Exercise 2: Problem 2 c) Solution

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Problem 2 c)

In this problem, we are asked to consider the following model in INLA:

```
mod <- inla(n.rain ~ f(day, model="rw1", constr=TRUE, hyper=hyper),
data=rain, Ntrials=n.years, control.compute=list(config = TRUE),
family="binomial", verbose=TRUE, control.inla=control.inla)
```

Want to find out:

- How is it different from the model in 2a mathematically?
- Are the predictions from this model significantly different from the model in 2a?
- Why?

Comparison with the model in 2a

- Model in 2a:

$$y_t|\tau_t \sim \mathrm{Bin}(n_t, \pi(\tau_t)), \quad \pi(\tau_t) = \frac{\exp(\tau_t)}{1+\exp(\tau_t)} = \frac{1}{1+\exp(-\tau_t)}.$$

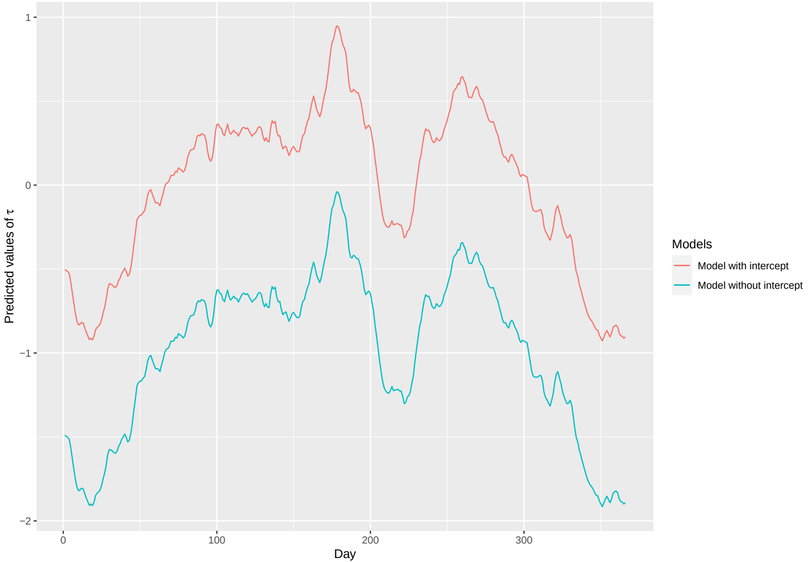
- The new model includes an intercept term:

$$y_t|\tau_t \sim \mathrm{Bin}(n_t, \pi(\eta_t)), \;\; \pi(\eta_t) = \frac{\exp(\eta_t)}{1+\exp(\eta_t)} = \frac{1}{1+\exp(-\eta_t)}.$$

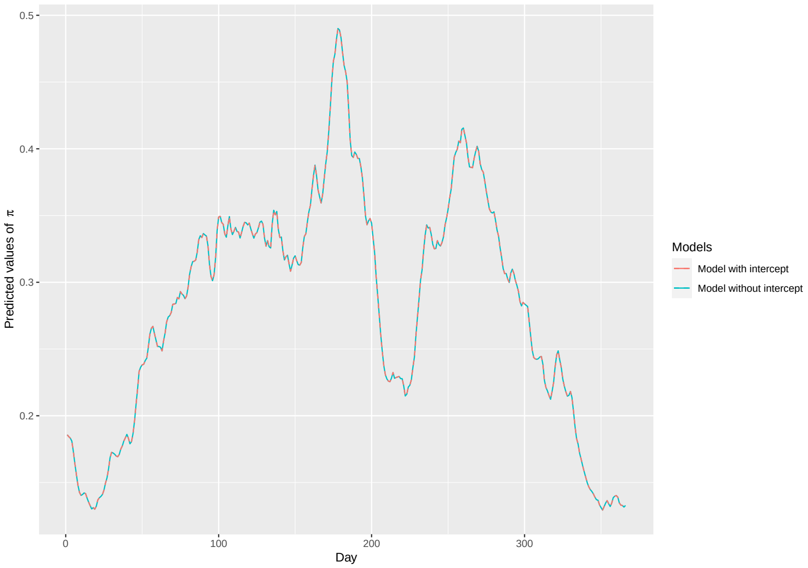
  where $\eta_t = \beta_0 + \tau_t$ and α is the intercept term.
- Default prior on β_0.
- Loggamma(α, β) prior on θ.
- Constr=TRUE $\implies$ Sum-to-zero constraint:

$$\sum_{t=1}^{T} \tau_t = 0.$$

Comparison of predictions of τ



Comparison of predictions of π



Comparison of predictions of π_{201} and π_{366}

- Predictions from the new model:

```
##                           mean         sd 0.025quant 0.5quant 0.975quant
## fitted.Predictor.201 0.3329546 0.02871596  0.2785427 0.332292  0.3911304
##                           mode
## fitted.Predictor.201 0.3309666
```

```
##                           mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.366 0.1328184 0.02346734 0.09103605 0.1313672  0.1828485
##                           mode
## fitted.Predictor.366 0.1284745
```

- Predictions from model in 2a:

```
##                           mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.201 0.3329556 0.02871629  0.2785433 0.3322929  0.3911323
##                           mode
## fitted.Predictor.201 0.3309674
```

```
##                           mean         sd 0.025quant  0.5quant 0.975quant
## fitted.Predictor.366 0.1328162 0.02346736 0.09103365 0.1313649  0.1828461
##                           mode
## fitted.Predictor.366 0.1284723
```

Explanantion

- Different constraints $\Longrightarrow$ different results for τ
- Same shape
- Adding the intercept gives almost identical predictions of π

Mathematically:

- Looking at the posterior distribution

Conclusions

- The model has an intercept term and sum-to-zero constraint

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- The model has an intercept term and sum-to-zero constraint
- No significant differences in predictions between the two models
- Adding the intercept term to the model with sum-to-zero constraint makes the model as flexible as the model in 2a

Questions?