Exercise 2: Problem 2 c) Solution

Christian Oppegård Moen and Erling Fause Steen

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Problem 2 c)

In this problem, we are asked to consider the following model in INLA:

```
mod <- inla(n.rain ~ f(day, model="rw1", constr=TRUE),
data=rain, Ntrials=n.years, control.compute=list(config = TRUE),
family="binomial", verbose=TRUE, control.inla=control.inla)
```

Want to find out:

- How is it different from the model in 2a mathematically?
- Are the predictions from this model significantly different from the model in 2a?
- Why?

Comparison with the model in 2a

- Model in 2a:

$$y_t|\tau_t \sim \text{Bin}(n_t, \pi(\tau_t)), \quad \pi(\tau_t) = \frac{\exp(\tau_t)}{1+\exp(\tau_t)} = \frac{1}{1+\exp(-\tau_t)}.$$

- The new model includes an intercept term:

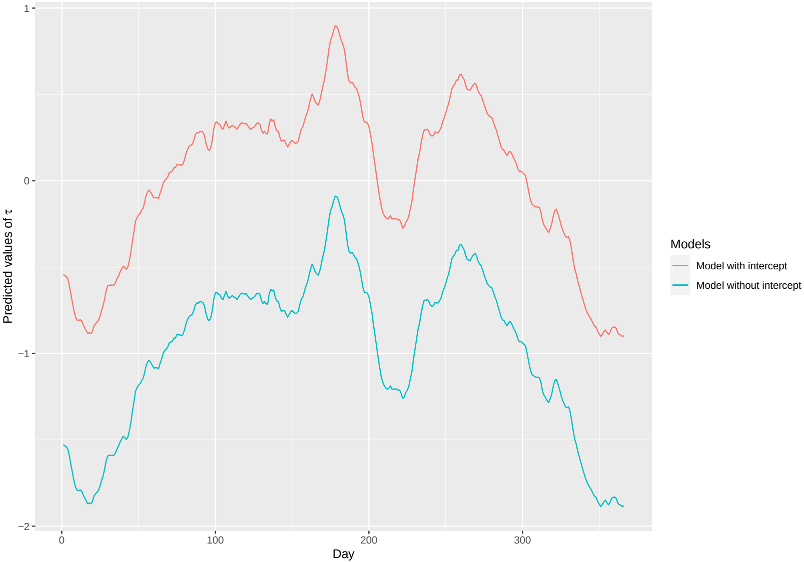
$$y_t|\tau_t \sim \text{Bin}(n_t, \pi(\eta_t)), \; \pi(\eta_t) = \frac{\exp(\eta_t)}{1+\exp(\eta_t)} = \frac{1}{1+\exp(-\eta_t)}.$$

  where $\eta_t = \alpha + \tau_t$ and α is the intercept term.

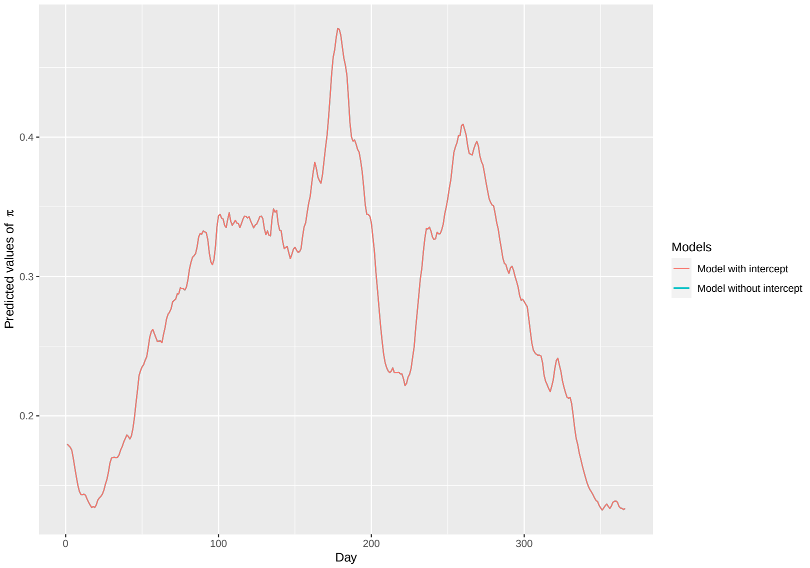
- Default prior on α.
- Loggamma(α, β) prior on θ.
- Constr=TRUE $\implies$ Sum-to-zero constraint:

$$\sum_{t=1}^{T} \tau_t = 0.$$

Comparison of predictions of τ



Comparison of predictions of π



Comparison of predictions of π_{201} and π_{366}

- Predictions from the new model:

```
##                            mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.201 0.3281998 0.02582527  0.2792408 0.3275999  0.3805736
##                            mode
## fitted.Predictor.201 0.3264117
```

```
##                            mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.366 0.1336206 0.02144614 0.09501938 0.1324285   0.179017
##                           mode
## fitted.Predictor.366 0.130071
```

- Predictions from model in 2a:

```
##                            mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.201 0.3282009 0.02582567  0.2792412 0.3276009  0.3805755
##                            mode
## fitted.Predictor.201 0.3264125
```

```
##                            mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.366 0.1336182 0.02144626 0.09501697 0.1324261   0.179015
##                            mode
## fitted.Predictor.366 0.1300685
```

Explanantion

- Different constraints $\implies$ different results for τ
- Same shape
- Adding the intercept gives almost identical predictions of π

Mathematically:

- Same posterior distribution

Conclusions

- The model has an intercept term and sum-to-zero constraint

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- The model has an intercept term and sum-to-zero constraint
- No significant differences in predictions between the two models
- Adding the intercept term to the model with sum-to-zero constraint makes the model as flexible as the model in 2a

Questions?