

# Group Meeting #7

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2024/8/9



# Renormalization Group and Matrix Product State

- The density matrix renormalization group (DMRG) algorithm can be implemented in two formulations:

**Physical:** Renormalization group (RG)

**Mathematical:** Matrix product state (MPS)

→ **variational matrix product state (VMPS)**

These two formulations are mathematically equivalent, but VMPS is more concise and easier to implement.

Catarina, G., and Bruno Murta. *The European Physical Journal B* 96.8 (2023): 111.

- VMPS can be used to find the non-equilibrium steady state (NESS) of the SIS model.

Merbis, Wout, Clélia de Mulatier, and Philippe Corboz. *arXiv preprint arXiv:2305.06815* (2023).



# VMPS Algorithms for the Transverse Ising Model

- Utilizing the VMPS algorithm to find the ground state of the **transverse Ising model**

Hamiltonian:  $\hat{H} = -J \sum_i \sigma_i^x \otimes \sigma_{i+1}^x - h \sum_i \sigma_i^z$

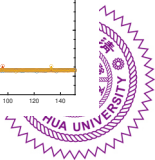
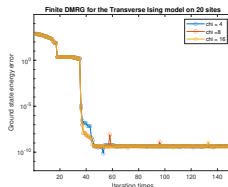
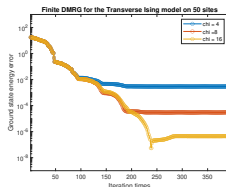
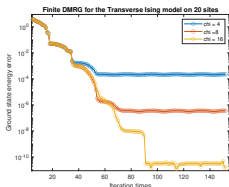
Critical point:  $J = h$

Ground state energy:  $\inf_{|\psi\rangle} \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | \psi \rangle}$ .

Optimization:  $\min_i \lambda_i(\hat{H})$

$\hat{H}$  hermitian  $\rightarrow$  **Lanczos algorithm**

<https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html>



# VMPS for the SIS Model

- The Hamiltonian of the SIS model:

Hamiltonian:

$$\hat{W} = \lambda \sum_{i=1}^{N-1} (\hat{n}_i \hat{w}_{i+1}^{0 \rightarrow 1} + \hat{w}_i^{0 \rightarrow 1} \hat{n}_{i+1}) + \sum_{i=1}^{N-1} \hat{w}_i^{1 \rightarrow 0} + \hat{W}_{\text{driv}}(\alpha)$$

- Diagonalizability:

Master equation:  $\partial_t |P(t)\rangle = \hat{W} |P(t)\rangle$

The state:  $|P(t)\rangle = e^{t\hat{W}} |P(0)\rangle$

Write  $\hat{W}$  in its Jordan form

→  $\hat{W}$  is **diagonalizable**.

- The spectrum:

$\hat{W}$  is diagonalizable

→ The eigenvalues are **negative or zero**.

- NESS:

Entries of  $\hat{W}$  are  $e^{\lambda_i t}$  goes to zero as  $t \rightarrow \infty$  if  $\lambda_i < 0$

→ the only important entry is  $e^{0t} = 1$

→ the NESS is the **eigenvector of  $\hat{W}$  with  $\lambda = 0$**



# VMPS for the SIS Model

- Utilizing the VMPS algorithm to find the non-equilibrium steady state (NESS) of the SIS model

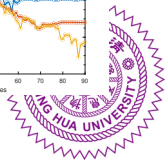
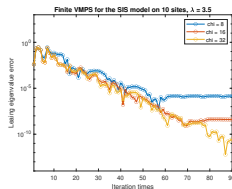
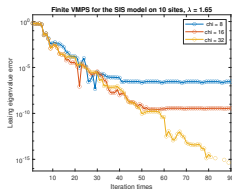
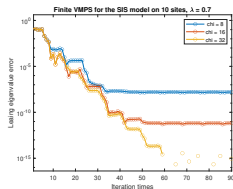
Eigenvalue of the NESS:  $\sup_{|\psi\rangle} \frac{\langle \psi | \hat{H} | \psi \rangle}{\langle \psi | \psi \rangle}$

Optimization:  $\max_i \lambda_i(\hat{H})$

$\hat{H}$  not hermitian  $\rightarrow$  Arnoldi algorithm

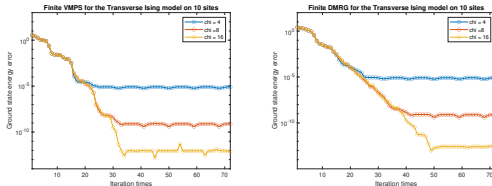
$\rightarrow$  More unstable

<https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html>



# VMPS for the SIS Model

- Two diagonalization may result in different results



The instability may increase

<https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html>

