\mathbf{K}_1$ is obtained from the orthogonal kinematic restrictions of all involved fixed conventional wheels for a 4WDS structure. In addition, the kinematic conditions for the centered steerable wheels are given in matrix $\mathbf{K}_2$. The resulting mobility capability by using the nonslip conditions yield a mobility degree of $\delta m=0$, and a steering degree of $\delta_s=3$.

$$\delta m=3-\mathrm{rank}\{\mathbf{K}_1\}$$

and

$$\delta s=\mathrm{rank}\{\mathbf{K}_2\}$$

The numeric value 3 arises from the number of degrees of freedom onto the robot's plane of mobility x,y,θ. In our particular case (i.e. 4WDS structure), it uniquely includes centered steerable wheels. So that, both matrixes are **K**1= **K**2. It is because the rank{**K**1}=3.

Furthermore, **u**(t) is the control input vector, so the general formulation for the posture kinematic model is $\dot{\mathbf{z}}=\mathbf{B}(\mathbf{z})\,\mathbf{u}(t)$ is obtained from the product of transpose of the orthogonal rotation matrix $\mathbf{R}(\theta)^T$ and the vector solutions (the nullspace vectors) of **K**2, which is described by Σ(**K**2) as,

$$\dot{\mathbf{z}}= \mathbf{R}(\theta)^T\Sigma(\mathbf{K}2)\mathbf{u} \qquad (3)$$

Since the rank(**K**2)=3 it is not possible to obtain the posture kinematic model because the null-space vector for **K**2 has dimension zero.

$$\Sigma(\mathbf{K}2)=\{\} \qquad (4)$$

Therefore, a solution for this linear algebra problem is tackled from different kinematic conditions in next section.

IV. 4WDS Rolling Constraints

This section contains the posture kinematic model obtained from the pure rolling kinematic conditions, which is orthogonal to restrictions seen in previous section. The motion components of all wheels in the rolling plane are depicted by figure 4. The kinematic equations are stated on wheels' motion contributions in X, Y and yaw degrees of freedom.

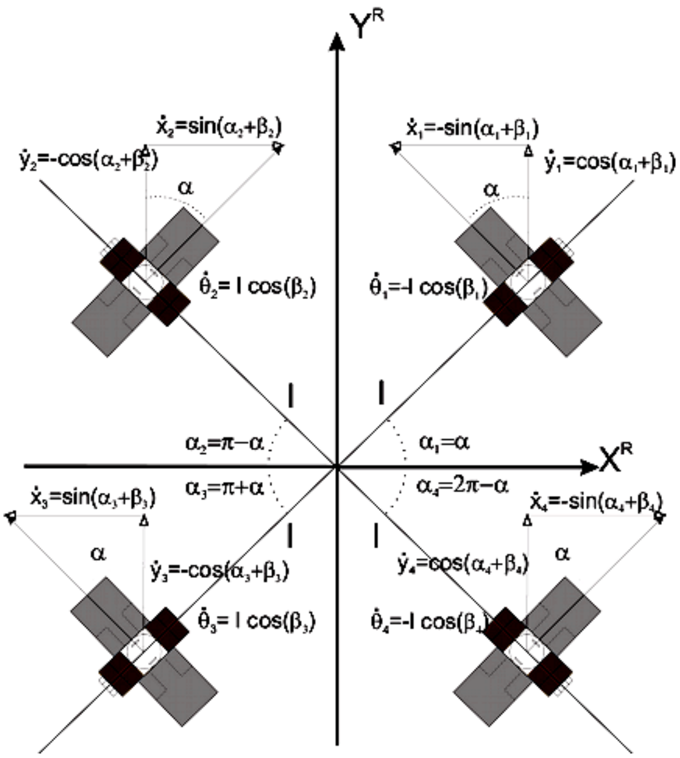

Figure 4. Rolling condition kinematic parameters.

We define each wheel's plane restriction vector k_i from figure 4 by,

$$\mathbf{k}_i \in \Re^3, \quad \mathbf{k}_i = (\dot{x} \quad \dot{y} \quad \dot{\theta})^T \qquad (5)$$

The wheel kinematic vector constraint that define its motion is given by (6),

$$\mathbf{k}_i = [-\sin(\alpha_i + \beta_i) \quad \cos(\alpha_i + \beta_i) \quad l\cos(\beta_i)]^T \qquad (6)$$

With the four wheels rolling condition vectors, the matrix **K**A formed containing the restrictions in terms of four column-vectors given by,

$$\mathbf{K}_A = (\mathbf{k}_1 \quad \mathbf{k}_2 \quad \mathbf{k}_3 \quad \mathbf{k}_4) \qquad (7)$$

In addition, the vector of wheels velocities **Φ** is expressed in terms of linear velocities as,

$$\mathbf{\Phi} = r\,(\dot{\varphi}_1 \quad \dot{\varphi}_2 \quad \dot{\varphi}_3 \quad \dot{\varphi}_4)^T \qquad (8)$$

The kinematic posture model for forward kinematics is the result of the matrix contributions tide to kinematic restrictions along the wheel plane given by **K**A in equation (14), and the column-vector of lineal velocities **Φ**, as described in equation (9) ,

$$\dot{\xi} = \mathbf{K}_A\mathbf{\Phi} \qquad (9)$$

By analyzing figure 4, the values l, α_n and β_n in each wheel are summarized in the table II as to depict how the steering angle β (in the robots local coordinates system), is comprised of the steer angle in whels coordinate system ϕ, and wheel's angle position α.

TABLE II. Values for α_n, β_n and l in the 4WDS structure.

Wheel	α_n	β_n	l
1_c	α	$-\alpha+\phi_t$	l
2_c	$\pi-\alpha$	$\alpha+\phi_t$	l
3_c	$\pi+\alpha$	$-\alpha+\phi_t$	l
4_c	$2\pi-\alpha$	$\alpha+\phi_t$	l

Therefore, according to figure 4, the values of each wheels rolling condition vector **k** are defined in following equations (10)-(13),

$$\mathbf{k}_1 = \begin{pmatrix} -\sin(\alpha_1+\beta_1) \\ \cos(\alpha_1+\beta_1) \\ -l_1\cos(\beta_1) \end{pmatrix} \qquad (10)$$

$$\mathbf{k}_2 = \begin{pmatrix} \sin(\alpha_2+\beta_2) \\ -\cos(\alpha_2+\beta_2) \\ l_2\cos(\beta_2) \end{pmatrix} \qquad (11)$$

$$\mathbf{k}_3 = \begin{pmatrix} \sin(\alpha_3+\beta_3) \\ -\cos(\alpha_3+\beta_3) \\ l_3\cos(\beta_3) \end{pmatrix} \tag{12}$$

$$\mathbf{k}_4 = \begin{pmatrix} -\sin(\alpha_4+\beta_4) \\ \cos(\alpha_4+\beta_4) \\ -l_4\cos(\beta_4) \end{pmatrix} \tag{13}$$

In the figures 3 and 4, all wheels' steering angles are controlled synchronously having same angles direction in time (as seen in Table II). In order to establish the steering angles in a common coordinate system (the attached robots inertial frame), we replace the values given in table II, with identities and algebraically arrange accordingly into the equations (10), (11), (12), y (13). Likewise, to simplify the equations length we use the following notation equivalences,

$$\begin{aligned} \mathrm{s}_{\phi_\mathrm{t}} &= \sin(\phi_\mathrm{t}) \\ \mathrm{c}_{\phi_\mathrm{t}} &= \cos(\phi_\mathrm{t}) \\ \mathrm{s}_{\alpha,\phi_\mathrm{t}} &= \sin(\alpha+\phi_\mathrm{t}) \\ \mathrm{c}_{\alpha,\phi_\mathrm{t}} &= \cos(\alpha+\phi_\mathrm{t}) \end{aligned}$$

No by substituting the column-vectors k_1, k_2, k_3, and k_4 into $\mathbf{K}_A$, the following rolling conditions matrix is yield,

$$\mathbf{K}_A = \frac{1}{4}\begin{pmatrix} -\mathrm{s}_{\phi_\mathrm{t}} & -\mathrm{s}_{\phi_\mathrm{t}} & -\mathrm{s}_{\phi_\mathrm{t}} & -\mathrm{s}_{\phi_\mathrm{t}} \\ \mathrm{c}_{\phi_\mathrm{t}} & \mathrm{c}_{\phi_\mathrm{t}} & \mathrm{c}_{\phi_\mathrm{t}} & \mathrm{c}_{\phi_\mathrm{t}} \\ -l\mathrm{s}_{\alpha,\phi_\mathrm{t}} & l\mathrm{c}_{\alpha,\phi_\mathrm{t}} & l\mathrm{s}_{\alpha,\phi_\mathrm{t}} & -l\mathrm{c}_{\alpha,\phi_\mathrm{t}} \end{pmatrix} \tag{14}$$

The robot's forward kinematics solution is given by solving the general equation (9). In equation (9) we substitute the matrix (14), and the resulting mathematical expression determines the robot's posture vector state with components $\dot{x}$, $\dot{y}$ and ω (yaw velocity). Thus, each state component equation is defined by (15), (16) and (17),

$$\begin{aligned} \dot{x} \;&=\; -\frac{1}{4}\sin(\phi_t)r\dot{\varphi}_1 - \frac{1}{4}\sin(\phi_t)r\dot{\varphi}_2 \\ &\quad -\frac{1}{4}\sin(\phi_t)r\dot{\varphi}_3 - \frac{1}{4}\sin(\phi_t)r\dot{\varphi}_4 \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{y} \;&=\; \frac{1}{4}\cos(\phi_t)r\dot{\varphi}_1 + \frac{1}{4}\cos(\phi_t)r\dot{\varphi}_2 \\ &\quad +\frac{1}{4}\cos(\phi_t)r\dot{\varphi}_3 + \frac{1}{4}\cos(\phi_t)r\dot{\varphi}_4 \end{aligned} \tag{16}$$

$$\begin{aligned} \dot{\theta} \;&=\; -\frac{l_1}{4}\sin(\frac{\pi}{4}+\phi_t)r\dot{\varphi}_1 + \frac{l_2}{4}\cos(\frac{\pi}{4}+\phi_t)r\dot{\varphi}_2 \\ &\quad +\frac{l_3}{4}\sin(\frac{\pi}{4}+\phi_t)r\dot{\varphi}_3 - \frac{l_4}{4}\cos(\frac{\pi}{4}+\phi_t)r\dot{\varphi}_4 \end{aligned} \tag{17}$$

Where *r* is the wheels' radius and is assumed to be constant and exactly the same for all wheel overtime. The angles ϕ_i are the wheels steering angle in wheel's local coordinate system. Its first derivative w.r.t. time dφ/dt represents the wheels angular velocity.

Furthermore, in order to obtain the inverse kinematics solution of the same system, we solved for the general linear equation (18). Where the vector of wheels velocities $\mathbf{\Phi}$ is now the unknown desired variable. So, by dropping $\mathbf{\Phi}$ off in equation (18).

$$\mathbf{\Phi} = \mathbf{K}_A^{-1}\dot{\xi} \tag{18}$$

The inverse solution for the rolling kinematic constrains $\mathbf{K}_A$ is obtained by obtaining its pseudoinverse, since it is not a squared matrix. Thus, the $\mathbf{K}_A^{-1}$ analytical solution is given by (19), where we have substituted $a=l(\cos(\alpha-\phi_t)^2+\cos(\alpha-\phi_t)^2)$ to simplify the expression.

$$\mathbf{K}_A^{-1} = \begin{pmatrix} -\frac{\sin(\phi_t)}{4} & \frac{\cos(\phi_t)}{4} & \frac{\cos(\alpha-\phi_t)}{2a} \\ -\frac{\sin(\phi_t)}{4} & \frac{\cos(\phi_t)}{4} & -\frac{\cos(\alpha-\phi_t)}{2a} \\ -\frac{\sin(\phi_t)}{4} & \frac{\cos(\phi_t)}{4} & -\frac{\cos(\alpha-\phi_t)}{2a} \\ -\frac{\sin(\phi_t)}{4} & \frac{\cos(\phi_t)}{4} & \frac{\cos(\alpha-\phi_t)}{2a} \end{pmatrix} \tag{19}$$

The value of ϕ_t assumed to be a feedback variable measured by using a particular type of sensor.

V. Numeric Results

In this section we present the simulation experiments to illustrate the 4WDS pose control laws by using the inverse and forward kinematics. In figure 5, the rover exhibits a control motion task. Only by changing the wheels' steering angle at same rate overtime (e.g. t1, t2,…,t5), the robot's trajectory exhibits a circular path. It is actually a depiction of its holonomic capability.

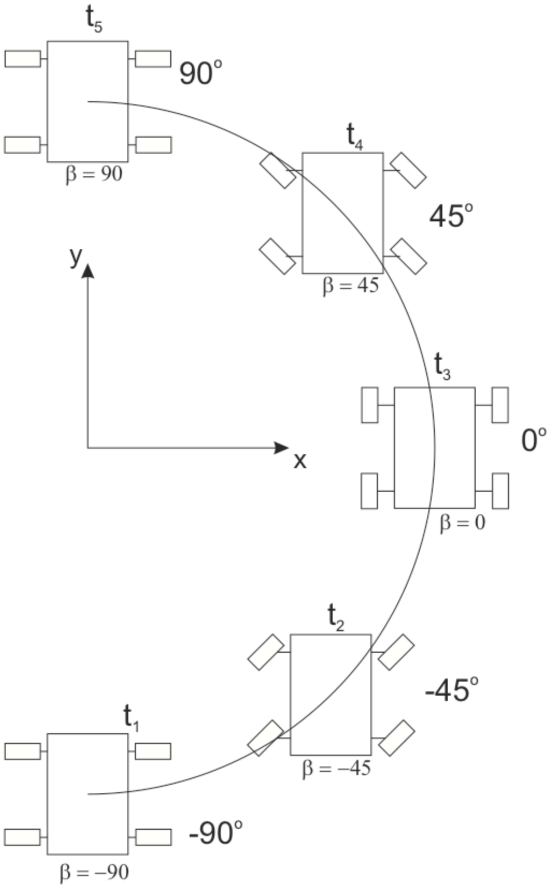

Figure 5. Holonomic mobility experiment varying only wheels steer angle β(t) at constant rate and wheels synchronized constant velocity .

In addition, the wheels velocities are for this case, synchronized at constant driving speed.

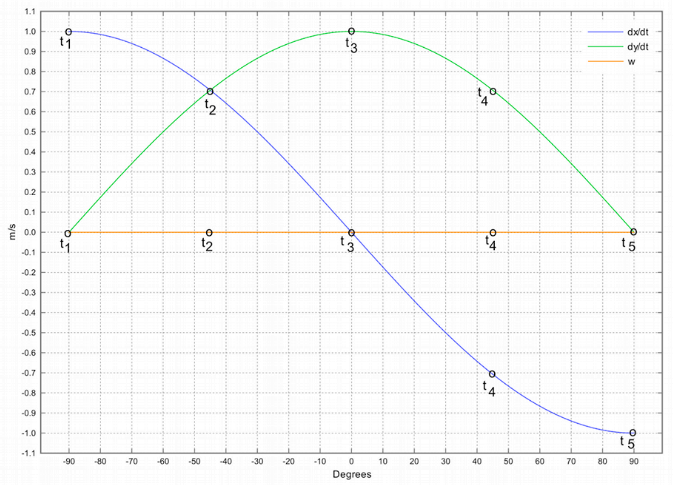

Figure 6. Resulting numeric values yielded by constant rate change for steer angle β in forward kinematics modality.

The numerical results for inverse kinematics are showed in figure 7. We input exactly the same resulting numeric values depicted in figure 6. Now, the resulting robot's wheels linear velocities are given with a constant vale in time.

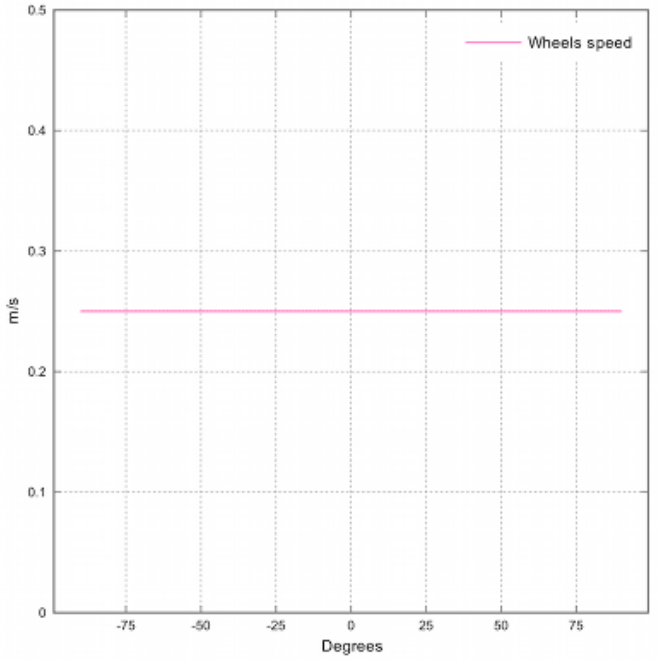

Figure 7. Inverse kinematic numerical results for averaged wheels tangential velocity.

CONCLUSIONS

By using the nonslip kinematic restrictions of centered steering wheels, the robot's 4WDS kinematic posture model generalized by $\dot{\mathbf{z}}=\mathbf{B}(\mathbf{z})\mathbf{u}$. It is usually obtained from the null-space solution for the matrix of kinematic configuration of wheels $\mathbf{K}_2$. However, we found out that for four-wheel structures (e.g. 4WD, 4WDS), such matrix is linearly dependent, and for its resulting null-space vector, its dimension is zero. Thus, by the traditional nonslip condition method, it is not possible to obtain a kinematic posture control law solution. Thus, we develop an algebraic solution by directly using the rolling kinematic restrictions (along wheels plane) with an algebraic arrangement of the wheels location in robot's coordinate frame. If all wheels are synchronized with respect to the steering angle β(t), its holonomic behavior is equivalent to motion without kinematic restrictions. Thus, nonslip conditions are not essential since holonomy substitutes the concept of non-slippage (assuming flat terrains, and no damper effects). In future works, we will extend the formulation to include asynchronous 4-steer variables, and spring-mass damper effects to enhance the rover's maneuverability and stability in a holonomic fashion.

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