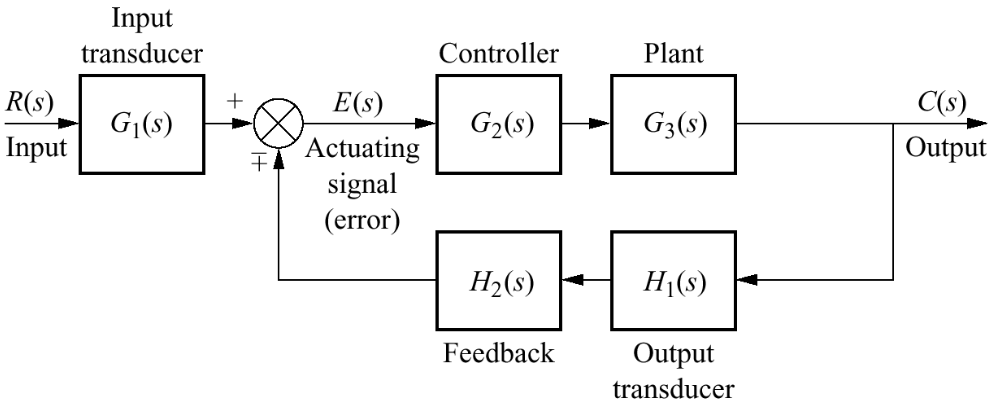
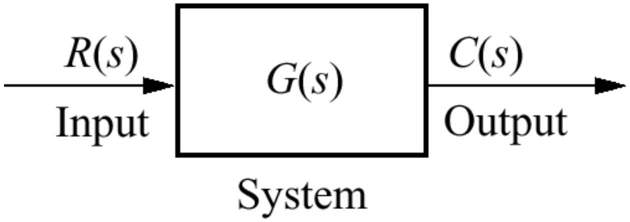
Reduction of Multiple Subsystems by Block Diagram Reduction and Signal Flow Graph

What is Multiple Subsystems?



Feedback Control System consists of process and subsystems

Our objective is to combine multiple subsystems into single system – thus the name reduction of multiple subsystems

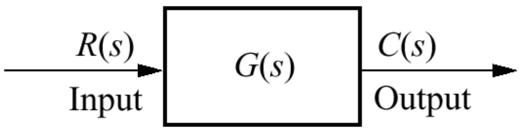


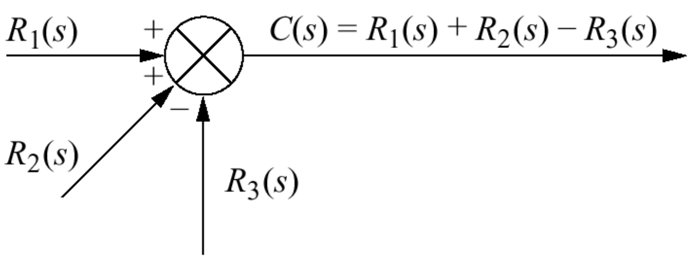
Reduction of Multiple Subsystems – Block Diagram Components

Components of block diagram:

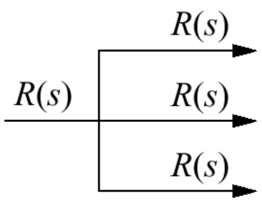
1. Signals
2. Subsystem
3. Summing junction
4. Pickoff point

Signals





Summing junction

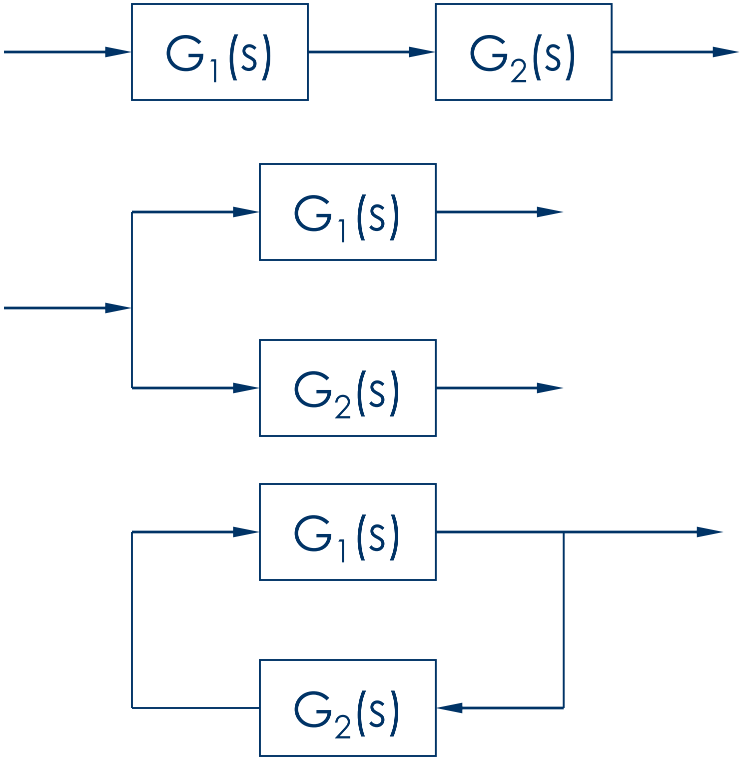


Pickoff point

Reduction of Multiple Subsystems – Block Diagram Topology

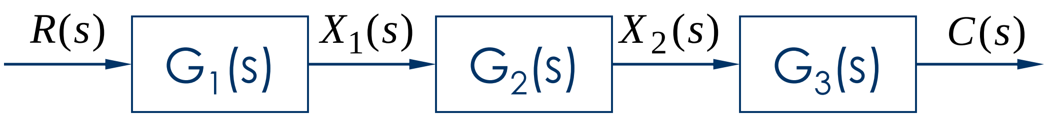
Block diagram has three topologies (or arrangements or forms):

1. Cascade form
2. Parallel form
3. Feedback form



Reduction of Multiple Subsystems – Block Diagram Topology

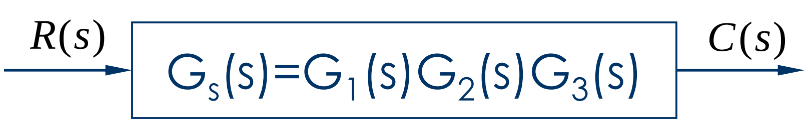
How to reduce subsystems in cascade form?



$$X_1(s) = R(s)G_1(s)$$

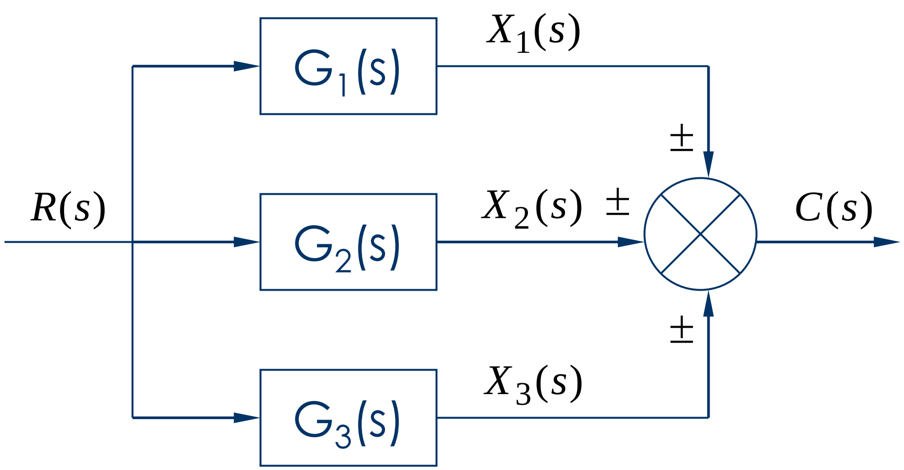
$$X_2(s) = X_1(s)G_2(s) = R(s)G_1(s)G_2(s)$$

$$C(s) = X_2(s)G_3(s) = R(s)G_1(s)G_2(s)G_3(s)$$

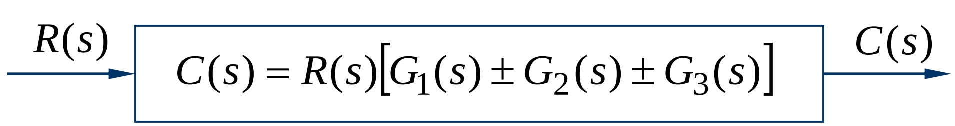


Reduction of Multiple Subsystems – Block Diagram Topology

How to reduce subsystems in parallel form?

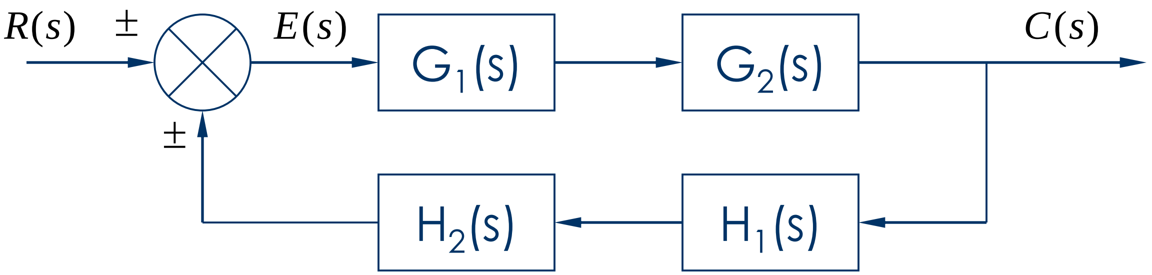


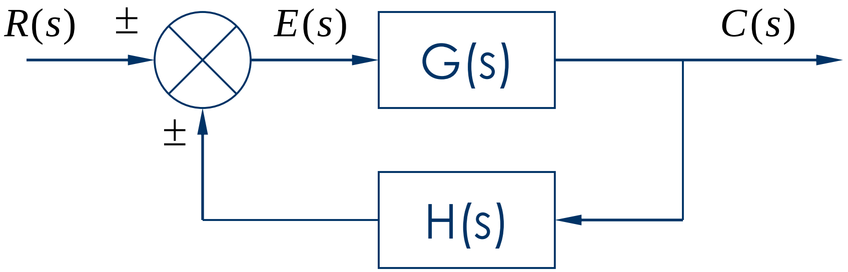
$$X_1(s) = R(s)G_1(s)$$
$$X_2(s) = R(s)G_2(s)$$
$$X_2(s) = R(s)G_3(s)$$
$$C(s) = R(s)G_1(s) \pm R(s)G_2(s) \pm R(s)G_3(s)$$



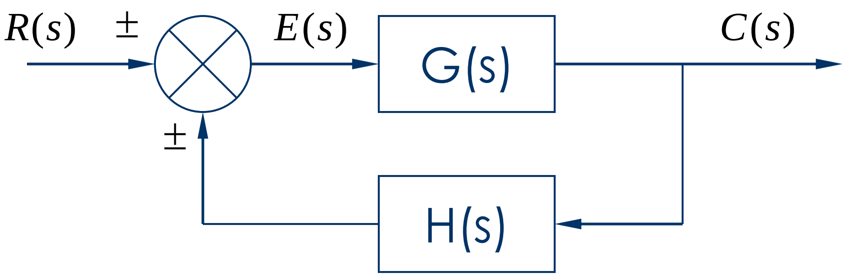
Reduction of Multiple Subsystems – Block Diagram Topology

How to reduce subsystems in feedback form?





Reduction of Multiple Subsystems – Block Diagram Topology



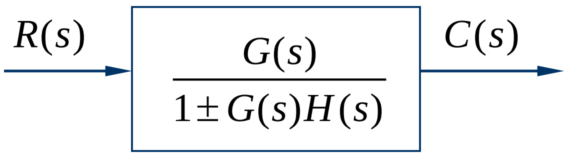
$$E(s) = R(s) \pm C(s)H(s)$$

$$C(s) = E(s)G(s)$$

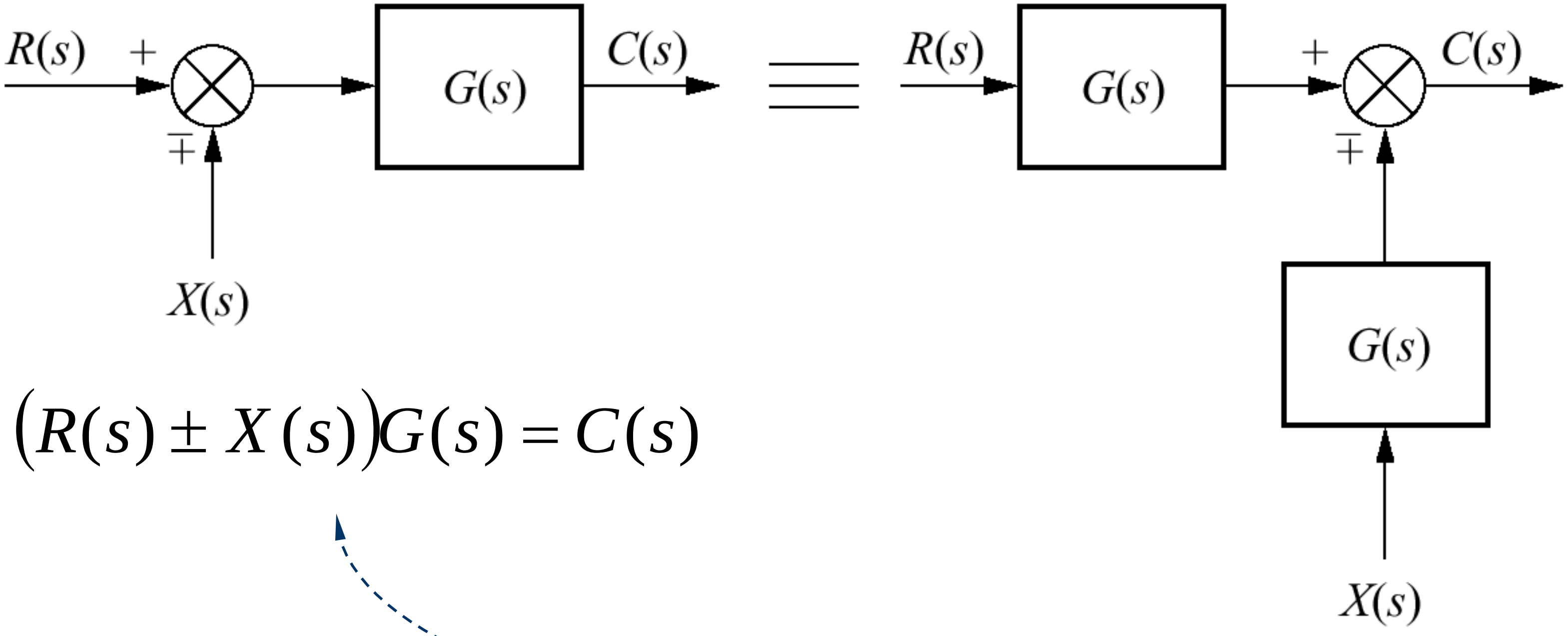
$$\frac{C(s)}{G(s)} = R(s) \pm C(s)H(s)$$

$$R(s) = \frac{C(s)}{G(s)} \pm C(s)H(s) = \left(\frac{1}{G(s)} \pm H(s)\right)C(s) = \left(\frac{1 \pm G(s)H(s)}{G(s)}\right)C(s)$$

$$C(s) = \left(\frac{G(s)}{1 \pm G(s)H(s)}\right)R(s)$$



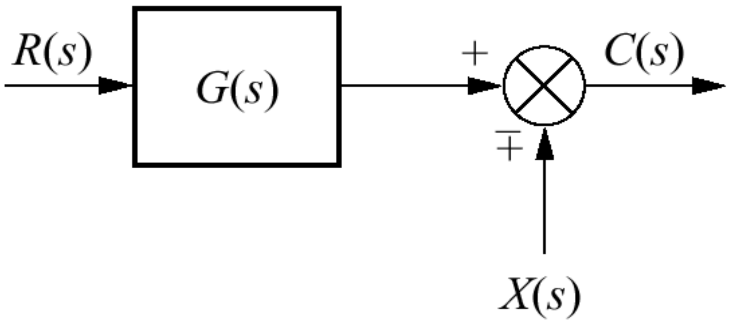
Reduction of Multiple Subsystems – Block Rearrangement

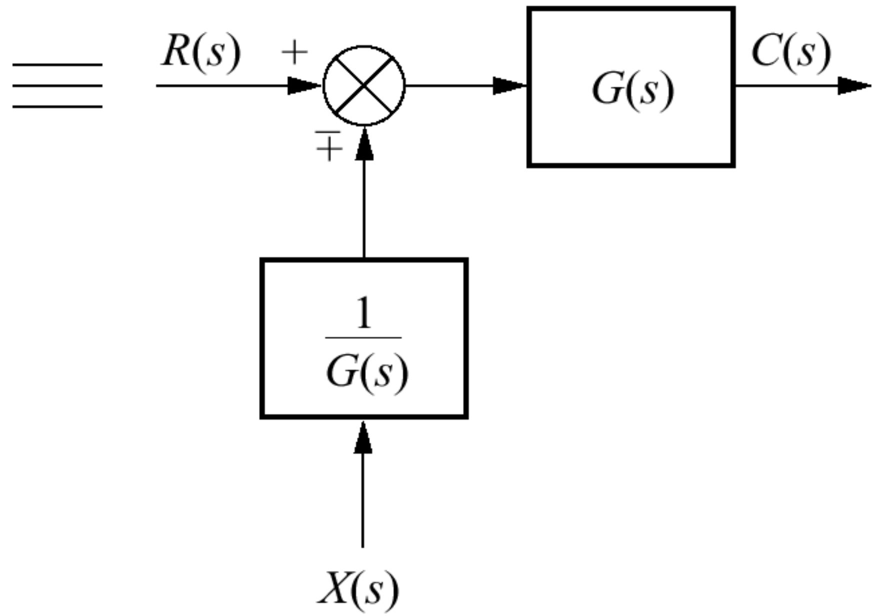


$$\left(R(s) \pm X(s)\right)G(s) = C(s)$$

$$R(s)G(s) \pm X(s)G(s) = C(s)$$

Reduction of Multiple Subsystems – Block Rearrangement

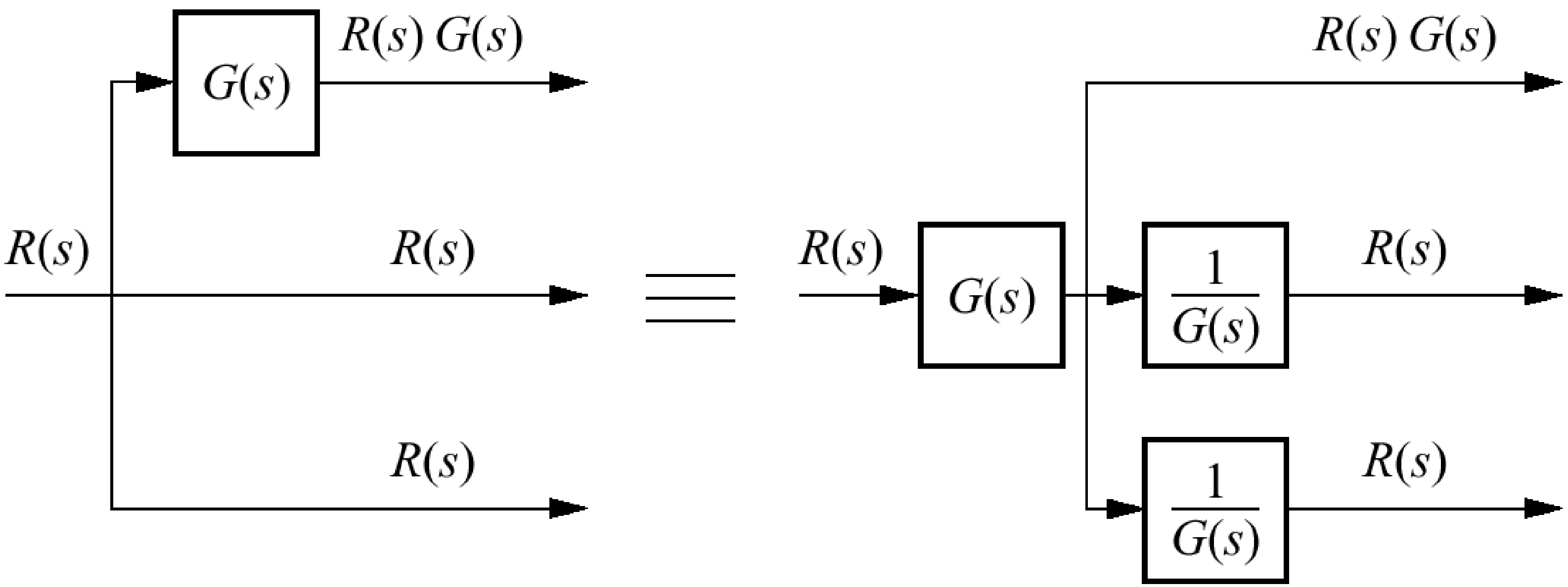




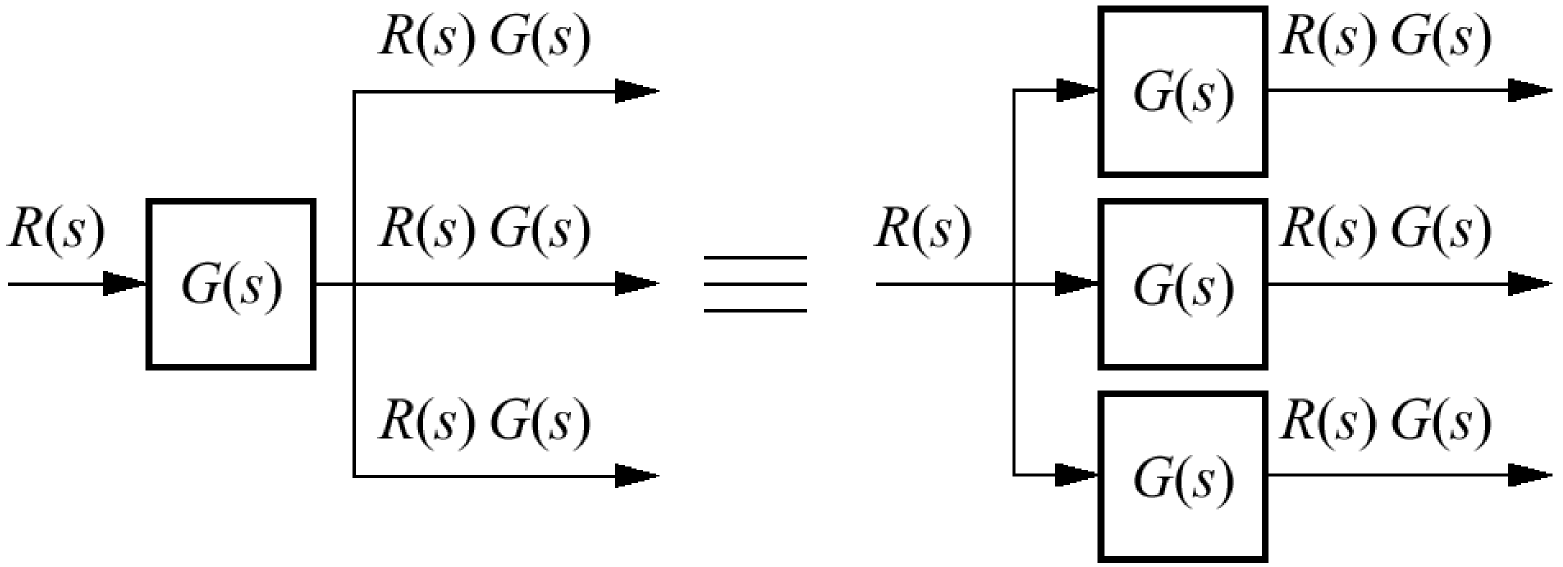
$$R(s)G(s) \pm X(s) = C(s)$$

$$(R(s) \pm \frac{X(s)}{G(s)})G(s) = C(s)$$

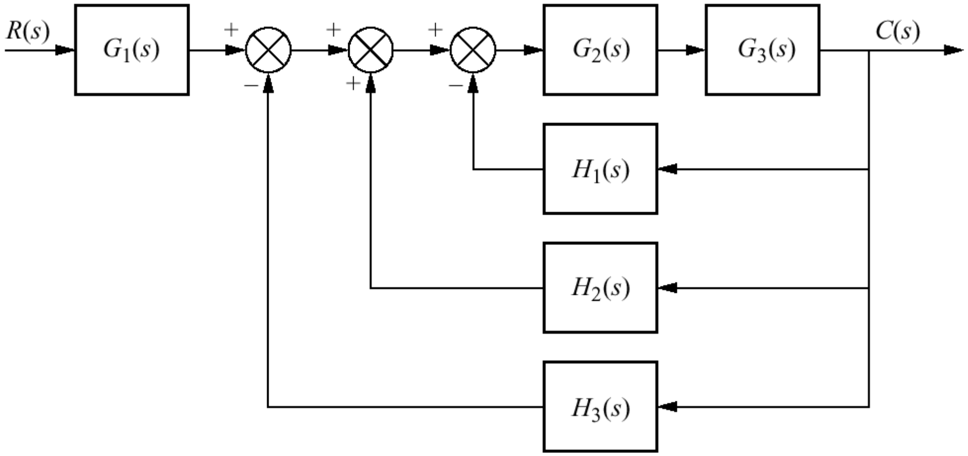
Reduction of Multiple Subsystems – Block Rearrangement

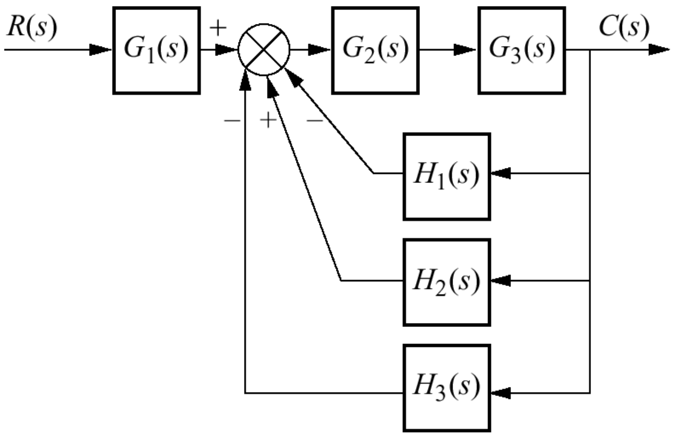


Reduction of Multiple Subsystems – Block Rearrangement

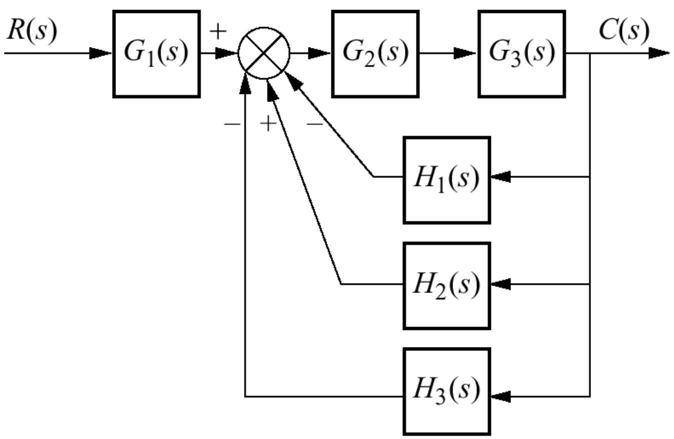


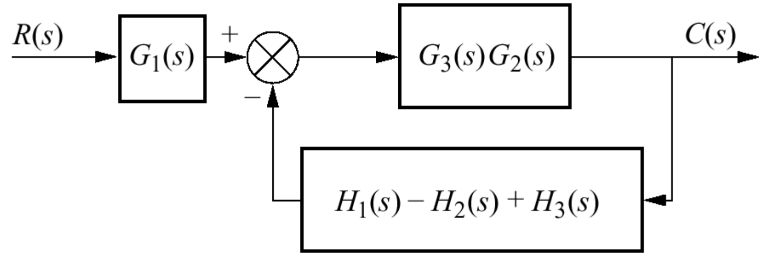
Reduction of Multiple Subsystems – Example

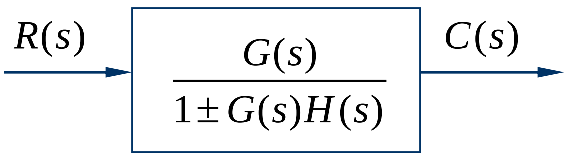


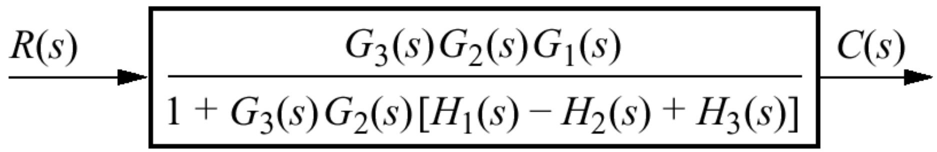


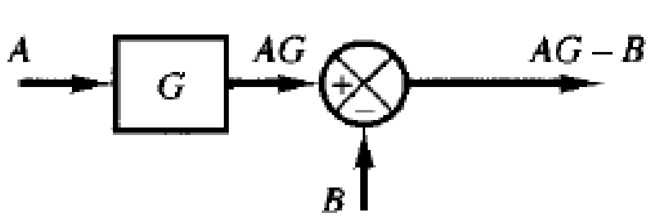
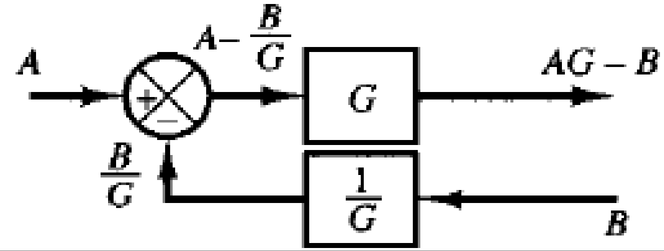
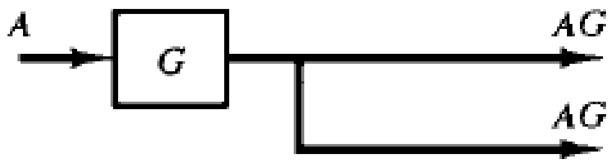
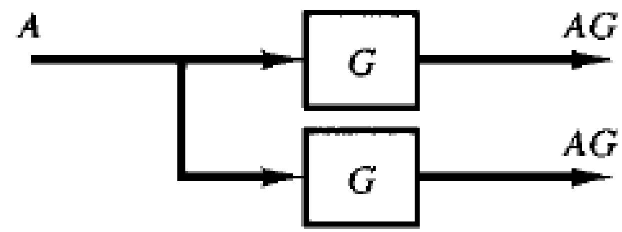
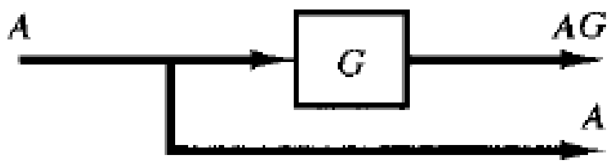
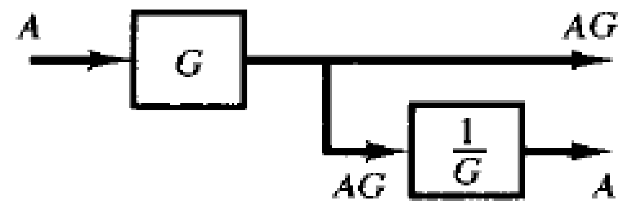
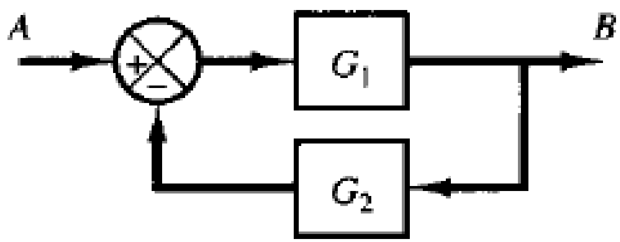
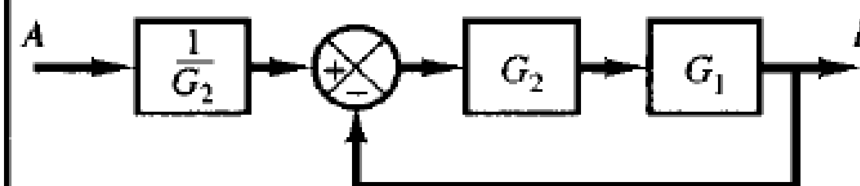
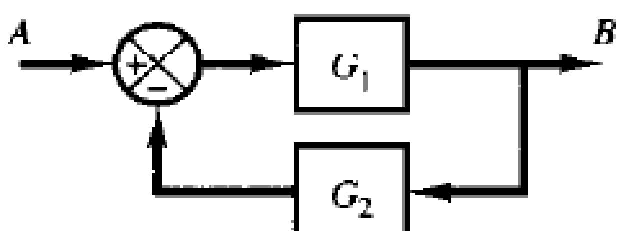
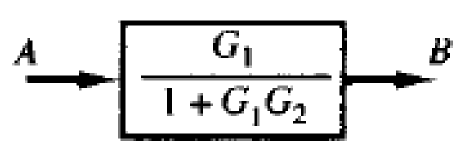
Reduction of Multiple Subsystems – Example





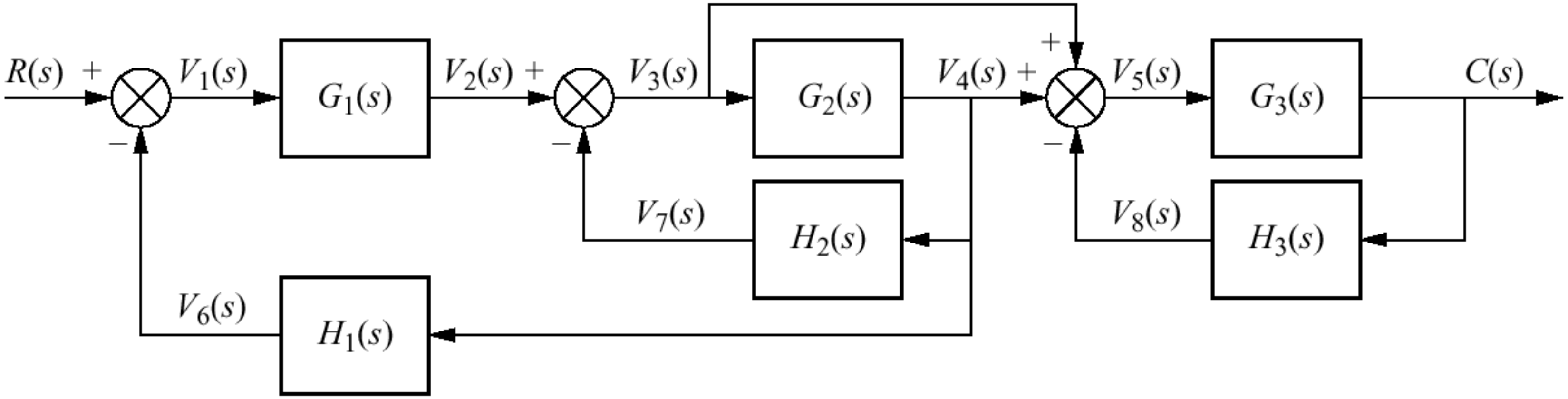




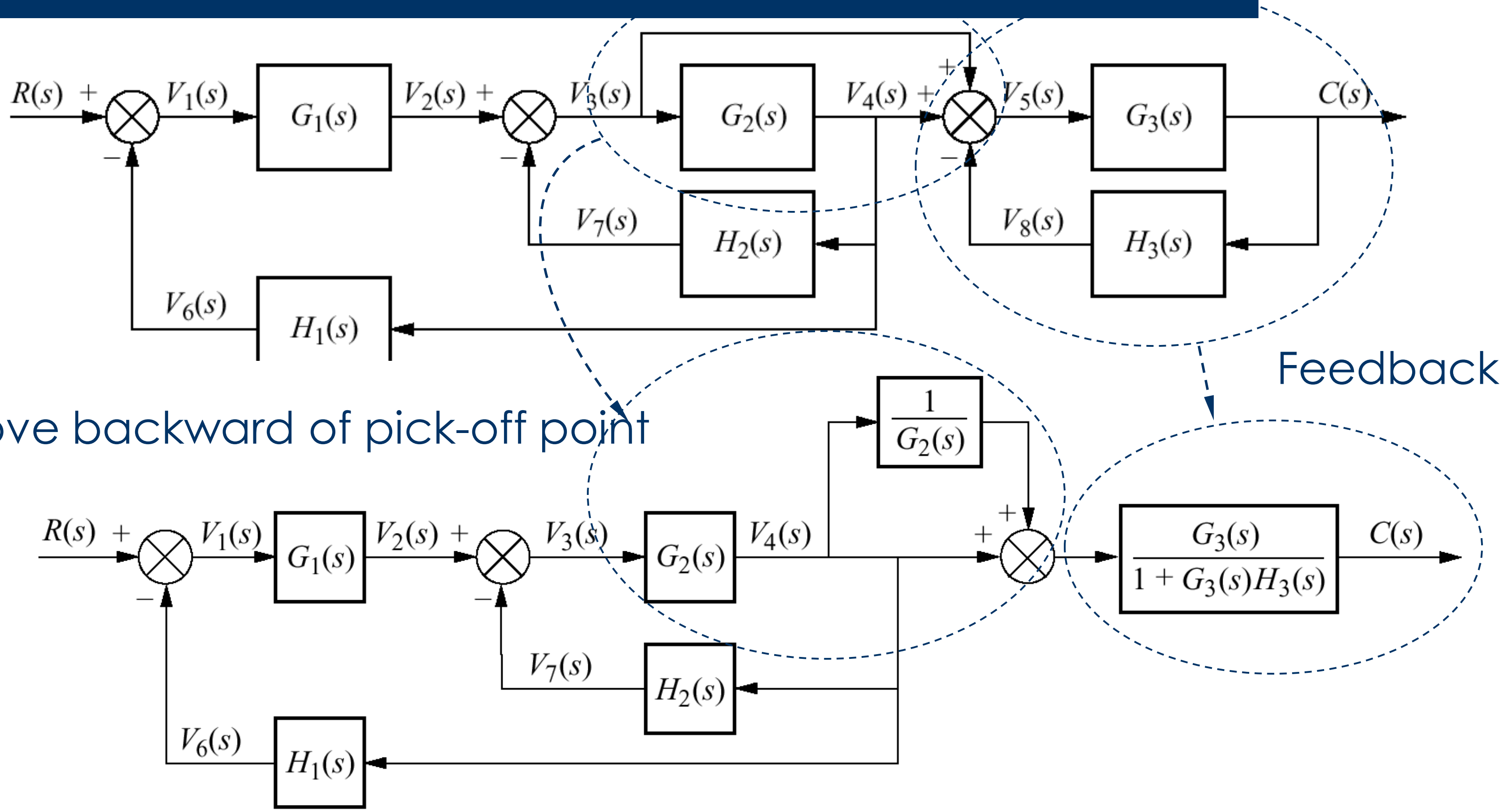
	Original Block Diagrams	Equivalent Block Diagrams
1		
2		
3		
4		
5		

Reduction of Multiple Subsystems – Example

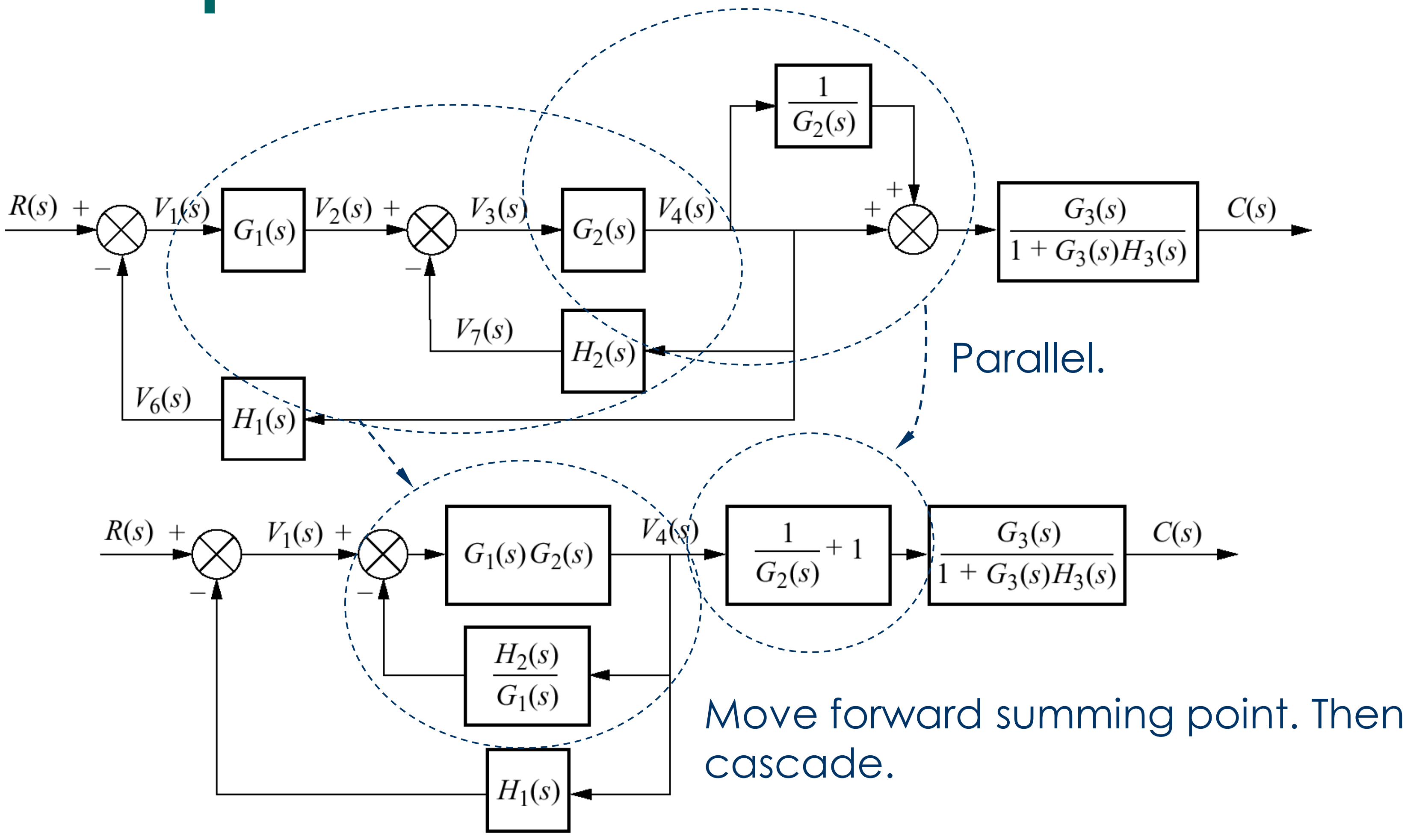
Reduce the multiple subsystems into one system



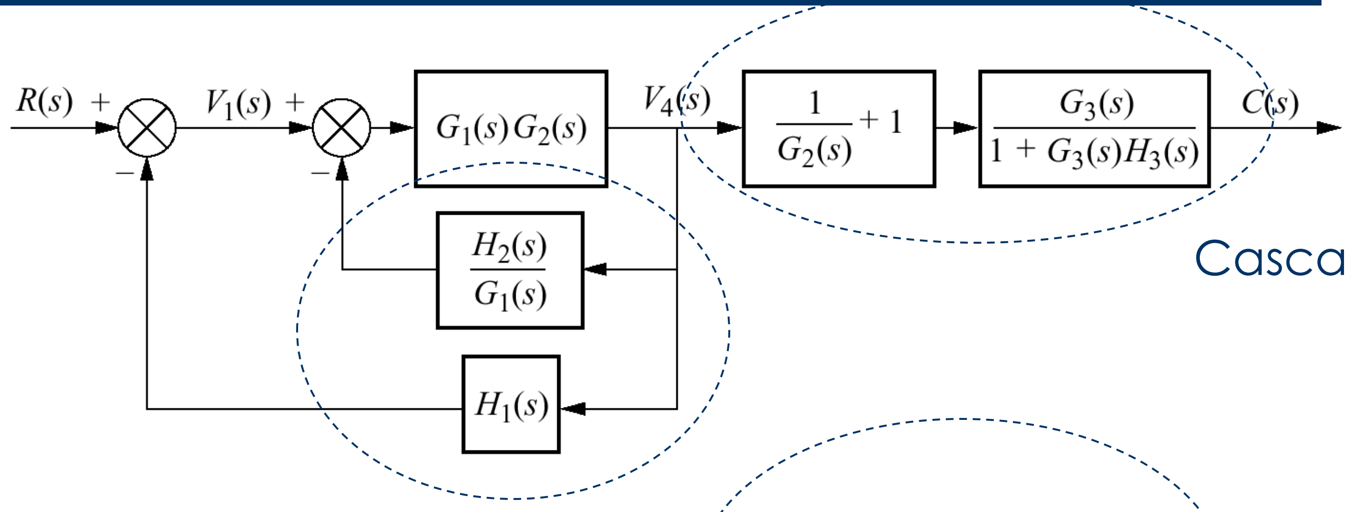
Reduction of Multiple Subsystems – Example



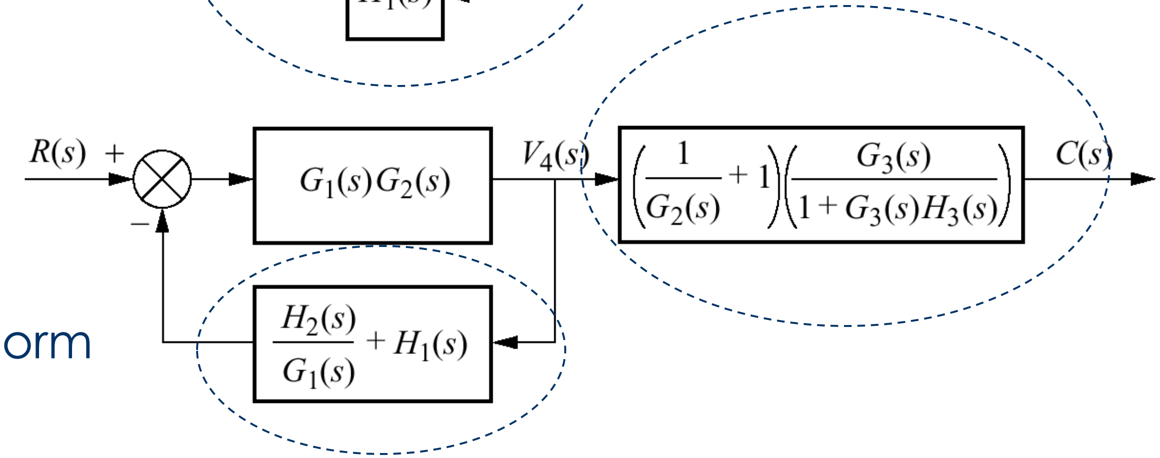
Reduction of Multiple Subsystems – Example



Reduction of Multiple Subsystems – Example



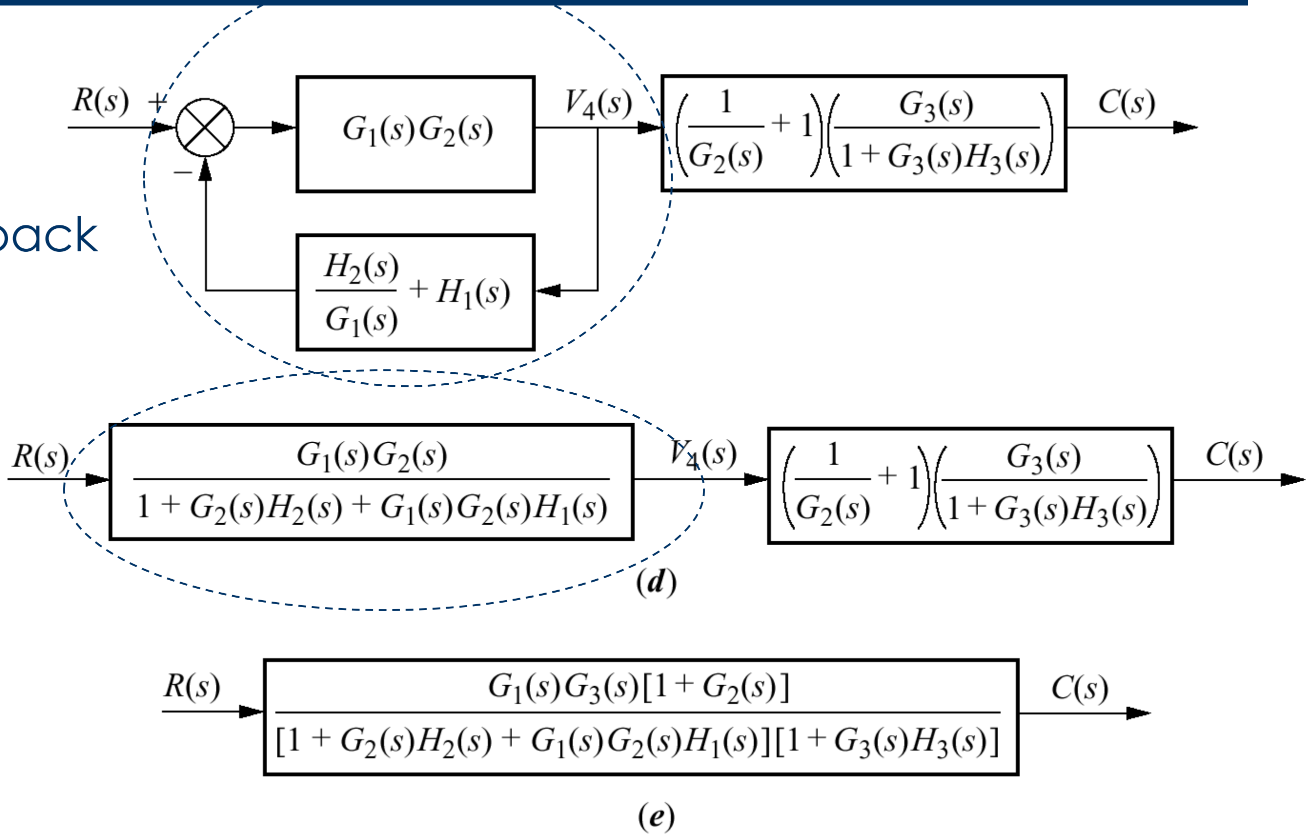
Cascade Form



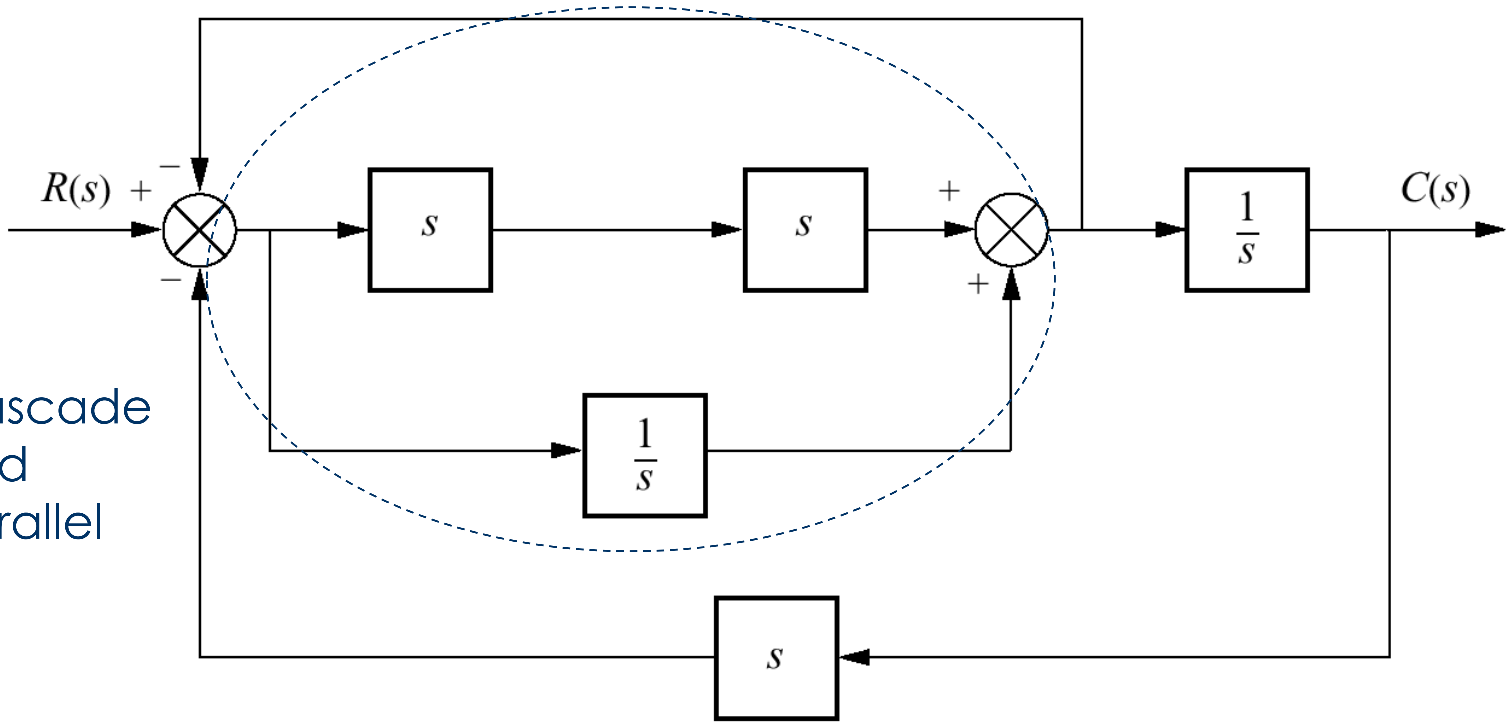
Parallel Form

Reduction of Multiple Subsystems – Example

Feedback form

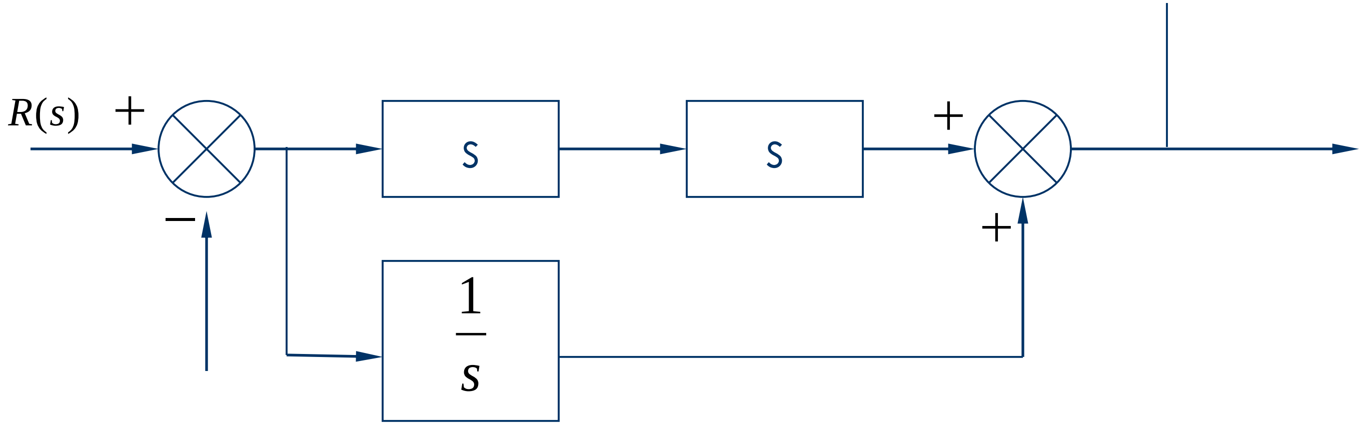


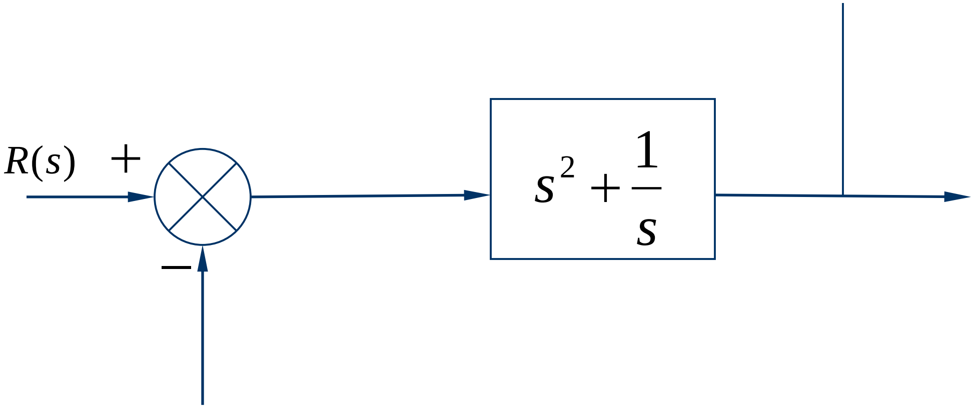
Reduction of Multiple Subsystems – Example



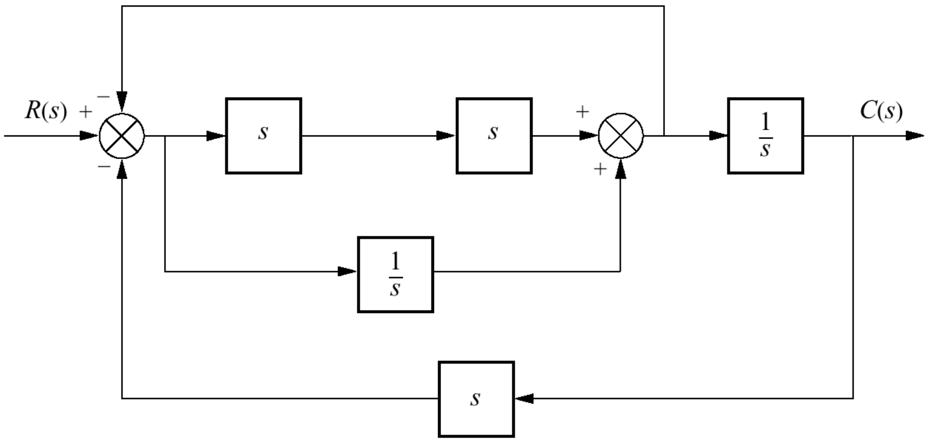
Cascade
and
Parallel

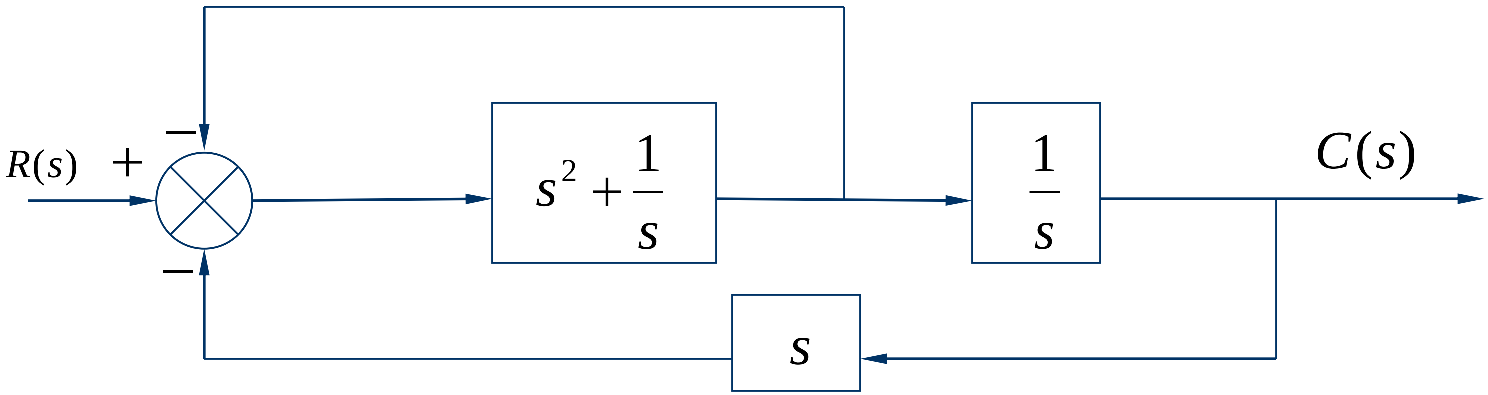
Reduction of Multiple Subsystems – Example



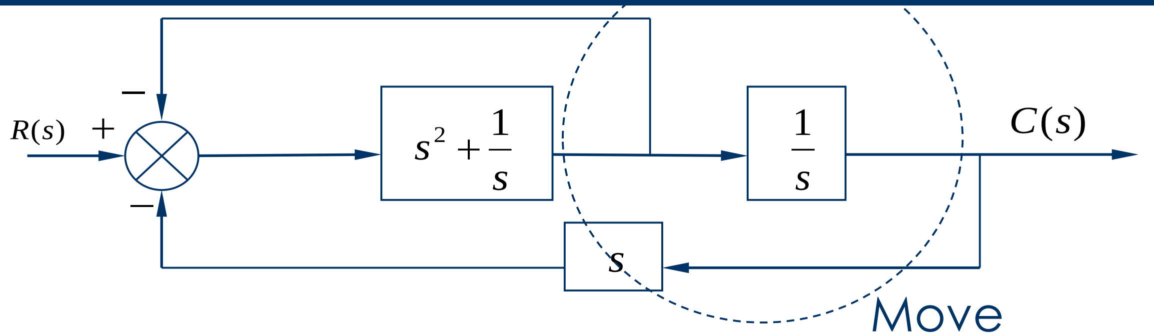


Reduction of Multiple Subsystems – Example

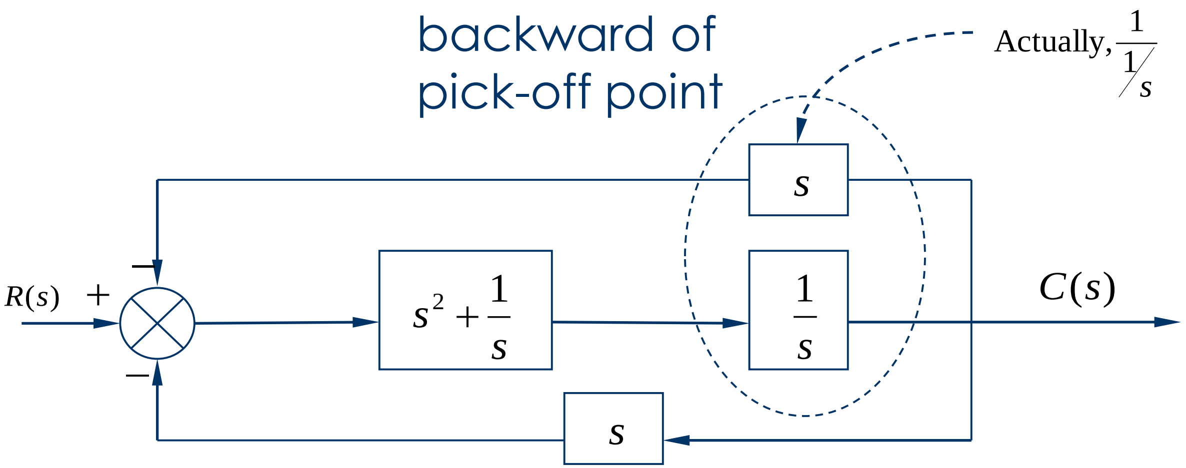




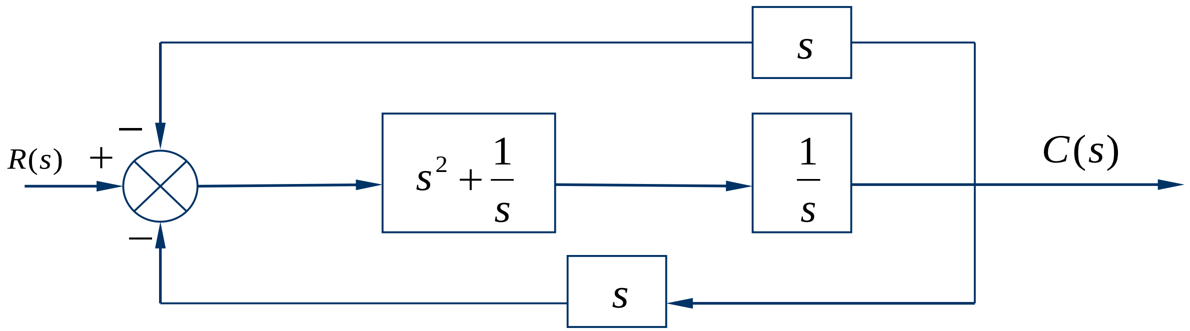
Reduction of Multiple Subsystems – Example



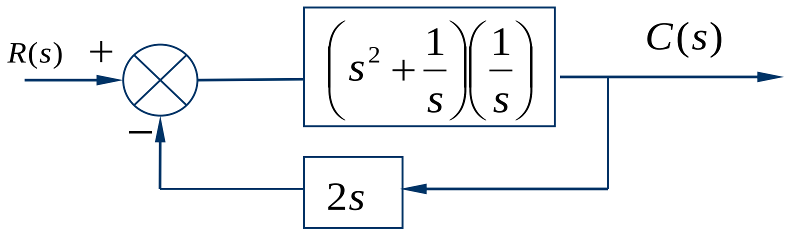
Move backward of pick-off point



Reduction of Multiple Subsystems – Example



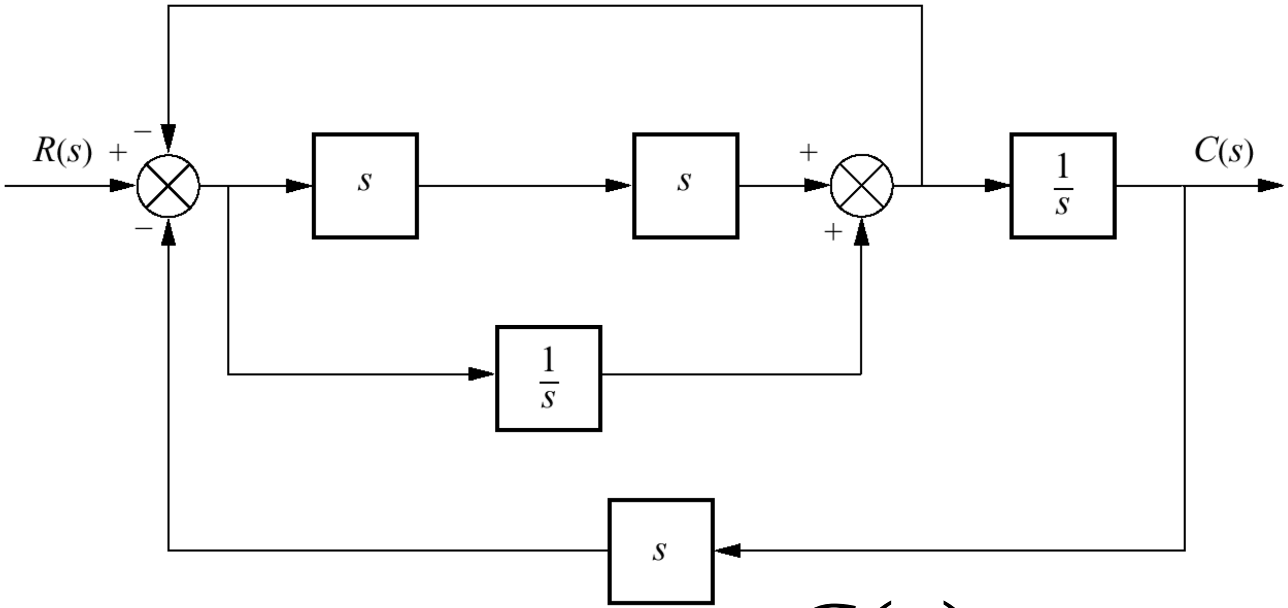
Parallel and Cascade



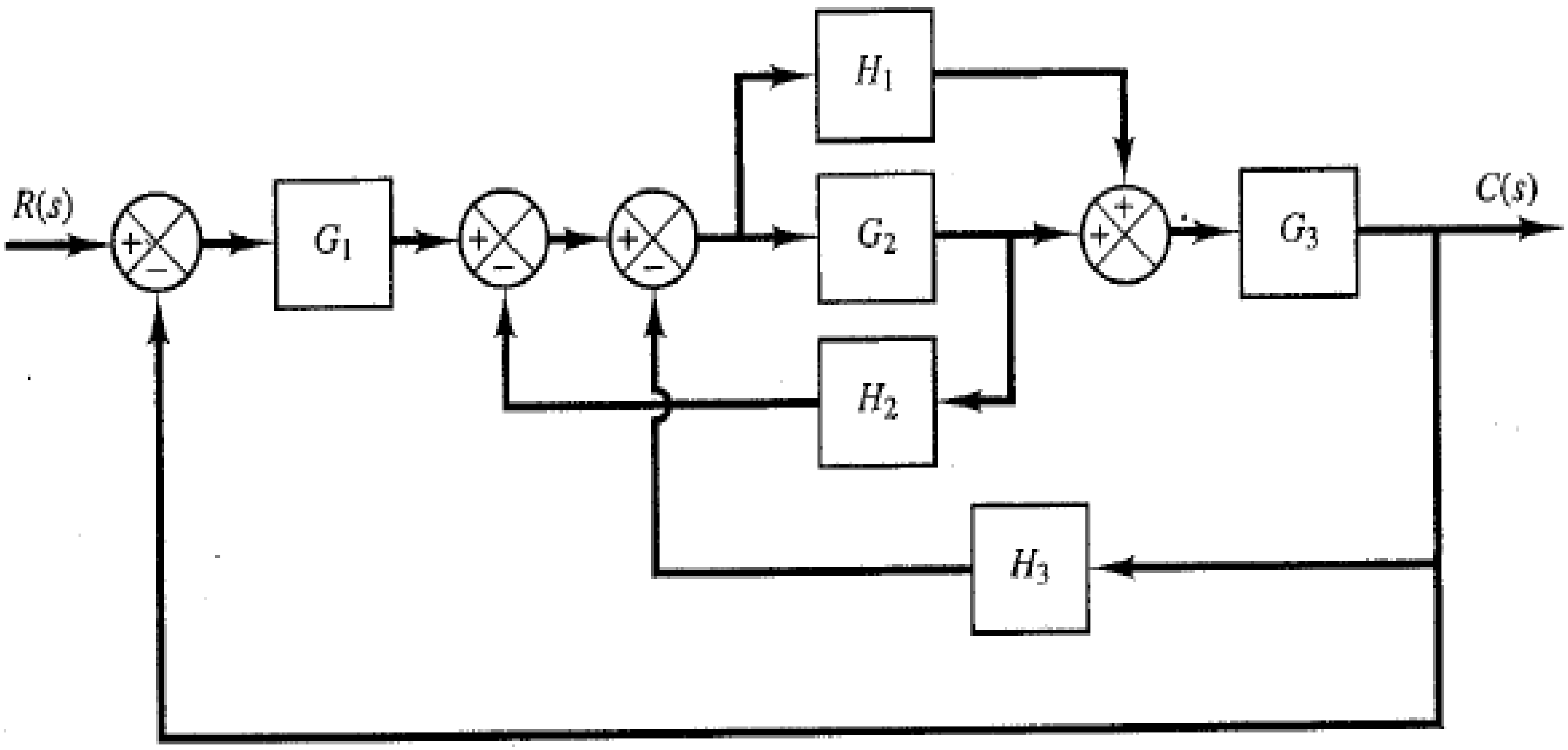
$$\frac{C(s)}{R(s)} = \frac{\left(s + \frac{1}{s^2}\right)}{1 + (2s)\left(s + \frac{1}{s^2}\right)} = \frac{\frac{s^3+1}{s^2}}{1 + \left(2s^2 + \frac{2}{s}\right)} = \frac{\frac{s^3+1}{s^2}}{\frac{s + 2s^3 + 2}{s}}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{s^3+1}{s}}{s + 2s^3 + 2} = \frac{s^3+1}{s(s + 2s^3 + 2)} = \frac{s^3+1}{s^2 + 2s^4 + 2s}$$

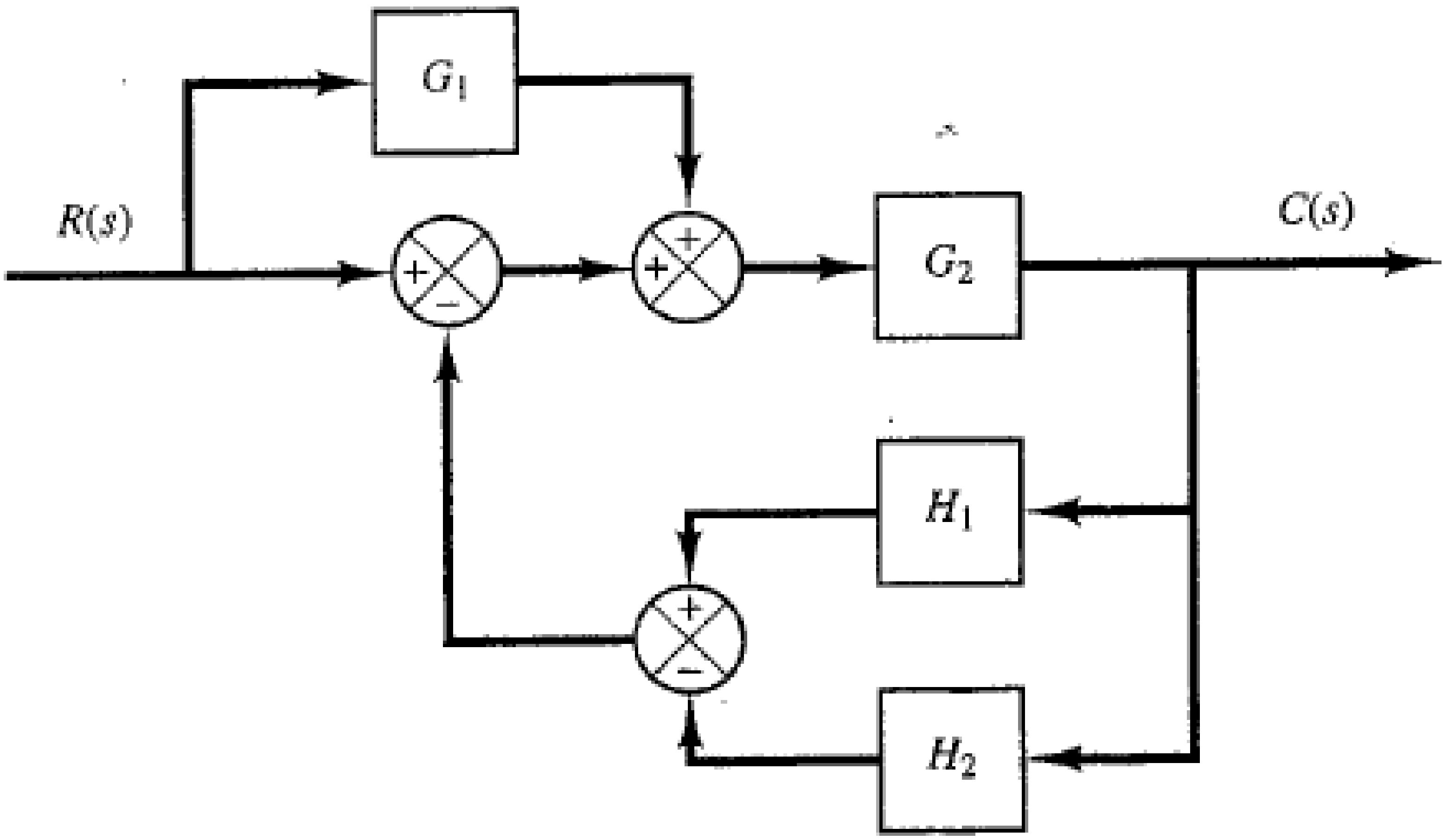
Reduction of Multiple Subsystems – Example



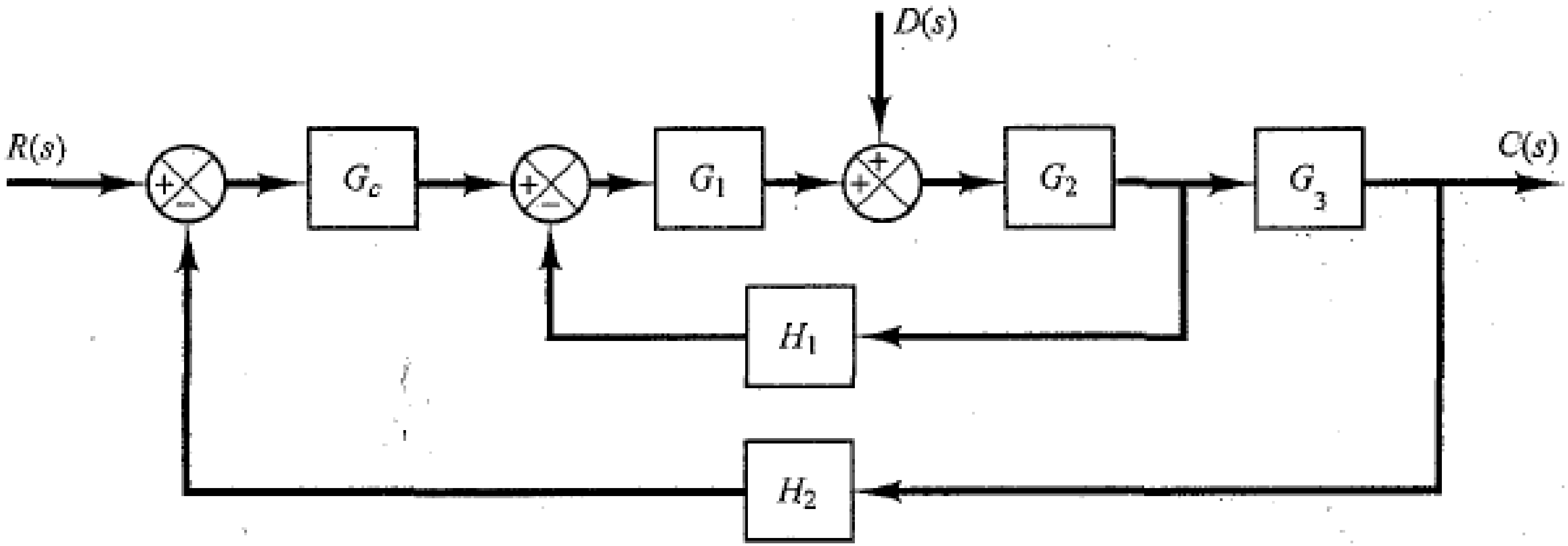
$$\frac{C(s)}{R(s)} = \frac{s^3 + 1}{s^2 + 2s^4 + 2s}$$



R(s)
+
−
G1
+
−
+
−
H1
G2
H2
+
+
G3
C(s)
H3



G_1
$R(s)$
$C(s)$
G_2
H_1
H_2
+
−
+
+
+
−


D(s)
R(s)
C(s)
+
−
+
−
+
+
Gc
G1
G2
G3
H1
H2

Signal Flow Graphs

Node. A node is a point representing a variable or signal.

Transmittance. The transmittance is a real gain or complex gain between two nodes. Such gains can be expressed in terms of the transfer function between two nodes.

Branch. A branch is a directed line segment joining two nodes. The gain of a branch is a transmittance.

Input node or source. An input node or source is a node that has only outgoing branches. This corresponds to an independent variable.

Output node or sink. An output node or sink is a node that has only incoming branches. This corresponds to a dependent variable.

Mixed node. A mixed node is a node that has both incoming and outgoing branches.

Path. A path is a traversal of connected branches in the direction of the branch arrows. If no node is crossed more than once, the path is open. If the path ends at the same node from which it began and does not cross any other node more than once, it is closed. If a path crosses some node more than once but ends at a different node from which it began, it is neither open nor closed.

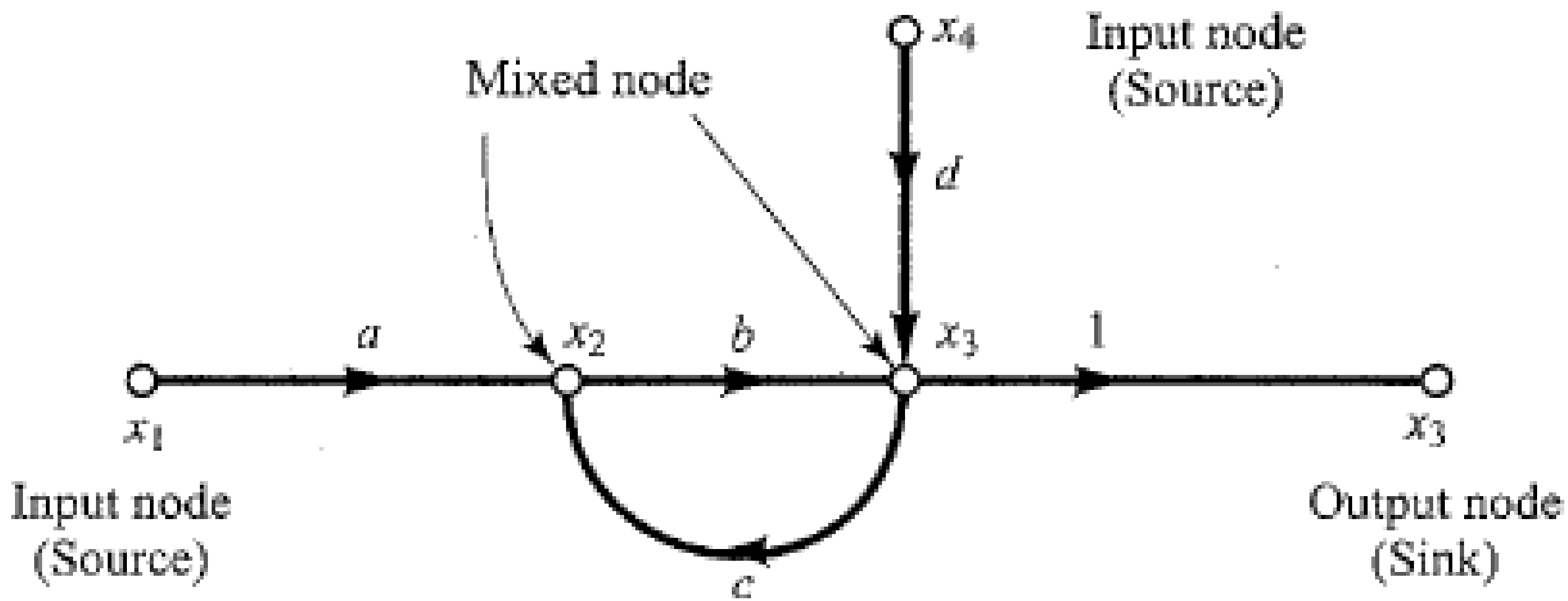
Loop. A loop is a closed path.

Loop gain. The loop gain is the product of the branch transmittances of a loop.

Nontouching loops. Loops are nontouching if they do not possess any common nodes.

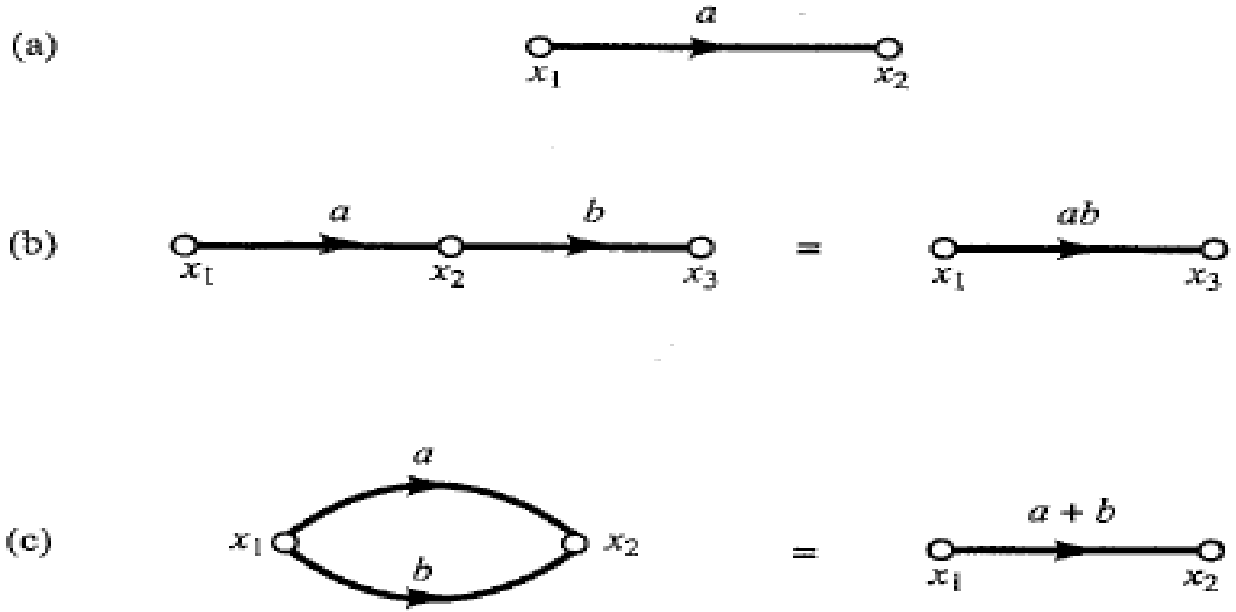
Forward path. A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.

Forward path gain. A forward path gain is the product of the branch transmittances of a forward path.

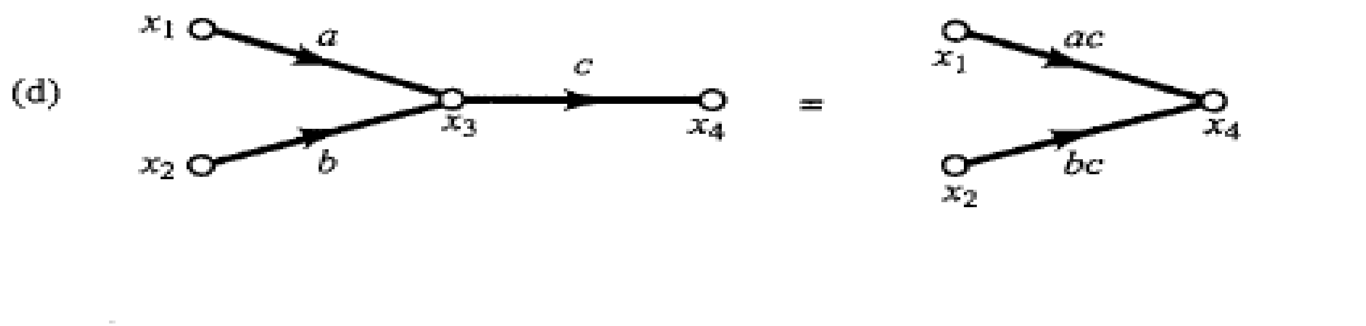


Properties of Signal Flow Graphs. A few important properties of signal flow graphs are as follows:

1. A branch indicates the functional dependence of one signal on another. A signal passes through only in the direction specified by the arrow of the branch.
2. A node adds the signals of all incoming branches and transmits this sum to all outgoing branches.
3. A mixed node, which has both incoming and outgoing branches, may be treated as an output node (sink) by adding an outgoing branch of unity transmittance. (See Figure 3–35. Notice that a branch with unity transmittance is directed from x_3 to another node, also denoted by x_3.) Note, however, that we cannot change a mixed node to a source by this method.
4. For a given system, a signal flow graph is not unique. Many different signal flow graphs can be drawn for a given system by writing the system equations differently.



Signal Flow Graph Algebra.



$$x_3 = abx_1 + bcx_3$$

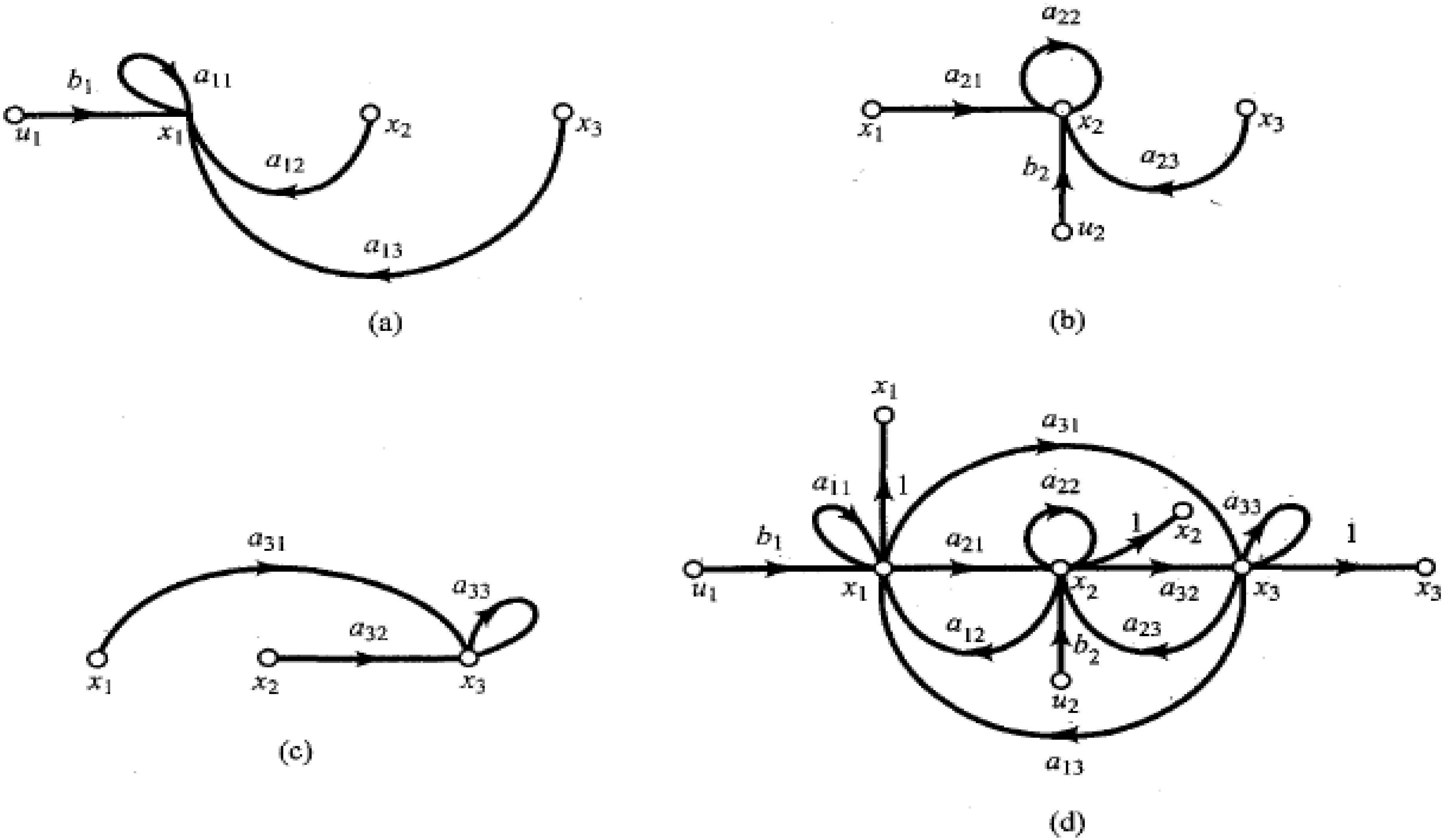
$$x_3 = \frac{ab}{1 - bc} x_1$$

Consider a system defined by the following set of equations:

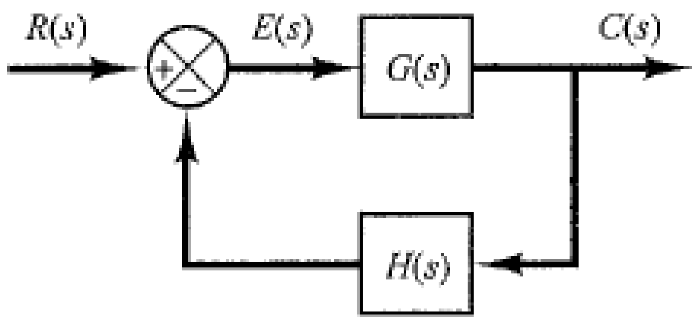
$$x_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1$$

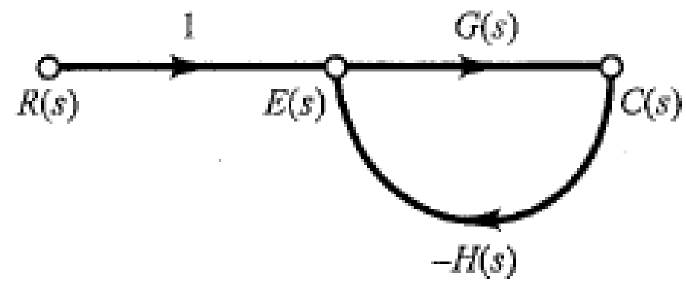
$$x_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2$$

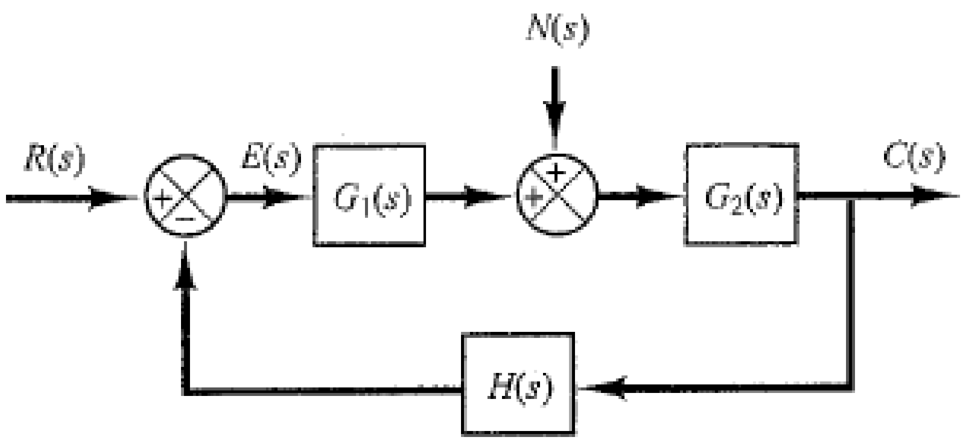
$$x_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3$$

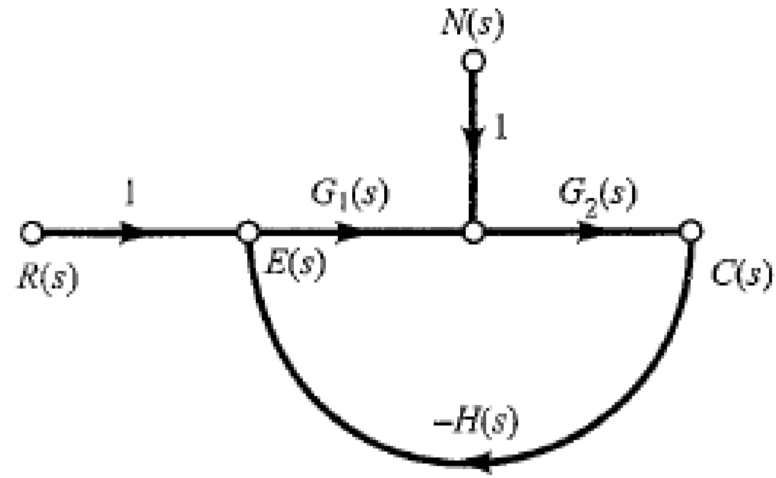


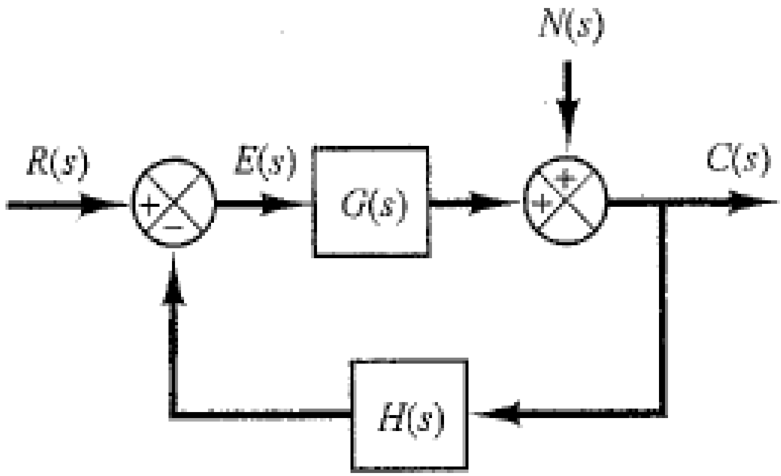
Block Diagrams and Corresponding SFG

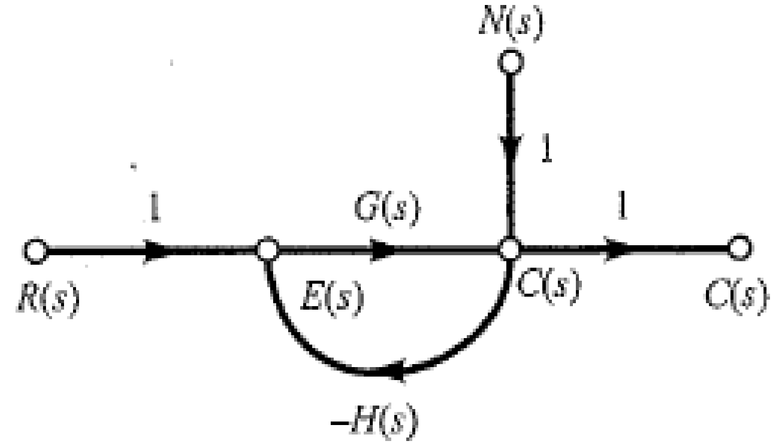


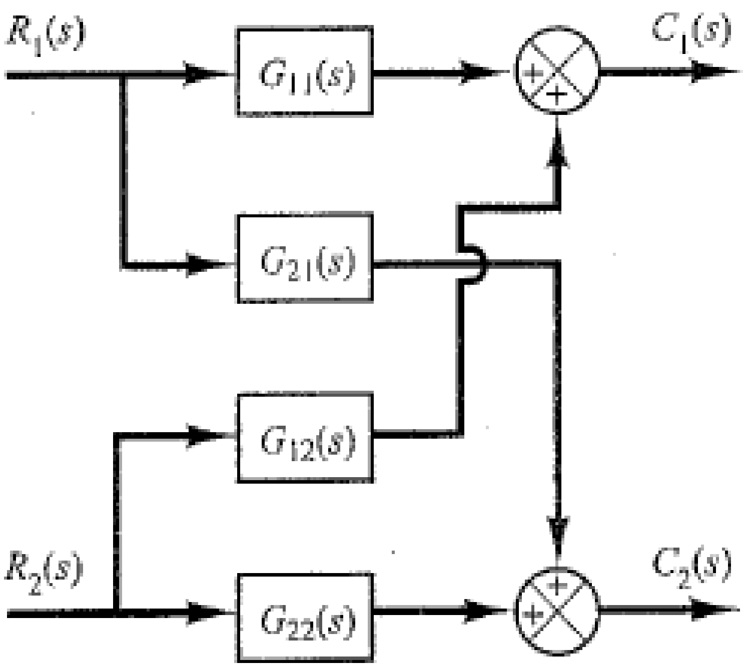


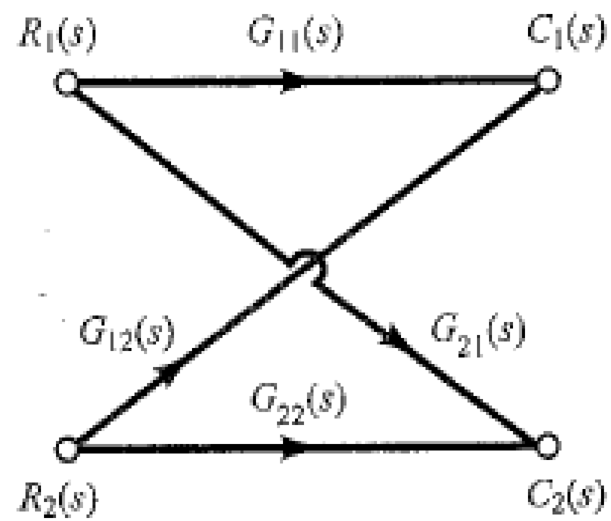





N(s)
R(s)
E(s)
G(s)
C(s)
H(s)


N(s)
1
1
G(s)
1
R(s)
E(s)
C(s)
C(s)
−H(s)


R1(s)
G11(s)
C1(s)
G21(s)
G12(s)
R2(s)
G22(s)
C2(s)


R1(s)
G11(s)
C1(s)
G12(s)
G21(s)
G22(s)
R2(s)
C2(s)

Masons Gain Formula

Mason's gain formula, which is applicable to the overall gain, is given by

$$P = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

where

P_k = path gain or transmittance of kth forward path

Δ = determinant of graph

= 1 − (sum of all individual loop gains) + (sum of gain products of all possible combinations of two nontouching loops) − (sum of gain products of all possible combinations of three nontouching loops) + ⋯

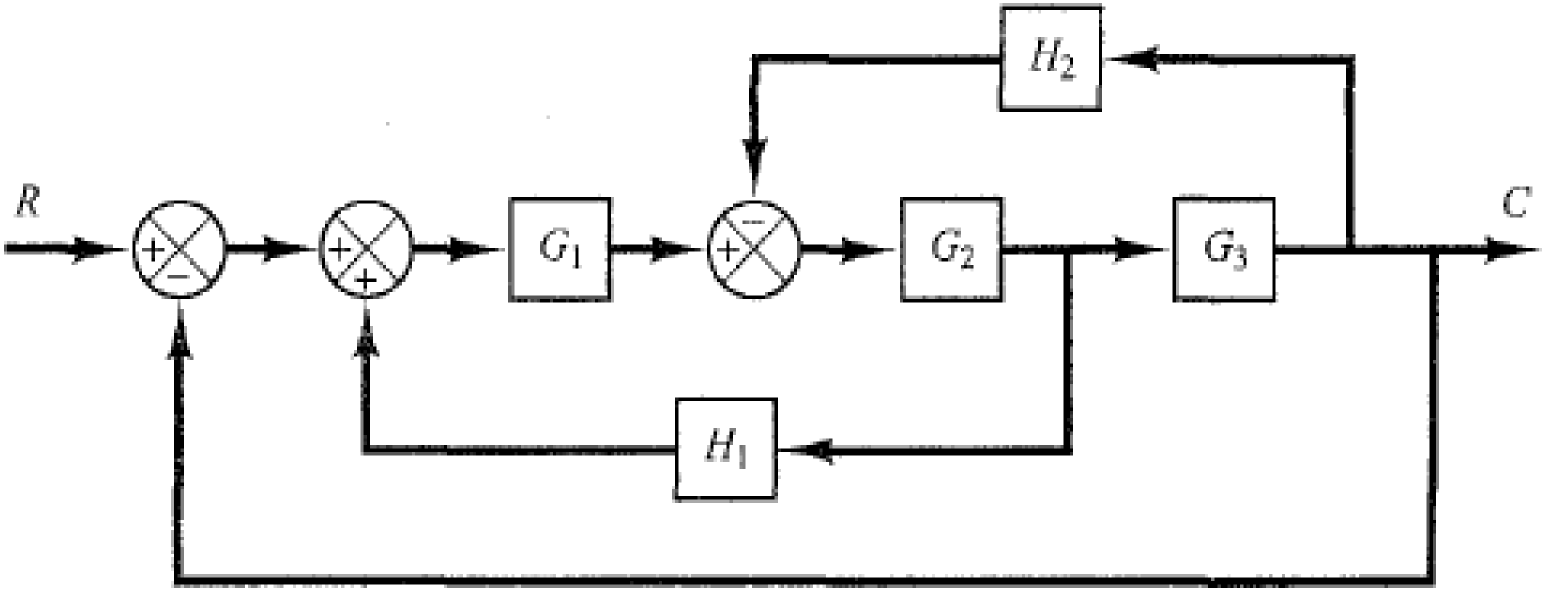
$$= 1 - \sum_a L_a + \sum_{b,c} L_b L_c - \sum_{d,e,f} L_d L_e L_f + \cdots$$

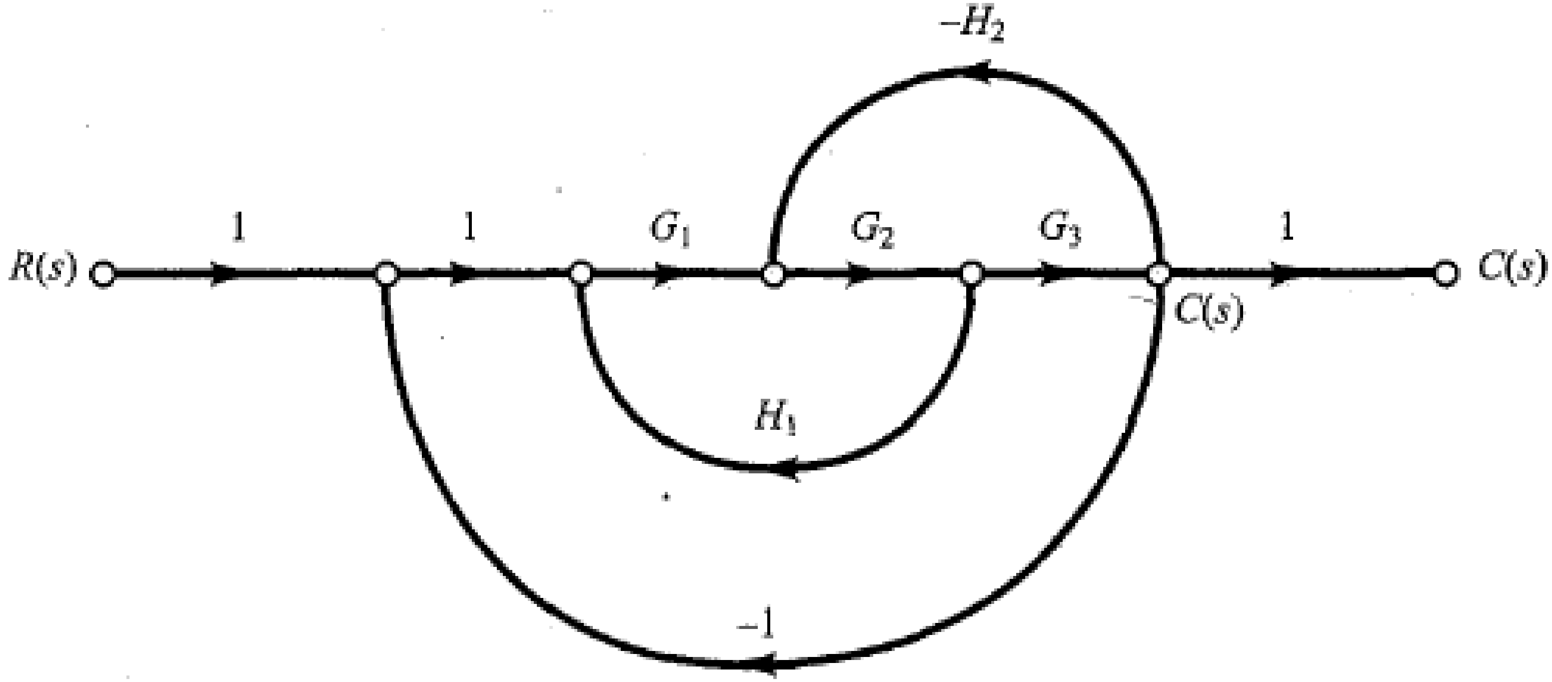
$\sum_a L_a$ = sum of all individual loop gains

$\sum_{b,c} L_b L_c$ = sum of gain products of all possible combinations of two nontouching loops

$\sum_{d,e,f} L_d L_e L_f$ = sum of gain products of all possible combinations of three nontouching loops

Δ_k = cofactor of the kth forward path determinant of the graph with the loops touching the kth forward path removed, that is, the cofactor Δ_k is obtained from Δ by removing the loops that touch path P_k


R
+
−
+
+
G1
−
+
G2
G3
C
H2
H1



$$P_1 = G_1G_2G_3 \qquad L_1 = G_1G_2H_1 \qquad \Delta_1 = 1$$

$$L_2 = -G_2G_3H_2$$

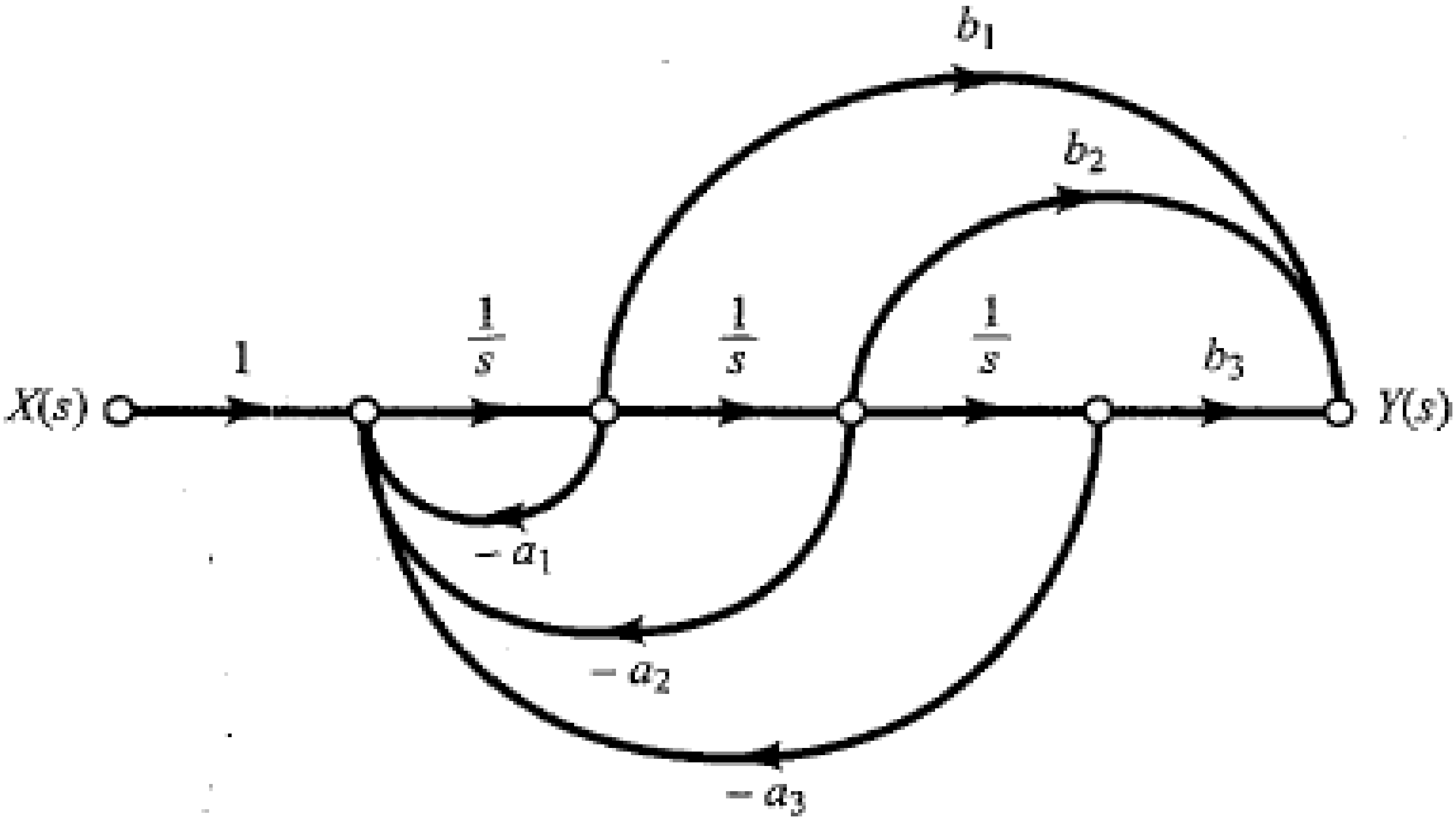
$$L_3 = -G_1G_2G_3$$

$$\begin{aligned}\Delta &= 1 - (L_1 + L_2 + L_3)\\ &= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3\end{aligned}$$

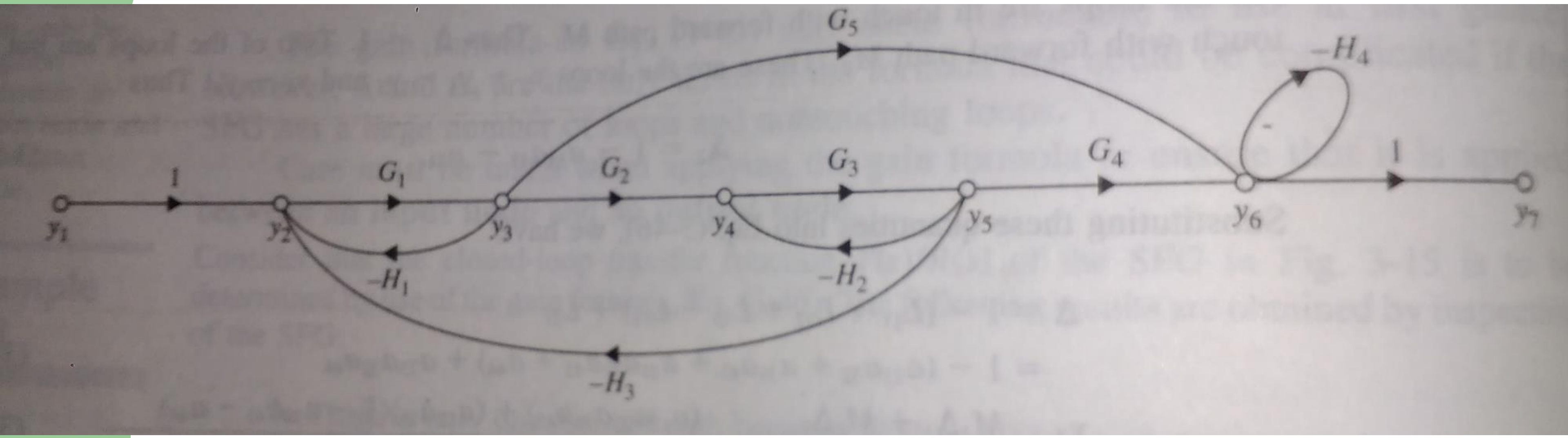
$$\frac{C(s)}{R(s)} = P = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

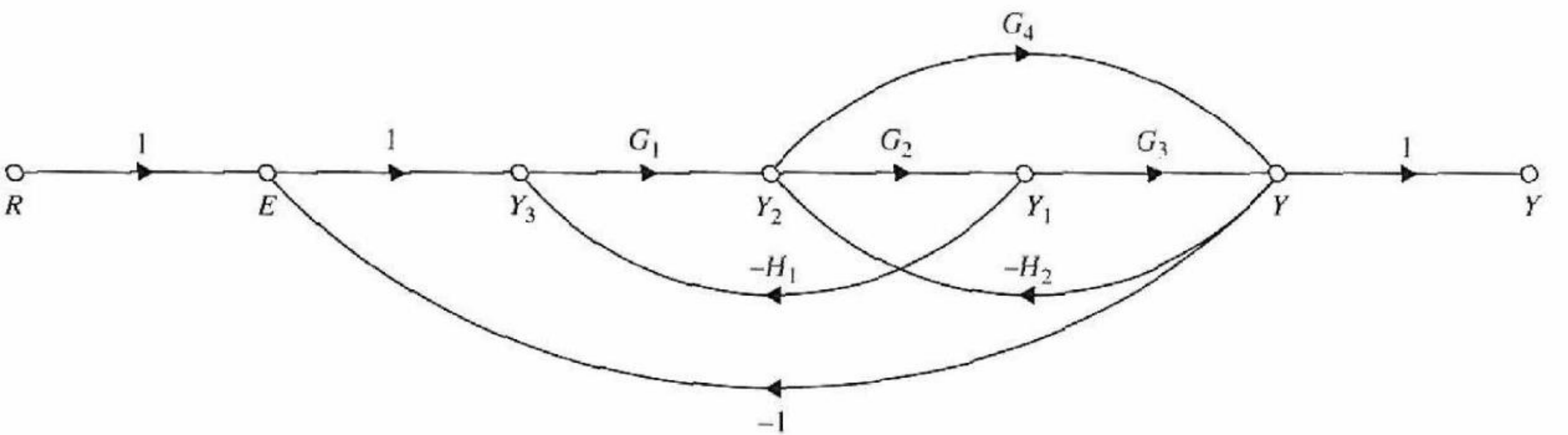
Problem 1



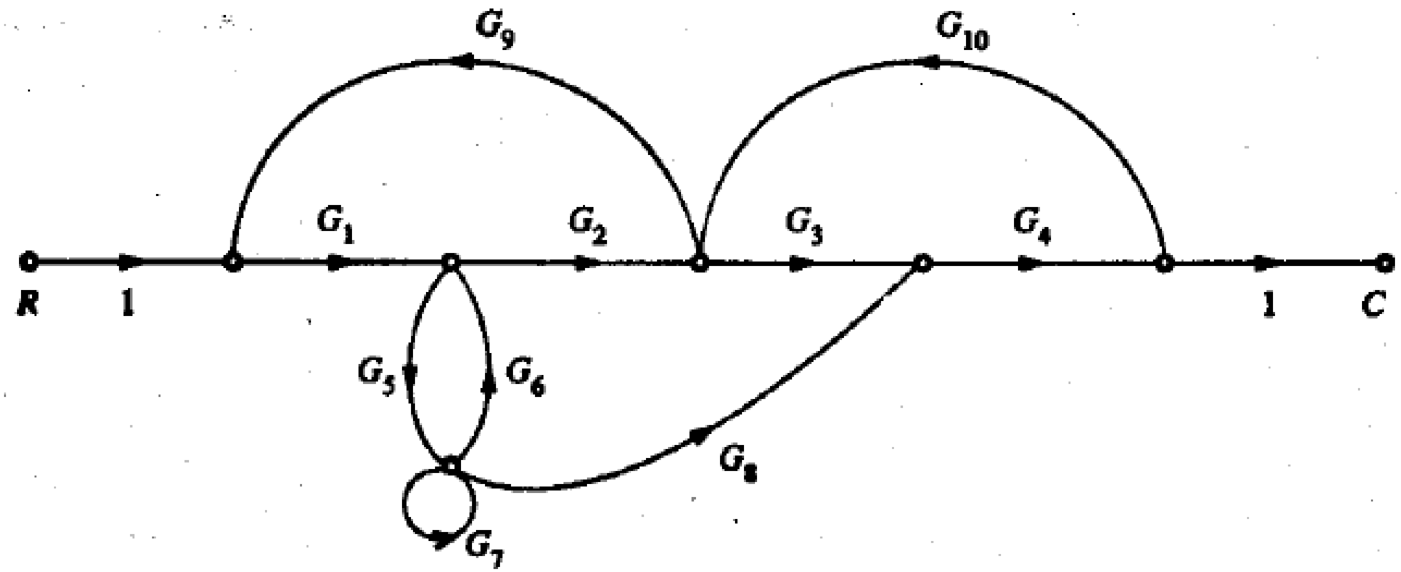
Problem 2



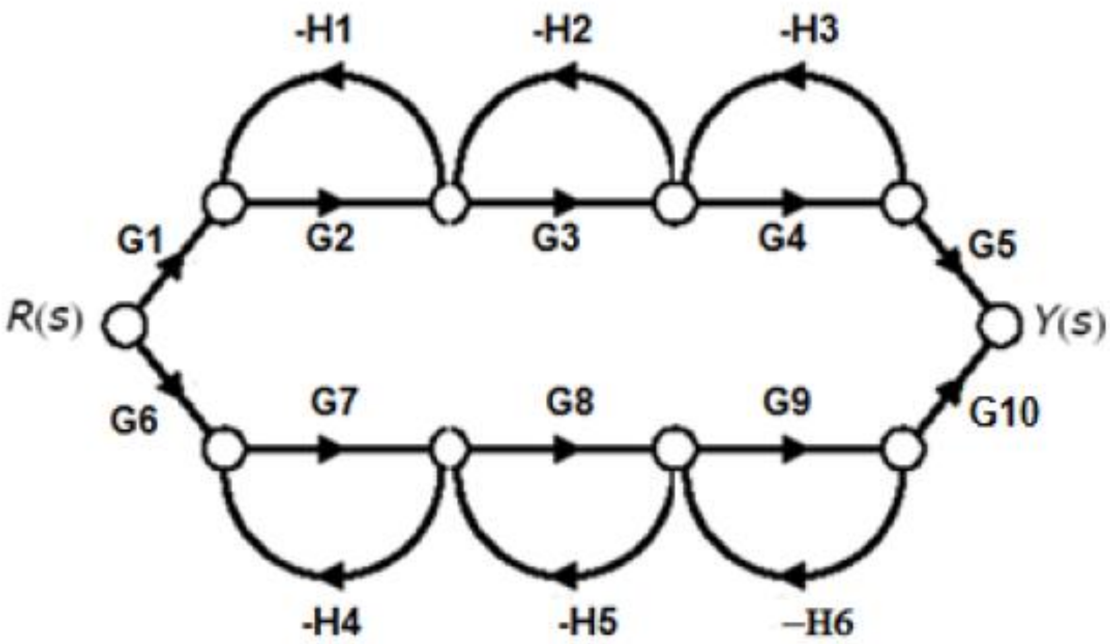
Problem 3



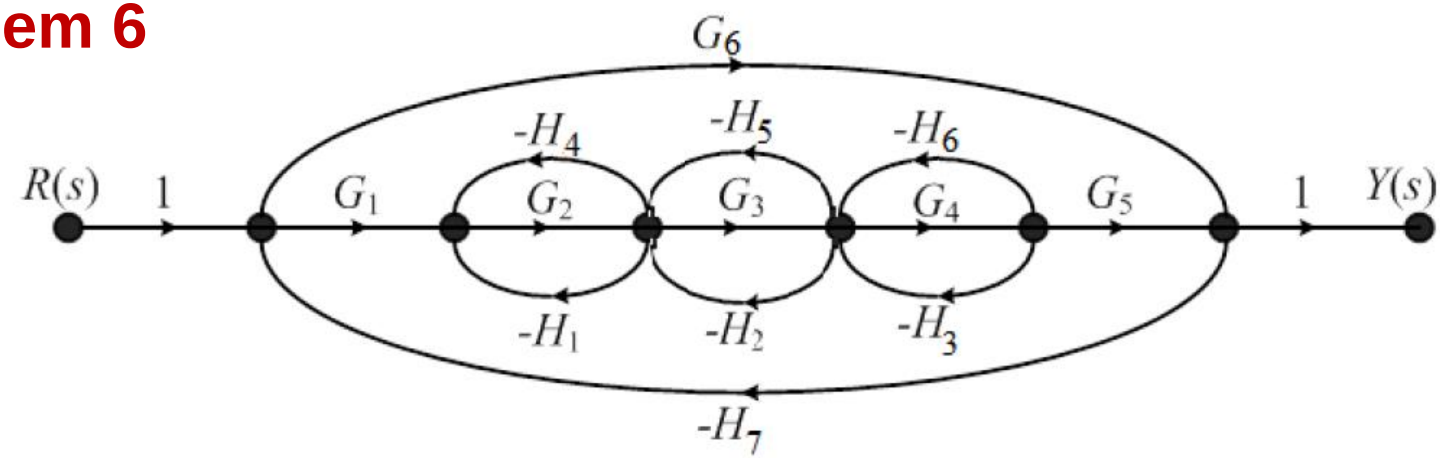
Problem 4



Problem 5



Problem 6



THANK YOU