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A Probabilistic Perspective of Classification

CSCI316 Big Data Mining Techniques and Implementation

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Contents

Bayesian Classification

- A probabilistic classifier: performs *probabilistic prediction, i.e.,* predicts class membership probabilities
- Foundation: Based on Bayes' Theorem.
- Performance: A simple Bayesian classifier, *naïve Bayesian classifier*, has comparable performance with decision tree and other classifiers
- Incremental: Each training example can incrementally increase/decrease the probability that a hypothesis is correct — prior knowledge can be combined with observed data
- Standard: Even though general Bayesian methods are computationally intractable, simple Bayesian methods can provide a baseline of optimal decision making against which other methods can be measured

Classification Concepts Recap

- Given a set of records, each of which is described by a sequence of attributes $X_1, \dots, X_n, Y$. The last one is an attribute of interest, called a **class**. The rest are called **features**.
- Given a new record where Y is unknown, the task of classification is to predict which class this record falls into.
- **Probabilistic classifier**: the output of prediction is a class together with a *probabilistic score*
  - to what extent the new record falls into the output class
  - Provides the likelihood instead of a hard decision

Probability and Uncertainty

- Our main tool is the probability theory, which assigns to a numerical degree of belief between *0* and *1* to each event.
  - It provides a way of characterizing the uncertainty
- Random variables:
  - Boolean random variables: cavity might be true or false
  - Discrete random variables: weather might be sunny, rainy, cloudy, snow
    - $P(weather = sunny)$
    - $P(weather = rainy)$
    - $P(weather = cloudy)$
    - $P(weather = snow)$
  - Continuous random variables: the temperature has continuous values
    - Discretization: < 10, $[10, 20]$, > 20
    - Probability density function: e.g., Normal distribution.

Prior and Posterior Probabilities

- Before the evidence is obtained; prior probability
  - $P(a)$ the prior probability that the proposition is true
  - $P(rain) = 0.1$
- After the evidence is obtained; posterior probability
  - $P(a \mid b)$
  - The probability of *a* given that all we know is *b* (i.e., **conditional probability**)
  - $P(rain \mid cloudy) = 0.8$

Bayes' Theorem (Simple)

- The conditional probability of event C occurring, given event A, is

$$P(C|A) = \frac{P(A \cap C)}{P(A)}$$

  – E.g. A is an attribute and C is the class.

- Bayes' theorem for two events:

$$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{P(C \cap A)}{P(A)} = \frac{P(A|C) \cdot P(C)}{P(A)}$$

  – It links the prior probabilities of two events and their posterior probabilities given each other.

Example

- Computing the probability that a patient carries a disease based on the result of a lab test.
- The test returns a positive result in 95% of the cases in which the disease is actually present, and it returns a positive result in 6% of the cases in which the disease is not present.
- Furthermore, 1% of the entire population has this disease.
- Let $C = \{\text{having the disease}\}$ and $A = \{\text{testing positive}\}$.
- From the above description, $P(C) = 0.01$, $P(\neg C) = 0.99$, $P(A|C) = 0.95$ and $P(A|\neg C) = 0.06$.

Reasoning with Bayes' Theorem

$$P(A) = P(A \cap C) + P(A \cap \neg C)$$
$$= P(C) \cdot P(A|C) + P(\neg C) \cdot P(A \mid \neg C)$$
$$= 0.01 \times 0.95 + 0.99 \times 0.06 = 0.0689$$

$$P(C|A) = \frac{P(A|C)P(C)}{P(A)} = \frac{0.95 \times 0.01}{0.0689} \approx 0.1379$$

Therefore, if some one has a test with positive result, he has 13.79% chance to carry the disease.

Bayes' Theorem (General)

- In a more general form, Bayes' theorem says that

$$P(Y|X_1, \dots, X_m) = \frac{P(X_1, \dots, X_m|Y) \cdot P(Y)}{P(X_1, \dots, X_m)}$$

- Linking it to classification: Y is the class and $X_1, \dots, X_m$ are attributes.
- E.g., attributes: *age, income, student_status, credit_rating*; class: *buys_computer*

RID	*age*	*income*	*student*	*credit_rating*	*Class: buys_computer*
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes

Naïve Bayes Classifiers

- To apply Bayes theorem to classification, one main problem is *the number of combinations of attribute values*
  - If there are m attributes and each attribute has k values, there are m^k combinations! Impractical to keep track of their join probabilities.
- Recall $P(Y|X_1,\dots,X_m) = \frac{P(X_1,\dots,X_m|Y)\cdot P(Y)}{P(X_1,\dots,X_m)}$
- We don't need to compute $P(X_1,\dots,X_m)$ since we just want to find out *which class (value of Y) has the highest score by comparison*.
  - E.g., given *age=youth*, *income=high*, *student=no*, and *credit_rating=fair*, is *buys_computer=yes* more likely?
    - In this case, we don't need to know the joint probability of *age=youth, income=high, student=no*, and *credit_rating=fair*
  - In other words, we just reply on

$$P(Y|X_1,\dots,X_m) \propto P(X_1,\dots,X_m|Y)\cdot P(Y)$$

where $\propto$ indicates "being propositional to".

Naïve Bayes Classifiers

- Still, $P(Y|X_1, \ldots, X_m) = \frac{P(X_1, \ldots, X_m|Y) \cdot P(Y)}{P(X_1, \ldots, X_m)}$
- We use the **conditional independence** assumption.
  - Each attribute is conditionally independent of every other attribute given a class label
  - Namely, $P(X_1, \ldots, X_m|Y) = P(X_1|Y) \cdots P(X_m|Y)$ which dramatically simplifies the computation of $P(X_1, \ldots, X_m|Y)$
- Therefore, we are concerned with

$$P(Y|X_1, \ldots, X_m) \propto P(X_1|Y) \cdots P(X_m|Y) \cdot P(Y)$$

Dataset Example

- Training tuples:

RID	*age*	*income*	*student*	*credit_rating*	*Class: buys_computer*
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

Illustration of Naïve Bayes Classifiers

- Let C_1 correspond to the class *buys_computer* = *yes* and C_2 correspond to *buys_computer* = *no*.
- Let $\boldsymbol{X}$ denote

  (*age* = *youth, income* = *medium, student* = *yes, credit rating* = *fair*)
- The objective is to maximize $P(\boldsymbol{X}|C_i)P(C_i)$ for $i = 1,2$
- First, the prior probability of each class can be computed based on the training tuples:

$$P(buys\_computer = yes) = 9/14 = 0.643$$
$$P(buys\_computer = no) = 5/14 = 0.357$$

Illustration of Naïve Bayes Classifiers

- Next, compute the conditional probabilities of attributes on the class labels:

$$
\begin{aligned}
P(age = youth \mid buys\_computer = yes) &= 2/9 = 0.222 \\
P(age = youth \mid buys\_computer = no) &= 3/5 = 0.600 \\
P(income = medium \mid buys\_computer = yes) &= 4/9 = 0.444 \\
P(income = medium \mid buys\_computer = no) &= 2/5 = 0.400 \\
P(student = yes \mid buys\_computer = yes) &= 6/9 = 0.667 \\
P(student = yes \mid buys\_computer = no) &= 1/5 = 0.200 \\
P(credit\_rating = fair \mid buys\_computer = yes) &= 6/9 = 0.667 \\
P(credit\_rating = fair \mid buys\_computer = no) &= 2/5 = 0.400
\end{aligned}
$$

Naïve Bayes Reasoning

- Using those probabilities, obtain:

$$\begin{aligned} P(\boldsymbol{X}|buys\_computer = yes) = {} & P(age = youth \mid buys\_computer = yes) \\ & \times P(income = medium \mid buys\_computer = yes) \\ & \times P(student = yes \mid buys\_computer = yes) \\ & \times P(credit\_rating = fair \mid buys\_computer = yes) \\ = {} & 0.222 \times 0.444 \times 0.667 \times 0.667 = 0.044. \end{aligned}$$

$$P(\boldsymbol{X}|buys\_computer = no) = 0.600 \times 0.400 \times 0.200 \times 0.400 = 0.019.$$

- Finally

$$P(\boldsymbol{X}|buys\_computer = yes)P(buys\_computer = yes) = 0.044 \times 0.643 = 0.028$$
$$P(\boldsymbol{X}|buys\_computer = no)P(buys\_computer = no) = 0.019 \times 0.357 = 0.007$$

- Therefore, the classifier predicts *buys_computer = yes*

A Simple NB Implementation

- Assume that our goal is to implement a Naïve Bayes classifier to predict whether a given text contains abusive content or not
  - represented by "1" or "0", respectively
- In this example, we generate the training dataset by ourselves:

```python
#   the training dataset
def loadDataSet():
    postingList = [['my', 'dog', 'has', 'flea',
                    'problems', 'help', 'please'],
                   ['maybe', 'not', 'take', 'him',
                    'to', 'dog', 'park', 'stupid'],
                   ['my', 'dalmation', 'is', 'so',
                    'cute', 'I', 'love', 'him'],
                   ['stop', 'posting', 'stupid',
                    'worthless', 'garbage'],
                   ['mr', 'licks', 'ate', 'my', 'steak',
                    'how', 'to', 'stop', 'him'],
                   ['quit', 'buying', 'worthless',
                    'dog', 'food', 'stupid']]
    classVec = [0, 1, 0, 1, 0, 1]  # 1 is abusive, 0 not
    return postingList, classVec
```

A Simple NB Implementation

- We generate a vocabulary of words and a feature matrix where each word is an attribute (i.e., column name)
  - Each row is a vector for a text; if the text contains some word, it has value 1 in the corresponding column.

```
from numpy import *
def createVocabList(dataSet):
    vocabSet = set([])  # create empty set
    for document in dataSet:
        vocabSet = vocabSet | set(document)
        # union of the two sets
    return list(vocabSet)

def setOfWords2Vec(vocabList, inputSet):
    returnVec = [0] * len(vocabList)
    for word in inputSet:
        if word in vocabList:
            returnVec[vocabList.index(word)] = 1
        else:
            print("word: %s is not in my Vocabulary!" % word)
    return returnVec
```

A Simple NB Implementation

- Check the two functions that we have just defined:

```
# call the two defined functions for illustration:
listOPosts, listClasses = loadDataSet()
myVocabList = createVocabList(listOPosts)
print(myVocabList)
# ['cute', 'love', 'help', 'garbage', 'quit', 'I',
# 'problems', 'is', 'park', 'stop', 'flea',
# 'dalmation', 'licks', 'food', 'not', 'him', 'buying',
# 'posting', 'has', 'worthless', 'ate', 'to', 'maybe',
# 'please', 'dog', 'how', 'stupid', 'so', 'take',
# 'mr', 'steak', 'my']
setOfWords2Vec(myVocabList, listOPosts[0])
#   [0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0,
# 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1]
```

- You can generate a training matrix by calling `setOfWords2Vec()` in a loop (for each element in `listOPosts`).

Pseudo-code of NB Training Function

- The pseudo-code of training function of a Naïve Bayes classifier is as follows:

```
calculate the number (or proportion) of documents in each class;
for every training document:
        for each class:
                if a token appears in the document then
                        increase the count for that token;
                increase the total count for tokens;
        for each class:
                for each token:
                        divide the token count by the total token count
                                to get conditional probabilities;
        return conditional probabilities for each class;
```

- Note that only discrete random variables (e.g., categorical attributes) are considered in this example.
  – Continuous random variables are considered later.

A Simple NB Implementation

```python
# build a naive bayes classifier: step 1
def trainNB0(trainMatrix, trainCategory):
    numTrainDocs = len(trainMatrix)
    numWords = len(trainMatrix[0])
    pAbusive = sum(trainCategory) / float(numTrainDocs)
    p0Num = zeros(numWords) # as numerator
    p1Num = zeros(numWords) # as numerator
    p0Denom = 0 # as denominator
    p1Denom = 0 # as denominator
    for i in range(numTrainDocs):
        if trainCategory[i] == 1:
            p1Num += trainMatrix[i]
            p1Denom += sum(trainMatrix[i])
        else:
            p0Num += trainMatrix[i]
            p0Denom += sum(trainMatrix[i])
    p1Vect = p1Num / p1Denom
    p0Vect = p0Num / p0Denom
    return p0Vect, p1Vect, pAbusive
```

A Simple NB Implementation

```
# build a naive bayes classifier: step 2
# vec2Classify the output vector of setOfWords2Vec, say,
[0, 1, 0,...]
# p0Vec, p1Vec, pAbusive are the output of trainBN0()
def classifyNB0(vec2Classify, p0Vec, p1Vec, pAbusive):
    # element-wise power computation
    p1 = prod(power(p1Vec, vec2Classify)) * pAbusive
    p0 = prod(power(p0Vec, vec2Classify)) * (1.0 -
        pAbusive)
    if p1 > p0:
        return 1
    else:
        return 0
```

Can you see this part is just the Bayesian classifier $P(Y|X_1, \ldots, X_m) \propto P(X_1|Y) \cdots P(X_m|Y) \cdot P(Y)$ **?**

A Simple NB Implementation

- Build an NB classifier and test it:

```
def testingNB0():
    # training
    listOPosts, listClasses = loadDataSet()
    myVocabList = createVocabList(listOPosts)
    trainMat = []
    for postinDoc in listOPosts:
        trainMat.append(setOfWords2Vec(myVocabList,postinDoc))
    p0V, p1V, pAb = trainNB0(array(trainMat),
                             array(listClasses))
    # classifying: case 1
    testEntry = [‘love’, ‘my’, ‘dalmation’]
    thisDoc = array(setOfWords2Vec(myVocabList, testEntry))
    print(testEntry, ‘classified as: ’,
          classifyNB0(thisDoc, p0V, p1V, pAb)) # out: 0
    # classifying: case 2
    testEntry = ['stupid', 'garbage']
    thisDoc = array(setOfWords2Vec(myVocabList, testEntry))
    print(testEntry, 'classified as: ',
          classifyNB0(thisDoc, p0V, p1V, pAb)) # out: 1
```

Multiple Occurrences

- In our simple NB implementation, we've treated the presence or absence of a word as a feature.
- But if a word appears more than once in a document, this information is not accounted for.
- Bag-of-words model: a *bag* of words can have multiple occurrences of each word, whereas a *set* of words can have only one occurrence of each word.

```
def bagOfWords2Vec(vocabList, inputSet):
    returnVec = [0] * len(vocabList)
    for word in inputSet:
        if word in vocabList:
            returnVec[vocabList.index(word)] += 1
                                                #   was "=" before
    return returnVec
```

Numerical Underflow

- If the number of attributes is large, the outputs of a Naïve Bayesian classifier are usually very small.
- In theory this is not a problem, because only the ratio between the outputs matters; however, in practical, the difference may be close or rounded off to 0 (this is unknown as the ***underflow*** problem).
- To avoid this, one widely used treatment is to manipulate a logarithm of a number rather than the number itself. Therefore,
- Thus $p_* = p_1 \cdots p_m$ becomes $\log(p_*) = \log(p_1) + \cdots + \log(p_m)$
  - The ratio between the output values of the classifier is not distorted!
  - As multiplication becomes $+$, the underflow is avoided.

Smoothing Zero Count

- Another problem is the ***zero count***: the count of records with a value of an attribute is zero when some class label is given
- If the zero count occurs, then one of $P(X_1|Y), \ldots, P(X_m|Y)$ is zero, and their multiplication is zero (no matter how large the rest are)
  - This is certainly counter-intuitive
  - Also, applying the log function to a zero probability, log(0) is negative infinite
- One common technique to overcome this is the ***Laplace smoothing*** (or add-one) technique: it adds 1 to all counts.
  - Because usually the training dataset is large (i.e., the total count is large), adding 1 to each count causes minimum effect
  - But if it would cause effect, add a very small number $\varepsilon > 0$ instead of 1.

Smoothing Zero Count

- Suppose that for the class *buys computer* = *yes* in some training database, *D*, containing 1000 tuples. We have 0 tuple with *income* = *low*, 990 tuples with *income* = *medium*, and 10 tuples with *income* = *high*.
- Without the Laplacian smoothing, the probabilities of those events are 0, 0.990 (from 990/1000) and 0.010 (from 10/1000), respectively.
- If a tuple has *income* = *low*, the probability of falling into the class *buys computer* = *yes* is 0, no matter what values for other attributes!
- With the Laplacian smoothing for the three quantities, adding 1 more tuple for each income value: the probabilities become 0.001 (from 1/1003), 0.988 (from 991/1003) and 0.011 (form 11/1003).
- The above phenomenon won't happen.

Continuous-Value Features

- We now consider an extension to Naïve Bayesian classifiers which are able to handle continuous-value features.
- If X is continuous, there are two common approaches to compute $P(X = a \mid Y = c)$:
  - ***Discretization/bucketing/binning***: The range of X is $(-\infty, a_1], [a_2, b_1], \dots, [a_k, b_{k-1}], [b_k, +\infty)$ for some k.
  - Assume that X has a **Gaussian distribution** (a.k.a. normal distribution).
- The following is the *probability density function* (PDF) of a Gaussian distribution with mean μ and variance σ^2:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

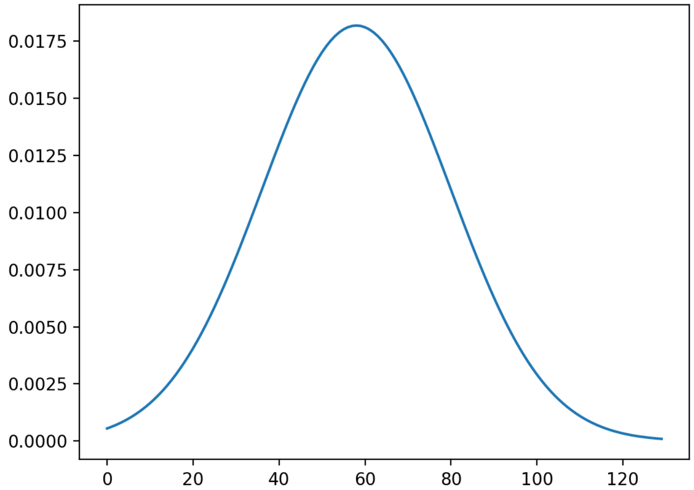
  - ➢ If we compute the mean value μ_0 and standard deviation σ_0 based on the training data for X when $Y = c$, then $P(X = x|Y = c)$ ***is*** $f(x, \mu_0, \sigma_0)$

Continuous-Value Features

- Estimation of mean and variance: Given observations $[x_1, \ldots, x_N]$
  - mean $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$
  - Variance $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2 = \left(\frac{1}{N}\sum_{i=1}^{N} {x_i}^2\right) - \mu^2$
- For example, if the incomes are not discretized in the costumer data and are 30, 36, 47, 50, 56, 60, 63, 70, 110 (K dollars) when *buys_computers = yes*, then
  - the mean is 58K and
  - the variance is 481.56
- Then

$P(income = 47 \mid buys\_computers = yes)$

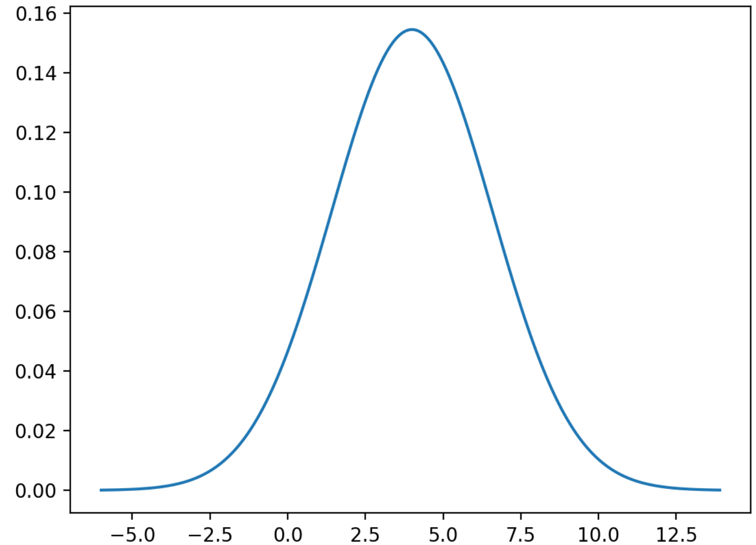
$is\ \frac{1}{\sqrt{2\pi}\cdot 21.94} e^{-\frac{(47-58)^2}{2\cdot 481.56}} = 0.016$



Continuous-Value Features

- To reason about PDF of the Gaussian distribution, we can use the norm package of the scipy.stats libarary :
  https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.norm.html#scipy.stats.norm

```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as pyplt
a = range(9)
mu = np.mean(a) # mean
sigma = (np.var(a))**0.5
    # standard deviation
x = np.arange(-6, 14, 0.1)
y = norm.pdf(x, mu, sigma)
pyplt.plot(x,y)
pyplt.show()
```



*Continuous-Value Features

- The previous example provides an interpretation is somehow "over simplistic", since the probability that a continuous random variable takes a particular value is zero.
- Instead, we should compute the conditional probability that X lies within some interval, say, $[r, r+\epsilon]$, where ϵ is a small constant:

$$P(r \leq X \leq r+\epsilon) = \int_{r}^{r+\epsilon} f\,(X, \mu, \sigma) dX \approx f(X, \mu, \sigma) \cdot \epsilon$$

- Since ϵ appears as a constant multiplicative factor for each class, it *cancels out* when normalizing the target probability, leaving just the $f\,(X, \mu, \sigma)$ part.

Comments and Summary

- Advantages
  - Easy to implement
  - Good results obtained in most of the cases
- Disadvantages
  - Assumption: class conditional independence, therefore loss of accuracy
  - Practically, dependencies exist among variables
    - E.g., Symptoms: fever, cough etc., Disease: lung cancer, diabetes, etc.
    - Dependencies among these cannot be modeled by Naïve Bayes Classifier
- How to deal with these dependencies?
  - Belief Bayesian Network
- From correlation to causality?