

# CSCI317 Database Performance Tuning

## Relational Algebra

Dr Janusz R. Getta

School of Computing and Information Technology -  
University of Wollongong

# Relational Algebra

## Outline

Relational algebra ? What is it ?

Operations

SELECT statement versus relational algebra

# Relational algebra ? What is it ?

**Relational algebra** is one two formal query languages associated with the relational database model

**Relational algebra** is a collection operations whose arguments are relational tables, sets of attributes, and formulas of propositional calculus

An operation of **relational algebra** accepts one or two relational tables as the arguments and it returns a relational table as the result

The other arguments of **relational algebra** operations are set of attributes (columns) and formulas of propositional calculus

What about the other formal language?

The other formal language is called as a **relational calculus** and it is based on the language of **First Order Logic**

# Relational Algebra

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# Operations

## Query

- A query expressed in the language of relational algebra is an expression built over the operations of relational algebra and the arguments like relational tables, sets of attributes and formulas of propositional calculus

## Operations

- Selection:  $\sigma_{\varphi}(r)$
- Projection:  $\pi_X(r)$
- Cross-product:  $r \times s$
- Join:  $r \bowtie_{\varphi} s$
- Antijoin:  $r \sim_{\varphi} t$
- Set operations:  $r \cup t$  (union),  $r \cap t$  (intersection)  $r - t$  (difference)

# Operations

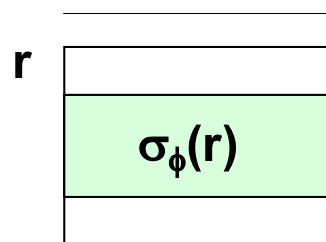
## Selection

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$

**Selection** operation  $\sigma$  applied to a relational table  $r$  is denoted by  $\sigma_{\phi}(r)$  and it returns a set of all rows  $v \in r$  such that evaluation of  $\phi(v)$  returns **TRUE**

For example  $\sigma_{\text{salary} > 80K}(\text{EMPLOYEE})$  returns all rows from **EMPLOYEE** table that have a value of attribute salary greater than **80K**

**Selection** operation **horizontally restricts** a relational table



# Operations

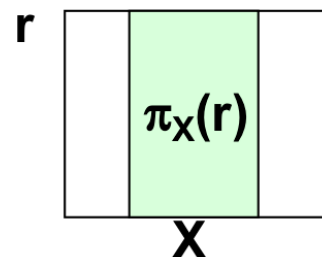
## Projection

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ ,  
let  $X \subseteq Y$

**Projection** operation  $\pi$  applied to a relational table  $r$  is denoted by  $\pi_X(r)$   
and it returns a set of all rows  $v$  such that there exists a row  $w \in r$  and  
 $v = w[X]$

For example  $\pi_{\{e\#, salary\}}(EMPLOYEE)$  returns a table built over a schema  
 $\{e\#, salary\}$  (two columns) such that each one of its rows is "included" in  
a relational table  $EMPLOYEE$

**Projection** operation **vertically restricts** a relational table



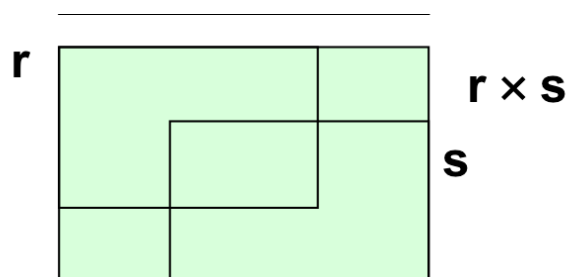
# Operations

## Cross-product

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $X$

**Cross-product** operator  $\times$  applied to the relational tables  $r$  and  $s$  is denoted by  $r \times s$  and it returns all rows  $v$  such that  $v = \text{concat}(u, w)$ ,  $u \in r$  and  $w \in s$

**Cross product** operation returns all possible concatenations of the rows from  $r$  and  $s$





# Operations

## Cross-product

For example, an expression

$\text{EMPLOYEE}(e\#, \text{name}, \text{salary}, \text{dname}) \times \text{DEPARTMENT}(\text{dname}, \text{budget})$

returns a relational table

$\text{ED}(e\#, \text{name}, \text{salary}, \text{E.dname}, \text{D.dname}, \text{budget})$

that consists of the concatenations of all rows from the relational tables

$\text{EMPLOYEE}$  and  $\text{DEPARTMENT}$

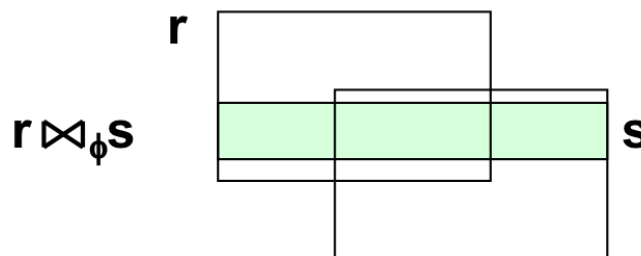
# Operations

## Join

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $X$ , let  $\varphi$  be a formula of propositional logic over  $X \cup Y$

**Join** operation  $\bowtie_{\varphi}$  applied to the relational tables  $r$  and  $s$  is denoted by  $r \bowtie_{\varphi} s$  and it returns a value of relational algebra expression  $\sigma_{\varphi}(r \times s)$

**Join** operation **concatenates** all pairs of rows  $(v, w)$ , where  $v \in r$  and  $w \in s$  that satisfy a given condition  $\varphi(v, w)$



# Operations

## Join

For example, an expression

$\text{EMP1}(\text{e\#, salary, dname}) \bowtie_{\text{EMP1.salary} > \text{EMP2.salary}} \text{EMP2}(\text{e\#, salary, dname})$

returns a relational table

$\text{EMP12}(\text{E1.e\#, E1.salary, E1.dname, E2.e\#, E2.salary, E2.dname})$

that contains the concatenations of all rows from  $\text{EMP1}$  and  $\text{EMP2}$  and such that  $\text{EMP1.salary} > \text{EMP2.salary}$

# Operations

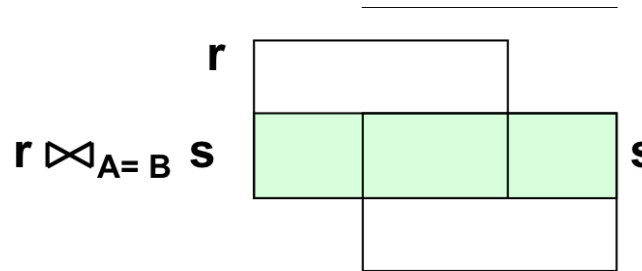
## Equijoin

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $X$ , let  $A \in Y$  and  $B \in X$

**Equijoin** operation  $\bowtie_{A=B}$  applied to the relational table  $r$  and  $s$  is denoted by

$r \bowtie_{A=B} s$  and it returns a value of relational algebra expression

$\sigma_{r.A=s.B}(r \times s)$



# Operations

## Equijoin

For example

$\text{EMP}(\text{e\#}, \text{name}, \text{salary}, \text{dname}) \bowtie_{\text{EMP.dname, DEPT.dname}} \text{DEPT}(\text{dname}, \text{budget})$

returns a relational table

$\text{ED}(\text{e\#}, \text{name}, \text{salary}, \text{E.dname}, \text{D.dname}, \text{budget})$

that consists of the concatenations of all rows the relational tables **EMP** and **DEPT** such that  $\text{EMP.dname} = \text{DEPT.dname}$

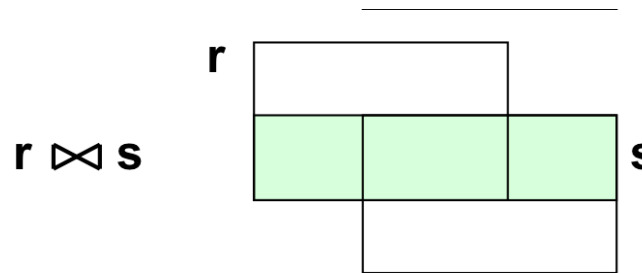
# Operations

## Natural join

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $X$ , let  $A = Y \cap X$

**Natural join** operation  $\bowtie$  applied to the relational tables  $r$  and  $s$  is denoted by  $r \bowtie s$  and it returns a value of relational algebra expression

$$\pi_{\text{schema}(r) \cup \text{schema}(s)} ( \sigma_{r.A=s.A} (r \times s) )$$



# Operations

## Natural join

For example, an expression

$\text{EMP}(\text{e\#}, \text{name}, \text{salary}, \text{dname}) \bowtie \text{DEPT}(\text{dname}, \text{budget})$

returns a relational table  $\text{ED}(\text{e\#}, \text{name}, \text{dname}, \text{budget})$

that consists of the concatenations of all rows from the relational tables

$\text{EMP}$  and  $\text{DEPT}$  and  $\text{EMP.dname} = \text{DEPT.dname} = \text{ED.dname}$

# Operations

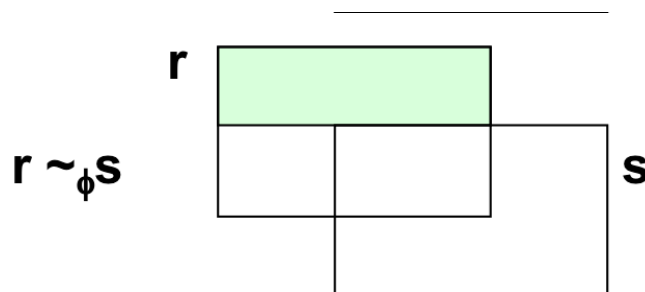
## Antijoin

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $X$ , let  $\phi$  be a propositional formula over  $X \cup Y$

**Antijoin** operation  $\sim_{\phi}$  applied to the relational tables  $r$  and  $s$  is denoted by

$r \sim_{\phi} s$  and it returns a value of relational algebra expression

$r - \pi_Y(\sigma_{\phi}(r \times s))$





# Operations

## Antijoin

For example, an expression

$\text{EMP1}(\text{e\#, salary, dname}) \sim_{\text{EMP1.salary} = \text{EMP2.salary}} \text{EMP2}(\text{e\#, salary, dname})$

returns a relational table  $\text{EMP12}(\text{E1.e\#,E1.salary,E1.dname})$

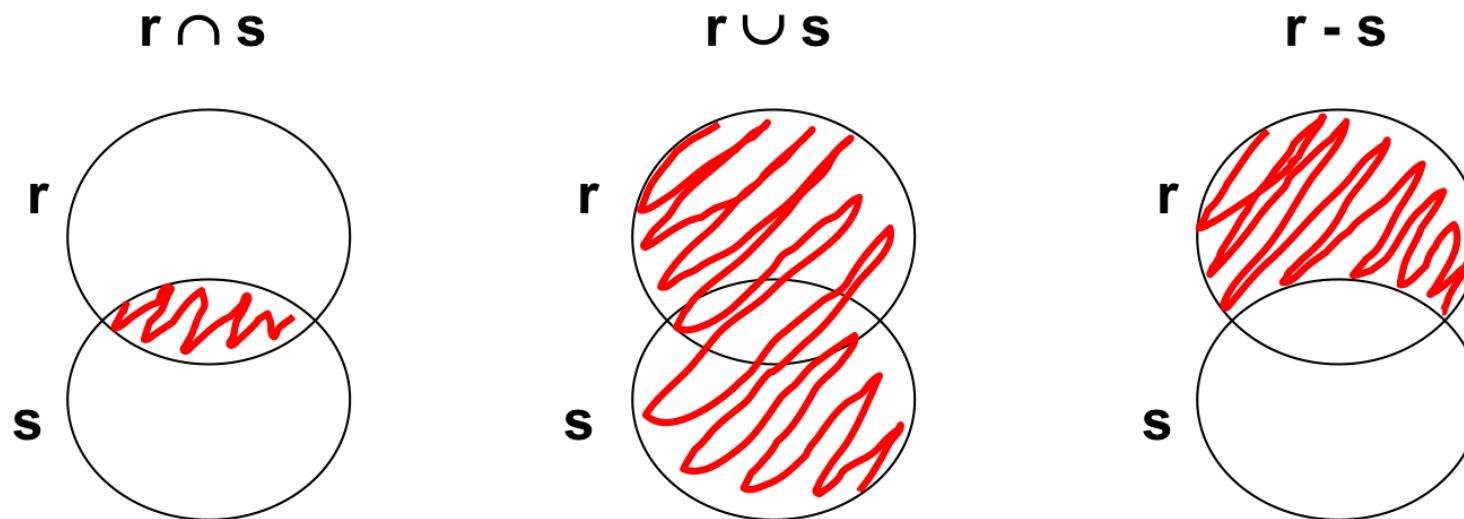
that contains all rows from a relational table  $\text{EMP1}$  such that for all values in  $\text{EMP1.salary}$  does not exist a value  $\text{EMP2.salary}$  where  $\text{EMP1.salary} = \text{EMP2.salary}$

# Operations

## Set operators

Let  $r$  be a relational table built over a schema (a set of attributes)  $Y$ , and  $s$  be a relational table built over a schema  $Y$

Visualizations of **set operations** are the following



# Relational Algebra

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# SELECT statement versus relational algebra

Any **SELECT** statement that does not contain **GROUP BY** clause and group functions can be translated into a relational algebra expression

```
SELECT S_NAME, S_ADDRESS  
FROM SUPPLIER  
WHERE S_ACCTBAL > 100;
```

SELECT statement with a selection and projection

$\pi_{S\_NAME, S\_ADDRESS}(\sigma_{S\_ACCTBAL > 100}(SUPPLIER))$

```
SELECT P_NAME  
FROM PART  
WHERE P_SIZE = 6  
INTERSECT  
SELECT P_NAME  
FROM PART  
WHERE P_TYPE = 'XYZ';
```

SELECT statement with a selection, projection, and INTERSECT operation

$\pi_{P\_NAME}(\sigma_{P\_SIZE=6}(PART)) \cap \pi_{P\_NAME}(\sigma_{P\_TYPE='XYZ'}(PART))$

# SELECT statement versus relational algebra

SELECT statement a selection, projection, and join operation

```
SELECT L_SHIPDATE
FROM LINEITEM, ORDERS
WHERE LINEITEM.L_ORDERKEY = ORDERS.O_ORDERKEY AND
      ORDERS.O_TOTALPRICE > 100;
```

$\pi_{L\_SHIPDATE} (\sigma_{O\_TOTALPRICE > 100} (ORDERS \bowtie_{L\_ORDERKEY = O\_ORDERKEY} LINEITEM))$

SELECT statement with a selection, projection, and two join operations

```
SELECT C_NAME, N_NAME
FROM CUSTOMER JOIN ORDERS
      ON CUSTOMER.C_CUSTKEY = ORDERS.O_CUSTKEY
      JOIN NATION
      ON CUSTOMER.C_NATIONKEY = NATION.N_NATIONKEY
WHERE C_ACCTBAL = 0;
```

$\pi_{C\_NAME, N\_NAME} ( \sigma_{C\_ACCTBAL=0} (CUSTOMER) \bowtie NATION) \bowtie ORDERS ) )$

# SELECT statement versus relational algebra

SELECT statement with a projection and left outer join operation

```
SELECT C_CUSTKEY, O_ORDERKEY
FROM CUSTOMER LEFT OUTER JOIN ORDERS
ON C_CUSTKEY = O_CUSTKEY;
```

$$((\pi_{C\_CUSTKEY}(CUSTOMER) - \pi_{O\_CUSTKEY}(ORDERS)) \times \{NULL\}) \cup \pi_{C\_CUSTKEY, O\_ORDERKEY}(CUSTOMER \bowtie_{C\_CUSTKEY=O\_CUSTKEY} ORDERS)$$

Nested SELECT statement

```
SELECT O_TOTALPRICE
FROM ORDERS
WHERE O_CUSTKEY IN (SELECT C_CUSTKEY
FROM CUSTOMER
WHERE C_ACCTBAL = 0);
```

$$\pi_{O\_TOTALPRICE}(ORDERS \bowtie_{O\_CUSTKEY=C\_CUSTKEY} (\sigma_{C\_ACCTBAL=0}(CUSTOMER)))$$

# SELECT statement versus relational algebra

Nested SELECT statement

```
SELECT O_TOTALPRICE
FROM ORDERS
WHERE O_CUSTKEY NOT IN (SELECT C_CUSTKEY
                        FROM CUSTOMER
                        WHERE C_ACCTBAL = 0);
```

$\pi_{O\_TOTALPRICE} (ORDERS \sim_{O\_CUSTKEY=C\_CUSTKEY} (\sigma_{C\_ACCTBAL=0} (CUSTOMER)))$

Nested SELECT statement with existential quantifier

```
SELECT O_TOTALPRICE
FROM ORDERS
WHERE EXISTS (SELECT C_CUSTKEY
              FROM CUSTOMER
              WHERE ORDERS.O_CUSTKEY = CUSTOMER.C_CUSTKEY AND C_ACCTBAL = 0);
```

$\pi_{O\_TOTALPRICE} (ORDERS \bowtie_{O\_CUSTKEY=C\_CUSTKEY} (\sigma_{C\_ACCTBAL=0} (CUSTOMER)))$

# SELECT statement versus relational algebra

Nested SELECT statement with negated existential quantifier

```
SELECT O_TOTALPRICE  
FROM ORDERS  
WHERE NOT EXISTS (SELECT C_CUSTKEY  
                  FROM CUSTOMER  
                  WHERE ORDERS.O_CUSTKEY = CUSTOMER.C_CUSTKEY AND C_ACCTBAL = 0);
```

$\pi_{O\_TOTALPRICE} (ORDERS \sim_{O\_CUSTKEY=C\_CUSTKEY} (\sigma_{C\_ACCTBAL=0} (CUSTOMER)))$



# References

R. Ramakrishnan and J. Gherke Database Management Systems, 3rd ed.  
Mc Graw-Hill, 2003,chapter 4.2