U O W

Classification by Splitting Data

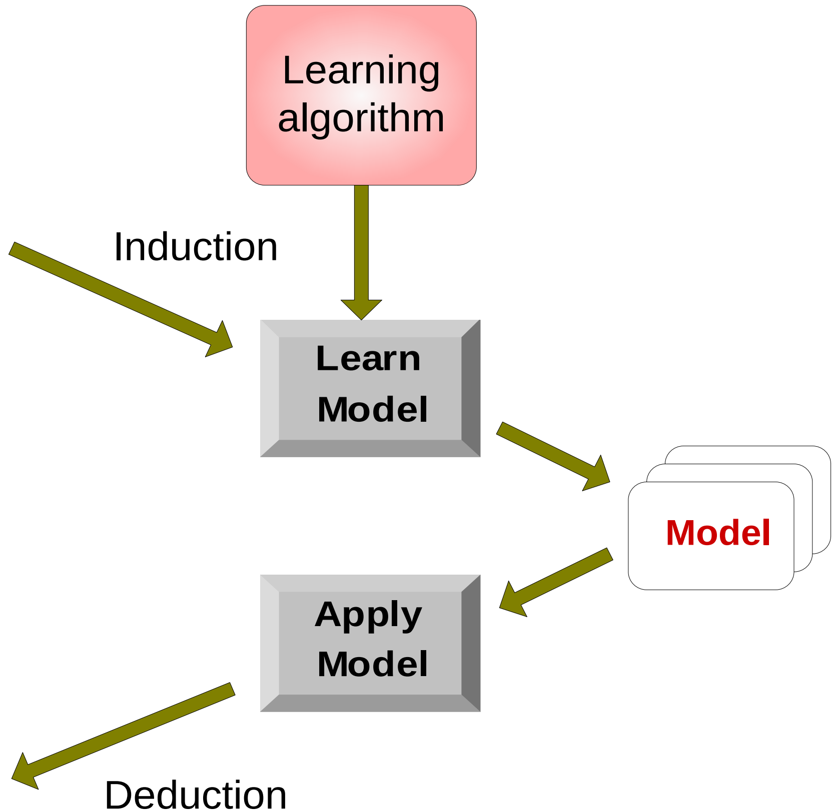
CSCI316: Big Data Mining Techniques and Implementation

UNIVERSITY
OF WOLLONGONG
AUSTRALIA

The Classification Problem: An Example

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

What is a Decision Tree

- A decision tree is a ***flowchart-like tree structure***
  - Each *internal node* (non-leaf node) denotes a test on an attribute
  - Each *branch* (i.e., subtree) represents an outcome of the test
  - Each *leaf node* (or terminal node) holds a class label
- It simulates the process of human decision-marking.
  - Thus, one advantage of decision trees is *understandability*

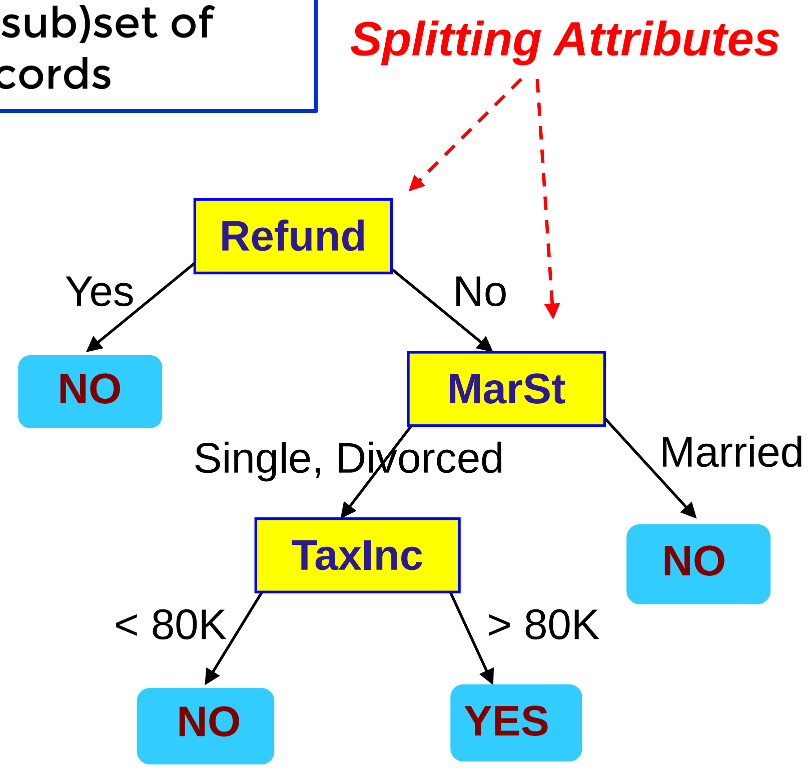
Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data

Each node is associated with a (sub)set of records

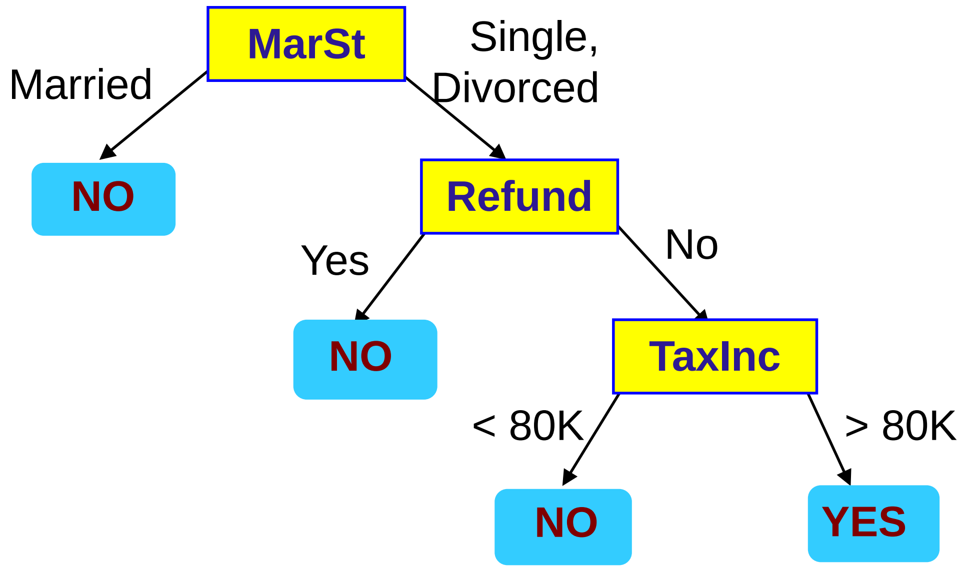


Model: Decision Tree

Another Example of Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



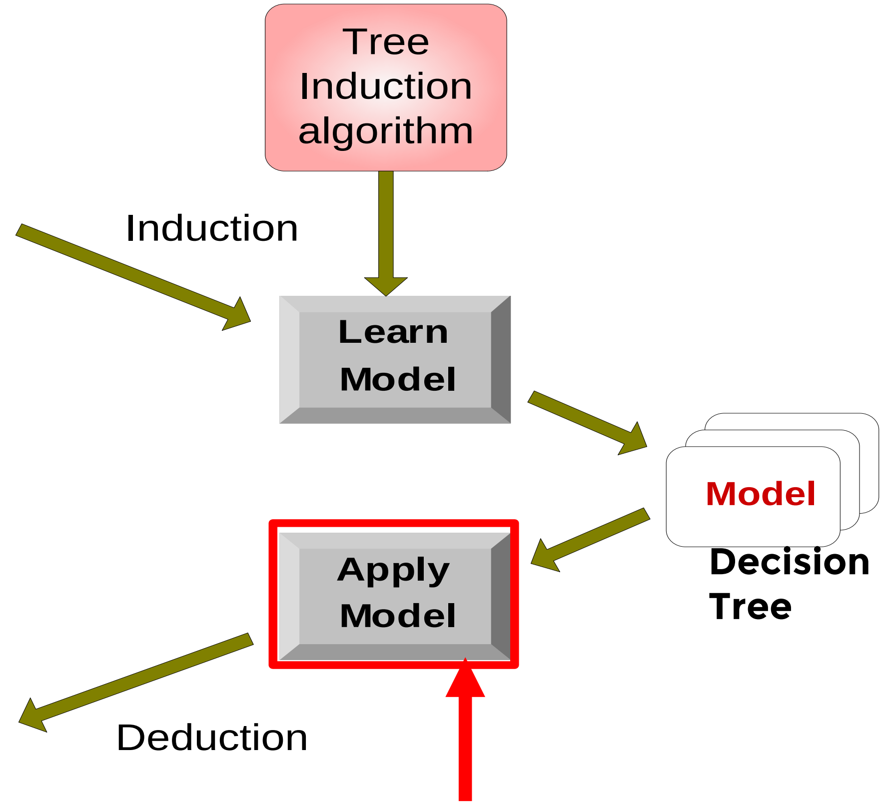
There could be multiple trees that fit the same data!

UNIVERSITY OF WOLLONGONG AUSTRALIA

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

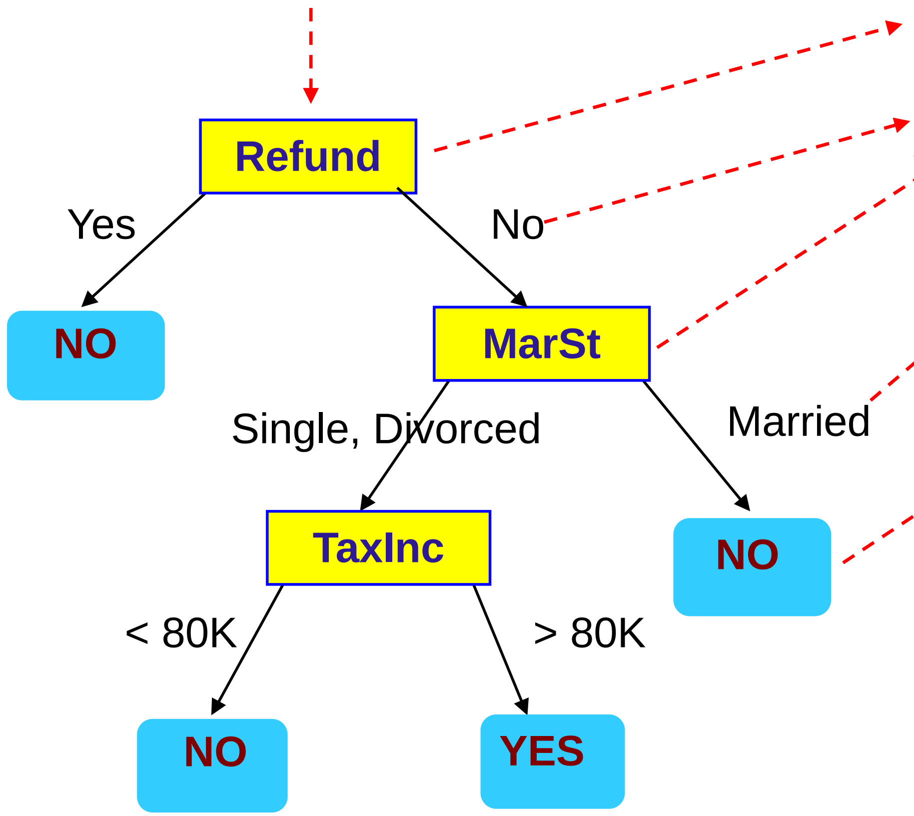


Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

Apply Model to Test Data

Start from the root of tree.



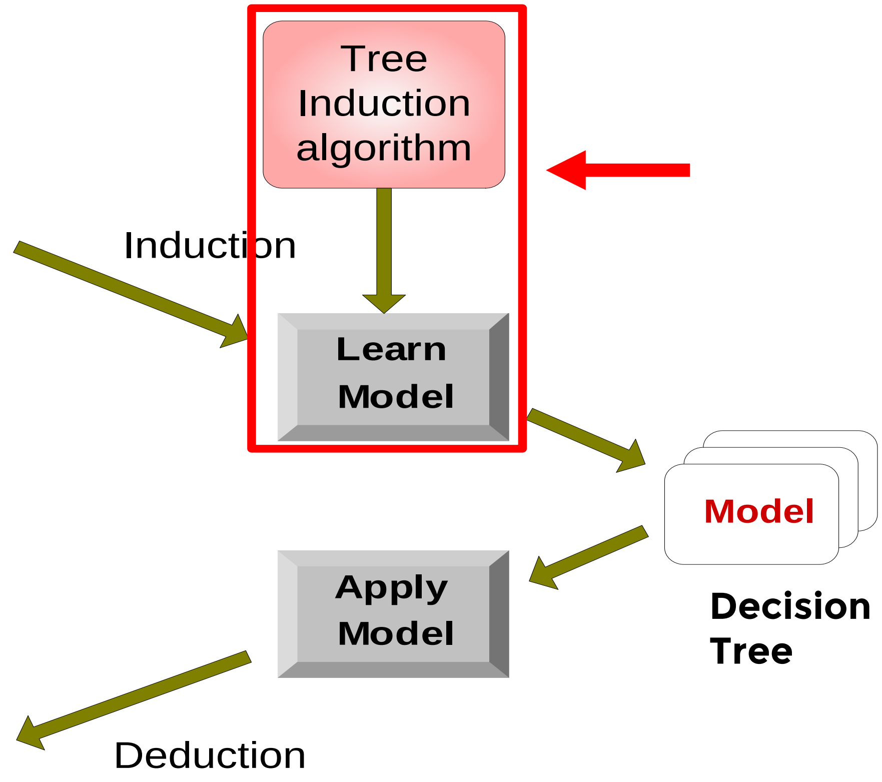
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	? **No**

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set



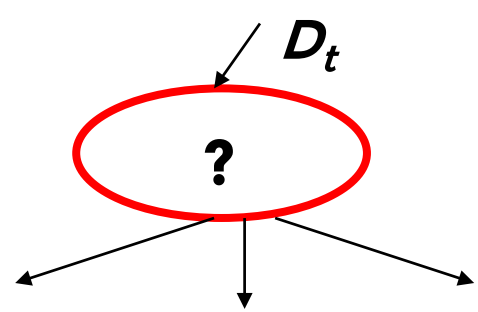
Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set

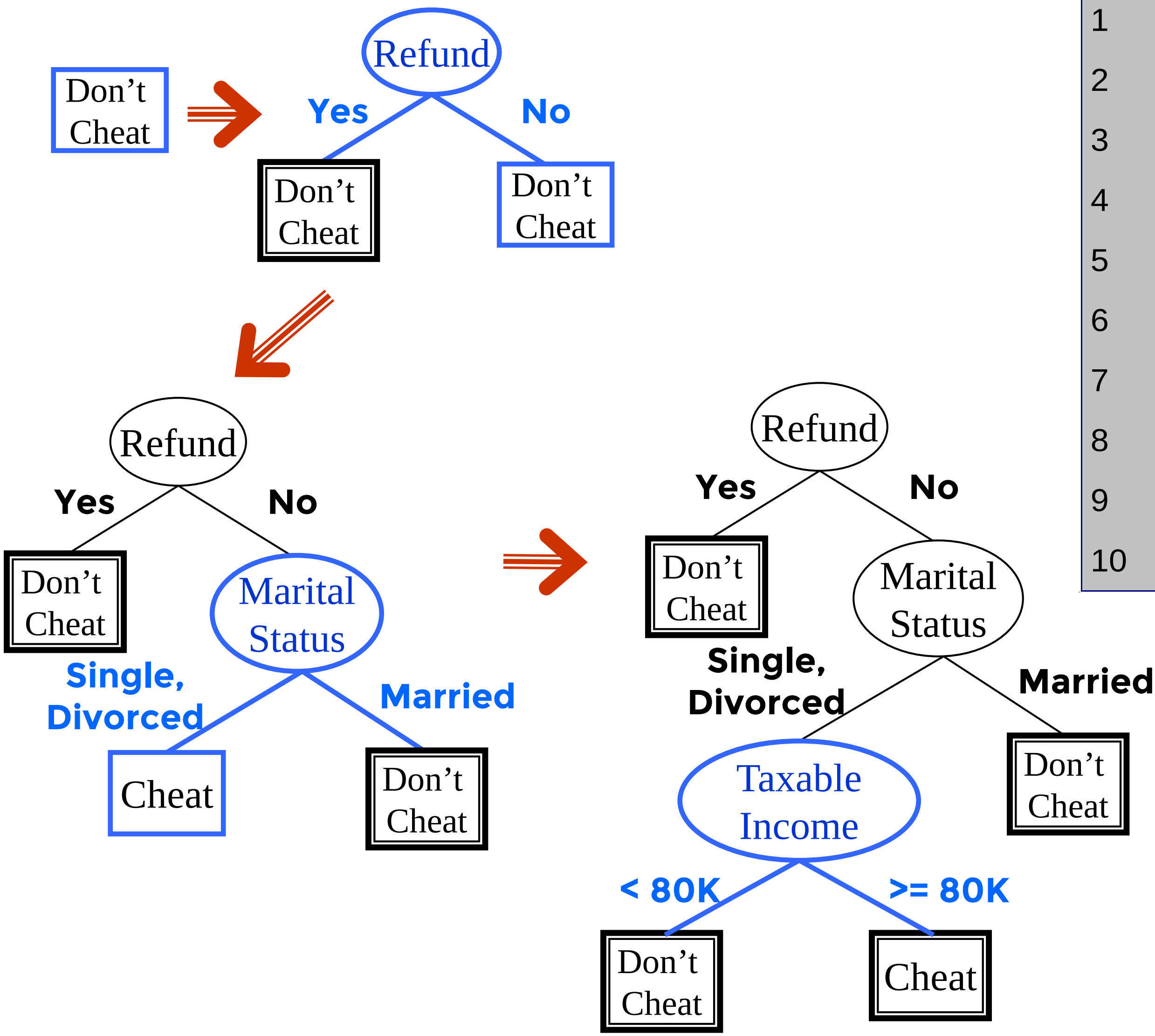
General Structure of Decision Tree Induction Algorithms

- Let D_t be the associated set of training records that reach a node t
- General Procedure:
  - If D_t contains records that belong the same class y_t, then t is a leaf node, labeled as y_t
  - If D_t is an empty set, then t is a leaf node, labeled as the same class as its parent node
  - If no more attributes to split D_t, then t is a leaf node, labeled as the *majority class*
  - Otherwise, ***split*** the dataset into smaller subsets, each of which is associated with a child node of the node t, and ***recursively*** apply the same procedure to child node

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



Hunt's Algorithm



Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition? (focus)
    - How to determine the best split?
  - Determine when to stop splitting

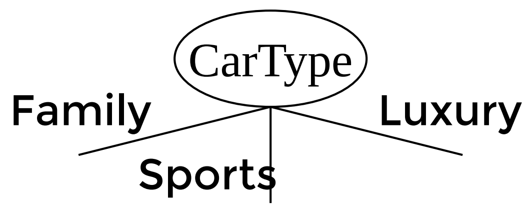
How to Specify Test Condition?

- Depends on the attribute types
  - Nominal/categorical
  - Ordinal
  - Continuous

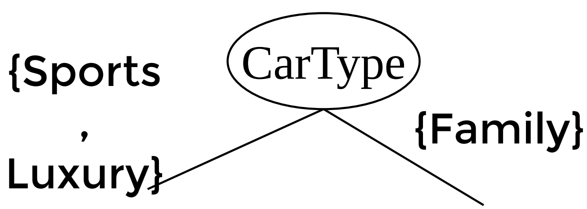
- Depends on the number of ways to split
  - 2-way split
  - Multi-way split

Splitting Based on Nominal Attributes

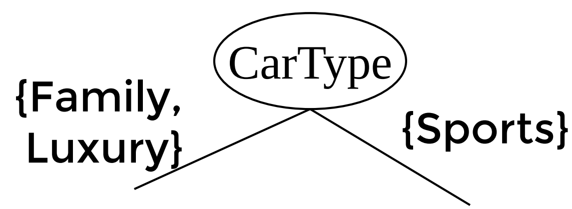
- Multi-way split: Use as many partitions as distinct values.



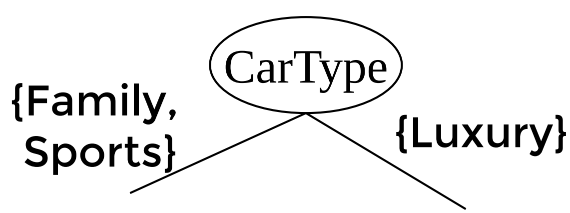
- Binary split: Divide values into two subsets.
  Need to find optimal partitioning.



OR



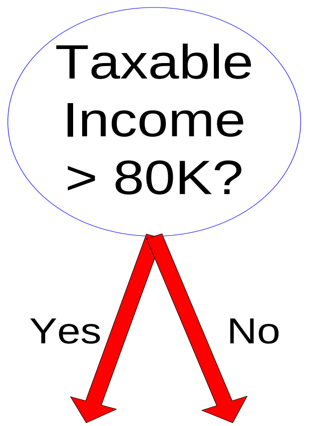
OR



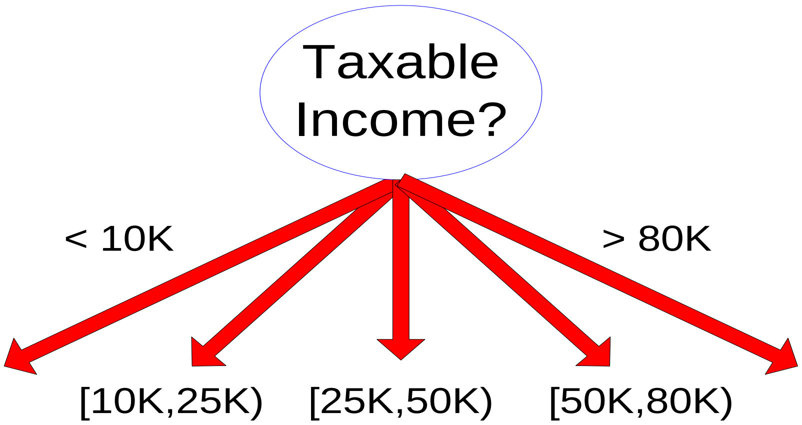
Splitting Based on Ordinal/Continuous Attributes

- Different ways of handling
  - Discretization to form an ordinal categorical attribute
    - Static – discretize once at the beginning
    - Dynamic – bucketing, percentiles, clustering…
  - Binary Decision: $(A < v)$ or $(A \geq v)$
    - consider all possible splits and finds the best cut
    - can be more computationally intensive

Splitting Based on Ordinal/Continuous Attributes



(i) Binary split



(ii) Multi-way split

UNIVERSITY OF WOLLONGONG AUSTRALIA

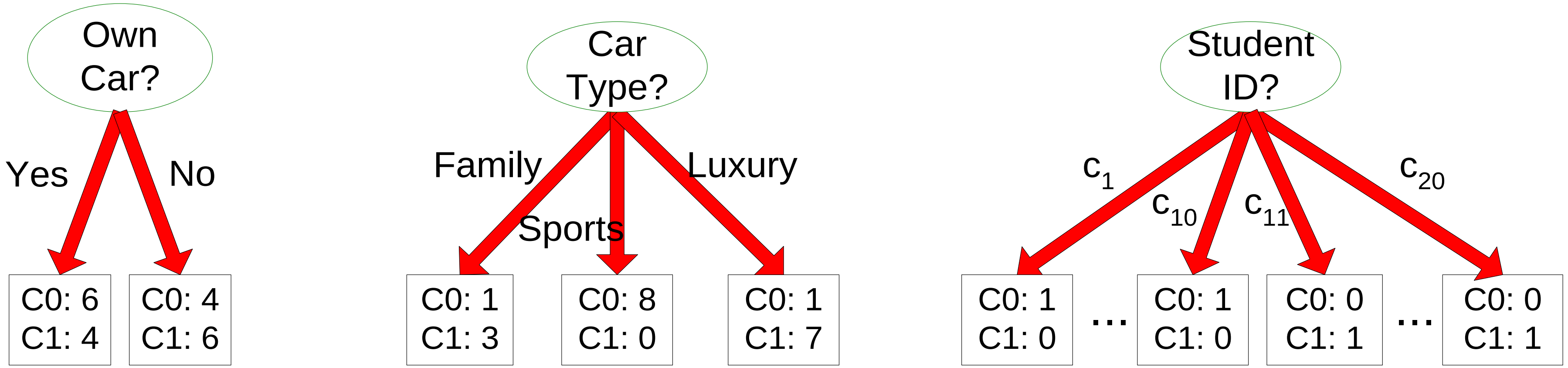
Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split? (focus)
  - Determine when to stop splitting

How to determine the Best Split

**Before Splitting: 10 records of class 0,
10 records of class 1**



Which test condition is the best?

How to determine the Best Split

- Greedy approach:
  - Nodes with homogeneous class distributions are preferred
- Need a measure of node **impurity** (or information **uncertainty**):

C0: 5	C0: 9
C1: 5	C1: 1

Non-homogeneous,

High degree of impurity

Homogeneous,

Low degree of impurity

Another way to look at Impurity and Uncertainty

- We flip two different coins: (0 is “head”, 1 is “tail”)
  - 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0…
  - 0 1 0 1 0 1 1 1 0 0 1 1 0 1 0 1 0 1…

- Question: *How to measure/quantify the information uncertainty with the two coins?*

UNIVERSITY OF WOLLONGONG AUSTRALIA

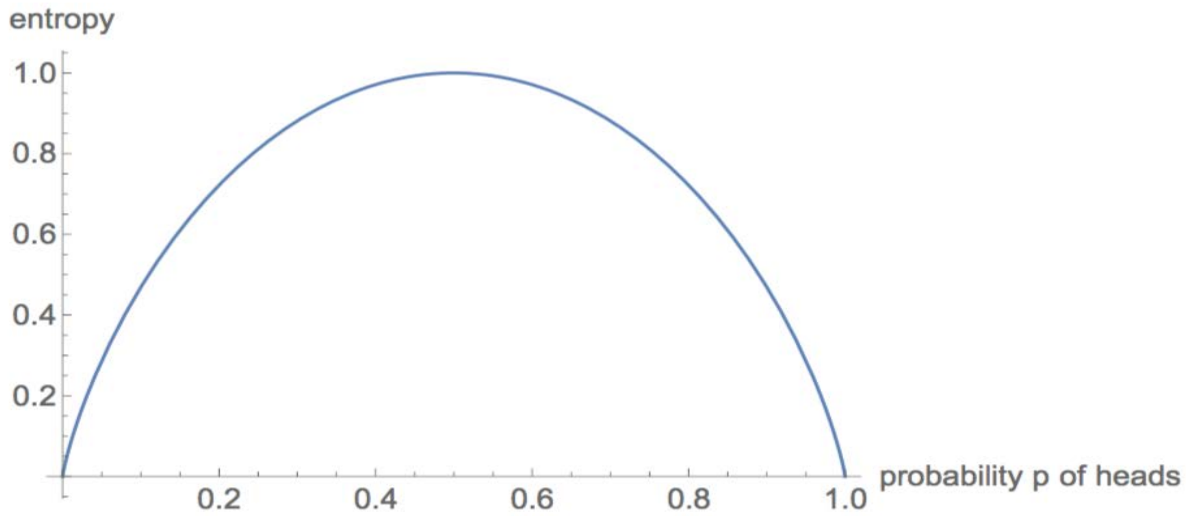
Different Measures of Impurity/Uncertainty

- Entropy (information gain)
- Gain ratio
- Gini Index
- Variance
- Others …

Shannon Entropy

- Logarithm: $y = \log_a x$
  - $2^3 = 8 \Leftrightarrow \log_2 8 = 3$
  - $2^{-1} = 0.5 \Leftrightarrow \log_2 0.5 = -1$
- Shannon Entropy:

$$H(X) = -\sum_{x \in X} p(x) \log_2 p(x)$$



UNIVERSITY OF WOLLONGONG AUSTRALIA

Conditional Entropy

- Example: X = {Raining, Not raining}, Y= {Cloudy, not cloudy}

	Cloudy	Not cloudy	Total
Is Raining	24/100	1/100	1/4
Not Raining	25/100	50/100	3/4
Total	49/100	51/100	

- What is the entropy of cloudiness, given the knowledge of whether or not it is raining?

$$\begin{aligned} H(Y|X) &= \sum_{x \in X} p(x) H(Y|X=x) \\ &= \frac{1}{4} H(\quad Y \quad |\text{is raining}) + \frac{3}{4} H(\quad Y \quad |\text{not raining}) \\ &\approx 0.75 \text{ bits} \end{aligned}$$

UNIVERSITY
OF WOLLONGONG
AUSTRALIA

Information Gain

- If I don't know whether it is raining or not, the entropy of cloudiness is $H(Y) \approx 1.00$ bit (*verifying this as an exercise*)
- How much information about cloudiness do we gain by discovering whether it is raining?
- The Shannon entropy tells $\text{InfoGain}(Y|X) = H(Y) - H(Y|X) \approx 0.25$ bit
- How do we make use of this measure to construct our decision tree?
  - E.g., to determine the best split of the dataset.

Splitting Based on InfoGain

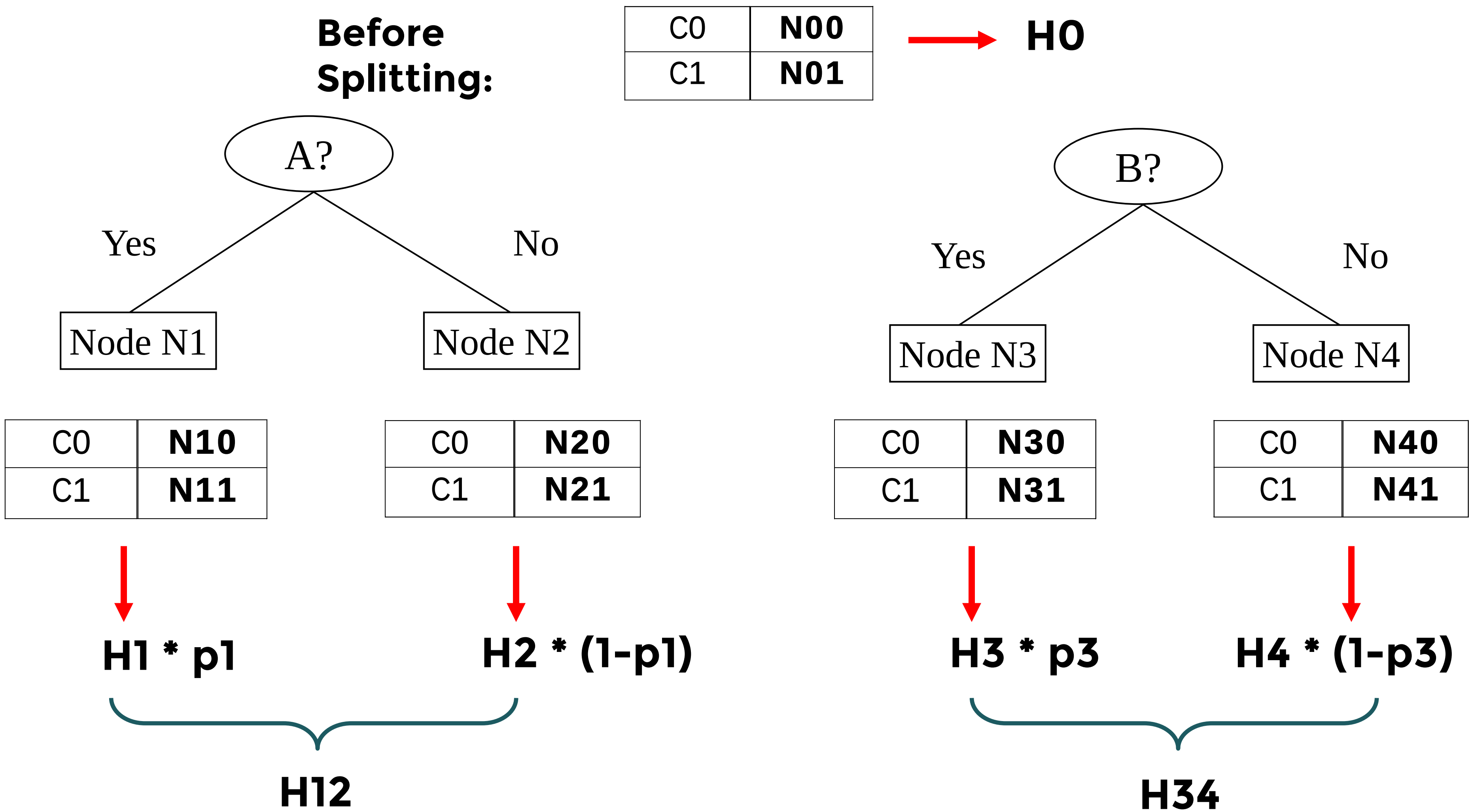
- Let D be the set of training records that reach a node
  - Compute the entropy $H(D)$ for D
- Let *Attribute_List* be a set of attributes associated with D
  - Each split with an attribute in *Attribute_List* produces a **partition** on $P = \{D_1, \dots, D_v\}$ on D
  - Compute the conditional entropy for each split and then calculate the InfoGain:

$$H_P(D) = \sum_{i=1}^{v} \frac{|D_i|}{|D|} H(D_i)$$

$$\text{InfoGain}(P) = H(D) - H_P(D)$$

- Select an attribute that gives the best split (one with the *largest* InfoGain)

How to Find the Best Split



Gain = H0 – H12 vs H0 – H34

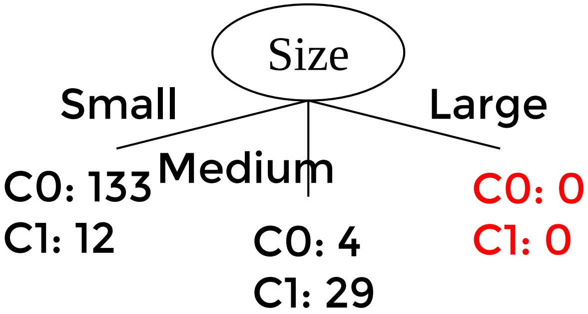
Tree Induction

- Greedy strategy.
  - Split the records based on an attribute test that optimizes certain criterion.

- Issues
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting (focus)

Stopping Criteria

1. No more attribute for splitting the dataset D_t
   - Majority vote: select the class label with most records to report
2. All tuples in D_t share the same class label
3. D_t is empty (no tuples)
4. Non-basic criteria
   - Tree pre-pruning (talked later), such as
     - set a threshold for the impurity measured
     - minimum dataset size
     - largest tree depth
     - etc.



Tree Induction Algorithm

Assumption: the training tuples contain categorical values only; multi-split is used.

Procedure: **generate_decision_tree**(*D*, *Attribute_List*).

- ❖ Generate a decision tree from a set of training tuples of *D*.

Input:

- – Dataset, *D*, which is a set of training tuples (each includes a tuple of feature values and one class label)
- – *Attribute_List*, the set of candidate attributes for split

Output: A decision tree

Tree Induction Algorithm

Pseudo-code:

(1) create a node *N*;

(2) **if** tuples in *D* are all of the same class, i.e. C, **then**

(3) **return** *N* as a (leaf) node labeled with the class C;

(4) **if** *Attribute_List* is empty **then**

(5) **return** *N* as a leaf node labeled with the majority class C_0

(6) find the *best_splitting_attribute* in *Attribute_List* to split *D*;

(7) *New_Attribute_List* ← *Attribute_List/{best_ splitting_attribute}*;

Tree Induction Algorithm

(8) **foreach** value *s* of *best_ splitting_attribute*;

(9) let D_s be a subset of *D* with *best_ splitting_attribute* being *s*;

(10) **if** D_s is empty **then**

(11) attach a (leaf) node labeled with the majority class in *D* to node *N*;

(12) **else** attach a new node, N_{child}, returned by applying **generate_decision_tree**(D_s, *New_Attribute_List*) to node *N*;

(13) **return** *N*;

Classification with Decision Trees

- Given a testing tuple, the classification with a decision tree is just by traversing the tree until a leaf is reached.
- Procedure: **classify**(*N*, *d*)
- Input: testing tuple *d*.
- Output: a class label C
- Pseudo-code:

(1) **if** *N* is a leaf node **then**
(2) **return** the class label C with *N*;
(3) **else** traverse to the child node N_{child} of *N* where the value of the *best_splitting_feature* matches the value in *d*;
(4) let C = **classify**(N_{child}, *d*);
(5) **return** C;

Python Implementation

- Python **dictionaries** are a convenient data structure to represent a decision tree
  - Each splitting feature is a node
  - For a multi-split tree with categorical features:

$$\textbf{tree} = \{<\text{index of a splitting feature}>: \{v_0: \textbf{tree}_1 \textbf{ or leaf}_1, \ldots, v_l: \textbf{tree}_l \textbf{ or leaf}_l\}\}$$

where each v is a value of the splitting feature.

  - For a binary-split tree with continuous features:

$$\textbf{tree} = \{\text{"spInd"}: <\text{index of a splitting feature}>, \text{"spVal"}: <\text{a float value}>, \text{"left"}: \textbf{tree1 or leaf1}, \text{"right"}: \textbf{tree2 or leaf2}\}$$

Python Implementation

- A **leaf** can just be a class label, say, C_i.
- Or, a leaf is represented by a NumPy array (i.e., vector) $ary = (q_1, \dots, q_m)$
  - such that $q_i = |D_{c_i}|$ is a class frequency where:
    - D is the set of training tuples associated with splitting_feature (as a node), and
    - $D_{c_i} \subseteq D$ contains all tuples in D that belong to class C_i
  - Note that a class label can be determined immediately from the vector ary.
    - E.g., just choose the class with the largest q_i

Python Implementation

- It is not hard to observe that both the tree induction and the classification involve a ***recursive function***.
- Recursive function example in Python:

```
def factorial(n):
    if n == 1:
        return 1
    else:
        return n * factorial(n-1)
```

- `factorial` is called within itself.
- Running:

  $4! = 4 * 3!$

  $3! = 3 * 2!$

  $2! = 2 * 1!$

  $1! = 1$

UNIVERSITY OF WOLLONGONG AUSTRALIA

Python Implementation

- To check whether a node in a tree (as a Python dictionary) is a leaf or grows a subtree:

```
# python3
isinstance(somenode, dict) == True #a subtree
# or
type(somenode).__name__=='dict' #a subtree
```

Sample Python Code (Compute Shannon Entropy)

```
# calculate Shannon Entropy of a dataset
def calcShannonEnt(dataSet):
    numEntries = len(dataSet) # number of tuples
    labelCounts = {}
    for featVec in dataSet:
        currentLabel = featVec[-1] # a class label is
the last element in each tuples
        if currentLabel not in labelCounts.keys():
            labelCounts[currentLabel] = 0
        labelCounts[currentLabel] += 1
    shannonEnt = 0.0
    for key in labelCounts:
        prob = float(labelCounts[key]) / numEntries
        shannonEnt -= prob * log(prob, 2)
    return shannonEnt
```

Sample Python Code (Multi-Split , Categorical Features)

```
def chooseBestMultiSplit(dataSet):
    numFeatures = len(dataSet[0]) - 1  # number of features
    baseEntropy = calcShannonEnt(dataSet)
    bestInfoGain = 0.0; bestFeature = -1
    for i in range(numFeatures): # iterate over all features
        uniqueVals = set([tuple[i] for tuple in dataSet])
        newEntropy = 0.0
        for value in uniqueVals:
# "splitDataSet" function, implemented elsewhere, filters
"dataset" such that the i-th feature equals to "value"
            subDataSet = splitDataSet(dataSet, i, value)
            prob = len(subDataSet) / float(len(dataSet))
            newEntropy += prob * calcShannonEnt(subDataSet)
        infoGain = baseEntropy - newEntropy
        if (infoGain > bestInfoGain):
            bestInfoGain = infoGain; bestFeature = i
    return bestFeature  # returns a feature index
```

How to Implement a Decision Tree Classifier

- How to represent/encode your decision tree?
  - Consider a Python dictionary (see previous slides)
- How to implement your tree induction algorithm based on the `calcShannonEnt` and `chooseBestMultiSplit` functions?
  - Consider a recursive Python function that calls the two functions
  - Address all basic stopping criteria
- How to classify (new) records with your decision tree?
  - Also consider a recursive function

Gain Ratio

- Disadvantage of InfoGain: Tends to prefer splits that result in large number of partitions, each being small but pure.
- Recall that each split on node results in a partition $P = \{D_1, \dots, D_v\}$ on D, the set of records associated with this node.
- $\text{SplitInfo}(P) = -\sum_{i=1}^{v} \frac{|D_i|}{|D|} \log\left(\frac{|D_i|}{|D|}\right)$
- $\text{GainRatio} = \text{InfoGain}(P)/\text{SplitInfo}(P)$

Gini Index

- Gini index (or Gini impurity) is a measure of how often a randomly chosen element from the set would be incorrectly labelled, if it was randomly labelled according to the distribution of labels in the subset.
  - Given D , a set of training tuples:

$$\mathrm{Gini}(D) = \sum_{i=1}^{m} p_i \sum_{j \neq i} p_j = 1 - \sum_{i=1}^{m} p_i^2$$

where $p_i = |D_{C_i}|/|D|$, i.e. the probability that a tuple in D belongs to class C_i. (Here D_{C_i} refers to a subset of D such that the tuple belongs to class C_i.)

Gini Index

- Gini index is suitable to binary split for continuous feature values.
  - If a binary split on some feature is a binary partition $P = \{D_1, D_2\}$ on D, the Gini index of D given this partitioning is
$$\mathrm{Gini}_P(D) = \frac{|D_1|}{|D|}\mathrm{Gini}(D_1) + \frac{|D_2|}{|D|}\mathrm{Gini}(D_2)$$
  - The reduction in impurity that would be incurred by the binary split is
$$\Delta\mathrm{Gini}_P = \mathrm{Gini}(D) - \mathrm{Gini}_P(D)$$

Variance

- Variance is the expectation of the squared deviation of a random variable from its mean.
  - is a simple error measure for binary classification (i.e., two class labels, often represented by 0 and 1)
  - Given D, a data partition or a set of training tuples:

$$\mathrm{Var}(D) = p(1 - p)$$

  where p is the probability that a tuple in D belongs to class C_0 and is estimated by $|D_{C_0}|/|D|$.

Comparison of Impurity Measures

- All impurity measures return good results in general, but
  - **Information gain**:
    - biased towards multivalued attributes
  - **Gain ratio:**
    - tends to prefer unbalanced splits in which one partition is much smaller than the others
  - **Gini index:**
    - biased to multivalued attributes
    - has difficulty when the number of classes is large
    - tends to favor tests that result in equal-sized partitions and purity in both partitions
  - **Variance:**
    - suitable to binary classification, even though extension is possible

Handling Numerical Attributes with Binary Splits

- We now consider the implementation of a decision tree with numerical (and continuous-valued) attributes
- Recall the use of a Python dictionary to represent a tree and a recursive function to build the tree (see previous slides).

$$\begin{aligned}\textbf{tree} = \{\text{"spInd"}: &< \text{index of a splitting feature} > , \\ &\text{"spVal"}: \text{<a float value>}, \\ &\text{"left"} : \textbf{tree1 or leaf1}, \\ &\text{"right"} : \textbf{tree2 or leaf2}\}\end{aligned}$$

- A key function is determining the best split.
  - In the following sample code, we implement such a function that uses the binary split and the variance criterion.

UNIVERSITY
OF WOLLONGONG
AUSTRALIA

Sample Python Code (Binary Split for Numerical Features)

```
from numpy import *
def chooseBestBiSplit(dataSet):
    _,n = shape(dataSet)
    S = var(dataSet[:,-1]) #variance of labels
    bestS = inf; bestIndex = 0; bestSplitValue = 0
    for featIndex in range(n-1):
        for splitVal in set(dataSet[:,featIndex]):
       # "bisplitDataSet" function should split "dataset" into
two subsets w.r.t. "featIndex" and "splitVal"
            subDS0, subDS1 = biSplitDataSet(dataSet,
               featIndex, splitVal)            p =
len(subDS0)/len(dataSet)
            newS = p*var(subDS0[:,-1]) + (1-p)*var(subDS1[:,-1])
            if newS < bestS:
                bestIndex = featIndex; bestSplitValue = splitVal
                bestS = newS
    if (S - bestS) < 0:
        return None, 0 # no need to split the dataset
    return bestIndex, bestSplitValue
```

In practice, this set may be large, so split may take a long time. In this case, use a subset of specific values (e.g., percentiles).

Advantages of Decision Tree Classifier

- Construction of the tree does not require any domain knowledge
- Can handle multidimensional data
- Representation of knowledge (as a decision tree) easy to assimilate by human
- The learning and classification steps are simple and fast
- Good accuracy in general.

Overfitting and Tree Pruning

- Overfitting: An induced tree may overfit the training data
  - Too many branches, some may reflect anomalies due to noise or outliers
  - Poor accuracy for unseen samples
- Two approaches to avoid overfitting
  - Pre-pruning: *Halt tree construction early*— do not split a node if this would result a measure falling below a threshold
    - Difficult to choose appropriate parameter thresholds
  - Post-pruning: *Merge branches* from a "fully grown" tree—get a sequence of progressively pruned trees
    - Use a set of data *different* from the training data to decide which is the "best pruned tree"

Tree Pre-Pruning

ops is an optional argument. If the variance decrement is small than ops[0] or the size of the split dataset is small than ops[1], stop the split process. By default, ops=(0.5,4).

```
def chooseBestBiSplit1(dataSet, ops=(0.5,4)):
    tolS = ops[0]; tolN = ops[1]
    _,n = shape(dataSet); S = var(dataSet[:,-1])
    bestS = inf; bestIndex = 0; bestValue = 0
    for featIndex in range(n-1):
        for splitVal in set(dataSet[:,featIndex]):
 # biSplitDataSet a function splitting according to the
split feature "tree['spInd']" and value "tree['spVal']" (see pp.44)
            D0, D1 = biSplitDataSet(dataSet,
                        featIndex, splitVal)
            if (shape(D0)[0] < tolN) or (shape(D1)[0] < tolN):
               continue
            p = float(len(D0))/len(dataSet)
            newS = p * var(D0) + (1-p) * var(D1)
            if newS < bestS:
                bestIndex = featIndex; bestValue = splitVal
                bestS = newS
    if (S - bestS) < tolS:
        return None, 0 #exit cond 1
    D0, D1 = biSplitDataSet(dataSet, bestIndex, bestValue)
    if (shape(D0)[0] < tolN) or (shape(D1)[0] < tolN):
        return None, 0 # exit cond 2
    return bestIndex, bestValue
```

Tree Post-Pruning

- We assume that *leaves of a trained decision tree are NumPy arrays*, representing the frequencies for all classes.
- To implement a post-pruning algorithm, we provide an auxiliary function that recursively propagates a vector of class frequencies from leaves into internal nodes:

```
def getSum(tree):
    if isinstance(tree['right'], dict):
        tree['right'] = getSum(tree['right'])
    if isinstance(tree['left'], dict):
        tree['left'] = getSum(tree['left'])
    return tree['left']+tree['right']
```

- When a subtree is removed from the current node (which therefore becomes a new leaf), `getSum()` will return a vector of class frequencies for this node.

Tree Post-Pruning Algorithm for binary-split trees

Assumption: for binary-split trees

Procedure: **prune**(*tree*, D_T).

Input:

- *tree*, a decision tree
- D_T, a set of testing tuples

Output: A pruned decision tree

Pseudo-code:

(1) traverse to a root node N of *tree*;

(2) if D_T is empty then

(3) return *getSum*(*tree*);

Tree Post-Pruning Algorithm

(4) split D_T based on the splitting feature and threshold at N;

(5) let D_{T1} and D_{T2} be the resulted subsets of D_T;

(6) **if** the left branch of N is not a leaf **then**

(7) apply **prune**(*tree*, D_{T1});

(8) **if** the right branch of N is not a leaf **then**

(9) apply **prune**(*tree*, D_{T2});

(10) **if** both the left and right branches of N are leaves **then**

(11) let errorMerge and errorNoMerge be the numbers of errors with and without merging the two branches, respectively;

(12) **if** errorMerge < errorNoMerge **then**

(13) **return** *getSum*(*tree*);

(14) **else return** *tree*;

Use a voting function in the array of class frequencies to get a predicted class, then count the errors.

Sample Python Code (Prune Function)

```
def prune(tree, testData):
    if shape(testData)[0] == 0:
        return getSum(tree) #if no test data collapse the tree
    lSet,rSet = biSplitDataSet(testData,tree['spInd'],tree['spVal'])
    if isinstance(tree['left'], dict) == True
        tree['left'] = prune(tree['left'], lSet)
    if isinstance(tree['right '], dict) == True
        tree['right'] =  prune(tree['right'], rSet)
    #if they are now both leaves, see if we can merge them
    if isinstance(tree['left'], dict) == False
       and isinstance(tree['right '], dict) == False :
    # a voting function “lab” returns predicted classes of leaves
        ll = lab(tree['left’]); lr = lab(tree['right’])
        treeSum = tree['left']+tree['right’]; ls = lab(treeSum)
    # count the errors (data whose class is predicted incorrectly)
        errorNoMerge = ...; errorMerge = ...
        if errorMerge < errorNoMerge:
            return treeSum # equals to getSum(tree)
    else: return tree
```

Summary

- Decision Tree Classifier
  - Theory
  - Implementation
  - Tree Pruning