Exercise 2: Problem 2 c) Solution

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Problem 2 c)

In this problem, we are asked to consider the following model in INLA:

```
mod <- inla(n.rain ~ f(day, model="rw1", constr=TRUE, hyper=hyper),
data=rain, Ntrials=n.years, control.compute=list(config = TRUE),
family="binomial", verbose=TRUE, control.inla=control.inla)
```

Want to find out:

- How is it different from the model in 2a mathematically?
- Are the predictions from this model significantly different from the model in 2a?
- Why?

Comparison with the model in 2a

- Model in 2a:

$$y_t|\tau_t \sim \text{Bin}(n_t, \pi(\tau_t)), \quad \pi(\tau_t) = \frac{\exp(\tau_t)}{1 + \exp(\tau_t)} = \frac{1}{1 + \exp(-\tau_t)}.$$

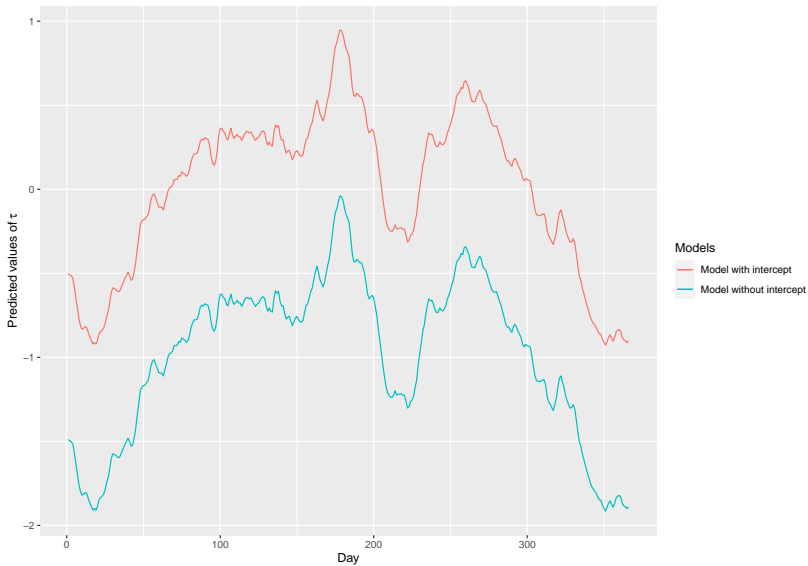- The new model includes an intercept term:

$$y_t|\tau_t \sim \text{Bin}(n_t, \pi(\eta_t)), \quad \pi(\eta_t) = \frac{\exp(\eta_t)}{1 + \exp(\eta_t)} = \frac{1}{1 + \exp(-\eta_t)}.$$

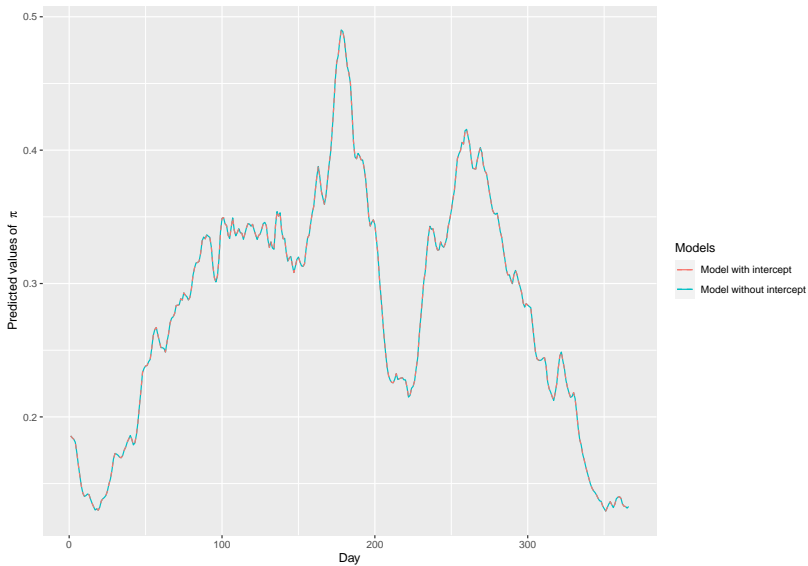 where $\eta_t = \alpha + \tau_t$ and α is the intercept term.

- Default prior on α.
- Loggamma(α, β) prior on θ.
- Constr=TRUE $\implies$ Sum-to-zero constraint:

$$\sum_{t=1}^{T} \tau_t = 0.$$

Comparison of predictions of τ

Comparison of predictions of π

Comparison of predictions of π_{201} and π_{366}

- Predictions from the new model:

```
##                              mean         sd 0.025quant 0.5quant 0.975quant
## fitted.Predictor.201    0.3329546 0.02871596  0.2785427 0.332292  0.3911304
##                              mode
## fitted.Predictor.201    0.3309666


##                              mean         sd 0.025quant 0.5quant 0.975quant
## fitted.Predictor.366    0.1328184 0.02346734 0.09103605 0.1313672  0.1828485
##                              mode
## fitted.Predictor.366    0.1284745
```

- Predictions from model in 2a:

```
##                              mean         sd 0.025quant 0.5quant 0.975quant
## fitted.Predictor.201    0.3329556 0.02871629  0.2785433 0.3322929  0.3911323
##                              mode
## fitted.Predictor.201    0.3309674


##                              mean         sd 0.025quant 0.5quant 0.975quant
## fitted.Predictor.366    0.1328162 0.02346736 0.09103365 0.1313649  0.1828461
##                              mode
## fitted.Predictor.366    0.1284723
```

Explanantion

- Different constraints $\implies$ different results for τ
- Same shape
- Adding the intercept gives almost identical predictions of π

Mathematically:

- Same posterior distribution

Conclusions

- The model has an intercept term and sum-to-zero constraint

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- No significant differences in predictions between the two models

- Adding the intercept term to the model with sum-to-zero constraint makes the model as flexible as the model in 2a

Questions?