Ollscoil Chathair
Bhaile Átha Cliath
Dublin City University

SEMESTER 1 EXAMINATIONS 2021/2022

MODULE: CA266 - Probability & Statistics

PROGRAMME(S):
CASE BSc in Computer Applications (Sft.Eng.)
ECSAO Study Abroad (Engineering & Computing)

YEAR OF STUDY: 2,O

EXAMINER(S):
Dr. Graham Healy (Internal) (Ext:7364)

TIME ALLOWED: 2 Hours

INSTRUCTIONS: **Answer 4 questions. All questions carry equal marks.**

PLEASE DO NOT TURN OVER THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.
The use of programmable or text storing calculators is expressly forbidden.
Please note that where a candidate answers more than the required number of questions, the examiner will mark all questions attempted and then select the highest scoring ones.

There are no additional requirements for this paper.

QUESTION 1 ***[TOTAL MARKS: 25]***

For each of these questions **explain your answer** i.e., show you reasoning.

Q 1(a) **[18 Marks]**

Suppose you have a box that contains 10 red balls, 5 green balls, and 5 purple balls. Let 5 balls be drawn at random **without replacement** from the box.

Please give the probabilities of the following events. Show your reasoning in each case.

a) The first ball is purple
b) None of the balls are the same colour
c) The second ball is green given the first ball was red
d) Either the first ball or second ball or both are purple
e) The first ball is red, the second ball is purple, the third ball is green, the fourth ball is red, and the fifth ball is red.
f) There are exactly 2 green balls

Q 1(b) **[7 Marks]**

Suppose in Q1(a) we were to sample **with replacement** and draw N balls at random.

a) What is the probability that the first red ball we draw is the 17^{th} ball given that the first 10 balls drawn contained no red balls?
b) How many balls on average would we need to draw until we got our first green ball?

[End of Question 1]

QUESTION 2 ***[TOTAL MARKS: 25]***

For each of these questions **<u>explain your answer</u>** i.e., show you reasoning.

The following table gives the percentage of staff in different departments of an organisation and the hours they work each day. There are 1,500 staff in total for the 3 departments.

		Hours				
		6	7	8	9	10
Department	Sales	5%	0%	10%	15%	15%
	Admin	0%	5%	10%	10%	5%
	Research	5%	5%	10%	0%	5%

Q 2(a) **[10 Marks]**

Give values for the following probabilities if an employee was selected at random:

(i) P(Department = Sales)
(ii) P(Hours = 7)
(iii) P(Hours > 8)
(iv) P(Hours = 9 **AND** Department = Admin)
(v) P(Hours = 6 **|** Department = Sales)
(vi) P(Department = Admin **OR** Hours = 7)
(vii) P(Department = Sales **|** Hours < 9)

Q 2(b) **[8 Marks]**

What are the mean and variance of the hours for the staff?

Q 2(c) **[7 Marks]**

What is the probability if you selected two employees at random, that both would be from different departments in the organisation?

[End of Question 2]

QUESTION 3 ***[TOTAL MARKS: 25]***

Every time Joe leaves his food unattended, either his cat or dog will eat some or all the food on the plate. When either the cat or dog only eats some of the food, Joe will not notice. The only way Joe knows the animals are eating his food, is because 1% of the time he returns to an empty plate i.e., one of the animals has eaten all his food.

Through some investigation and experimentation, Joe has observed the following probabilities:

- If the dog eats the food, the probability that the plate will be empty afterwards is .89
- If Joe finds an empty plate, the probability that the cat ate all the food is .05

Note: Never will both the cat and dog eat from the same plate. It's always only one of the animals.

For each of these questions **explain your answer** i.e., show you reasoning.

Q 3(a) **[4 Marks]**

If Joe returns and finds an empty plate, what is the probability that the dog ate all the food?

Q 3(b) **[8 Marks]**

If Joe leaves his plate of food unattended, what is the probability that the cat will eat any of the food while he is gone? note: It does not matter if the plate will be empty or not after the cat is finished eating.

Q 3(c) **[10 Marks]**

If Joe returns to find that his plate still has food on it, what is the probability that the dog ate some of the food but did not eat it all?

Q 3(d) **[3 Marks]**

If Joe returns to find that his plate still has food on it, what is the probability that the cat ate some of the food but did not eat it all?

[End of Question 3]

QUESTION 4 ***[TOTAL MARKS: 25]***

A call centre receives on average 5 calls per hour. 20% of the calls received are for a printer problem and 80% are for a computer problem. Assume the probability of receiving calls follows a Poisson distribution.

Note: In the following questions, you may express some of your answers in terms of logarithms, if you wish.

For each of these questions **explain your answer** i.e., show you reasoning.

Q 4(a) **[2 Marks]**
What is the mean number of minutes between calls?

Q 4(b) **[4 Marks]**
What is the probability that 5 calls about printer problems will be received in an hour?

Q 4(c) **[4 Marks]**
What is the probability of receiving 3 calls in a 15-minute period?

Q 4(d) **[5 Marks]**
What is the probability of there being at least 10 calls about a computer problem before a call about a printer problem?

Q 4(e) **[5 Marks]**
What is the probability of having to wait less than 15 minutes for a call about a printer problem?

Q 4(f) **[5 Marks]**
For what duration of time (in minutes) would the probability of receiving no calls about a printer problem be .15?

[End of Question 4]

QUESTION 5 ***[TOTAL MARKS: 25]***

For each of these questions **explain your answer** i.e., show you reasoning.

Answer any 2 of the following questions i.e., Q5(a), Q5(b) or Q5(c). There are equal marks for both questions. **For each answered question explain your answers i.e., show you reasoning.**

Q 5(a) **[12.5 Marks]**

For a certain population of people, you are told the mean salary is €44,000 and its standard deviation is €16,000. You can assume the salaries are normally distributed.

(i) What is the probability of a person having a salary less than €44,000?
(ii) What is the probability of a person having a salary less than €28,000?
(iii) What is the probability of a person having a salary greater than €60,000, if you are told their salary is greater than €28,000?

Q 5(b) **[12.5 Marks]**

Suppose you toss a fair coin 4 times.

If you get two heads in a row, you get 4 euro.
If you get three heads in a row, you get 8 euro.
If you get four heads in a row, you get 16 euro.

Let X be the total number of euros you win after 4 tosses.

(i) What is the probability of each value of X?
(ii) If exactly 3 of the 4 coin tosses are heads, what is the probability $X > 6$?

Q 5(c) **[12.5 Marks]**

In a computer installation, there are 10 servers. The probability that any given server is busy is .6

(i) What is the probability that at a given moment all the servers are busy?
(ii) What is the probability that at a given moment less than 4 servers are busy?
(iii) What is the minimum number of servers needed so that there is less than a .3 probability that all servers are busy?

[End of Question 5]

[END OF EXAM]