Group Meeting #7

Hao-Yang Yen

NTHU

2024/8/9

Renormalization Group and Matrix Product State

- The density matrix renormalization group (DMRG) algorithm can be implemented in two formulations:

  Physical: Renormalization group (RG)

  Mathematical: Matrix product state (MPS)

  → **variational matrix product state (VMPS)**

  These two formulations are mathematically equivalent, but VMPS is more concise and easier to implement.

  Catarina, G., and Bruno Murta. The European Physical Journal B 96.8 (2023): 111.

- VMPS can be used to find the non-equilibrium steady state (NESS) of the SIS model.

  Merbis, Wout, Clélia de Mulatier, and Philippe Corboz. arXiv preprint arXiv:2305.06815 (2023).

VMPS Algorithms for the Transverse Ising Model

- Utilizing the VMPS algorithm to find the ground state of the **transverse Ising model**

  Hamiltonian: $\hat{H} = -J \sum_i \sigma_i^x \otimes \sigma_{i+1}^x - h \sum_i \sigma_i^z$

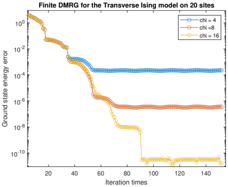
  Critical point: $J = h$

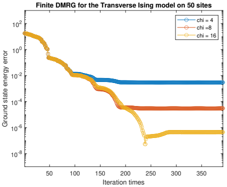
  Ground state energy: $\inf_{|\Psi\rangle} \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$.

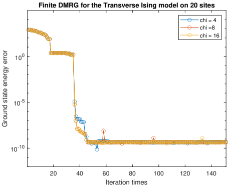
  Optimization: $\min_i \lambda_i(\hat{H})$

  $\hat{H}$ hermitian $\rightarrow$ **Lanczos algorithm**

https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html







VMPS for the SIS Model

- The Hamiltonian of the SIS model:

  Hamiltonian:
  $\hat{W} = \lambda \sum_{i=1}^{N-1} (\hat{n}_i \hat{w}_{i+1}^{0\to 1} + \hat{w}_i^{0\to 1} \hat{n}_{i+1}) + \sum_{i=1}^{N-1} \hat{w}_i^{1\to 0} + \hat{W}_{driv}(\alpha)$

- Diagonalizability:

  Master equation: $\partial_t |P(t)\rangle = \hat{W} |P(t)\rangle$
  The state: $|P(t)\rangle = e^{t\hat{W}} |P(0)\rangle$
  Write $\hat{W}$ in its Jordan form
  $\rightarrow$ $\hat{W}$ is **diagonalizable** or the state will diverge.

- The spectrum:

  $\hat{W}$ is diagonalizable
  $\rightarrow$ The eigenvalues are **negative or zero** .

- NESS:

  Entries of $\hat{W}$ are $e^{\lambda_i t}$ goes to zero as $t \to \infty$ if $\lambda_i < 0$
  $\rightarrow$ the only important entry is $e^{0t} = 1$
  $\rightarrow$ the NESS is the **eigenvector of $\hat{W}$ with $\lambda = 0$**

VMPS for the SIS Model

- Utilizing the VMPS algorithm to find the non-equilibrium steady state (NESS) of the SIS model

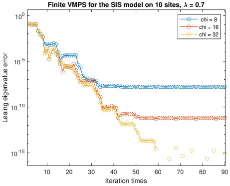
  Eigenvalue of the NESS: $\sup_{|\Psi\rangle} \frac{\langle\Psi|\hat{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$

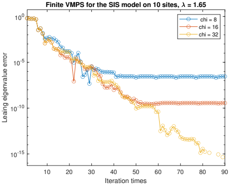
  Optimization: $\max_i \lambda_i(\hat{H})$

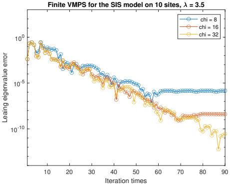
  $\hat{H}$ not hermitian $\rightarrow$ **Restarted Arnolid algorithm**

  $\rightarrow$**More unstable**

https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html

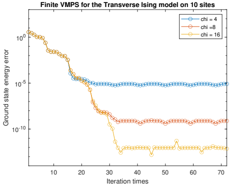


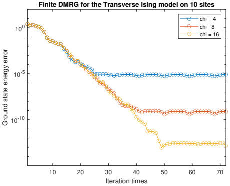




VMPS for the SIS Model

- Two diagonalization may result in different results





The instability may increase

https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html