

# Capacity Expansion Model

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# 1 Overview

This document presents the formulation of the Capacity Expansion (CE) optimization model. I tried the definitions of variables and equations as close as possible to the way they are defined in the `.gams` file. This way it should be easier to debug the optimization problem. For example, instead of defining a single variable  $n_{(\cdot)}$  for the number of new generators in each pair (location, type), I used the same definitions as the `.gams` file and created three variables:  $n_{c,j}^{(c)}$ ,  $n_{z,j}^{(\bar{c})}$  and  $n_{z,j}^{(r)}$  (which refer, respectively, to thermal generators that can be curtailed, thermal generators that cannot be curtailed and renewable generators).

# 2 Definitions

Table 1: Decision Variables

Set	Definition
$n_{c,j}^{(c)}$	number of new thermal generators of type $j$ in the class that CAN be curtailed (the $(c)$ superscript) built in CELL $c$
$n_{z,j}^{(\bar{c})}$	number of new thermal generators of type $j$ in the class that CANNOT be curtailed (the $(\bar{c})$ superscript) built in ZONE $z$
$n_{z,j}^{(r)}$	number of new generators of type $j$ in the class RENEWABLE (the $(r)$ superscript) built in ZONE $z$
$p_{c,j,t}^{(c)}$	electricity generation (GWh) at time $t$ of new generators of type $j$ in the class that CAN be curtailed (the $(c)$ superscript) built in CELL $c$
$p_{z,j,t}^{(\bar{c})}$	electricity generation (GWh) at time $t$ of new generators of type $j$ in the class that CANNOT be curtailed (the $(\bar{c})$ superscript) built in ZONE $z$
$p_{z,j,t}^{(r)}$	electricity generation (GWh) at time $t$ of new generators of type $j$ in the class RENEWABLE (the $(r)$ superscript) built in ZONE $z$
$p_{i,t}^{(e)}$	electricity generation (GWh) at time $t$ of existing (the $(e)$ superscript) generator of index $i$
$\text{flow}_{\ell,t}$	flow on line $\ell$ (GW) in hour $t$

Table 2: Sets

Set	Definition
$\mathcal{B}$	set of user-defined time blocks. These are needed for computational purposes. $\mathcal{B} = \{\text{peak-hours, winter, summer, spring, fall, special periods}\}$
$\mathcal{I}$	set of existing generators in the fleet.
$\mathcal{I}(z)$	subset of existing generators that are located in zone $z$ . $\mathcal{I}(z) \subseteq \mathcal{I}$
$\mathcal{C}$	set of grid cells that new techs can be placed in.
$\mathcal{C}(z)$	subset of grid cells that new techs can be placed in that are located in zone $z$ . $\mathcal{C}(z) \subseteq \mathcal{C}$
$\mathcal{J}$	set of candidate plant types for new construction
$\mathcal{J}^{(c)}$	subset of plant types for new construction that can be curtailed. $\mathcal{J}^{(c)} \subseteq \mathcal{J}$
$\mathcal{J}^{(\bar{c})}$	subset of plant types for new construction that CANNOT be curtailed. $\mathcal{J}^{(\bar{c})} \subseteq \mathcal{J}$
$\mathcal{J}^{(r)}$	subset of plant types for new construction that are renewable. $\mathcal{J}^{(r)} \subseteq \mathcal{J}$
$\mathcal{L}$	set with transmission lines between load zones
$\mathcal{Z}$	set with user defined load zones

Table 3: Parameters

Parameter	Definition
$P_{c,j,t}^{MAX}$	Maximum electricity generation capacity, accounting for de-ratings, of plant type $j \in \mathcal{J}^{(c)}$ at cell grid $c$ at time $t$ (MWh)
$P_j^{NP}$	Nameplate electricity generation capacity of plant type $j \in \mathcal{J}$ (MWh)
$P_{i,t}^{MAX}$	Maximum electricity generation capacity, accounting for de-ratings, of existing generator $i$ (non solar and non wind) at time $t$ (MWh)
$P_{solar,t}^{MAX}$	Maximum electricity generation by all existing solar generators at time $t$ (MWh)
$P_{wind,t}^{MAX}$	Maximum electricity generation by all existing wind generators at time $t$ (MWh)
$\overline{\text{flow}}_\ell$	Upper bound of transmission line $\ell$ (GW)
$FOM_j$	Annual fixed operation and maintenance costs of plant type $j$ (\$/MW)
$OCC_j$	Overnight capital cost of plant type $j$ (\$/MW)
$OC_j$	Operating cost of plant type $j$ (\$/MWh)
$OC_i$	Operating cost of existing plant $i$ (\$/MWh)
$M$	Planning reserve margin as fraction (%) of demand
$Q$	Discount rate
$D_j$	lifetime (years) of candidate plant of type $j$

Table 4: Indices

Indices	Definition
$b$	Time blocks representing peak-hours, winter, summer, spring, fall, special periods. $b \in \mathcal{B}$
$c$	grid cells that new techs can be placed in. $c \in \mathcal{C}$
$\ell$	Transmission Lines. $\ell \in \mathcal{L}$
$i$	existing generators in fleet. $i \in \mathcal{I}$
$z$	sub regions of SERC. $z \in \mathcal{Z}$

### 3 Objective Function

$$\begin{aligned}
TC = & \sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}^{(c)}} n_{c,j}^{(c)} \times P_j^{NP} \times (FOM_j + OCC_j \times CRF_j) \\
& + \sum_{z \in \mathcal{Z}} \sum_{j \in \mathcal{J}^{(\bar{c})}} n_{z,j}^{(\bar{c})} \times P_j^{NP} \times (FOM_j + OCC_j \times CRF_j) \\
& + \sum_{z \in \mathcal{Z}} \sum_{j \in \mathcal{J}^{(r)}} n_{z,j}^{(r)} \times P_j^{NP} \times (FOM_j + OCC_j \times CRF_j) \\
& + \sum_b \left( W_b \sum_{t_b \in T_b} \left( \sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}^{(c)}} p_{c,j,t_b}^{(c)} \times OC_{j,t_b} + \sum_{z \in \mathcal{Z}} \sum_{j \in \mathcal{J}^{(\bar{c})}} p_{z,j,t_b}^{(\bar{c})} \times OC_{j,t_b} \right. \right. \\
& \quad \left. \left. + \sum_{z \in \mathcal{Z}} \sum_{j \in \mathcal{J}^{(r)}} p_{z,j,t_b}^{(r)} \times OC_{j,t_b} + \sum_i p_{i,t_b} \times OC_{i,t_b} \right) \right)
\end{aligned} \tag{1}$$

$CRF_j$  is the capital recovery ratio of each technology  $j$  and is defined as:

$$CRF_j = \frac{Q}{1 - (1/(1+Q)^{D_j})} \tag{2}$$

The variable operating cost  $OC$  (in \$/MWh) for new and existing generators is equal:

$$OC_j = VOM_j + HR_j \times FC_j \quad \forall j \in \mathcal{J} \quad (\text{new generators}) \tag{3}$$

$$OC_i = VOM_i + HR_i \times FC_i \quad \forall i \in \mathcal{I} \quad (\text{existing generators}) \tag{4}$$

NOTE: I followed the same convention as the GAMS code, where the operating cost  $OC$  does not change by time  $t$  or location  $c$ .

### 4 Supply vs Demand constraint

$$\begin{aligned}
P_{t,z}^D = & \sum_{i \in \mathcal{I}(z)} p_{i,t} + \sum_{c \in \mathcal{C}(z)} \sum_{j \in \mathcal{J}^{(c)}} p_{c,j,t_b}^{(c)} + \sum_{j \in \mathcal{J}^{(\bar{c})}} p_{z,j,t_b}^{(\bar{c})} + \sum_{j \in \mathcal{J}^{(r)}} p_{z,j,t_b}^{(r)} \\
& + \sum_{\ell: \text{end}(\ell)=z} \text{flow}_{\ell,t} - \sum_{\ell: \text{begin}(\ell)=z} \text{flow}_{\ell,t}
\end{aligned} \tag{5}$$

### 5 Reserve margin constraint

$$\begin{aligned}
(1+M) \times P_{t,z}^D \leq & \sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}^{(c)}} P_{c,j,t}^{MAX} \times n_{c,j}^{(c)} + \sum_{j \in \mathcal{J}^{(r)}} P_{z,j,t}^{MAX} \times n_{z,j}^{(r)} \times CF_{j,t} \\
& + \sum_{i \in \mathcal{I} \setminus \{\mathcal{I}_w \cup \mathcal{I}_s\}} P_{i,t}^{MAX} + P_{\text{solar},t}^{MAX} + P_{\text{wind},t}^{MAX}
\end{aligned} \tag{6}$$

## 6 Maximum generation constraints

$$\sum_{i \in \mathcal{I}_s} p_{i,t} \leq P_{\text{solar},t}^{MAX} \quad \forall t \quad (7)$$

$$\sum_{i \in \mathcal{I}_w} p_{i,t} \leq P_{\text{wind},t}^{MAX} \quad \forall t \quad (8)$$

$$p_{i,t} \leq P_{i,t}^{MAX} \quad \forall t, \forall i \in \mathcal{I} \setminus \{\mathcal{I}_w \cup \mathcal{I}_s\} \quad (9)$$

$$p_{c,j,t}^{(c)} \leq P_{c,j,t}^{MAX} \times n_{c,j}^{(c)} \quad \forall c, t \text{ and } \forall j \in \mathcal{J}^{(c)} \quad (10)$$

$$p_{z,j,t}^{(\bar{c})} \leq P_{z,j,t}^{MAX} \times n_{z,j}^{(\bar{c})} \quad \forall z, t \text{ and } \forall j \in \mathcal{J}^{(\bar{c})} \quad (11)$$

$$p_{z,j,t}^{(r)} \leq n_{z,j}^{(r)} \times P_j^{NP} \times CF_{j,t} \quad \forall z, t \text{ and } \forall j \in \mathcal{J}^{(r)} \quad (12)$$

## 7 Transmission Constraint

$$0 \leq \text{flow}_{\ell,t} \leq \overline{\text{flow}_{\ell}} \quad \forall \ell, t \quad (13)$$