Computer Graphics 1

3 Geometry

Summer Semester 2022

Ludwig-Maximilians-Universität München

Tutorial 3: Geometry

- Geometric Representations
- Bézier Curve
- Polygon-based Surface Representation
- Summary

Geometric Representations

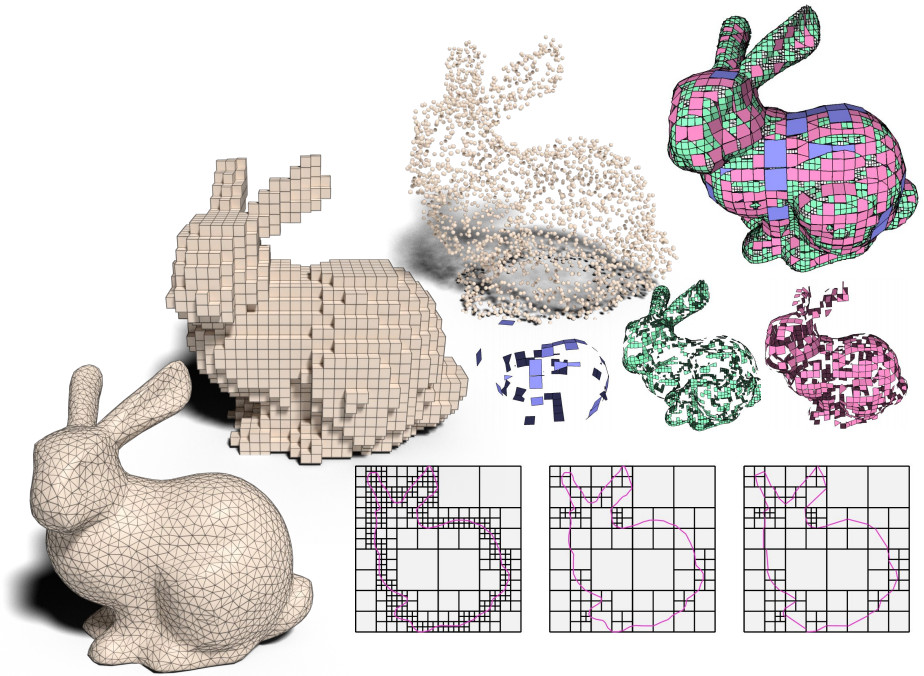
Geometry is the ***foundation*** of all graphics, and its representation gives the *language* for describing shape

- **Boundary representation:** deeply embed into modern graphics, and their algorithms are rich and mature
  - **Curve**: Bézier curves, B-splines...
  - **Surface**
    - Bézier surface
    - Polygon mesh: Triangles, quads, etc.

In active research:

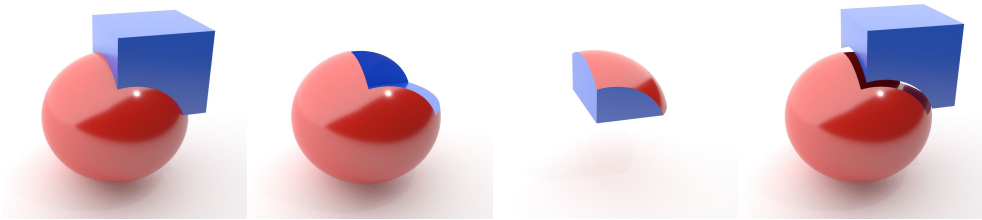
- Volumetric representation, e.g. Voxel, tetrahedron, etc.
- Parametric representation
- Procedural/generative models

And there are more geometry representations of course!



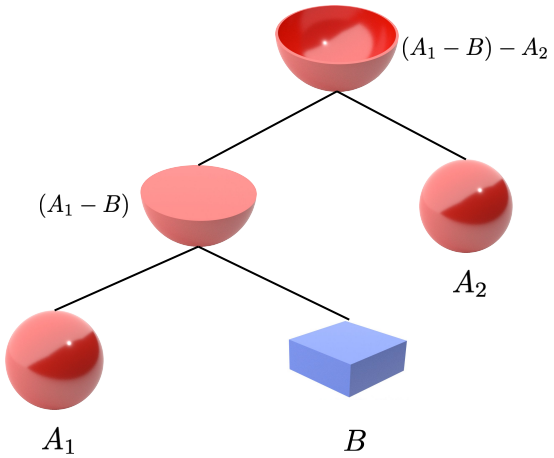
Example: Constructive Solid Geometry (CSG)

CSG is a implicit geometric representation that allows to represent complex models as a series of **boolean operations** between **primitives**.



union (OR) $A \cup B$ | difference (NOT) $A - B$ | intersection (AND) $A \cap B$ | exclusive or (XOR) $A \oplus B$

CSG objects can be represented by binary trees, where leaves represent primitives and nodes represent operations



Example: Advantages and Disadvantages of CSG

- Advantage
  - Minimum steps: represent solid objects as hierarchy of boolean operations
  - Easy to express a complex implicit surface
  - Low storage space needed: due to the simple tree structure and primitives
  - Easy to convert a CSG model to a polygon mesh (but not vise versa)
  - ...
- Disadvantage
  - Impossible to construct non-solid shape, e.g., organic models
  - High computational power needed to derive boundaries, faces and edges ⇒ needed for interactive manipulation
  - ...

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  - de Casteljau Algorithm
- Polygon-based Surface Representation
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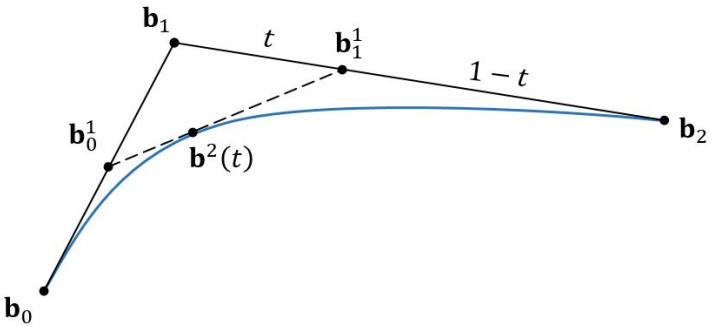
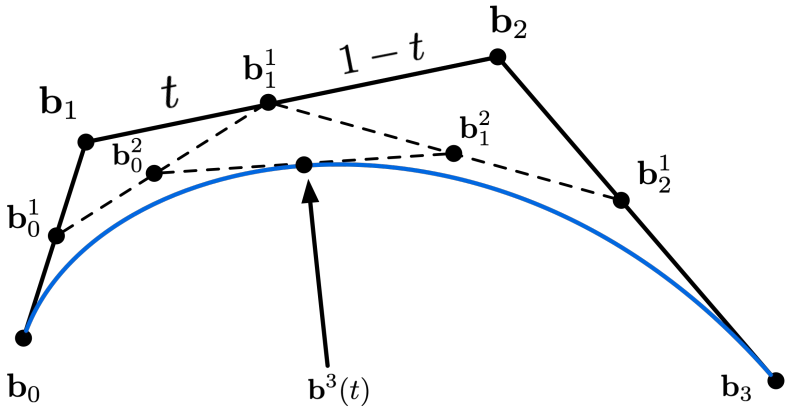
Bernstein-Bézier Curve

- (Bernstein-)Bézier curve is a *parametric* curve representation and the de facto standard for graphics design
- It has many important properties such as the *de Casteljau algorithm* and elegant geometric *interpolation*s
- Applications
  - Describe camera paths to control camera movements
  - Describe animation curves to control object movements
  - ...



Quadratic Bézier
3 control points

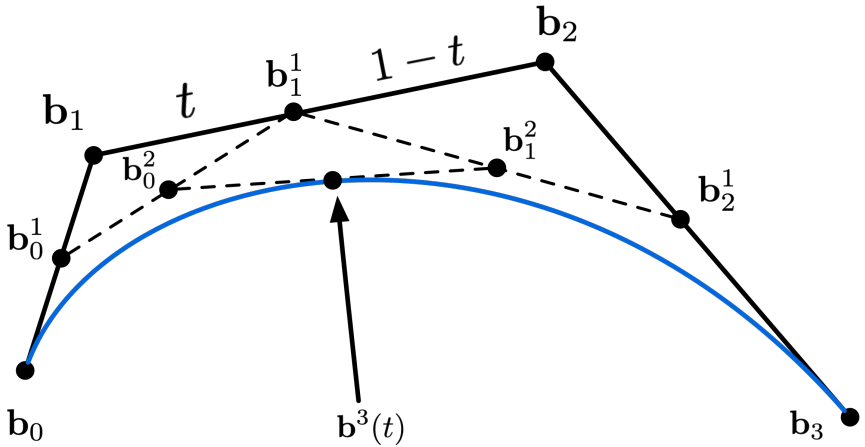


Cubic Bézier
4 control points

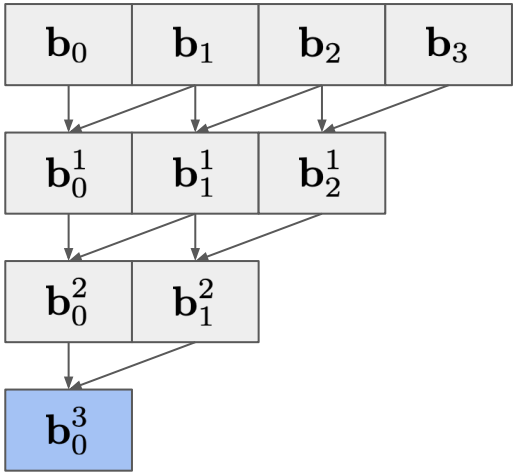
de Casteljau Algorithm

The de Casteljau algorithm offers the most intuitive way to describe a Bézier curve, but requires more computation.

Consider four points (cubic Bézier) as an example:

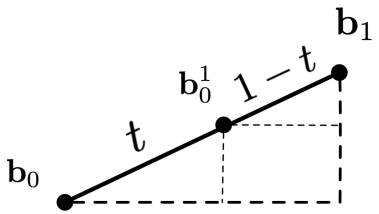


Control Points



Implement de Casteljau Algorithm: Interpolation

The coordinates of $\mathbf{b}_0^1$ is linearly interpolated via parameter t, i.e.:



$$\frac{|\mathbf{b}_0\mathbf{b}_0^1|}{|\mathbf{b}_0\mathbf{b}_1|} = t$$

Let $\mathbf{b}_0^1 = (x, y)^\top$, and $\mathbf{b}_0 = (x_0, y_0)^\top, \mathbf{b}_1 = (x_1, y_1)^\top$. Therefore, we have:

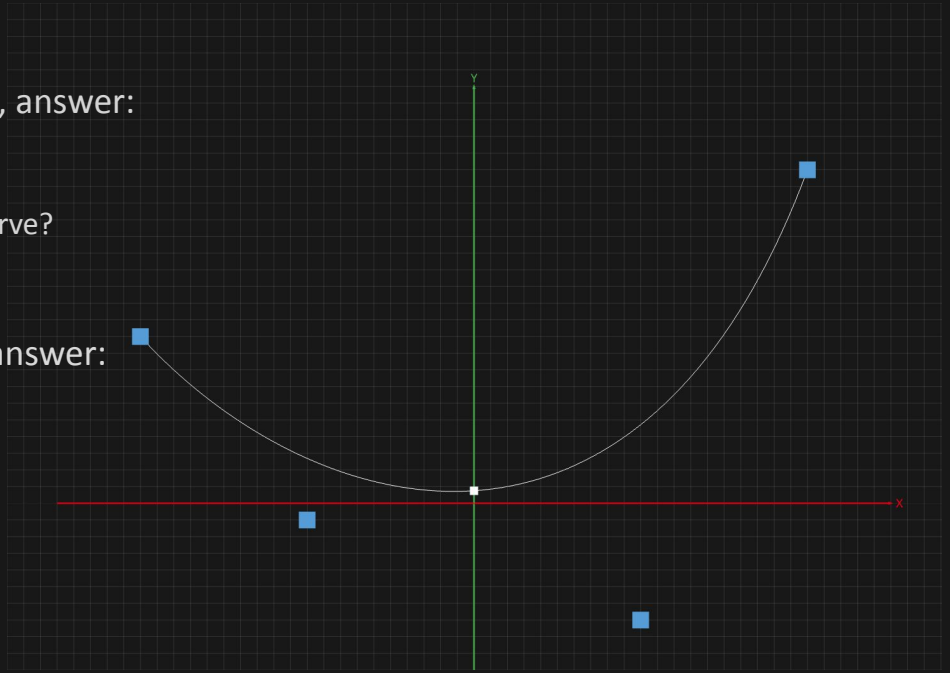
$$t = \frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0}$$

$$\begin{aligned} \Longrightarrow \quad x &= x_0 + t(x_1 - x_0) = (1-t)x_0 + tx_1 \\ y &= y_0 + t(y_1 - y_0) = (1-t)y_0 + ty_1 \end{aligned}$$

Breakout 1: Implement de Casteljau Algorithm

Open the provided code skeleton "bazier".

1. Look for TODO comment in the `src/main.ts`, and implement the `interpolate` function for the de Casteljau algorithm



2. Change the `sample` slider and see how Bézier is sampled, answer:
   - What happens when `sample` is below 5?
   - How many sample points are good enough to show the Bézier curve?

3. Toggle the `show` checkbox and change the parameter `t`, answer:
   - What happens when `t = 0` and `t = 1`?

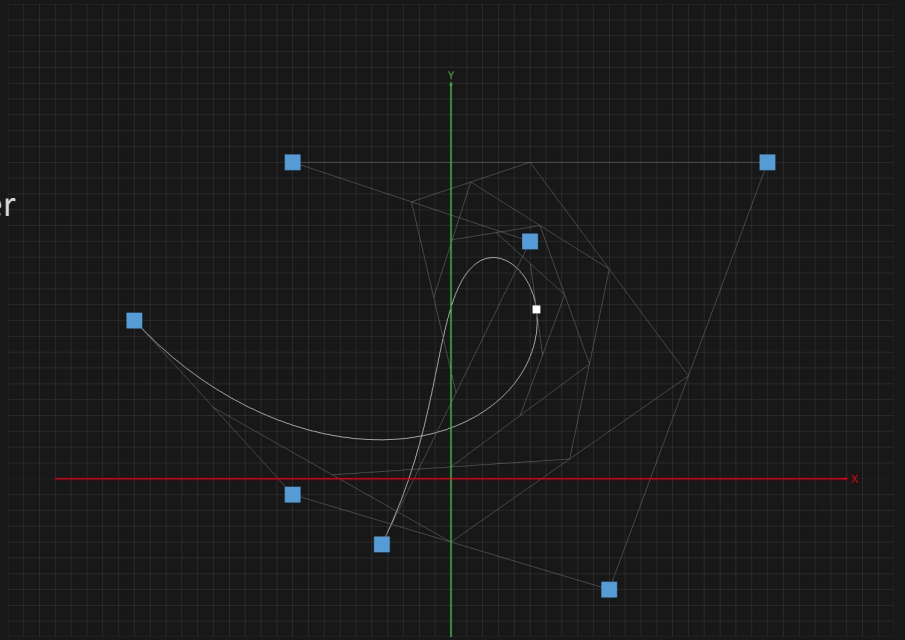
Breakout 1: Experiment with Bézier Curve

Play around with a Bézier curve created by many control points.

4. Look for TODO comment in the `src/main.ts` and add new points or remove existing ones from `this.controlPoints`.

5. Drag the control points directly from the visualization.

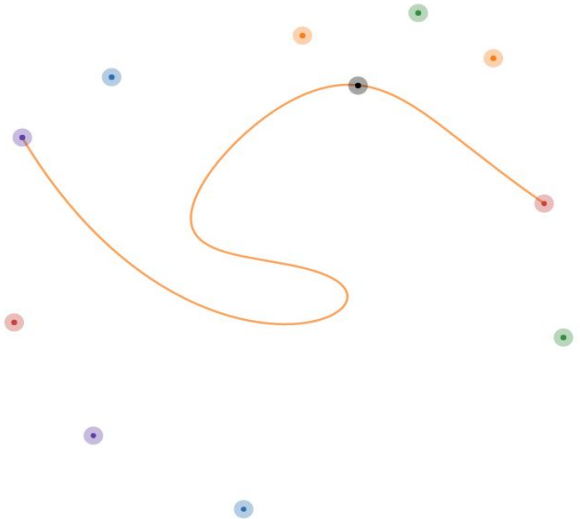
6. Spend a maximum of 2 minutes to try to reproduce the Bézier curve on the right side.



Higher-order Bézier Curves

Key issue: Very hard to control!

Can you imagine which control point changes which part of the curve?



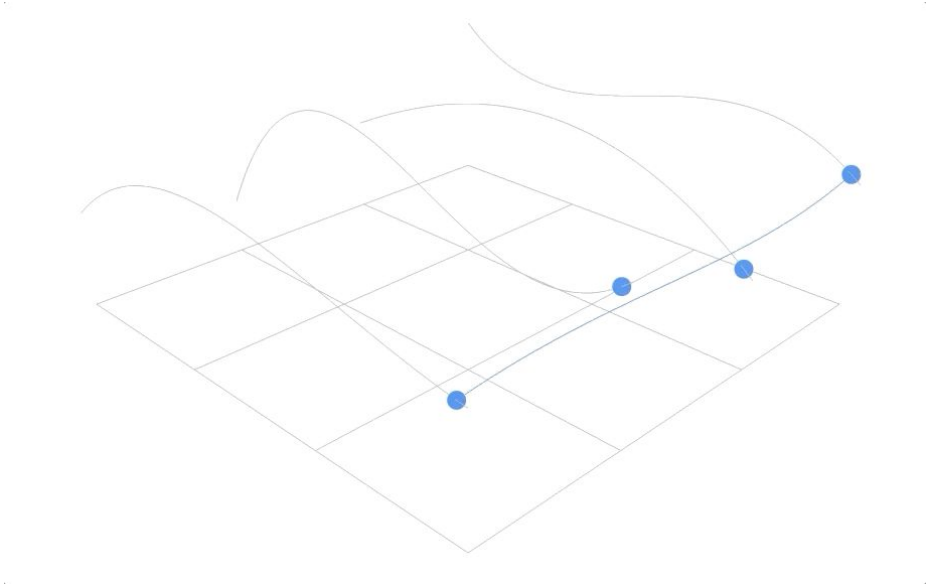
N-order Bézier Curve Playground: https://www.desmos.com/calculator/xlpbe9bgll

Bicubic Bézier Surface (Patch)

4 cubic Bézier curves determine a bicubic Bézier surface:

Each cubic Bézier curve needs 4 control points, with 4 curves, 4x4 = 16 control points in total.

Then, on an orthogonal direction, each Bézier curve contributes one control point.



http://acko.net/blog/making-mathbox/

Tutorial 3: Geometry

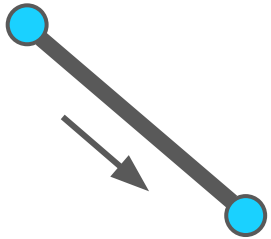
- Geometric Representations
- Bézier Curve
- Polygon-based Surface Representation
  - Meshes and Wavefront OBJ format
  - Geometry Buffers
- Summary

Linear Geometric Primitives

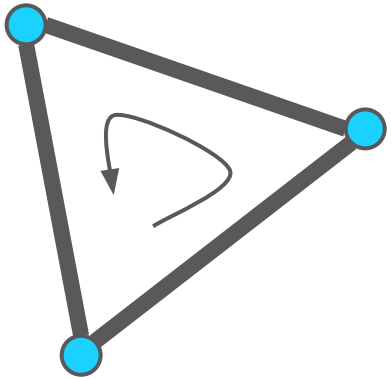
Vertex, edge, and face are the basic geometric primitives for constructing a polygonal-based surface

- A vertex is a point abstraction, and it does not only represent position, but can also contain other information
- An edge represents an *oriented* connectivity of two vertices
- A face is an *oriented* closed edge loop that can be either a triangle, quadrilateral, or arbitrary polygon

Vertex



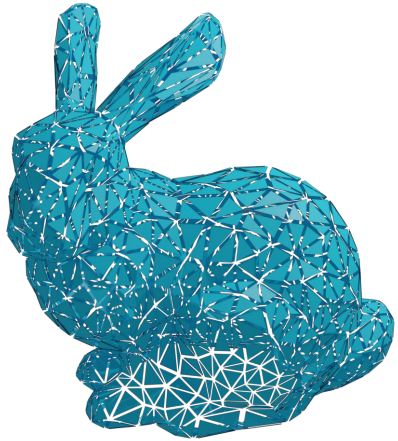
Edge



Face

Polygon-based Surface

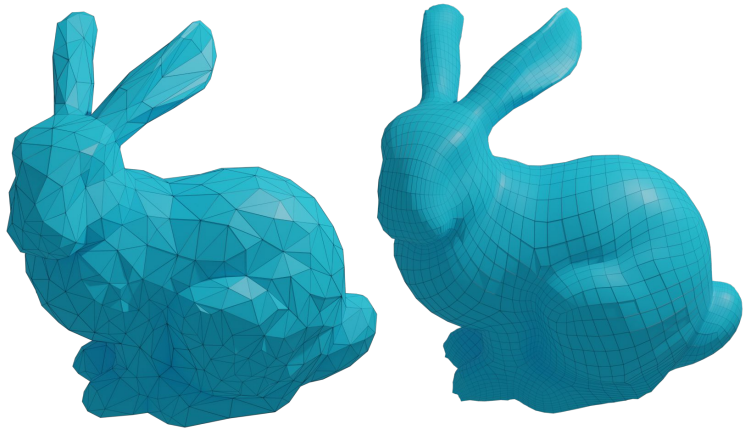
To represent a smooth surface in discrete settings, one can use a collection of polygons, which often refer to *polygon soup*. In a polygon soup, an edge only connects to a single face.



polygon soup

Polygon mesh adds more constraints on a polygon soup where an edge connects multiple faces, such as:

- Triangle mesh
- Quadrilateral mesh (or just quad mesh)
- Quad-dominant Mesh (often refer to a mixture of triangle and quad mesh but mostly quads)
- ...

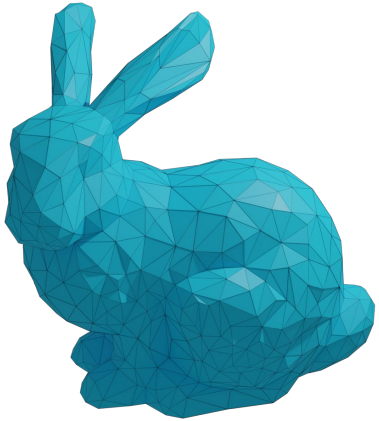


triangle mesh

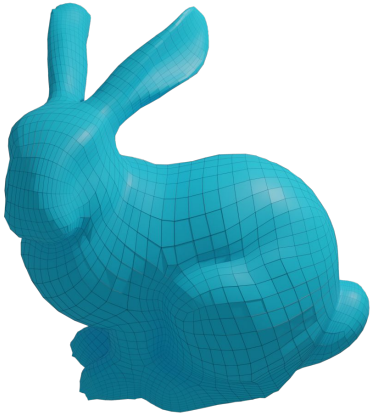
quad mesh

Triangle vs. Quad Mesh

- Triangle Mesh
  - a triangle is the simplest polygon, and other polygons can be turned into triangles
  - a triangle is guaranteed to be planar (linear element)
  - a triangle has well-defined interior (Q: How to check if a point is inside a triangle?)
  - it is easy to compute interactions between a triangle mesh and rays (***later in ray tracing***)
- Quad Mesh
  - quad meshes are much easier for modeling smooth and deformable surface
  - converting a quad mesh to a triangle mesh is a simple process (Q: Why?)
  - quad meshes have many sub-regions with grid-like connectivity (flow line or edge loop)
  - quad meshes are better for subdivisions than tri-meshes



triangle mesh



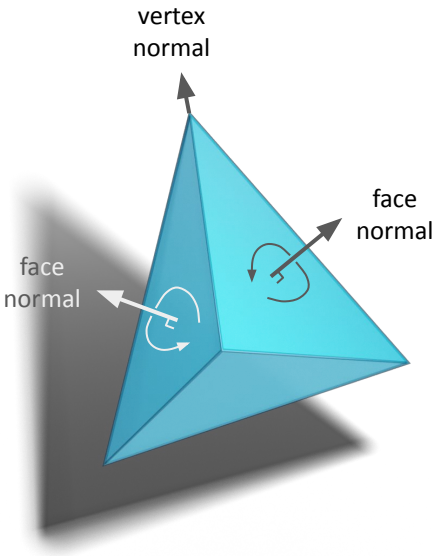
quad mesh

Normals

Normals are an important property on a continuous surface.

In discrete settings, there are two types of normals for a triangle mesh:

- Face normal: has *unit* length and is *orthogonal* with the given triangle
  - Face normal of a triangle is well defined (Q: why and how to compute?)
- Vertex normal: is an interpolation from the surrounding face normals
  - There are multiple (different) definitions for vertex normals
  - A possible definition is the average of the surrounding face normals
  - Vertex normals can also be manually defined, i.e. ground truth vertex normals

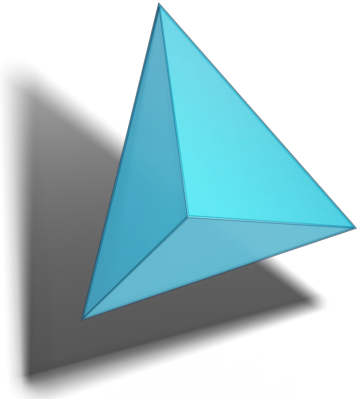


The orientation of a face describes a normal either inward-pointing or outward-pointing. Depending on left- or right handed system, we assume:

- Outward-pointing normals are determined by right-hand rule
- Inward-pointing normals point to the opposite direction of outward-pointing normals

The Wavefront Object File Format (.obj)

- The Wavefront object file format is one of the earliest developed polygon-based surface geometry definitions
- 3D softwares (eg. Blender) allow user to manually change geometry and exports the final result to a .obj file with predefined specification
- The format stores geometric information such as vertex *positions*, vertex *normals*, vertex *UV coordinates* (2D vector, later discuss in texture session) in an array together with a face *adjacency list*, that contains *oriented* vertex indices
- See an example of *tetrahedron* on the right side:



```
1   v -0.363322 -0.387725 0.859330
2   v -0.550290 -0.387725 -0.682297
3   v -0.038214 0.990508 -0.126177
4   v 0.951827 -0.215059 -0.050857
```
Vertices

```
1   vt 0.436598 0.753560
2   vt 0.833648 0.512884
3   vt 0.833648 1.000000
4   vt 0.436598 0.464299
5   vt 0.000000 0.195168
6   vt 0.436598 0.000000
7   vt 0.000000 0.685842
8   vt 0.423825 0.464299
9   vt 0.423825 0.925956
10  vt 0.436598 0.251320
11  vt 0.823853 0.000000
12  vt 0.823853 0.512884
```
UVs (later)

```
1   vn 0.3538 0.2340 -0.9056
2   vn 0.4727 0.4361 0.7658
3   vn 0.1202 -0.9926 -0.0146
4   vn -0.9454 0.3050 0.1147
```
Normals

```
1   f 3/1/1 4/2/1 2/3/1
2   f 3/4/2 1/5/2 4/6/2
3   f 4/7/3 1/8/3 2/9/3
4   f 2/10/4 1/11/4 3/12/4
```
Faces

OBJ File Format: Vertex Data

All vertices are ordered where the vertex index (starts from 1) is implied from their order in the file.

Lines starting with **v** represent vertex positions*:

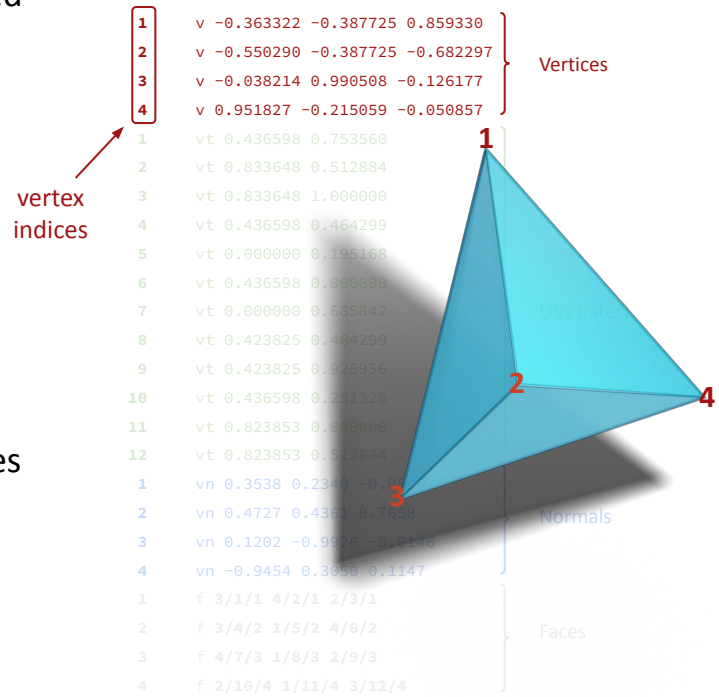
`v x y z`

where x, y, z are the position coordinates

Lines starting with **vt** represent vertex UV coordinates and **vn** describes vertex normal coordinates:

`vt x y`

`vn x y z`



*The actual OBJ file format contains more details such as a vertex position can use homogeneous representation, see here for a full format specification. For simplicity, we assume not using homogeneous representation in .obj file format.

OBJ File Format: Face Data

Lines starting with **f** represent a single face described by a list of vertices.

Each vertex concatenates its information using slash (/)

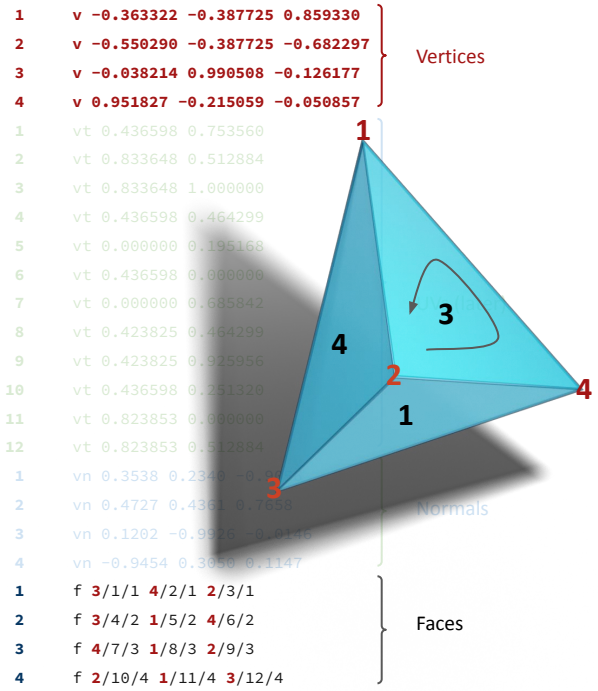
- Triangle face:

```
f v/vt/vn v/vt/vn v/vt/vn
```

- Quad face:

```
f v/vt/vn v/vt/vn v/vt/vn v/vt/vn
```

With more group of vertices, a face can be ngon (polygon with n edges.)



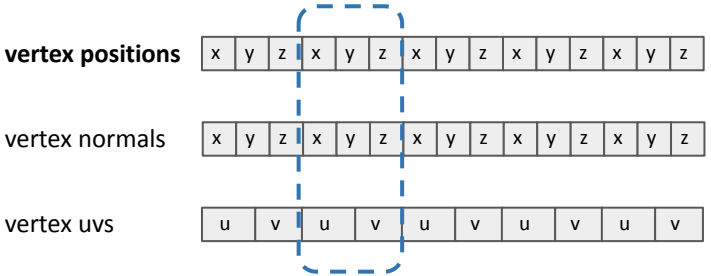
Vertex Buffer

Modern GPUs store a triangle mesh using a dense memory buffer, i.e. vertex buffer.

To create a geometry, one must interpret a vertex buffer using indices.

In three.js, BufferGeometry is the way of representing all (polygon-based) geometry. All geometry data is stored using BufferAttributes, and each BufferAttribute represents an array of one type of data: positions, normals, UVs, etc

By default, the vertex index starts from 0 (Note that this is different from an .obj file, where the index starts from 1)



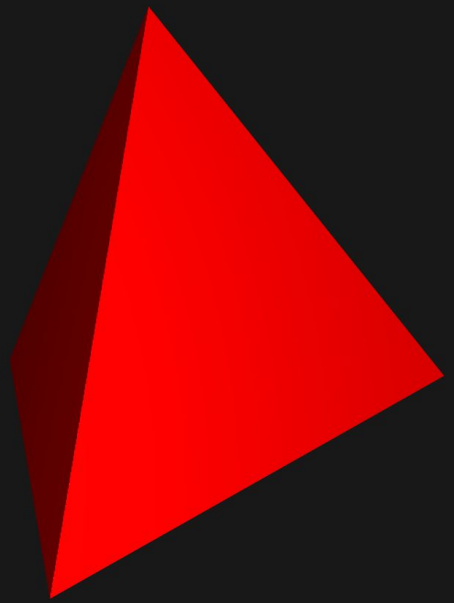
Breakout 2: Visualize Tetrahedron using BufferGeometry

Open the provided code skeleton "buffers".

Complete the vertex indices that constructs the faces of a tetrahedron below the TODO comment.

```
// vertex buffer object
const vbo = new Float32Array([
  -0.363322, -0.387725, 0.85933,
  ...
]);
// create a buffer geometry
const g = new BufferGeometry();
g.setIndex([
  // TODO: fill the vertex indices

]);
g.setAttribute('position', new BufferAttribute(vbo, 3));
```



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Summary

We discussed:

- Different geometric representations and CSG as an example of implicit geometry representation
- How to use the de Casteljau algorithm compute Bézier curves
- Polygon-based mesh surface, the wavefront object file format, and vertex buffer