Introduction to Hamiltonian Dynamical Systems and The N-Body Problem
(by KR Meyer)

Exercise Solutions

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Problem 1.7

Solution.

Suppose that $z(t)$ is a solution for $\dot{z} = J\nabla H(z)$, then since H is conservative, we have

$$\frac{\mathrm{d}H(z(t))}{\mathrm{d}t} = 0. \tag{1}$$

Thus, $H(z(t)) = E$ for all $t \in \mathbb{R}$, where E is a constant.

> Since $H(z) \to \infty$ as $\|z\| \to \infty$, $\|z(t)\|$ must be finite for all $t \in \mathbb{R}$, that is, there exists $R \in \mathbb{R}$ so that $\|z(t)\| < R$ for all $t \in \mathbb{R}$, or $H(z(t))$ diverges, which is a contradiction. As a consequence, $z(t)$ is bounded.