Wireless Netw (2014) 20:681–694
DOI 10.1007/s11276-013-0630-9

Power efficient range-free localization algorithm for wireless sensor networks

Shrawan Kumar · D. K. Lobiyal

Published online: 31 August 2013

Abstract Considering energy consumption, hardware requirements, and the need of high localization accuracy, we proposed a power efficient range-free localization algorithm for wireless sensor networks. In the proposed algorithm, anchor node communicates to unknown nodes only one time by which anchor nodes inform about their coordinates to unknown nodes. By calculating hop-size of anchor nodes at unknown nodes one complete communication between anchor node and unknown node is eliminated which drastically reduce the energy consumption of nodes. Further, unknown node refines estimated hop-size for better estimation of distance from the anchor nodes. Moreover, using average hop-size of anchor nodes, unknown node calculates distance from all anchor nodes. To reduce error propagation, involved in solving for location of unknown node, a new procedure is adopted. Further, unknown node upgrades its location by exploiting the obtained information in solving the system of equations. In mathematical analysis we prove that proposed algorithm has lesser propagation error than distance vector-hop (DV-Hop) and other considered improved DV-Hop algorithms. Simulation experiments show that our proposed algorithm has better localization performance, and is more computationally efficient than DV-Hop and other compared improved DV-Hop algorithms.

S. Kumar (✉) · D. K. Lobiyal
School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi 110067, India
e-mail: shrawankatiyar@gmail.com

D. K. Lobiyal
e-mail: lobiyal@gmail.com

Keywords Wireless sensor network · Localization · Error propagation · Power efficiency · Communication cost

1 Introduction

Wireless sensor network (WSN) is a group of sensor nodes which form multi-hop, self-configured network by wireless communication. In many applications of WSNs, such as target tracking [1], remote control of dangerous region, reconnaissance [2], unapproachable places monitoring [3], geographic routing protocols [4] and so forth, location knowledge of sensor nodes is necessary. Since, gathered information is meaningless without knowing the location from where the information is obtained. This makes localization a significant issue in WSNs researches. Global positioning system (GPS) [5] is the easiest way of determining location of nodes. But, GPS equipped sensor nodes are costlier and consume more energy which is a constraint of WSNs. It also has problem of line-of-sight. Therefore, GPS is infeasible for resource starved large scale WSNs.

Under the constraint of less hardware requirement and low energy consumption, many localization algorithms were developed. These localization algorithms are divided into two categories: range-based and range-free [6–8]. Range-based algorithms use absolute point to point distance estimates or angle estimates for location estimation [9, 10]. Range-free localization algorithms are based on nodes connectivity and hop information. Some range-free approaches are approximate point-in triangle test (APIT) [11], distance vector-hop (DV-Hop) [6], Centroid [12], Amorphous [13] etc. Localization accuracy of range-free algorithms is lower than range-based approaches, but these are easier to implement with less hardware requirement. In the above range-free algorithms, DV-Hop localization algorithm is the most popular because of its facility, feasibility, and good coverage quality. Major drawback of DV-Hop algorithm is its poor location accuracy. Therefore, in designing of localization algorithms two major elements energy efficiency and localization accuracy are considered.

Considering energy constraints and the need of high location accuracy, in this paper we proposed a power efficient range-free localization algorithm for wireless sensor networks (PERLA). The proposed algorithm has three steps. In the Step 1, by communicating to the unknown nodes, anchor node informs to unknown nodes about their coordinates and minimum number of hops from every anchor node. In Step 2, by calculating hop-size of anchor nodes at unknown nodes one complete communication between anchor node and unknown node is eliminated which drastically reduce the energy consumption of nodes. Further, unknown node refines estimated hop-size for better estimation of distance from the anchor nodes. Moreover, using average hop-size of anchor nodes, unknown node calculates distance from all anchor nodes. In Step 3, to reduce error propagation, involved in solving for location of unknown node, smallest distance equation of nearest anchor node is divided by every distance equation one at a time and using method of least square unknown node estimates its location. Further unknown node upgrades its location by exploiting the obtained information in solving the system of equations. In mathematical analysis we prove that proposed algorithm has lesser propagation error than DV-Hop and other improved DV-Hop algorithms. Simulation experiments show that PERLA has better localization performance, and is more computationally efficient than DV-Hop and other improved DV-Hop algorithms in all considered scenarios.

The rest of the paper is organized as follows: Sect. 2 describes related work. Our proposed algorithm power efficient range-free localization algorithm has been described in Sect. 3. Section 4 discusses mathematical analysis of error propagation. In Sect. 5, the simulation results are discussed for localization performances and power efficiency of the algorithms. Finally in Sect. 6, we conclude the paper.

2 Related work

Many range-free localization algorithms have been proposed in the literature, in which DV-Hop localization algorithm received more attention of researchers because of its easy implementation, feasibility, and less hardware requirements.

2.1 DV-Hop algorithm

In the DV-Hop algorithm, location of unknown nodes (sensor nodes that do not know its location) is calculated by using some anchor nodes (nodes that know their location by GPS). Readers interested in details of DV-Hop algorithm are advised to refer [6]. DV-Hop algorithm consists of three steps. In the Step 1, each anchor node broadcasts its location information (coordinate) and hop count value initialized by 1, this ensures that all nodes in the network obtain the coordinates of the anchor nodes and minimum hop count from every anchor nodes. In Step 2, each anchor node calculates the average size of one hop (hop-size) and floods it in the controlled manner. An unknown node receives the hop-size of the nearest anchor node and calculates the distance from anchor nodes. In the Step 3, reformulate distance equations into linear form and least squares method is used to estimate unknown node's location.

2.2 Error analysis of DV-Hop algorithm

In the DV-Hop algorithm, it is assumed that the minimum hop path between nodes is similar to a straight line, but in practical applications, it is not so. The communication range of each node in the network is not a standard circle ideally because it is anomalistic polygon as shown in Fig. 1. This is associated with the influence of network topology that makes every hop distance much different from others. If the average distance of per hop is used to estimate the distance between an anchor node and an unknown node, the estimated distance is different from the true distance. The difference between the estimated and true distance is called ranging error, which is the cause of poor localization [14]. Example 1 explains the generating process of ranging error [15, 16].

Example 1 In Fig. 2, $A1$, $A2$, and $A3$ are anchor nodes. U is the unknown node that needs to be localized. These anchor nodes know the distance from each other; in Fig. 2, the distance is 40, 30, and 30. The number of hops between U and $A1$ is 1, between U and $A2$ is 3, and between U and

Fig. 1 Irregular radio pattern of sensor

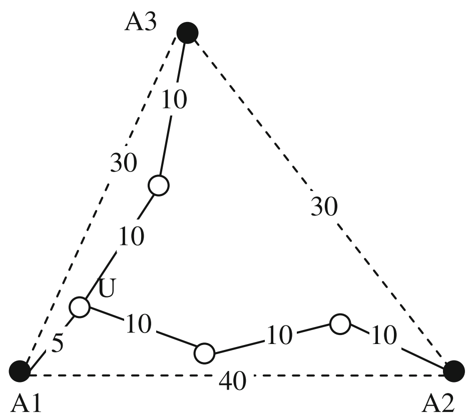

Fig. 2 Error analysis of DV-Hop algorithm

$A3$ is 2. The distance between $A1$ and U is 5. Let the length of other each hop is 10. In DV-Hop algorithm, anchor nodes $A1$, $A2$, and $A3$ calculate the average distance of per hop (hop-size) as follows:

$$A1: \quad (40+30)/(4+3) = 10;$$
$$A2: \quad (40+30)/(4+5) = 7.77;$$
$$A3: \quad (30+30)/(3+5) = 7.5;$$

After calculating the hop-size, anchor nodes broadcast it in the network. The unknown node saves the first received value of hop-size. In Example 1, anchor nodes $A1$, $A2$, and $A3$ will broadcast their hop-size 10, 7.77, and 7.5, respectively. The unknown node U is only one hop away from the anchor node $A1$; therefore, it will receives the first message from $A1$. The hop-size of the unknown node U is 10. The unknown node U estimates the distance from anchor nodes. The distance between unknown node U and $A1$ is 10, between U and $A2$ is 30, and between U and $A3$ is 20. The true distance between U and $A1$ is 5, whereas the estimated distance is 10. Then ranging error for $A1$ is $10 - 5 = 5$, which is 50 % of estimated distance. Due to this error, estimated location of unknown node U will be erroneous.

On the basis of the above analysis, we conclude that the localization accuracy of the DV-Hop algorithm is affected by ranging errors.

Therefore, various improvements such as [17–24], etc. are proposed to improve location accuracy of DV-hop algorithm.

2.3 Existing improved DV-Hop algorithm

Bao et al. [18] proposed an improved DV-Hop localization algorithm for WSNs. In the simulation, we compare this algorithm to our proposed algorithm. Hence, for simplicity we call it by EIDV-Hop2. In this algorithm, firstly, anchor nodes modify their estimated hop-size (one hop distance) by reducing the error in hop-size. Secondly, unknown node refines its hop-size by averaging the hop-size of all anchor nodes and using weight of the received hop-size of anchor nodes. Finally, by using multilateration unknown node estimates its location. Localization accuracy of this algorithm is better than DV-Hop, but on receiving and forwarding beacon packets of every anchor nodes (containing hop-size) by the unknown node, extra communication and computation effort are required. In doing this additional effort, node consumes more energy which is a constraint of WSNs.

Ying et al. [19] proposed an improvement of DV-Hop algorithm, in which, localization accuracy of DV-Hop algorithm is enhanced by three different methods. Firstly, using DV-Hop method anchor node localize the unknown nodes which are one hop away from anchor node then these localized unknown nodes work like anchor nodes and help in localizing to remaining unknown nodes. Secondly, if unknown node is more than one hop away from anchor nodes, unknown node estimates the distance from anchor nodes by multiplying hop-size and number of hops. In this distance estimation number of hops is 1 less than real hop's number and in estimated distance, last hop distance is added separately. Location of unknown nodes is estimated by trilateration. Finally, in the last third method, using Max–Min method unknown node is localized. All these three method enhance the localization accuracy but communications between nodes also increases. Due to this increased communication life of nodes in the network decreased.

In [20], if unknown node is one hop away from anchor node it calculates distance from anchor node by Received Signal Strength Indicator (RSSI) technique; Otherwise distance is estimated by DV-Hop method. Finally, using triangulation method unknown node estimates it location. Use of RSSI for distance estimation requires additional hardware which increases the cost of network that is main drawback of this algorithm.

In [22], accuracy of DV-Hop is improved by two ways. First, unknown node refines the hop-size by taking the weighted mean of hop-size of anchor nodes. Second, anchor node refines hop-size by using angle information between anchor node and unknown node. Drawback of this algorithm is that unknown node receives and forwards the hop-size information of at least three anchor nodes. Anchor and unknown node communicate to each other many times for calculating the angle between them. More communication between anchor and unknown nodes require more energy.

In [23], improved DV-Hop localization method for WSNs is similar to DV-Hop method. We compare this algorithm to our proposed algorithm. Therefore, for easiness we call it by EIDV-Hop1 through out the paper. In the algorithm, to improve the localization accuracy of DV-Hop algorithm a correction term is added in the estimated distance between anchor node and unknown node. The improvement in the localization accuracy of this method is very little which does not satisfied the requirement of many applications.

Chen et al. [24] proposed Improved DV-Hop Node Localization Algorithm in WSNs. We make comparison of this algorithm to our proposed algorithm; therefore, for simplicity we call it EIDV-Hop3. This algorithm has four steps. In the Step 1, some anchor nodes are deployed at the border of the monitoring area. In the Step 2, anchor nodes modify their estimated hop-size and broadcast it in the network. On receiving the modified hop-size, unknown nodes again modify the hop-size for themselves. In the Step 3, each unknown node calculates distance from anchor nodes, and estimates its location using 2-D Hyperbolic location algorithm [25]. In Step 4, particle swarm optimization algorithm is used to refine the location of unknown nodes as estimated in Step 3.

Main drawback of EIDV-Hop3 algorithm is that it requires special effort for the deployment of some anchor nodes at the boundary of monitoring area. In refining the location of unknown nodes, use of particle swarm optimization algorithm make heavy computation for weak processor of sensor nodes and utilize more energy that increases computational complexity.

3 Power efficient range-free localization algorithm (PERLA)

In DV-Hop and EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3 algorithms, anchor node floods packets two times. At first, node broadcasts their coordinates and then floods their hop-size contortedly. In the proposed algorithm, PERLA, anchor node floods packet just once, containing their coordinate. Hop-size of the anchor nodes is calculated at the unknown node. Therefore, PERLA eliminate complete one communication between anchor nodes and unknown nodes. Unknown node refines the hop-size of anchor nodes and by using average hop-size of anchor nodes estimates distance from the anchor nodes. To reduce the propagation of error in calculation and improving the localization accuracy, divided the distance equation of nearest anchor node by remaining distance equations one at a time. Finally, unknown node upgrades its location by exploiting the obtained information from the system of equations. The PERLA consist of three steps:

Step 1 of proposed algorithm is like the Step 1 of DV-Hop algorithm. In this step, each anchor node broadcasts a beacon packet with its location (coordinate) and hop count value initialized to 1. When a node receives beacon packets, it maintains a table (x_i, y_i, hop_i) for every anchor nodes, where (x_i, y_i) is the coordinate of anchor node i and hop_i is the minimum number of hops from anchor node i. If a received packet contains lesser hop count value to a particular anchor node, hop count value of the table is replaced with hop count value of received packet and this packet is forwarded in the network with increased hop count value by 1; otherwise this packet is discarded. By this mechanism, all unknown nodes in the network obtain minimum hop count value from every anchor node and coordinates of anchor nodes.

Step 2: unknown nodes obtained coordinates of anchor nodes from the Step 1. Other information, for example hop-size of anchor nodes which are available to other algorithms by the virtue of extra communication is not available to unknown node in the proposed algorithm. Using available limited information of anchor nodes, unknown node has to calculate hop-size of anchor nodes. Further, since hop-size is always lesser or equal to the communication range (R), therefore it is intuitive to estimate number of hops as ceiling of ratio of Euclidean distance to R for each pair of anchor nodes using (1)

$$hops_{ij} = \left\lceil \frac{d_{ij}}{R} \right\rceil, \tag{1}$$

where $\lceil . \rceil$ is the ceiling function, d_{ij} is the distance between anchor nodes i and j, R is the communication radius of the sensor nodes.

After calculating the number of hops between anchor nodes by using (1), unknown node calculates the hop-size for the anchor nodes using the number of hops and distance information between anchor nodes. To make hop-size (one hop distance) close to actual hop-size, unknown node take the average of the ratio of distance and number of hops between anchor nodes. This hop-size of anchor node calculated by unknown node is reflected by several anchor nodes. Therefore, unknown node calculates hop-size of ith anchor node by (2)

$$HopSize_i = \frac{\sum_{i \neq j} (d_{ij}/hops_{ij})}{n}, \tag{2}$$

where $hops_{ij}$ is the number of hops between anchor nodes i and j, and n is the number of anchor nodes.

By the Step 1, unknown node knows the coordinate of anchor nodes. Therefore, true distance between anchor nodes i and j is $d_{ij}^{true} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, where ($x_i$, y_i) and (x_i, y_i) are the coordinate of anchor nodes i and j. The estimated distance between anchor node i and j is

$$d_{ij}^{est} = HopSize_i \times hops_{ij}$$

Therefore, error in distance between anchor nodes i and j is

$$e_{ij} = d_{ij}^{est} - d_{ij}^{true}$$

Hence, per hop error is $e_{ij}^{ph} = (d_{ij}^{est} - d_{ij}^{true})/hop_{ij}$.

Using per hop error, unknown node refines hop-size of anchor nodes by (3)

$$HopSize_i^{ref} = HopSize_i - e_{ij}^{ph}. \tag{3}$$

Further, unknown node calculates average hop-size of anchor nodes by (4)

$$HopSize_{avg} = \left(\sum_{i=1}^{n} HopSize_i^{ref} / n \right). \tag{4}$$

Once an unknown node calculates the average hop-size, it estimates its distance from anchor nodes by using average hop-size and minimum hop count from respective anchor node (which is obtained in the Step 1). Hence, unknown node estimates distance from ith anchor node by using (5). In this distance estimation, unknown node use average hop-size to obtain distance close to actual distance.

$$d_i = HopSize_{avg} \times hops_i, \tag{5}$$

where $hops_i$ is the minimum number of hops from ith anchor node.

In Step 3, unknown node estimates its location. Let location (coordinate) of unknown node p is (x, y), location of ith anchor node is (x_i, y_i), and the distance estimated by (5) between the anchor node i and unknown node p is d_i. Therefore, distance of unknown node p from all n anchor nodes is given by (6)

$$\left.\begin{array}{r} \sqrt{(x-x_1)^2 + (y-y_1)^2} = d_1 \\ \sqrt{(x-x_2)^2 + (y-y_2)^2} = d_2 \\ \vdots \\ \sqrt{(x-x_n)^2 + (y-y_n)^2} = d_n \end{array}\right\} \tag{6}$$

Divide the distance equation of nearest anchor node (whose d_i is the smallest) by remaining equations one at a time and simplifying, we obtain a system of $n-1$ equations. Let kth anchor node is the nearest anchor node to the unknown node p. Therefore, dividing kth equation of (6) by remaining equations one at a time.

$$\frac{(x-x_k)^2 + (y-y_k)^2}{(x-x_i)^2 + (y-y_i)^2} = \frac{d_k^2}{d_i^2}, \quad \forall \quad i = 1, 2 \ldots n \quad \text{and} \quad i \neq k \tag{7}$$

Simplifying (7), we obtain a system of $n-1$ equations

$$\left.\begin{array}{l} -2(x_k - x_1 d_{k,1})x - 2(y_k - y_1 d_{k,1})y + (1 - d_{k,1})s = d_{k,1}(x_1^2 + y_1^2) - (x_k^2 + y_k^2) \\ -2(x_k - x_2 d_{k,2})x - 2(y_k - y_2 d_{k,2})y + (1 - d_{k,2})s = d_{k,2}(x_2^2 + y_2^2) - (x_k^2 + y_k^2) \\ \vdots \\ -2(x_k - x_{n-1} d_{k,n-1})x - 2(y_k - y_{n-1} d_{k,n-1})y + (1 - d_{k,n-1})s = d_{k,n-1}(x_{n-1}^2 + y_{n-1}^2) - (x_k^2 + y_k^2) \end{array}\right\},$$

where $d_{k,i} = d_k^2/d_i^2$ and $s = x^2 + y^2$.

Its matrix form be

$$GZ = D, \tag{8}$$

where G, D and Z are defined as

$$G = \begin{bmatrix} -2(x_k - x_1 d_{k,1}) & -2(y_k - y_1 d_{k,1}) & (1 - d_{k,1}) \\ -2(x_k - x_2 d_{k,2}) & -2(y_k - y_2 d_{k,2}) & (1 - d_{k,2}) \\ \vdots & \vdots & \vdots \\ -2(x_k - x_{n-1} d_{k,n-1}) & -2(y_k - y_{n-1} d_{k,n-1}) & (1 - d_{k,n-1}) \end{bmatrix}, D = \begin{bmatrix} d_{k,1}(x_1^2 + y_1^2) - (x_k^2 + y_k^2) \\ d_{k,2}(x_2^2 + y_2^2) - (x_k^2 + y_k^2) \\ \vdots \\ d_{k,n-1}(x_{n-1}^2 + y_{n-1}^2) - (x_k^2 + y_k^2) \end{bmatrix} \quad \text{and} \quad Z = \begin{bmatrix} x \\ y \\ s \end{bmatrix}$$

Now solving the above system of equation (8) applying method of least square, we obtain

$$Z = (G'G)^{-1} G'D, \tag{9}$$

where G' is the transpose of G.

From (9), unknown nodes obtain their location.

3.1 Upgrading the location of unknown nodes

Though, unknown node location, obtained by (9) has good accuracy, but using the information ($s = x^2 + y^2$) obtained by (9), we propose to update unknown node location to improve its localization accuracy.

Let solving the equation (9), unknown node estimates its location (x_u, y_u) and together with it also obtains the value of $s = x^2 + y^2$. But, (x_u, y_u) does not satisfy $s = x^2 + y^2$, since estimated distance between anchor and unknown node has the error. Hence, unknown node upgrades its location by exploiting the information $s = x^2 + y^2$. In upgrading the location, unknown node will have twice weight of its estimated location (x_u, y_u) than calculated value of its location (x_s, y_s) using $s = x^2 + y^2$.

The estimated location of unknown node by (9) is (x_u, y_u) shown in Fig. 3, and $s = x^2 + y^2$.

Then, square root of the ratio of $s = x^2 + y^2$ and $x_u^2 + y_u^2$ is denoted by t and define as:

$$t = \sqrt{\frac{s}{x_u^2 + y_u^2}}$$

Hence, unknown node location (x_s, y_s) calculated by exploiting the information $s = x^2 + y^2$ is

$$\left.\begin{aligned} x_s &= t \times x_u \\ y_s &= t \times y_u \end{aligned}\right\} \tag{10}$$

Further, unknown node upgrades its location by taking the average of its locations (x_u, y_u) and (x_s, y_s) obtained by (9) and (10) respectively. In averaging, location (x_u, y_u) has twice weight than location (x_s, y_s).

Therefore, upgraded location of unknown node is obtained by (11)

$$\left.\begin{aligned} x &= \frac{2x_u + x_s}{3} \\ y &= \frac{2y_u + y_s}{3} \end{aligned}\right\} \tag{11}$$

In the Fig. 3, A is the estimated location of unknown node obtained by solving the equation (9), B and C are the calculated location of unknown node using the value of $s = x^2 + y^2$ when $s > x_u^2 + y_u^2$ and $s < x_u^2 + y_u^2$, respectively.

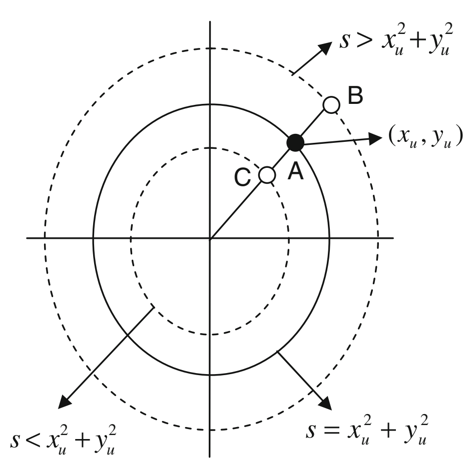

Fig. 3 Estimated Location of Unknown node

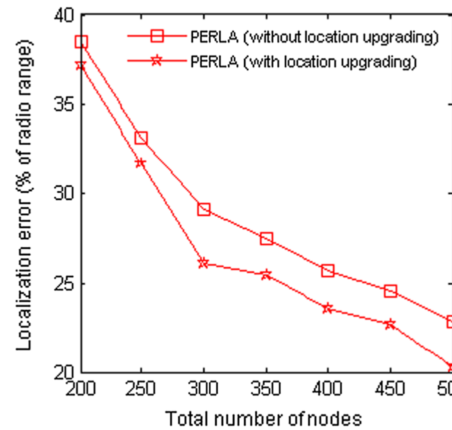

Fig. 4 Effect of location up-gradation of unknown node on localization error

In the Fig. 4, we show the effect of location up-gradation of unknown nodes on localization accuracy.

4 Mathematical analysis of error propagation

In the DV-Hop, EIDV-Hop1, and EIDV-Hop2 algorithms, in location estimation process of unknown node, each distance equation is subtracted by last equation, but in PERLA, distance equation of nearest anchor is divided by all distance equations one at a time. In this section, we prove mathematically that propagation of error for division operation (when numerator is lesser than denominator) is lesser than subtraction operation for two quantities. Further, we also prove that propagation of error in PERLA is lesser than EIDV-Hop3.

4.1 Propagation of error in subtraction and division operation of two measured quantities

Let us suppose that there are two measured values a and b $(a < b)$, with uncertainties (error) δa, and δb, respectively, and let result of their difference is c.

Therefore, $c = a - b$.

Let error in c is δc, then by [26], propagation of error δc in c, the difference of a and b is

$$\delta c = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{12}$$

Let for the same measured value a and b, with same uncertainties (error) δa, and δb, respectively, their division is denoted by $c1$.

Therefore, $c1 = a/b$.

Let error in $c1$ is $\delta c1$, then by [26], propagation of error $\delta c1$ in division $c1$ of a and b is

$$\delta c1 = |c1|\sqrt{(\delta a/|a|)^2 + (\delta b/|b|)^2} \tag{13}$$

Now, we prove that propagation of error $\delta c1$ (due to division operation) is lesser than propagation of error δc (due to subtraction operation).

i.e. to prove $\delta c1 < \delta c$.

Proof In our case, $a < b$, $a > 1$ and $b > 1$ (in our case a and b are the distance between unknown node and anchor nodes which is nearly equal to communication radius (R) that is far greater than 1).

Therefore,

$$\frac{\delta a}{|a|} < \delta a \tag{14}$$

And,

$$\frac{\delta b}{|b|} < \delta b \tag{15}$$

Now by (14) and (15)

$$\frac{(\delta a)^2}{|a|^2} + \frac{(\delta b)^2}{|b|^2} < (\delta a)^2 + (\delta b)^2,$$

using (12) and (13) we obtain

$$\frac{(\delta c1)^2}{|c1|^2} < (\delta c)^2$$
$$\Rightarrow \quad \delta c1 < |c1|(\delta c)$$

We know that

$$a < b \Rightarrow \frac{a}{b} < 1$$
$$\Rightarrow c1 < 1$$
$$\Rightarrow |c1|(\delta c) < \delta c$$

Thus,

$$\delta c1 < |c1|(\delta c) < \delta c$$
$$\Rightarrow \delta c1 < \delta c$$

Hence proved.

This proof shows that propagation of error in our proposed method is lesser than propagation of error in DV-Hop, EIDV-Hop1, and EIDV-Hop2 algorithms.

Now we prove that propagation of error in PERLA is lesser than EIDV-Hop3. In EIDV-Hop3, location of unknown nodes is estimated by 2-D Hyperbolic location method in which no one distance equation is subtracted by any distance equation like DV-Hop, EIDV-Hop1, and EIDV-Hop2. Therefore, in EIDV-Hop3, error in distance equations propagates as is.

As we assumed above that error in a and b ($a < b$) is δa, and δb, respectively. In our case a and b are the estimated distance between unknown node and anchor nodes. This estimated distances are always greater than $\sqrt{2}$ because distances between unknown node and anchor nodes are greater than or equal to one hop distance which is nearly equal to R; therefore, $b > \sqrt{2}$. In EIDV-Hop3, there are no intermediate operations, therefore error δa of a propagates in the system as is but in PERLA error $\delta c1$ given in (13) propagates in the system. Hence we prove that $\delta c1 < \delta a$ to show that propagation of error in PERLA is lesser than EIDV-Hop3.

Proof $\left(\frac{\delta a}{a}\right)^2$ and $\left(\frac{\delta b}{b}\right)^2$ are very small quantity.

Let us suppose that $\left(\frac{\delta a}{a}\right)^2 \approx \left(\frac{\delta b}{b}\right)^2$

By (13) $\delta c1 = |c1|\sqrt{(\delta a/|a|)^2 + (\delta b/|b|)^2}$

$$\Rightarrow \delta c1 = |c1|\sqrt{2(\delta a/|a|)^2}$$
$$\Rightarrow \delta c1 = \sqrt{2}|a/b|(\delta a/|a|)$$
$$\Rightarrow \delta c1 = \sqrt{2}(\delta a/|b|) < \delta a \text{ as } b > \sqrt{2}$$
$$\Rightarrow \delta c1 < \delta a$$

Hence proved.

5 Simulation experiment

This section provides simulation results and analysis of the results for localization error and computational cost. In order to verify performance of proposed algorithm PERLA against other algorithms viz. DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3, simulations are conducted on MATLAB 2008b. The simulations of all the algorithms including our proposed algorithm run on 100 randomly generated sensor node deployment scenarios and the average values are used for comparison.

In all the experiments, the experimental region is assumed to be square area of fixed size 100 m x 100 m. The sensor nodes are randomly distributed in the region. Each node (unknown or anchor) has the same communication radius.

In the simulation, the localization error is defined as the Average error function illustrated in (16).

$$Localization\,Error\,(LE) = \frac{\sum_{i=n+1}^{N}\sqrt{(x_i^e - x_i^a)^2 + (y_i^e - y_i^a)^2}}{R \times (N-n)}, \tag{16}$$

where (x_i^a, y_i^a) is the true coordinate of the unknown node i, the estimated coordinate of the unknown node i is (x_i^e, y_i^e), R is the communication radius of the sensor nodes, N is the total number of the nodes in the sensor field and n is the number of anchor nodes.

In the Sect. 3.1, we upgraded the location of unknown nodes to increase the localization accuracy. Therefore, to show the effect of location up-gradation of unknown node on localization error, we conduct ‘Experiment 1’ which is given below.

It can be noticed from (16) that number of unknown nodes, number of anchor nodes and communication radius of sensor nodes affects localization error. Hence, simulation experiments Experiment 2, Experiment 3, and Experiment 4 are conducted to analyze the behavior of localization error.

- Experiment 1: Effect of location up-gradation of unknown node on localization error
- Experiment 2: Effect of total number of nodes
- Experiment 3: Effect of percentage of anchor nodes
- Experiment 4: Effect of communication radius of sensor nodes.

In the real scenario, radio signals of sensor nodes are affected by environment. Therefore, communicating radius of the sensor nodes do not make standard circle, it is an anomalous polygon. In each experiment, Experiment 2, Experiment 3, and Experiment 4, we consider three different scenarios of communication ranging error 0–10 %, 0–20 % and 0–30 % to evaluate and compare the performance of PERLA with DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3. For all experiments, in EIDV-Hop3, the value of parameters for the Particle Swarm Optimization (PSO) algorithm are taken as $c1 = c2 = 2.05$, $w_{\max} = 0.9$, $w_{\min} = 0.4$, $iter_{\max} = 20$, the number of particles is 20, and $V_{\max} = 10$.

- Experiment 1: Effect of location up-gradation of unknown node on localization error

In this simulation experiment total number of node varies from 200 to 500, anchor nodes are 10 % of total nodes and communication radius is 15 m.

From Fig. 4 it is clear that on upgrading the location of unknown nodes localization error decreases on average 3 %. For each value of total number of nodes, localization error of PERLA after location up-gradation is lesser than before location up-gradation.

On applying for change in percentage of anchor nodes and change in communication range, localization error decreases in similar manner as in change in total number of nodes.

- Experiment 2: Effect of total number of nodes

In this experiment, total number of nodes varies from 200 to 500, for three different scenario of communication ranging error 0–10 %, 0–20 % and 0–30 %. Anchor nodes are 10 % of total number of nodes and communication radius is taken 15 m.

In the Fig. 5(a)–(c) we observed that as total number of nodes increases from 200 to 500, localization error of all the algorithms decreases. Localization error of PERLA is lesser than DV-hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3 for each value of total nodes. The decrement in localization error for PERLA is greater than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3 in all three cases of communication ranging errors 0–10 %, 0–20 % and 0–30 %. With increasing the communication ranging errors from 10 to 20 to 30 %, PERLA has very less effect for localization error than DV-Hop, EIDV-Hop1, and EIDV-Hop2.

In Fig. 5(a)–(c), the proposed algorithm PERLA has on average about 12, 13, and 19 % lesser localization error compared with DV-Hop and 8, 9, and 12 % lesser than EIDV-Hop1 and EIDV-Hop2 for communication ranging error 0–10 %, 0–20 %, and 0–30 %, respectively. While, the average localization error of PERLA is about 5, 5, and 6 % lesser compared with EIDV-Hop3. These results show that as the communication ranging error increases, the localization error of all algorithms also increases, but this increment is lesser for the proposed approaches as compared with other algorithms.

- Experiment 3: Effect of percentage of anchor nodes

In experiment 3, total number of nodes kept fixed 300. Anchor nodes are varies from 5 to 35 % for three different scenario of communication ranging error 0–10 %, 0–20 % and 0–30 %. Communication radius is taken 15 m.

Fig. 6(a)–(c), shows that as percentage of anchor nodes varies from 5 % to 35 %, localization error of algorithms DV-Hop, EIDV-Hop1, EIDV-Hop2, EIDV-Hop3, and PERLA decreases. The PERLA has lesser localization error than others for each ratio of anchor nodes. The decrement in localization error of PERLA is greater than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3 in all three cases of communication ranging errors 0–10 %, 0–20 % and 0–30 %. Proposed algorithm PERLA has lesser effect than DV-Hop, EIDV-Hop1, and EIDV-Hop2 for localization error with increasing the communication ranging errors from 10 to 20 to 30 %.

In Fig. 6(a)–(c), the proposed algorithm PERLA has on average about 14, 14 and 15 % lesser localization error compared with DV-Hop and 10, 11, and 12 % lesser than EIDV-Hop1 and EIDV-Hop2 for communication ranging error 0–10 %, 0–20 %, and 0–30 %, respectively. On the other hand, average localization error of proposed algorithm is about 7, 7, and 10 % lesser compared with EIDV-Hop3. These experiments show that as communication ranging error increases, the localization error of all algorithms also increases, but increment in the localization error is lesser for the proposed approach than other compared algorithms.

- Experiment 4: Effect of communication radius of sensor nodes

To study the effect of communication radius on localization error, we set total number of nodes 300 and anchor

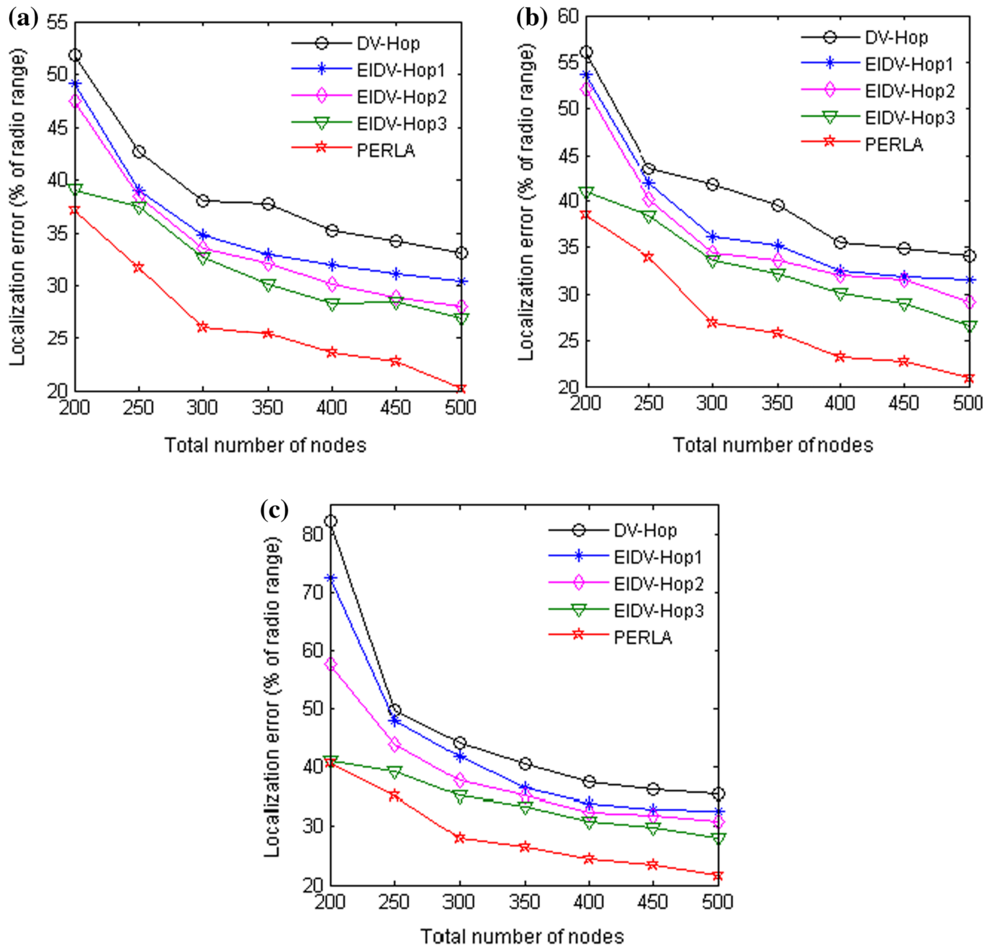

Fig. 5 Total number of nodes versus localization error with **a** 0–10 %, **b** 0–20 %, and **c** 0–30 % communication ranging error

nodes are 10 % of total nodes. Communication radius of sensor nodes changes from 15 to 40 m, for three different scenario of communication ranging error 0–10 %, 0–20 % and 0–30 %.

In Fig. 7(a)–(c), localization error of all the algorithms decreases with increasing communication radius of sensor nodes from 15 to 40 m. For each value of communication radius, localization error of the PERLA is lesser than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3. The proposed algorithm seems to be robust with variation in communication ranging error as with increase in the communication ranging errors from 10 to 20 to 30 %. PERLA has very less effect for localization error than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3.

The Fig. 7(a)–(c) show that proposed algorithm PERLA has on average about 8, 8 and 8 % lesser localization error than DV-Hop and 5, 6, and 5 % lesser than EIDV-Hop1 and EIDV-Hop2 for communication ranging error 0–10 %, 0–20 %, and 0–30 %, respectively. At the same time PERLA has on average about 2, 2, and 4 % lesser localization error than EIDV-Hop3.

5.1 Practical analysis on localization error

In this section, we analyze on localization error by conducting a simulation for estimating localization error distribution. The simulation run on 1,000 randomly generated sensor node deployment scenarios for all algorithms including PERLA and error distribution graph of localization error is constructed to analyze the distribution of localization error.

In this simulation experiment simulation parameters are taken as: total number of nodes is 250, anchor nodes ratio is 10 % of total nodes, and communication radius of sensor node is 15 m.

Figure 8 shows that for the given parameters, PERLA has most of its frequency of localization error in the 25–30, and 30–35 interval of localization error. While for DV-

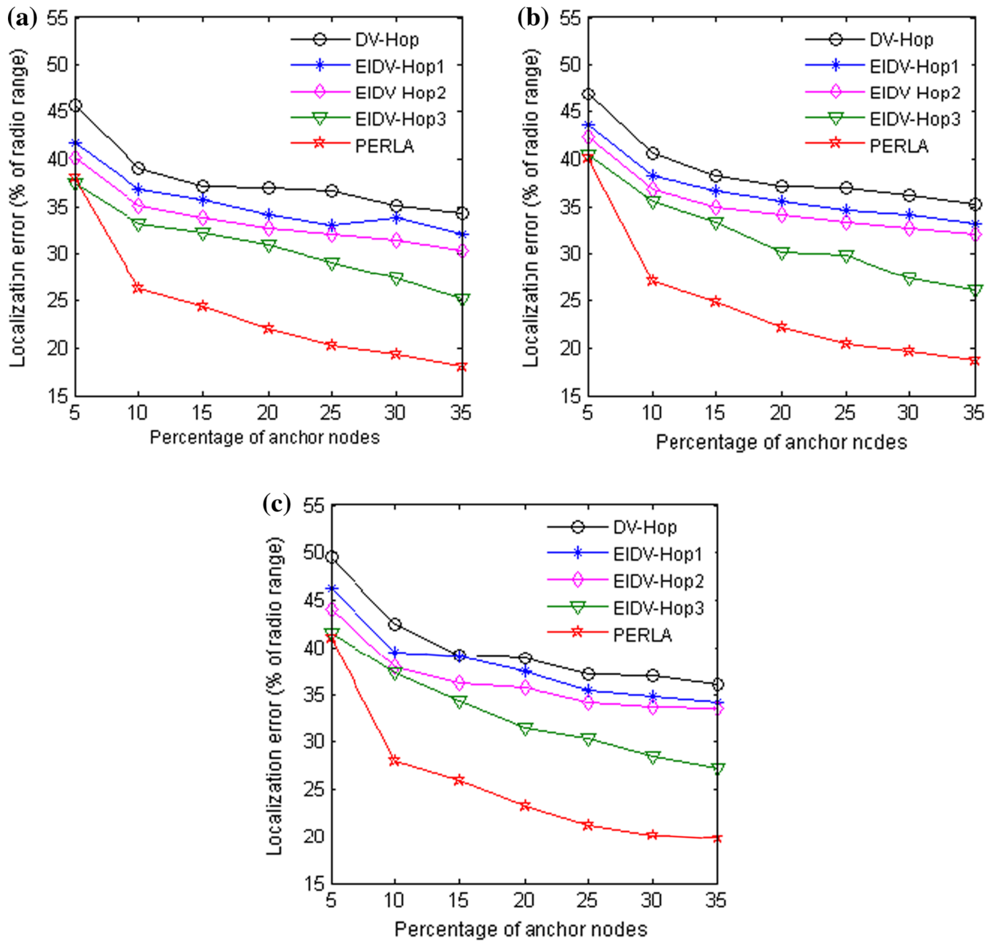

Fig. 6 Percentage of anchor nodes versus localization error with **a** 0–10 %, **b** 0–20 %, and **c** 0–30 % communication ranging error

Hop, EIDV-Hop1, and EIDV-Hop2 a large amount of frequency of localization error lies in the 35–40 and 40–45 interval of localization error. In EIDV-Hop3, the majority of localization error frequency lies the interval 30–35, and 35–40.

This analysis clearly states that PERLA has lesser localization error than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3.

5.2 Communication cost and computational efficiency

This section describes the communication cost and computational efficiency of the algorithms

5.2.1 Communication cost

The communication cost of the algorithms is represented by the number of transmitting and receiving packets by the nodes in the localization process. In the algorithms, DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3 anchor node communicates to unknown nodes one time in the Step 1 to broadcast its coordinate and one time in Step 2 to floods its calculated hop-size. In the Step 2 of DV-Hop, and EIDV-Hop1, unknown node receives and forwards only first arrived hop-size information while in EIDV-Hop2, and EIDV-Hop3 unknown node receives and forwards all arrived hop-size messages. In PERLA, hop-size of anchor nodes is calculated at unknown node. Therefore, in the Step 2 of PERLA communication is not required between anchor node and unknown node. Thus, PERLA is able to reduce one complete communication in the network between anchor node and unknown node. This reduction in the communication is a significant achievement in terms of energy which is a valuable constraint of resource starved WSNs. The given Table 1 shows that PERLA has lesser number of transmitted and received packets between nodes than DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3.

Let us suppose that total number of nodes (unknown and anchor nodes) is N, number of anchor nodes is n, and average connectivity of node is C_{avg}, the communication

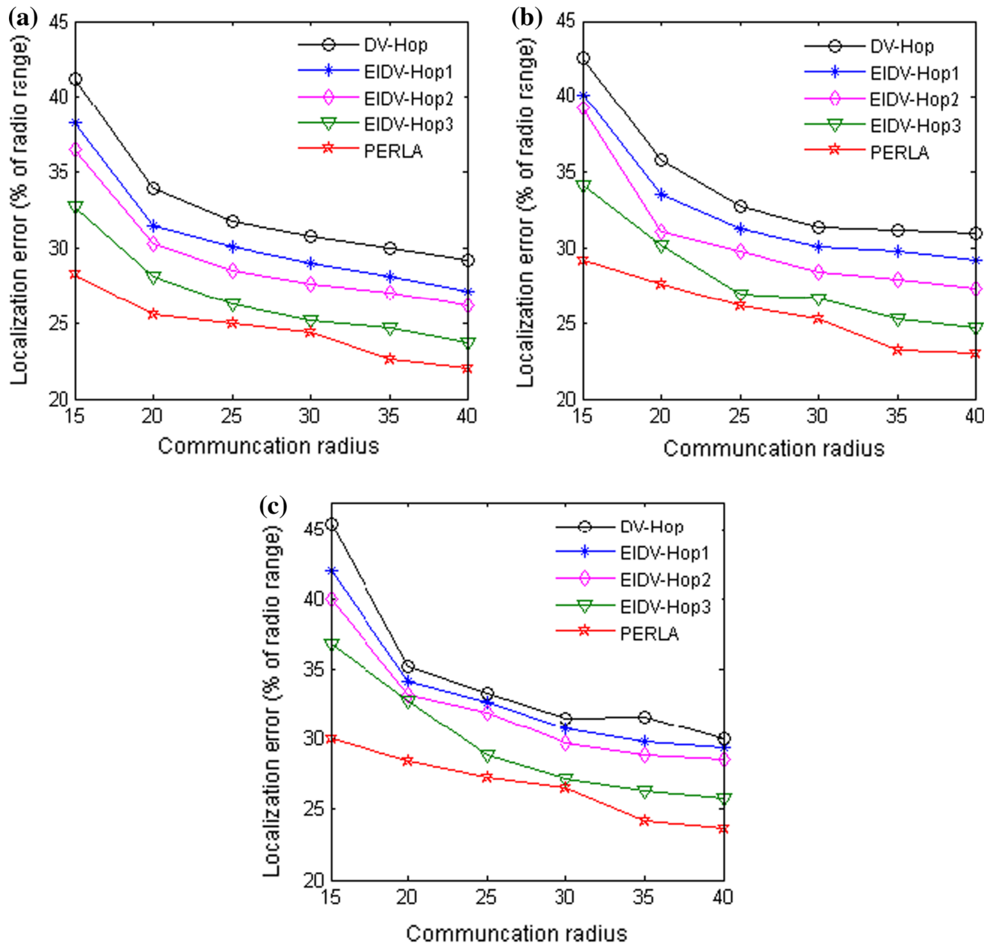

Fig. 7 Communication radius versus localization error with **a** 0–10 %, **b** 0–20 %, and **c** 0–30 % communication ranging error

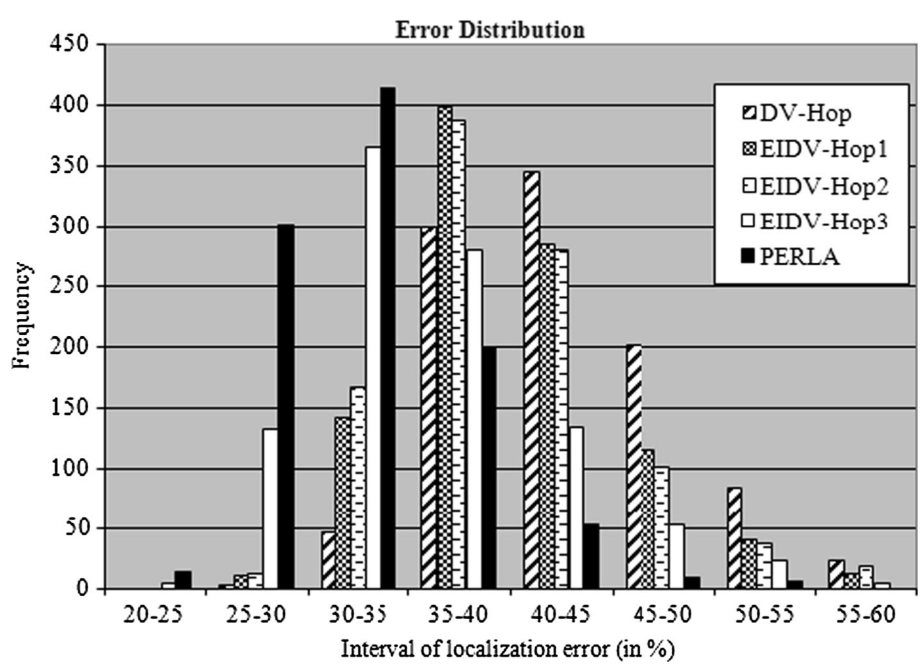

Fig. 8 Distribution of localization error

Table 1 Number of transmitted and received packets in the algorithms

Algorithm		Step 1	Step 2	Total number of transmitted and received packets (TNP)
DV-Hop	Broadcast packet	$N \times n$	N	$N \times n \times (1 + C_{avg}) - n \times C_{avg} + 2N - n$
	Received packet	$n \times (N-1) \times C_{avg}$	$N - n$	
	Total	$n \times [N + (N-1) \times C_{avg}]$	$2N - n$	
EIDV-Hop1	Broadcast packet	$N \times n$	N	$N \times n \times (1 + C_{avg}) - n \times C_{avg} + 2N - n$
	Received packet	$n \times (N-1) \times C_{avg}$	$N - n$	
	Total	$n \times [N + (N-1) \times C_{avg}]$	$2N - n$	
EIDV-Hop2	Broadcast packet	$N \times n$	$n + (N-n) \times n$	$N \times n \times (1 + C_{avg}) - n \times C_{avg} + n + (N-n) \times (n + C_{avg})$
	Received packet	$n \times (N-1) \times C_{avg}$	$(N-n) \times C_{avg}$	
	Total	$n \times [N + (N-1) \times C_{avg}]$	$n + (N-n) \times (n + C_{avg})$	
EIDV-Hop3	Broadcast packet	$N \times n$	$n + (N-n) \times n$	$N \times n \times (1 + C_{avg}) - n \times C_{avg} + n + (N-n) \times (n + C_{avg})$
	Received packet	$n \times (N-1) \times C_{avg}$	$(N-n) \times C_{avg}$	
	Total	$n \times [N + (N-1) \times C_{avg}]$	$n + (N-n) \times (n + C_{avg})$	
PERLA	Broadcast packet	$N \times n$	NIL	$N(n + n \times C_{avg}) - n \times C_{avg}$
	Received packet	$n \times (N-1) \times C_{avg}$	NIL	
	Total	$n \times [N + (N-1) \times C_{avg}]$	NIL	

Table 2 Localization Time (in second) of various algorithms for variation in total number of nodes (anchor nodes are 10 % of total nodes and communication radius is 15 m), bold figures shows minimum time

Total number of nodes	Localization algorithm				
	DV-Hop	EIDV-Hop1	EIDV-Hop2	EIDV-Hop3	PERLA
200	0.024	0.026	0.029	3.176	**0.022**
250	0.037	0.040	0.042	4.946	**0.031**
300	0.053	0.057	0.061	7.084	**0.043**
350	0.073	0.079	0.082	9.720	**0.058**
400	0.098	0.105	0.110	12.667	**0.077**
450	0.124	0.131	0.139	16.202	**0.095**
500	0.155	0.165	0.174	20.133	**0.119**

cost of algorithms in terms of transmitted and received packets is given in Table 1.

5.2.2 Computational efficiency

Computational cost of the proposed algorithm and DV-Hop, EIDV-Hop1, and EIDV-Hop2 algorithms is $O(n^2)$ where n is number anchor nodes. And EIDV-Hop3 algorithm uses particle swarm optimization (PSO) with computational complexity depending on number of particles taken $O(n)$ per iteration per particle of PSO. But it requires many iterations and particles to converge to descent accuracy.

Thus, comparing computational complexity in terms order does not gives clear idea about efficiency of different algorithms, so we considered comparing time taken by each algorithm to localize, and this does not include the communicational time. Localization time of nodes for the algorithms is discussed for change in total number of nodes and percentage of anchor node is fixed.

Results of the Table 2 show that the computation involved in proposed algorithm is little faster than DV-Hop, EIDV-Hop1, and EIDV-Hop2, while it is approx 100 time faster than EIDV-Hop3, which is a significant achievement with lesser communication. This proves computational efficiency of the proposed algorithms over DV-Hop, EIDV-Hop1, EIDV-Hop2, and EIDV-Hop3.

6 Conclusion

To achieve better localization accuracy with lesser communication, lesser hardware requirement and lower energy consumption, we proposed PERLA for wireless sensor networks. In the proposed algorithm hop-size for the anchor nodes is calculated at the unknown node that reduces one complete communication between anchor and unknown node. Unknown node refines hop-size of anchor nodes and takes average of the refined hop-size of anchor nodes for better estimation of distance between itself and anchor nodes. Nearest anchor node distance equation is divided by remaining distance equations one at a time to reduce the propagation of error in solving the system of equations. Further, unknown node estimates its location using method of least square and upgrades its location using information obtained by solving system of equations. Finally, we mathematically prove that propagation of error in our method is lesser than error propagates in DV-hop and other compared improved DV-Hop algorithms. Simulation results show that our proposed algorithm is more computationally efficient and localization performance is superior to DV-Hop and other compared improved DV-Hop algorithms.

Acknowledgments Authors, gratefully acknowledge financial support given by Council of Scientific and Industrial Research, India as scholarship (Senior Research fellowship).

References

1. Chen, C. C., & Liao, C.-H. (2011). Model-based object tracking in wireless sensor networks. *Wireless Networks, 17*(2), 549–565.
2. Xiao-gang, Q., & Chen-xi, Q. (2010). An Improvement of GAF for lifetime elongation in wireless sensor networks. *Journal of Convergence Information Technology, 5*(7), 112–119.
3. Akyildiz, I. F., Su, W., Sankarasubramaniam, Y., & Cayirci, E. (2002). Wireless sensor networks: a survey. *Computer Networks, 38*, 393–422.
4. Zeng, K., Ren, K., Lou, W., & Moran, P. J. (2009). Energy aware efficient geographic routing in lossy wireless sensor networks with environmental energy supply. *Wireless Networks, 15*(1), 39–51.
5. Hofmann-Wellenhof, B., Lichtenegger, H., & Collins, J. (1993). *Global positioning system: Theory and practice* (2nd ed.). New York: Springer.
6. Niculescu, D., & Nath, B. (2001). Ad-hoc positioning system. *Global telecommunications conference (GlobeCom), IEEE*, Vol. 5, pp. 2926–2931.
7. Priyantha, N. B., Balakrishnan, H., Demaine, E., & Teller, S. (2003). Anchor-free distributed localization in sensor networks. Technical report 892, MIT Laboratory for Computer Science.
8. Bulusu, N., Heidemann, J., & Estrin, D. (2000). GPS-less low cost outdoor localization for very small devices. *IEEE Personal Communications Magazine, 7*(5), 28–34.
9. Moore, D., Leonard, J., Rus, D., & Teller, S. (2004). Robust distributed network localization with noisy range measurements. In *Proceeding of the 2nd international conference on embedded networked sensor systems*, pp. 50–61.
10. Neal, P., & Alfred, O. H. (2003). Using proximity and quantized RSS for sensor localization in wireless networks. In *Proceedings of the 2nd ACM international conference on wireless sensor networks applications*, pp. 20–29.
11. He, T., Huang, C., Blum, B. M., Stankovic, J. A., & Abdelzaher, T. (2003). Range-free localization schemes for large scale sensor networks. In *Proceedings of the 9th annual international conference on mobile computing and networking*, pp. 81–95.
12. Capkun, S., Hamdi, M., & Hubaux, J.-P. (2001). GPS-free positioning in mobile Ad Hoc networks. In *Proceedings of the 34th annual hawaii international conference on system sciences*, pp. 3481–3490.
13. Nagpal, R. (1999). Organizing a global coordinate system from local information on an amorphous computer. A.I. Memo 1666, MIT A.I. Laboratory.
14. Gui, L., Val, T., & Wei, A. (2011). A novel two-class localization algorithm in wireless sensor networks. *The international journal network protocols and algorithms*, Vol. 3, No. 3, pp. 1–16.
15. Hou, S., Zhou, X., & Liu, X. (2010). *A novel DV-Hop localization algorithm for asymmetry distributed WSNs*. 3rd IEEE international conference on computer science and information technology (ICCSIT), Vol. 4, pp. 243–248.
16. Qian, Q., Shen, X., & Chen, H. (2011). An improved node localization algorithm based on DV-Hop for wireless sensor networks. *Computer Science and Information Systems, 8*(4), 953–972.
17. Chen, H., SeZaki, K., Deng, P., & CheungSo, H. (2008). An improved DV-Hop localization algorithm for wireless sensor networks. In *3rd IEEE international conference on industrial electronics and application (ICIEA)*, pp. 1557–1561.
18. Bao, X., Bao, F., Zhang, S., & Liu, L. (2010). An improved DV-Hop localization algorithm for wireless sensor networks. In *6th international conference on wireless communications networking and mobile computing (WiCOM),* pp. 1–4.
19. Ying, D., Jianping, W., & Chongwei, Z. (2010). Improvement of DV-Hop localization algorithms for wireless sensor networks. In *6th international conference on wireless communications networking and mobile computing (WiCOM),* pp. 1–4.
20. Fang, W., & Yang, G. (2011). Improvement based on DV-Hop localization algorithm of wireless sensor network. *International conference on mechatronic science, electric engineering and computer (MEC).* pp. 2421–2424.
21. Li, Y. Y. (2011). Improved DV-Hop localization algorithm based on local estimating and dynamic correction in location for WSNs. *International Journal of Digital Content Technology and its Applications, 5*(8), 196–202.
22. Dengyi, Z., & Feng, L. (2012). Improvement of DV-Hop localization algorithms in wireless sensor networks. *International Symposium on instrumentation & measurement, sensor network and automation (IMSNA).* pp. 567–569.
23. Yu, W., & Li, H. (2012). An improved DV-Hop localization method in wireless sensor networks. *International conference on computer science and automation engineering (CSAE).* pp. 199–202.
24. Chen, X., & Zhang, B. (2012). Improved DV-Hop node localization algorithm in wireless sensor networks. *International Journal of Distributed Sensor Networks.* doi:10.1155/2012/213980.
25. Chan, Y. T., & Ho, K. C. (1994). A simple and efficient estimator for hyperbolic location. *IEEE Transactions on Signal Processing, 42*(8), 1905–1915.
26. Taylor, J. R. (1997). *An introduction to error analysis* (2nd ed.). California: University Science Books Sausalito.

Author Biographies

Shrawan Kumar received his masters (M.Sc.) in Mathematics from Kanpur University. He also received M.Tech. (Computer Science) from School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi, India in 2009. Now he is pursuing Ph.D in Computer Science from School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi, India. His research interest includes Wireless Sensor Networks, Mobile Ad hoc Networks.

D. K. Lobiyal received his Ph.D and M. Tech (Computer Science) from School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi, India in 1996 and 1991, respectively, and B. Tech. (Computer Science and Engineering) from Lucknow University, India in 1988. He is currently working as Associate Professor at School of Computer and Systems Sciences, Jawaharlal Nehru University, New Delhi, India. His research interest includes Mobile Ad hoc Networks, VoD, Bioinformatics and Natural Language Processing. Dr. Lobiyal has published papers in International journals and conferences including IEEE, Wiley & Sons, Springer, Inder Science, WSEAS, IGI Global, and ACTA Press.