Interest rate swap pricing using Vacisek model

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Abstract

In this paper I will attempt to price a fictional interest rate swap.
Thank you for reading.

1 Introduction

An interest rate swap is a contract which allows 2 institutions to exchange the interest rate payments on their loans. As the name suggests, interest rate swap exchanges only the streams of interest payments over a fixed period, typically one paying a fixed rate and the other a floating rate based on a benchmark like LIBOR or SOFR. These days LIBOR is getting replaced.

These swaps are commonly used by banks, corporations, or asset managers to manage interest rate risk—e.g., a company with floating-rate debt might enter a swap to pay fixed instead, stabilizing its future payments.

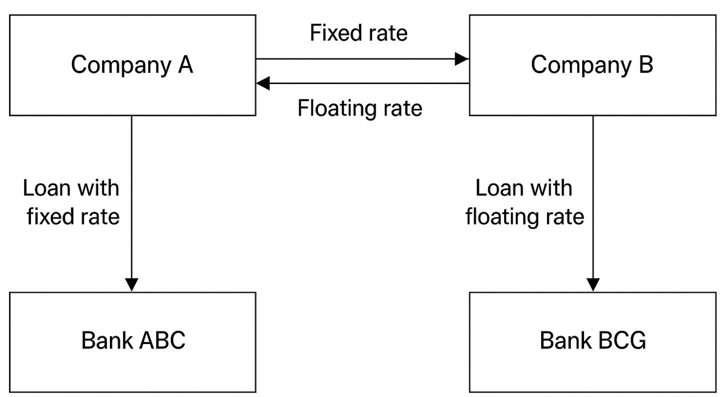

Figure 1: Plain vanila interest rate swap

Figure 1 shows the exchange of interest rate swaps between two companies with different loans, exchanging interest rate payments. We have 2 banks giving out loans to the subsequent companies. Company A thinks that a floating rate is safer as they believe rates will fall , while Company B stands on the opposite opinion. Lets find out what happens!

Interest Rate Swap Setup

Assumptions and Parameters TO BE FIXED

Parameter	Value	Notes
Notional	\$100 million	Swap principal, not exchanged
Swap type	Payer-fixed	Pay fixed, receive floating
Fixed rate (R_{fixed})	5.00%	Annualized, paid once per year
Floating index	LIBOR (simplified)	Assumed flat for this example
Payment frequency	Annual	Payments at $t_1 = 1$, $t_2 = 2$, $t_3 = 3$
Day count fraction (Δ)	1.0	Based on annual payments
Tenor	3 years	Total duration of the swap
Discount curve rate (r)	4.00%	Flat curve, discrete compounding
Discount formula	$P(0,t) = \frac{1}{(1+r)^t}$	Discrete compounding assumption
Valuation date	$t = 0$	Pricing is done today
Floating leg valuation	$1 - P(0, t_n)$	Assumes par-par at reset

Table 1: Plain Vanilla Interest Rate Swap Assumptions

1.1 Discount Factor Calculation Using Zero-Coupon Bond Formula

We assume the interest rate is compounded **discretely**, with a flat annual rate of:

$$r = 4\% = 0.04$$

The discount factor for time t is computed using the zero-coupon bond pricing formula:

$$P(0,t) = \frac{1}{(1+r)^t}$$

Swap Parameters (from Setup)

Parameter	Value	Notes
Notional	\$100 million	Fixed/floating amounts scaled by this
Fixed rate (R_{fixed})	5.00%	Annualized rate
Tenor	3 years	Payments at $t_1 = 1$, $t_2 = 2$, $t_3 = 3$
Discount rate (r)	4.00%	Discrete compounding
Day count fraction (Δ)	1.0	Annual convention

Table 2: Interest Rate Swap Setup

Discount Factor Calculations

$$P(0,1) = \frac{1}{(1+0.04)^1} = \frac{1}{1.04} = 0.961538$$
$$P(0,2) = \frac{1}{(1+0.04)^2} = \frac{1}{1.0816} = 0.924556$$
$$P(0,3) = \frac{1}{(1+0.04)^3} = \frac{1}{1.124864} = 0.889000$$

Present Value Calculation for the Fixed Leg

Given Parameters

- Notional: \$100,000,000
- Fixed Rate: $R_{\text{fixed}} = 5\% = 0.05$
- Day Count Fraction: $\Delta = 1.0$
- Discount Rate: $r = 4\%$
- Discount Factors:

$$
\begin{aligned}
P(0,1) &= \frac{1}{(1+0.04)^1} = 0.961538 \\
P(0,2) &= \frac{1}{(1+0.04)^2} = 0.924556 \\
P(0,3) &= \frac{1}{(1+0.04)^3} = 0.8900
\end{aligned}
$$

Formula for Fixed Leg PV

$$
\text{PV}_{\text{fixed}} = \sum_{i=1}^{3} (\text{Notional} \times R_{\text{fixed}} \times \Delta \times P(0,t_i))
$$

Calculation

$$
\begin{aligned}
\text{PV}_{\text{fixed}} &= 100{,}000{,}000 \times 0.05 \times (P(0,1) + P(0,2) + P(0,3)) \\
&= 5{,}000{,}000 \times (0.961538 + 0.924556 + 0.889000) \\
&= 5{,}000{,}000 \times 2.775094 \\
&= \boxed{\$13{,}875{,}470}
\end{aligned}
$$

1.2 Floating Rate

The floating rate in an interest rate swap is a variable interest rate that resets periodically based on a reference rate, such as **LIBOR**, **SOFR**, or **EURIBOR**. At each reset date, the floating payment is calculated as:

$$\text{Floating Payment} = \text{Notional} \times \text{Floating Rate} \times \Delta$$

where Δ is the day count fraction for the period. The rate reflects current market conditions, which makes the floating leg sensitive to interest rate movements and often viewed as having a value close to par at each reset.

Since I am trying to model the rate , I will use a stochastic process - Vasicek short rate model for modelling the interest rate.

1.3 Continuous-Time OU Process

The continuous-time Ornstein-Uhlenbeck (OU) process is given by the stochastic differential equation (SDE):

$$dX(t) = \theta(\mu - X(t))dt + \sigma dW(t)$$

Integrating over the time interval $[t, t+1]$:

$$X(t+1) - X(t) = \int_t^{t+1} \theta(\mu - X(s))ds + \int_t^{t+1} \sigma dW(s)$$

Using the Euler-Maruyama approximation $X(s) \approx X(t)$ for $s \in [t, t+1]$:

$$X(t+1) - X(t) \approx \theta(\mu - X(t)) \int_t^{t+1} ds + \sigma \int_t^{t+1} dW(s)$$

Evaluating the integrals, with $\int_t^{t+1} ds = 1$ and $\int_t^{t+1} dW(s) = W(t+1) - W(t) \sim \mathcal{N}(0,1) = \varepsilon_t$:

$$X(t+1) - X(t) \approx \theta(\mu - X(t)) \cdot 1 + \sigma\varepsilon_t$$

Rearranging to get the discrete-time OU process for rate R_t:

$$R_{t+1} = R_t + \theta(\mu - R_t) + \sigma\varepsilon_t$$

1.4 Discretized OU Process

The discretized version for rate R_t and year t (with $\Delta t = 1$) is:

$$R_{t+1} = R_t + \theta(\mu - R_t) + \sigma\varepsilon_t$$

where ε_t is a standard normal random variable.

1.5 Vasicek Model Simulation Parameters

The following parameters were used for the Vasicek interest rate simulation:

Quantile Analysis of Simulated Rates

After simulating the paths, we analyze the distribution of the interest rates at a specific future time, typically the final time T.

1. **Extracting Final Rates:** We consider the set of simulated interest rates at time T:

$$S = \{r_T^{(1)}, r_T^{(2)}, \ldots, r_T^{(n_{sim})}\}$$

This set S represents an empirical sample from the probability distribution of r_T.

Table 3: Vasicek Model and Simulation Parameters

Parameter	**Symbol**	**Value**
Speed of Mean Reversion	κ (a)	0.1
Long-Term Mean	θ (b)	0.05
Volatility	σ	0.01
Initial Interest Rate	r_0	0.03
Total Time (Years)	T	1.0
Time Step (Years)	Δt (dt)	$1/252 \approx 0.0040$
Number of Steps	N	252
Number of Simulations	n_{sim}	10,000

2. **Defining Quantile Probabilities:** We choose a set of probabilities, $q_1, q_2, \ldots, q_m$, where $0 < q_j < 1$. These correspond to the percentiles we want to extract (e.g., $0.001, 0.01, 0.05, \ldots, 0.999$).

3. **Calculating Quantile Values:** For each chosen probability q_j, we estimate the q_j-th quantile, Q_{q_j}, from the empirical sample S. This value Q_{q_j} is such that approximately $q_j \times 100\%$ of the observations in S are less than or equal to Q_{q_j}.

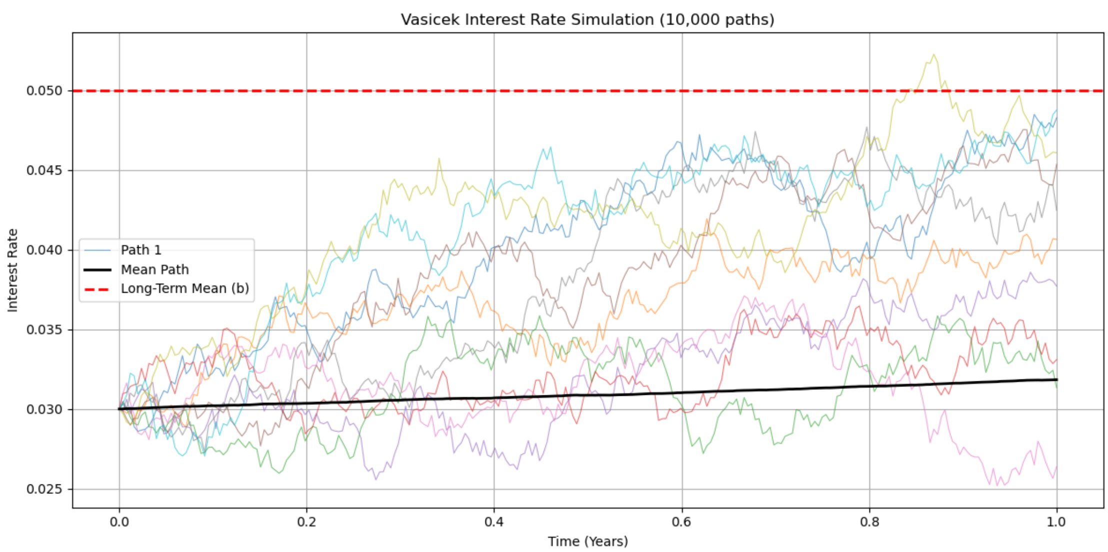

Figure 2: Vacisek Simulation first 10 paths from 10 000 paths

The calculation typically involves:

(a) **Sorting:** The sample S is conceptually (or internally by the function) sorted in non-decreasing order:

$$r_{(1)} \leq r_{(2)} \leq \ldots \leq r_{(n_{sim})}$$

(b) **Index Calculation:** For each q_j, a continuous index k_j is calculated:

$$k_j = q_j \cdot (n_{sim} - 1)$$

(c) **Interpolation:**

- If k_j is an integer, the q_j-th quantile is the value at index k_j (using 0-based indexing): $Q_{q_j} = r_{(k_j+1)}$.

- If k_j is not an integer, let $i = \lfloor k_j \rfloor$ (the integer part) and $f = k_j - i$ (the fractional part). The q_j-th quantile is typically estimated by linear interpolation between the two nearest sorted data points:

$$Q_{q_j} = (1-f) \cdot r_{(i+1)} + f \cdot r_{(i+2)}$$

(Adjusting for 0-based array indexing, this would be $(1-f) \cdot r_{(i)} + f \cdot r_{(i+1)}$).

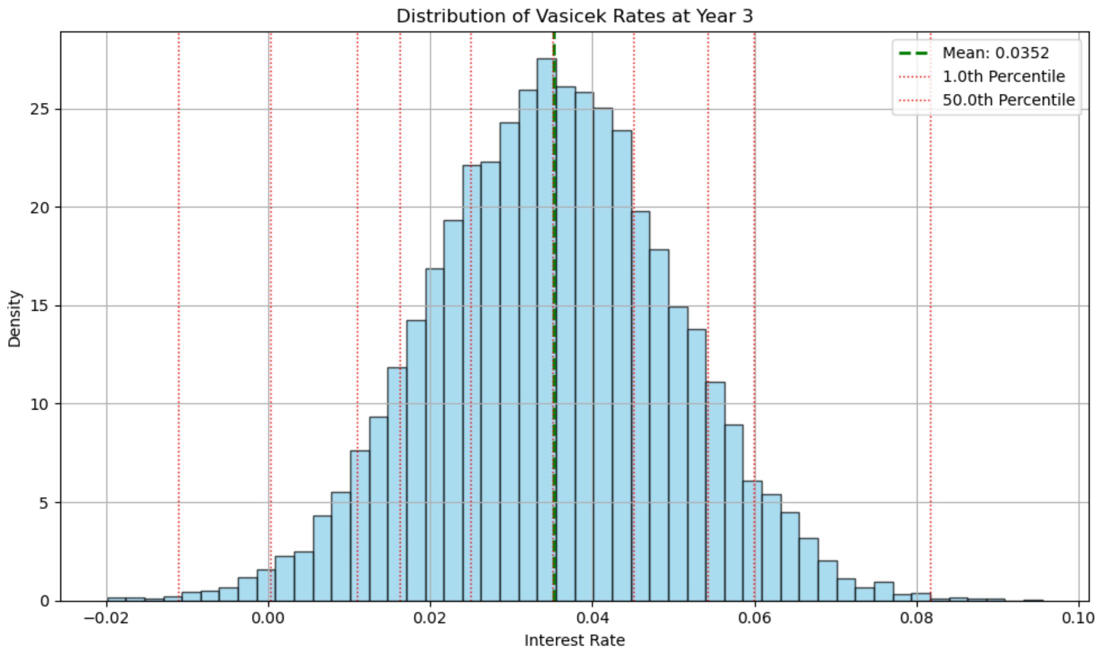

Figure 3: Histogram of the 10 rates picked based on quantile

The following table summarizes the statistical properties and quantile values of the simulated interest rates at the end of the simulation period ($T = 3$ year):

Table 4: Quantile Analysis of Interest Rates at Years 1, 2, and 3

Quantile / Statistic	**Year 1**	**Year 2**	**Year 3**
Mean rate	0.0318	0.0336	0.0352
Standard deviation	0.0095	0.0128	0.0149
0.1th percentile	0.0033	-0.0063	-0.0111
1.0th percentile	0.0096	0.0038	0.0003
5.0th percentile	0.0163	0.0125	0.0109
10.0th percentile	0.0196	0.0174	0.0162
25.0th percentile	0.0254	0.0250	0.0250
50.0th percentile (Median)	0.0319	0.0335	0.0351
75.0th percentile	0.0383	0.0422	0.0451
90.0th percentile	0.0440	0.0500	0.0543
95.0th percentile	0.0474	0.0548	0.0600
99.9th percentile	0.0606	0.0735	0.0817

The calculated Q_{q_j} values provide specific points within the empirical distribution of r_T, providing information on its shape, spread, and tail behavior (e.g. Value-at-Risk).

After simulation we have 10,000 paths. By taking the endpoints of each path, we have 10 000 points.

From there we use a histogram and order them by size, as in the example above. The percentile value we take the value of the simulation. For example, on the 1st percentile, the value of the rate is 0.0096.

4. **Descriptive Statistics:** The empirical mean and standard deviation of the final rates are also calculated:

$$\text{Mean}(S) = \bar{r}_T = \frac{1}{n_{sim}} \sum_{j=1}^{n_{sim}} r_T^{(j)}$$

$$\text{StdDev}(S) = \sqrt{\frac{1}{n_{sim}-1} \sum_{j=1}^{n_{sim}} (r_T^{(j)} - \bar{r}_T)^2}$$

These provide measures of central tendency and dispersion of the simulated rates at time T.

1.6 Floating Leg Present Value Calculation

Given the quantile analysis of floating rates and a notation of \$100 million with annual payments over 3 years, the floating leg payment at each year t is:

$$\text{Floating Payment}_t = \text{Notional} \times r_t \times \Delta,$$

where $\Delta = 1$ year.
Discount factors using a flat discount rate of 4% are:

$$P(0,1) = 0.9615, \quad P(0,2) = 0.9246, \quad P(0,3) = 0.8890.$$

Using the mean floating rates at each year from the Vasicek simulation:

$$\text{PV}_1 = 100,000,000 \times 0.0318 \times 0.9615 = 3,055,770$$
$$\text{PV}_2 = 100,000,000 \times 0.0336 \times 0.9246 = 3,106,656$$
$$\text{PV}_3 = 100,000,000 \times 0.0352 \times 0.8890 = 3,129,280$$

Summing these gives the total present value of the floating leg:

$$\text{PV}_{\text{floating}} = 3,055,770 + 3,106,656 + 3,129,280 = 9,291,706 \approx \boxed{\$9.29 \text{ million}}.$$

Similarly, this calculation can be performed for any quantile rate by replacing r_t with the respective quantile value at year t.

Net Swap Value and Interpretation

Net Present Value (NPV) of the Swap:

$$\text{NPV}_{\text{swap}} = \text{PV}_{\text{fixed}} - \text{PV}_{\text{floating}} = 13{,}875{,}470 - 9{,}291{,}706 = 4{,}583{,}764$$

Interpretation:

The positive net present value indicates that the payer of the fixed leg benefits from the swap under current interest rate expectations. Specifically, the fixed leg payments exceed the expected floating leg payments in present value terms by approximately \$4.58 million. This suggests that entering into this swap is advantageous for the fixed leg payer, assuming the mean interest rate path is realized.

References

[1] Mark S. Joshi, *The Concepts and Practice of Mathematical Finance*, 2nd Edition, Cambridge University Press, 2014. Available at: https://www.amazon.co.uk/dp/B00NBKTGPG

[2] Chris Kelliher, *Quantitative Finance with Python: Engineering Derivatives with Advanced Mathematics*, Independently published, 2024. Available at: https://www.amazon.co.uk/dp/B0D4FC5VXW