Tutorial 5 (Week 6) - Linear Regression

Learning Objectives

After completing this tutorial, you should be able to:

- Use `statsmodels` to perform linear regression
- Implement Simple Linear Regression
- Estimate coefficients and analyze the model
- Apply the model for hypothesis testing
- Implement Multiple Linear Regression

This tutorial uses the case study covered in the textbook "An Introduction to Statistical Learning: with Applications in R" by James G. et al. (link (https://link-springer-com.libproxy1.nus.edu.sg/book/10.1007/978-1-4614-7138-7)). The codes are written using this notebook (https://github.com/justmarkham/DAT4/blob/master/notebooks/08_linear_regression.ipynb) and this notebook (https://nbviewer.jupyter.org/github/JWarmenhoven/ISL-python/blob/master/Notebooks/Chapter%203.ipynb).

In [2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

%matplotlib inline
#plt.style.use('seaborn-v0_8-white')
```

Dataset

The Advertising data set consists of the *sales* (in thousands on units) of a certain product in 200 different markets, along with *advertising budgets* (in thousands of dollars) for the product in each of those markets for three different media: TV, radio, and newspaper.

Suppose you are a data analyst hired to provide advice on how to improve sales of this product.

It is not possible for your client to directly increase sales of the product. On the other hand, they can control the advertising expenditure in each of the three media. Therefore, if you determine that there is an association between advertising and sales, then you can instruct your client to adjust advertising budgets, thereby indirectly increasing sales. In other words, your goal is to develop an **accurate model that can be used to predict sales on the basis of the three media budgets**.

As usual, load the data into pandas, and obtain a summary statistics to start exploring. You can specify `index_col` to treat the first column as index values.

In [4]:
```
# TODO
advertising = pd.read_csv( 'Advertising-T5.csv', index_col=0 )
advertising.describe()
```

Out[4]:

	TV	Radio	Newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
mean	147.042500	23.264000	30.554000	14.022500
std	85.854236	14.846809	21.778621	5.217457
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	10.375000
50%	149.750000	22.900000	25.750000	12.900000
75%	218.825000	36.525000	45.100000	17.400000
max	296.400000	49.600000	114.000000	27.000000

Introduction to Statsmodels

As the name implies, `statsmodels` is a Python library built specifically for statistics. It is built on top of NumPy, SciPy, and matplotlib, but it contains more advanced functions for statistical testing and modeling that you won't find in numerical libraries like NumPy or SciPy. To know more about this package, follow the tutorials in this link (https://www.statsmodels.org/stable/user-guide.html).

The `statsmodels` package is included in the Anaconda distribution.

In [5]:
```
import statsmodels.api as sm
import statsmodels.formula.api as smf
```

`statsmodels` allows users to fit statistical models using R-style formulas. The `formula.api` module is a convenience interface for specifying models using formula strings and DataFrames. It hosts lowercase counterparts for many of the same functions found in the `api` module (e.g., `ols` corresponding to `api.OLS`, `glm` corresponding to `api.GLM`). In general, the lowercase models accept `formula` and `df` arguments, whereas uppercase ones take `endog` and `exog` (https://www.statsmodels.org/stable/endog_exog.html) design matrices.

Simple Linear Regression

Simple Linear Regression (SLR) predicts a response *Y* on the basis of a single predictor variable *X*. It assumes an approximately linear relationship between *X* and *Y*.

We can regress sales (Y) onto TV (X) by fitting the model: $sales \approx \beta_0 + \beta_1 \times TV$

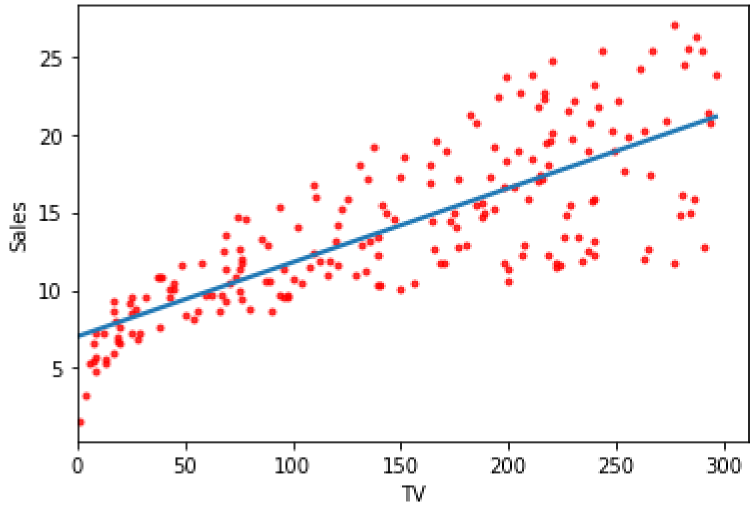
β_0 (*intercept*) and β_1 (*coefficient of X*) are the model coefficients. We must estimate their values, using our data, to be able to make predictions with the model.

Let us first plot the regression of sales onto TV, using seaborn `regplot`.

In [6]:
```
# TODO
sns.regplot( x=advertising.TV, y=advertising.Sales, ci=None, scatter_kws={'color':'

# Start the x-axis at 0 to make the intercept clear
plt.xlim( xmin=0 )
```

Out[6]: (0.0, 311.185)



We will use the *least squares* method to estimate the coefficients. We look at two ways to perform the regression, one using the uppercase model `OLS` from `api` and the other using the lowercase model `ols` from `formula.api`.

`api.OLS`

The required inputs to [OLS (https://www.statsmodels.org/stable/generated/statsmodels.regression.linear_model.OLS.html)](https://www.statsmodels.org/stable/generated/statsmodels.regression.linear_model.OLS.html) are `endog` (the response y) and `exog` (the predictor x).

An intercept is not included in `exog` by default and should be added by users. This is achieved by calling `statsmodels` function `add_constant`, which adds a column of ones to the `exog` array.

In [7]:
```
X = advertising[['TV']]
y = advertising[['Sales']]

est = sm.OLS( y, sm.add_constant(X) ).fit()
est.summary()
```

Out[7]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.612
Model:	OLS	**Adj. R-squared:**	0.610
Method:	Least Squares	**F-statistic:**	312.1
Date:	Fri, 06 Oct 2023	**Prob (F-statistic):**	1.47e-42
Time:	15:08:22	**Log-Likelihood:**	-519.05
No. Observations:	200	**AIC:**	1042.
Df Residuals:	198	**BIC:**	1049.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	7.0326	0.458	15.360	0.000	6.130	7.935
TV	0.0475	0.003	17.668	0.000	0.042	0.053

Omnibus:	0.531	**Durbin-Watson:**	1.935
Prob(Omnibus):	0.767	**Jarque-Bera (JB):**	0.669
Skew:	-0.089	**Prob(JB):**	0.716
Kurtosis:	2.779	**Cond. No.**	338.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

`formula.api.ols`

The required inputs to `ols` (https://www.statsmodels.org/stable/generated/statsmodels.formula.api.ols.html) are the formula string and the data. The formula syntax follows the framework of the `patsy` (https://patsy.readthedocs.io/en/latest/formulas.html#the-formula-language) package.

In [8]:
```
est = smf.ols( formula='Sales ~ TV', data=advertising ).fit()
est.summary()
```

Out[8]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.612
Model:	OLS	**Adj. R-squared:**	0.610
Method:	Least Squares	**F-statistic:**	312.1
Date:	Fri, 06 Oct 2023	**Prob (F-statistic):**	1.47e-42
Time:	15:08:25	**Log-Likelihood:**	-519.05
No. Observations:	200	**AIC:**	1042.
Df Residuals:	198	**BIC:**	1049.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	7.0326	0.458	15.360	0.000	6.130	7.935
TV	0.0475	0.003	17.668	0.000	0.042	0.053

Omnibus:	0.531	**Durbin-Watson:**	1.935
Prob(Omnibus):	0.767	**Jarque-Bera (JB):**	0.669
Skew:	-0.089	**Prob(JB):**	0.716
Kurtosis:	2.779	**Cond. No.**	338.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

From the results obtained above, we can see the coefficient estimates as follows:

- $\overset{\wedge}{\beta_0} = 7.03$
- $\overset{\wedge}{\beta_1} = 0.0475$

In other words, an additional 1,000 dollars spent on TV advertising is associated with selling approximately 47.5 additional units of the product. This is because β_1 is the slope - the average increase in sales associated with a one-unit increase in TV budget (which is in thousands of dollars).

In [9]:
```
est.params
```

Out[9]:
```
Intercept    7.032594
TV           0.047537
dtype: float64
```

We can use the model coefficients to manually make a sales prediction. Suppose the company spends $25,000 in a new market, how much sales can we predict that market will

In [10]:
```
# TODO
est.params[0] + est.params[1] * 25
```

Out[10]:
```
8.221009559953188
```

You can refer to the `OLSResults` [reference (https://www.statsmodels.org/devel/generated/statsmodels.regression.linear_model.OLSResults](https://www.statsmodels.org/devel/generated/statsmodels.regression.linear_model.OLSResults) for other accessible attributes.

Residuals and Assessing Model Accuracy

Residual and RSS

Residual ($e_i = y_i - \hat{y}_i$) is the difference between the i^{th} observed response value and the i^{th} response value that is predicted by our linear model.

The least squares approach chooses the values for $\hat{\beta}_0$ and $\hat{\beta}_1$ to minimize the **residual sum of squares (RSS)** $(e_1^2 + e_2^2 + \ldots)$.

We can use the model coefficients to manually calculate the RSS, by summing the squared residuals. Recall that the predicted value $\hat{y}_i$ is $\beta_0 + \beta_1 \times TV_i$.

In [11]:
```
# TODO
((advertising.Sales - (est.params[0] + est.params[1] * advertising.TV))**2).sum()
```

Out[11]:
```
2102.5305831313512
```

We can check our calculation against the RSS value computed by OLS, which is given by the attribute `ssr` (sum of squared residuals) in the OLS result.

In [12]:
```
est.ssr
```

Out[12]:
```
2102.5305831313512
```

Residual Standard Error

The RSS value can be used to compute the residual standard error (RSE), which is given by the formula: $RSE = \sqrt{\frac{RSS}{n-2}}$ where n is the number of samples.

In [13]:
```
# TODO
np.sqrt( est.ssr / (len(advertising.TV) - 2) )
```

Out[13]:
```
3.2586563686504624
```

The RSE is found to be 3.26. In other words, actual sales in each market deviate from the true regression line by approximately 3.26 units, on average.

The OLS results also show us the standard errors of the model coefficients, which are the standard deviation over these coefficients learned from the samples. They are shown in the summary and also accessible from the attribute `bse` .

In [15]:
```
# TODO
est.bse
```

Out[15]:
```
Intercept     0.457843
TV            0.002691
dtype: float64
```

Confidence Interval and Hypothesis Testing

We can call the function [conf_int](https://www.statsmodels.org/devel/generated/statsmodels.regression.linear_model.OLSResults (https://www.statsmodels.org/devel/generated/statsmodels.regression.linear_model.OLSResults) from the OLS results to get the 95% confidence interval of the model coefficients.

In [16]:
```
# TODO
est.conf_int( alpha = .05 )
```

Out[16]:

	0	1
Intercept	6.129719	7.935468
TV	0.042231	0.052843

As it relates to model coefficients, the conventional hypothesis test is as follows:

- Null hypothesis: There is no relationship between TV ads and Sales (that is, $\beta_1 = 0$)
- Alternative hypothesis: There is a relationship between TV ads and Sales (that is, $\beta_1 \neq 0$)

Intuitively, we reject the null hypothesis if the 95% confidence interval of β_1 does not include zero. We can check the p-value, the probability that the coefficient is actually zero, from the OLS results as well.

In [17]:
```
# TODO
est.pvalues
```

Out[17]:
```
Intercept     1.406300e-35
TV            1.467390e-42
dtype: float64
```

As the p-value for TV is less than 0.05, we reject the null hypothesis that there is no relationship between TV ads and Sales.

Note that the p-value for the intercept is generally not used.

R-squared statistic

The R^2 statistic provides an alternative measure of fit. It takes the form of a proportion—the proportion of variance explained—and so it always takes on a value between 0 and 1, and is independent of the scale of Y.

$$R^2 = 1 - \frac{RSS}{TSS}$$

The **Total Sum of Squares (TSS)** measures the total variance in the response Y, and can be thought of as the amount of variability inherent in the response before the regression is performed. In contrast, RSS measures the amount of variability that is left unexplained after performing the regression. An R^2 statistic that is close to 1 indicates that a large proportion of the variability in the response has been explained by the regression.

The OLS result already provides us with the R^2 value, and we can access it from its attributes.

In [18]:
```
# TODO
est.rsquared
```

Out[18]:
```
0.611875050850071
```

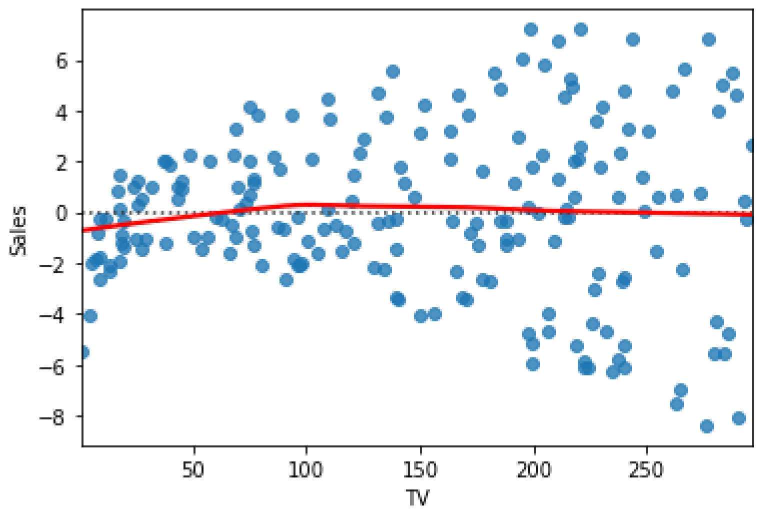
Now let us plot the residuals with respect to the predicted values.

For this plot, we can use `sns.residplot` (https://seaborn.pydata.org/generated/seaborn.residplot.html) which will regress y on x and then draw a scatterplot of the residuals. So the graph we will obtain will be a scatterplot for the residuals with the predictor variable.

We can set the `lowess` parameter to add a LOWESS (Locally Weighted Scatterplot Smoothing) curve, to help emphasize structure in the scatterplot.

In [19]:
```
# TODO
sns.residplot( x=advertising['TV'], y=advertising['Sales'], lowess=True, order=1, l
```

Out[19]:
```
<AxesSubplot:xlabel='TV', ylabel='Sales'>
```



Ideally, this should look like a random scatter plot with zero mean and constant variance for all predicted values. Our residual plot doesn't look like an ideal residual plot. This is because the mean line of the residuals is not close to zero for all values of $\hat{y}$ and the variance seems to be increasing with the predictor, TV. This means that our model assumptions are being violated.

Among many possible reasons for this violation, the following are usual suspects:

- There is another feature other than TV that affects sales.
- The relation between TV and sales is not linear.

We will implement the first fix using Multiple Linear Regression. But first let's break the model linearity assumption.

Polynomial Regression and Heteroscedasticity

As we see, the assumption of linearity between TV and sales might not hold. However, from the residual plot, we observe more clearly that the standard deviation of residual keeps on increasing as the magnitude of predicted response increases. This issue is called **heteroscedasticity**.

To address this issue, we could transform our response variable y with functions such as $\sqrt{y}$, $\log(y)$ or represent a new variable, TV^2.

To differentiate the arithmetic operators we want to perform on our variables (such as TV^2) from Patsy operators inside the formula, we can enclose the operation in Patsy's built-in identity function `I()` .

In [20]:
```
poly_est = smf.ols( 'np.log(Sales) ~ TV + I(TV**2)', data=advertising ).fit()
poly_est.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	np.log(Sales)	**R-squared:**	0.671
Model:	OLS	**Adj. R-squared:**	0.668
Method:	Least Squares	**F-statistic:**	201.1
Date:	Fri, 06 Oct 2023	**Prob (F-statistic):**	2.60e-48
Time:	15:24:18	**Log-Likelihood:**	4.1605
No. Observations:	200	**AIC:**	-2.321
Df Residuals:	197	**BIC:**	7.574
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.8039	0.049	37.091	0.000	1.708	1.900
TV	0.0082	0.001	10.431	0.000	0.007	0.010
I(TV ** 2)	-1.515e-05	2.62e-06	-5.773	0.000	-2.03e-05	-9.98e-06

Omnibus:	54.051	**Durbin-Watson:**	1.842
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	161.339
Skew:	-1.106	**Prob(JB):**	9.24e-36
Kurtosis:	6.804	**Cond. No.**	1.11e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.11e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

To see the values in the model:

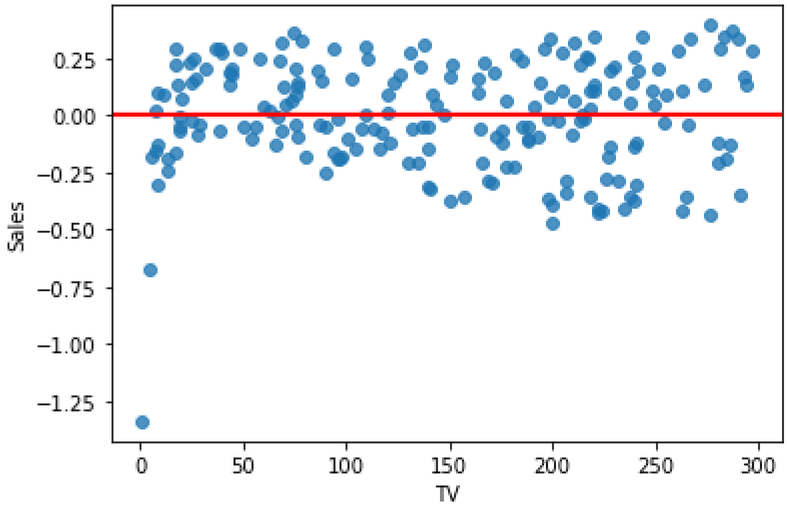
In [21]:
```
smf.ols( 'np.log(Sales) ~ TV + I(TV**2)', data=advertising ).exog[:5,]
```

Out[21]:
```
array([[1.000000e+00, 2.301000e+02, 5.294601e+04],
       [1.000000e+00, 4.450000e+01, 1.980250e+03],
       [1.000000e+00, 1.720000e+01, 2.958400e+02],
       [1.000000e+00, 1.515000e+02, 2.295225e+04],
       [1.000000e+00, 1.808000e+02, 3.268864e+04]])
```

Plotting the residuals, note that the order is 2.

In [24]:
```
sns.residplot( x=advertising["TV"], y=np.log(advertising['Sales'] ), lowess=True, o
```

Out[24]: <AxesSubplot:xlabel='TV', ylabel='Sales'>



Performing Model Prediction

In an earlier section, we manually calculated the sales prediction for a TV ad spending of $25,000. We can use the multiple linear regression model we have estimated to do the prediction for us.

NOTE: The sklearn library also has implementation of linear regression which can be used for such prediction. The API is the same one you are already familiar with - the `fit` , `transform` and `fit_transform` methods for the `sklearn.linear_model.LinearRegression` .

In [25]:
```
# Construct a DataFrame, which is the format expected by the statsmodels interface
X_test = pd.DataFrame( {'TV': [25]} )

est.predict( X_test )
```

Out[25]:
```
0    8.22101
dtype: float64
```

Linear models rely upon a lot of assumptions (such as the features being independent), and if those assumptions are violated (which they usually are), R-squared and p-values are less reliable. R-squared will always increase as we add more features to the model, even if they are unrelated to the response. Thus, selecting the model with the highest R-squared is not a reliable approach for choosing the best linear model. There is an alternative to R-squared called *adjusted R-squared* that penalizes model complexity (to control for overfitting).

In general, to select the best model to use for prediction, you might want to resort to the classical *cross validation*. This is easily possible using the `sklearn` API.

Multiple Linear Regression

When we have more than one predictor variables, we use multiple linear regression. It essentially gives each predictor a separate slope coefficient in a single model. So, if we want to analyze whether radio ads and newspaper ads are also associated with sales, we can do a multiple linear regression as:

$sales \approx \beta_0 + \beta_1 \times TV + \beta_2 \times radio + \beta_3 \times newspaper$

Let us first do a simple linear regression using `ols` on these features independently.

In [26]:
```
# TODO: Regress sales onto radio
est = smf.ols( formula='Sales ~ Radio', data=advertising ).fit()
est.summary().tables[1]
```

Out[26]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	9.3116	0.563	16.542	0.000	8.202	10.422
Radio	0.2025	0.020	9.921	0.000	0.162	0.243

In [27]:
```
# TODO: Regress sales onto newspaper
est = smf.ols( formula='Sales ~ Newspaper', data=advertising ).fit()
est.summary().tables[1]
```

Out[27]:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	12.3514	0.621	19.876	0.000	11.126	13.577
Newspaper	0.0547	0.017	3.300	0.001	0.022	0.087

From the simple linear regression, we can say that a 1,000-dollars increase in spending on radio advertising is associated with an average increase in sales by around 203 units, while the same increase in spending on newspaper advertising is associated with an average increase in sales by around 55 units.

Now let us use `ols` to create a model with all three features, that is, the union of (TV + radio + newspaper).

In [28]:
```
# TODO
mul_est = smf.ols( formula='Sales ~ TV + Radio + Newspaper', data=advertising ).fit(
mul_est.summary()
```

Out[28]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.897
Model:	OLS	**Adj. R-squared:**	0.896
Method:	Least Squares	**F-statistic:**	570.3
Date:	Fri, 06 Oct 2023	**Prob (F-statistic):**	1.58e-96
Time:	15:29:38	**Log-Likelihood:**	-386.18
No. Observations:	200	**AIC:**	780.4
Df Residuals:	196	**BIC:**	793.6
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2.9389	0.312	9.422	0.000	2.324	3.554
TV	0.0458	0.001	32.809	0.000	0.043	0.049
Radio	0.1885	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.0010	0.006	-0.177	0.860	-0.013	0.011

Omnibus:	60.414	**Durbin-Watson:**	2.084
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	151.241
Skew:	-1.327	**Prob(JB):**	1.44e-33
Kurtosis:	6.332	**Cond. No.**	454.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

We notice that the multiple regression coefficient estimates for TV and Radio are pretty similar to the simple linear regression coefficients. However, while the newspaper regression coefficient estimate in SLR was significantly non-zero, the coefficient estimate for newspaper in the multiple regression model is close to zero, and the corresponding p-value is no longer significant, with a value around 0.86.

Let us check the correlation matrix of the Advertising data.

In [ ]:
```
advertising.corr()
```

From the correlation matrix, we notice that the correlation between radio and newspaper is 0.35. This reveals a tendency to spend more on newspaper advertising in markets where more is spent on radio advertising.

Now suppose that the multiple regression is correct and newspaper advertising has no direct impact on sales, but radio advertising does increase sales. Then in markets where we spend more on radio our sales will tend to be higher, and as our correlation matrix shows, we also tend to spend more on newspaper advertising in those same markets. Hence, in a simple linear regression which only examines sales versus newspaper, we will observe that higher values of newspaper tend to be associated with higher values of sales, *even though*

F-statistic

The F value is the ratio of the mean regression sum of squares divided by the mean error sum of squares. Its value will range from zero to an arbitrarily large number. In multiple linear regression, the null hypothesis we look at is if there is a relationship between the response and predictor.

- $H_0 : \beta_1 = \beta_2 = \beta_3 = \ldots = 0$

We can assess the accuracy of the multiple regression model.

In [29]:
```
print( "RSE:", mul_est.resid.std() )
print( "R squared:", mul_est.rsquared )
print( "F-statistic:", mul_est.fvalue )
print( "F-test pvalue:", mul_est.f_pvalue )
```

```
RSE: 1.6727572743844112
R squared: 0.8972106381789522
F-statistic: 570.2707036590942
F-test pvalue: 1.575227256092437e-96
```

The large F-statistic suggests that *at least one of the advertising media must be related to sales* (at least one is non-zero). We see that the probability of the f-statistic is close to zero, so we have extremely strong evidence that at least one of the media is associated with increased sales.

Suppose our null hypothesis is that specific coefficients are zero. The t-statistic for each predictor provide information about whether each individual predictor is related to the response, after adjusting for the other predictors.

Residual Plots for the Multiple Linear Regression

Let us again plot the residual plots to visualize the trend in residuals as we did for simple linear regression.

As the `residplot` function is not designed for multiple linear regression, but only for simple linear regression, we will write a helper function `get_vis_dataframe` to get predictions from the model to plot.

We make use of the function [`wls_prediction_std` (https://www.adamsmith.haus/python/docs/statsmodels.graphics.regressionplots.wls_prediction](https://www.adamsmith.haus/python/docs/statsmodels.graphics.regressionplots.wls_prediction) from `statsmodels` sandbox module, which takes the regression result object and provides the lower and upper values within which the prediction will lie with 95% confidence.

In [30]:
```
from statsmodels.sandbox.regression.predstd import wls_prediction_std

def get_vis_dataframe( est, X, y, ylabel='yobs', yhlabel='ypred' ):

    # Prepare the dataframe: copy the predictors
    rvis = X.copy()

    # Observed response and predicted response
    rvis[ylabel], rvis[yhlabel] = y, est.predict( X )

    # Residuals obtained from OLS
    rvis['resid'] = est.resid

    # Obtain lower und upper confidence bounds using wls_prediction_std (standard er
    std, upper, lower = wls_prediction_std( est )
    rvis['upper'], rvis['lower'] = upper, lower

    return rvis
```

Applying it on the three features:

In [31]:
```
X = advertising[["TV", "Radio", "Newspaper"]]
y = advertising[["Sales"]]

mulvis = get_vis_dataframe( mul_est, X, y )
mulvis.head()
```

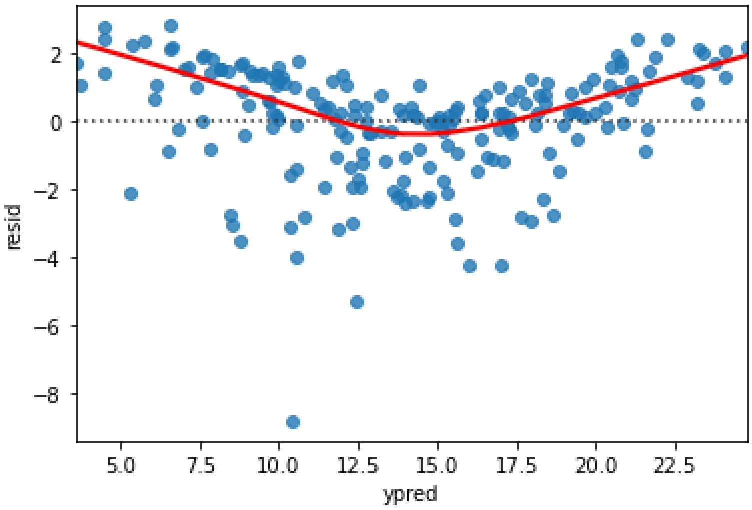
Out[31]:

	TV	Radio	Newspaper	yobs	ypred	resid	upper	lower
1	230.1	37.8	69.2	22.1	20.523974	1.576026	17.158283	23.889666
2	44.5	39.3	45.1	10.4	12.337855	-1.937855	8.981672	15.694038
3	17.2	45.9	69.3	9.3	12.307671	-3.007671	8.919038	15.696303
4	151.5	41.3	58.5	18.5	17.597830	0.902170	14.246273	20.949386
5	180.8	10.8	58.4	12.9	13.188672	-0.288672	9.825762	16.551582

Now using `residplot` to plot the residuals:

In [33]:
```
sns.residplot( x=mulvis["ypred"], y=mulvis.resid, lowess=True, order=1, line_kws=di
```

Out[33]: <AxesSubplot:xlabel='ypred', ylabel='resid'>



Multiple Polynomial Regression

Lastly, let us add polynomial terms to the multiple linear regression. We use a loop to construct the polynomial terms over the three features.

In [34]:
```
predictors = ["TV", "Radio", "Newspaper"]

poly2_predictors = []
for p in predictors:
    poly2_predictors.append( '{} + I({}**2)'.format(p, p) )

print( poly2_predictors )
```

```
['TV + I(TV**2)', 'Radio + I(Radio**2)', 'Newspaper + I(Newspaper**2)']
```

We can then construct the formula to pass to `ols` .

In [35]:
```
poly2_formula = 'Sales ~ '+ ' + '.join( poly2_predictors )
poly2_formula
```

Out[35]: 'Sales ~ TV + I(TV**2) + Radio + I(Radio**2) + Newspaper + I(Newspaper**2)'

Now run `ols` with this formula.

In [36]:
```
# TODO
poly_mul_est = smf.ols( formula=poly2_formula, data=advertising ).fit()
poly_mul_est.summary()
```

Out[36]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.918
Model:	OLS	**Adj. R-squared:**	0.915
Method:	Least Squares	**F-statistic:**	358.1
Date:	Fri, 06 Oct 2023	**Prob (F-statistic):**	1.01e-101
Time:	15:34:45	**Log-Likelihood:**	-364.09
No. Observations:	200	**AIC:**	742.2
Df Residuals:	193	**BIC:**	765.3
Df Model:	6		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	1.4111	0.457	3.085	0.002	0.509	2.313
TV	0.0781	0.005	15.497	0.000	0.068	0.088
I(TV ** 2)	-0.0001	1.7e-05	-6.619	0.000	-0.000	-7.89e-05
Radio	0.1595	0.028	5.611	0.000	0.103	0.216
I(Radio ** 2)	0.0007	0.001	1.219	0.224	-0.000	0.002
Newspaper	0.0101	0.015	0.693	0.489	-0.019	0.039
I(Newspaper ** 2)	-0.0001	0.000	-0.717	0.474	-0.000	0.000

Omnibus:	24.915	**Durbin-Watson:**	2.123
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	51.506
Skew:	-0.592	**Prob(JB):**	6.54e-12
Kurtosis:	5.186	**Cond. No.**	1.64e+05

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.64e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

We can again use our `get_vis_dataframe` function and `residplot` to plot the residual plot of the multiple polynomial regression.

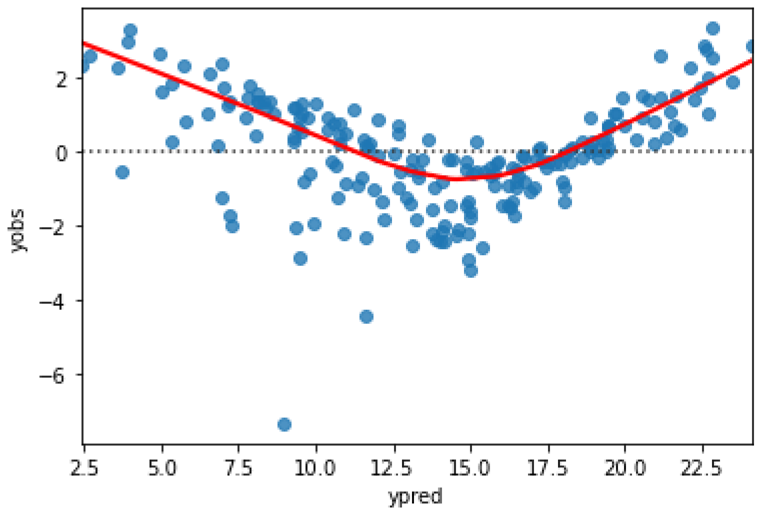
In [37]:
```
# TODO
mulvis = get_vis_dataframe( poly_mul_est, X, y )
mulvis.head()
```

Out[37]:

	TV	Radio	Newspaper	yobs	ypred	resid	upper	lower
1	230.1	37.8	69.2	22.1	20.577400	1.522600	17.537951	23.616849
2	44.5	39.3	45.1	10.4	12.223167	-1.823167	9.187231	15.259104
3	17.2	45.9	69.3	9.3	11.633162	-2.333162	8.553412	14.712912
4	151.5	41.3	58.5	18.5	18.620470	-0.120470	15.581589	21.659351
5	180.8	10.8	58.4	12.9	13.835436	-0.935436	10.792764	16.878108

In [38]:
```
# TODO
sns.residplot( x=mulvis["ypred"], y=mulvis["yobs"], lowess=True, line_kws=dict(colo
```

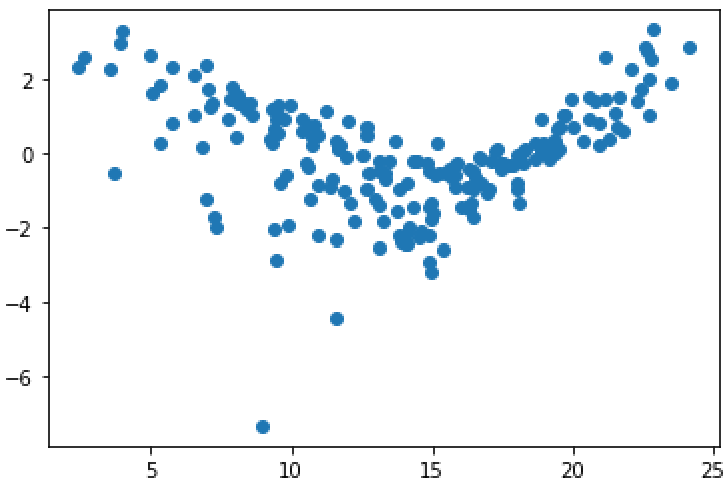
Out[38]: <AxesSubplot:xlabel='ypred', ylabel='yobs'>



This plot is equivalent to:

In [39]:
```
plt.scatter( mulvis.ypred, mulvis.resid )
```

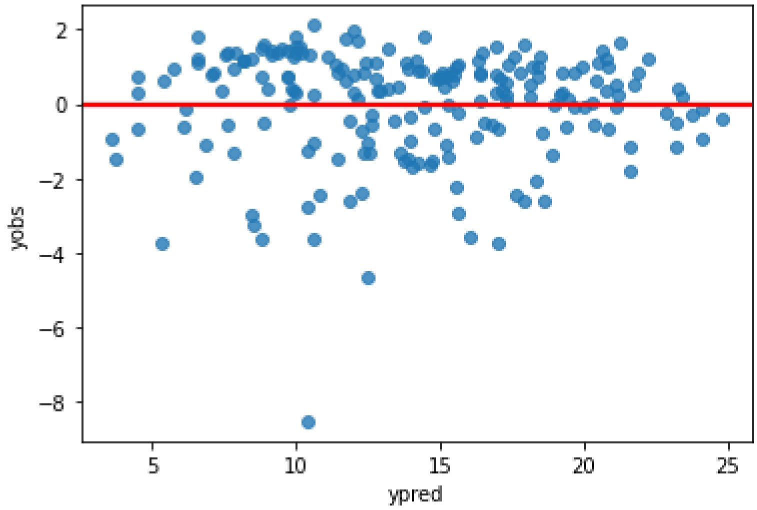
Out[39]: <matplotlib.collections.PathCollection at 0x27c038ef9a0>



We also plot the residual plot of the multiple linear regression. Will the residual plot with `order = 2` be the same as that of the polynomial regression above?

In [40]:
```
mulvis = get_vis_dataframe( mul_est, X, y )
sns.residplot( x=mulvis["ypred"], y=mulvis["yobs"], lowess=True, order=2, line_kws=
```

Out[40]: <AxesSubplot:xlabel='ypred', ylabel='yobs'>



This plot is equivalent to:

In [ ]:
```
values = mulvis.resid - np.polyval( np.polyfit(mulvis.ypred, mulvis.resid, 2), mulvi

plt.scatter( mulvis.ypred, values )
```