Assignment - 2

1. Lossy medium.

Phase const : $\beta = 1.6$ rad/m

freq : $f = 10^7$ Hz

Magnitude decreases by 60% for every 2m.

After 2m, $I_{out} = [1 - 0.6] I_{in}$

$= 0.4 I_{in}$

$$\frac{I_{out}}{I_{in}} = e^{-\alpha L}$$

$[L = 2m]$

or, $\frac{0.4 I_{in}}{I_{in}} = e^{-\alpha L}$

or, $-\alpha L = \ln(0.4)$

or, $\alpha = -\frac{1}{2}(-0.9162)$

or, $\alpha = 0.4581 \ m^{-1}$

$\therefore$ Skin depth $= \delta = \frac{1}{\alpha} = \underline{\underline{2.1827 \ m}}$.

Speed of the wave $\Rightarrow u = f\lambda$

$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta}$ $\quad \therefore u = 10^7 \times \frac{2\pi}{1.6}$

Speed $= \underline{\underline{3.925 \times 10^7 \ m/s}}$.

2. Conductivity of Copper wire: $\sigma = 5.6 \times 10^7$ S/m

$\epsilon_r = 1$; $\mu_r = 1$

Radius(a) = 1.2 mm
Length(l) = 600 m

a) DC Resistance $= \dfrac{l}{\sigma A}$

$$= \frac{600}{5.6 \times 10^7 \times \pi \times (1.2 \times 10^{-3})^2}$$

$$\boxed{R_{dc} = 2.3695\ \Omega}.$$

b) AC Resistance $= \dfrac{l}{\sigma \delta\, 2\pi a}$

$f = 100$ MHz

$$= \frac{600 \sqrt{\pi \times 100 \times 10^6 \times \mu_0 \times 5.6 \times 10^7}}{5.6 \times 10^7 \times 2 \times \pi \times (1.2 \times 10^{-3})}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \qquad \boxed{R_{ac} = 211.28\ \Omega.}$$

c) When, $R_{dc} = \dfrac{1}{10} R_{ac}$; $f = ?$

$$\frac{R_{ac}}{R_{dc}} = \frac{\dfrac{l}{\sigma\delta\, 2\pi a}}{\dfrac{l}{\sigma \pi a^2}} = \frac{a}{2\delta} = \frac{a}{2}\sqrt{\pi f \mu \sigma}$$

Now, $\dfrac{a}{2}\sqrt{\pi f \mu \sigma} = 10$ $\Rightarrow f = 1.257 \times 10^6$ Hz

$$\boxed{f = 1.257 \text{ MHz}.}$$

3. Relaxation Time $\rightarrow \frac{\epsilon}{\sigma}$

a) Mica $\rightarrow$ $\sigma = 10^{-15}$ s/m

$\epsilon_r = 6$

$$T = \frac{\frac{6 \times 10^{-9}}{36\pi}}{10^{-15}} = \underline{\underline{5.3 \times 10^{4} \text{ sec.}}}$$

b) Hard Rubber $\rightarrow$ $\sigma = 10^{-15}$ s/m

$\epsilon_r = 3.1$

$$T = \frac{\frac{3.1 \times 10^{-9}}{36\pi}}{10^{-15}} = \underline{\underline{2.741 \times 10^{4}}} \text{ sec}$$

c) Distilled water $\rightarrow$ $\sigma = 10^{-4}$ s/m

$\epsilon_r = 80$

$$T = \frac{\frac{80 \times 10^{-9}}{36\pi}}{10^{-4}} = 7.07 \times 10^{-6} \text{ sec}$$

$$= \underline{\underline{7.07 \ \mu s}}.$$

4. $\rightarrow$ Option (d).

Linear and homogeneous..

$\boxed{D = \epsilon E}$ $\boxed{\nabla \cdot J = \sigma \nabla \cdot E}$

5. Option (a) $\rightarrow$ Normal components of $\vec{D}$: continuous across a dielectric boundary.

6. length(l) = 10 m

(a) Inner radius(r_i) = 1.5 cm

~~Outer~~ Outer copper thickness = 0.5 cm.

Outer radius (r_o) = (1.5 + 0.5) cm = 2 cm.

$\rho_i \leftarrow \rho_{copper} = 1.77 \times 10^{-8} \ \Omega \cdot m$

$\rho_o \leftarrow \rho_{steel} = 11.8 \times 10^{-8} \ \Omega \cdot m.$

$Area_i = \pi r^2$

$S_i = \pi (1.5 \times 10^{-2})^2$

$= 2.25\pi \times 10^{-4} \ m^2$

$Area_{out} = \pi (r_o^2 - r_i)^2$

$S_o = \pi (4 - 2.25) \times 10^{-4} \ m^2$

$= \pi \times 1.75 \times 10^{-4} \ m^2$

$$R = \frac{R_i R_o}{R_o + R_i}$$ (Resistance in parallel).

$$= \left[\frac{\frac{\rho_i}{S_i} \frac{\rho_o}{S_o}}{\frac{\rho_i}{S_i} + \frac{\rho_o}{S_o}} \right] \times l$$

Substituting all the values.

$$\boxed{R = 0.27 \ m\Omega}$$

(b) Total current = 60 A

$$V = I_i R_i = I_o R_o.$$

$$\frac{I_i}{I_o} = \frac{R_o}{R_i} = \frac{0.3219}{1.669} = 0.1929$$

$$\therefore I_i + I_o = 1.1929 I_o$$

$$1.1929 I_o = 60 \text{ Amp.}$$

$$\Rightarrow \boxed{\begin{array}{l} I_o = 50.3\text{A} \longrightarrow \text{Copper.} \\ I_i = 9.7\text{ A} \longrightarrow \text{Steel.} \end{array}}$$

(c)

$$l = 10 \text{ m.}$$

$$\rho = 1.77 \times 10^{-8}. \quad \Omega \cdot \text{m}$$

$$S_o = 1.75\pi \times 10^{-4} \quad \text{m}^2$$

$$\therefore R = \frac{10 \times 1.77 \times 10^{-8}}{1.75\pi \times 10^{-4}}$$

$$\boxed{R = 0.322 \text{ m}\Omega}$$

7. r.i. of glass = 1.5

r.i. of air = 1

$$\frac{\text{Reflected Power}}{\text{Incident Power}} = |\Gamma|^2$$

$\Gamma \rightarrow$ Reflection Coeff. @ medium interface.

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$\eta_1 \rightarrow$ intrinsic impedence of medium-1

$\eta_2 \rightarrow$ " " " " -2.

(r.i) $n_2 = c\sqrt{\mu_2 \epsilon_2} = 1.5$.

$\mu_2 \rightarrow$ Permeability of glass $= \mu_0$.

$\epsilon_2 \rightarrow$ Permittivity of glass $= \epsilon_r \epsilon_0$.

$$3 \times 10^8 \sqrt{4\pi \times 10^{-7} \times \epsilon_r \times \frac{1}{36\pi} \times 10^{-9}} = 1.5$$

$$\Rightarrow \sqrt{\epsilon_r} = 1.5.$$

and $\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{\eta_0}{1.5}$.

$$\therefore \Gamma = \frac{\frac{\eta_0}{1.5} - \eta_0}{\frac{\eta_0}{1.5} + \eta_0} = \frac{1-1.5}{1+1.5} = -\frac{1}{5}$$

Air.

$[\eta_0 = \eta_0]$

$\therefore$ Reflected Power $= |\Gamma|^2 \times$ Incident Power.

$$= \left(\frac{1}{5}\right)^2 \times P_i$$

$$\therefore \frac{P_r}{P_i} = 0.04$$

$$= 4\%.$$

Option-(B) is correct.

8). $\mu_1 = \mu_2 = \mu_0$

$R \rightarrow$ reflection coeff

$T \rightarrow$ transmission coeff.

for avg. powers.

$$P_{r,avg} = R\, P_{i,avg}$$

$$P_{t,avg} = T\, P_{i,avg}$$

$$P_{i,avg} = \frac{E_{io}^2}{2\eta_1}; \quad P_{r,avg} = \frac{E_{ro}^2}{2\eta_1}; \quad P_{t,avg} = \frac{E_{to}^2}{2\eta_2}$$

$$R = \frac{P_{r,avg}}{P_{i,avg}} = \left[\frac{E_{ro}}{E_{io}}\right]^2 = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right]^2$$

$$= \left[\frac{\sqrt{\frac{\mu_0}{\epsilon_2}} - \sqrt{\frac{\mu_0}{\epsilon_1}}}{\sqrt{\frac{\mu_0}{\epsilon_2}} + \sqrt{\frac{\mu_0}{\epsilon_1}}} \right]^2 = \left[\frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right]^2 \times \mu_0$$

$$n_1 = c\sqrt{\mu_1 \epsilon_1} = c\sqrt{\mu_0 \epsilon_1}$$

$$n_2 = c\sqrt{\mu_0 \epsilon_2}$$

Thus $\boxed{R = \left[\frac{n_1 - n_2}{n_1 + n_2} \right]^2}$

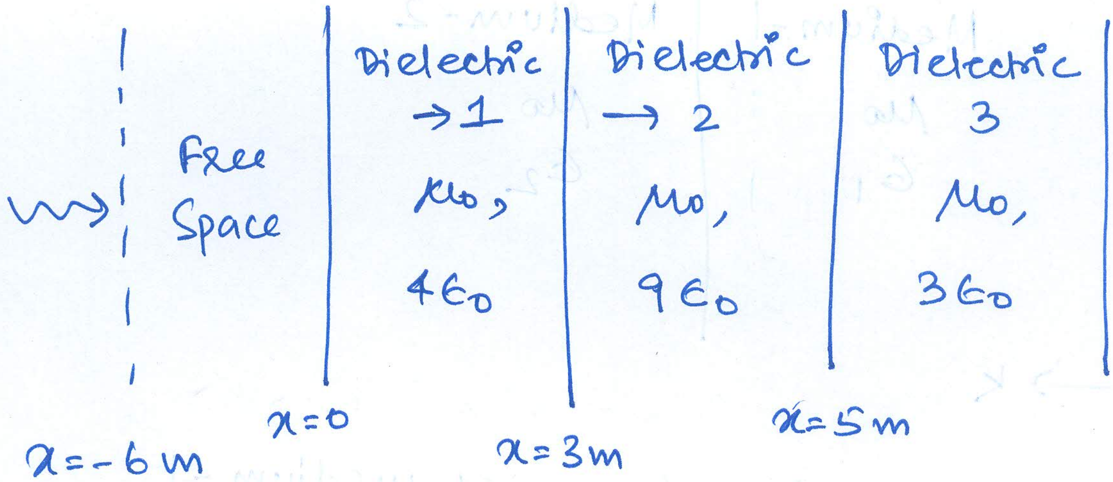
Similarly, $$T = \frac{P_{t,avg}}{P_{i,avg}} = \frac{\eta_1 E_{to}^2}{\eta_2 E_{io}^2}$$

$$= \frac{\eta_1}{\eta_2} \frac{4\eta_2^2}{(\eta_1 + \eta_2)^2}$$

$$\boxed{T = \frac{4 n_1 n_2}{(n_1 + n_2)^2}}$$

Remember $\rightarrow \left[\eta \propto \frac{1}{n} \right]$

9.



Velocity of wave in free space $\rightarrow c = \sqrt{\frac{\mu_0}{\epsilon_0}}$

$= 3 \times 10^8$ m/s

Velocity of wave in Dielectric - 1 is

$$u_{p1} = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{c}{2}$$

Velocity of wave in Dielectric - 2 is

$$u_{p2} = \sqrt{\frac{\mu_0}{9\epsilon_0}} = \frac{c}{3}$$

Velocity of wave in Dielectric - 3 is

$$u_{p3} = \sqrt{\frac{\mu_0}{3\epsilon_0}} = \frac{c}{\sqrt{3}}$$

$\therefore$ Time 't' is taken by the wave to strike the Interface @ $x = 5$m :

$$t = t_1 + t_2 + t_3$$

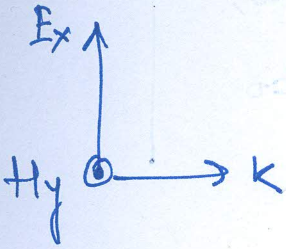
$$= \frac{6}{3 \times 10^8} + \frac{3}{(3\times10^8)/2} + \frac{2}{(3\times10^8)/3}$$

$$= (0.02 + 0.02 + 0.02) \times 10^{-6}$$

$$\boxed{t = 0.06\ \mu s.}$$

10.

Medium-1	Medium-2
μ_0	μ_0
ϵ_1	ϵ_2



Intrinsic impedance of medium-1

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$$

Intrinsic impedance of medium-2

$$\eta_2 = \sqrt{\frac{\mu_0}{\epsilon_2}}$$

$\therefore$ Reflection co-eff @ interface of two medium:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$= \frac{\sqrt{\frac{\mu_0}{\epsilon_2}} - \sqrt{\frac{\mu_0}{\epsilon_1}}}{\sqrt{\frac{\mu_0}{\epsilon_2}} + \sqrt{\frac{\mu_0}{\epsilon_1}}}$$

Given:
Ref $= \frac{1}{5}$ Inci.
$\Rightarrow \Gamma = 1/5$

$$\frac{1}{5} = \left\{\frac{\sqrt{1/\epsilon_2} - \sqrt{1/\sqrt{\epsilon_1}}}{1/\sqrt{\epsilon_2} + 1/\sqrt{\epsilon_1}}\right\}$$

$$\frac{1}{5} = \frac{1 - \sqrt{\frac{\epsilon_2}{\epsilon_1}}}{1 + \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

$$\frac{S+1}{S-1} = \frac{2}{2\sqrt{\epsilon_2/\epsilon_1}} \qquad \text{[By Rationalization]}$$

$$\Rightarrow \quad \frac{6}{4} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

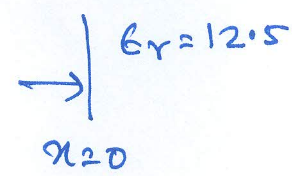
$$\Rightarrow \quad \frac{\epsilon_1}{\epsilon_2} = \frac{9}{4} = 2.25$$

$$\boxed{\epsilon_1/\epsilon_2 = 2.25}$$

11.

$$\vec{E} = 3.6 \cos(\omega t - 5x)\hat{a}_y \text{ V/m}$$

$$\vec{H}_r = \overset{1.25}{\cancel{12.5}} \cos(\omega t + 3x)\hat{a}_z \text{ mA/m}$$



$$\eta_1 = \eta_0 = 120\pi$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

$$\frac{E_{ro}}{E_{io}} = \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{——— (1).}$$

But $\quad E_{ro} = \eta_0 H_{ro}$. ——— (2).

Combining (1) and (2),

$$E_{ro} = \eta_0 H_{ro} = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right] E_{io}.$$

$$\Rightarrow \eta_0 = \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right] \frac{E_{io}}{H_{ro}}.$$

Now, $$\frac{E_{io}}{H_{ro}} = \frac{3.6}{1.25 \times 10^{-3}} = 3000.$$

$$\eta_0 = 3000 \left[\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right]$$

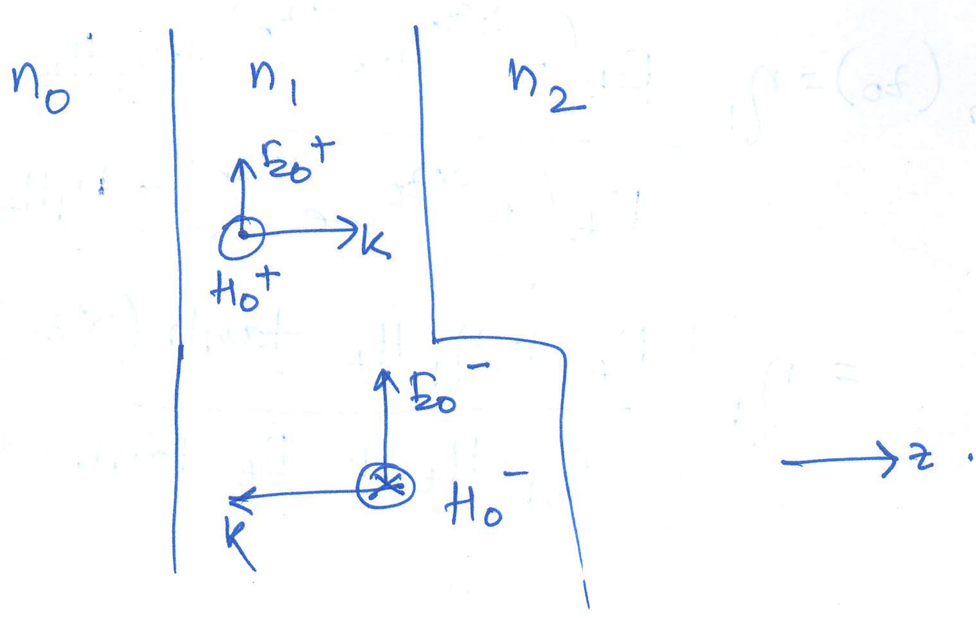
$$\Rightarrow 377 = 3000 \left[\frac{\eta_2 - 377}{\eta_2 + 377}\right]$$

$$\Rightarrow \eta_2 = 485.37.$$

$$\mu_2 = \epsilon_0 \epsilon_r \eta_2^2$$

$$\boxed{\mu_2 = 2.604 \times 10^{-5} \ H/m.}$$

12.



Field inside is :

$$E(z) = E_0^+ e^{-\gamma z} + E_0^- e^{\gamma z}$$

$$H(z) = \frac{E_0^+}{\eta_1} e^{-\gamma z} - \frac{E_0^-}{\eta} e^{\gamma z}$$

Now at the second interface,

$$E_t = E(L) = E_0^+ e^{-\gamma L} + E_0^- e^{\gamma L}$$

$$H_t = H(L) = \frac{E_0^+}{\eta_1} e^{-\gamma L} - \frac{E_0^-}{\eta_1} e^{\gamma L}$$

Input Impedance;

$$\eta_{in}(z = z_0) = \frac{E(L - z_0)}{H(L - z_0)}$$

$$= \eta_1 \left[\frac{E_0^+ e^{-\gamma(L - z_0)} + E_0^- e^{\gamma(L - z_0)}}{E_0^+ e^{-\gamma(L - z_0)} - E_0^- e^{\gamma(L - z_0)}} \right]$$

$$\eta_{in}(z_0) = \eta_1 \frac{E_t\left(e^{-\gamma z_0} + e^{\gamma z_0}\right) + \eta_1 H_t \left(e^{-\gamma z_0} - e^{\gamma z_0}\right)}{E_t\left(e^{-\gamma z_0} - e^{\gamma z_0}\right) - \eta_1 H_t \left(e^{-\gamma z_0} + e^{\gamma z_0}\right)}$$

$$= \eta_1 \left[\frac{E_t + \eta_1 H_t \tanh(\gamma z_0)}{\eta_1 H_t + E_t \tanh(\gamma z_0)} \right]$$

$$\eta_1(z) = \eta_1 \left[\frac{\eta_2 + \eta_1 \tanh(\gamma z_0)}{\eta_1 + \eta_2 \tanh(\gamma z_0)} \right]$$

Here, $\sigma = 0$

$$\gamma = j\omega\sqrt{\mu\epsilon} \qquad \sqrt{\mu\epsilon} = \frac{n_1}{c}$$

$$\eta_1(z) = \eta_1 \left[\frac{\eta_2 + j\eta_1 \tan(\omega\sqrt{\mu\epsilon}\, z)}{\eta_1 + j\eta_2 \tan(\omega\sqrt{\mu\epsilon}\, z)} \right]$$

$$\eta_{in}(L) = \eta_1 \left[\frac{\eta_2 + j\eta_1 \tan\left(\frac{2\pi}{\lambda} c \frac{n_1}{c} \cdot \frac{\lambda}{4n_1}\right)}{\eta_1 + j\eta_2 \tan\left(\frac{2\pi}{\lambda} \cdot c \cdot \frac{n_1}{c} \cdot \frac{\lambda}{4n_1}\right)} \right]$$

$$= \eta_1 \frac{j\eta_1}{j\eta_2} = \frac{\eta_1^2}{\eta_2}$$

$$\eta_i = \sqrt{\frac{\mu_i}{\epsilon_i}}$$

As dielectric medium,

$$\eta_i = \sqrt{\frac{\mu_0}{\epsilon_i}} \qquad n_i = \sqrt{\mu_0 \epsilon_i}$$

$$\eta_i n_i = \mu_0 .$$

$$\eta_i = \frac{\mu_0}{n_i}$$

$$\eta_{in}(L) = \left(\frac{\mu_0}{n_1}\right)^2 \frac{n_2}{\mu_0} = \frac{\mu_0}{n_0}$$

$$\eta_{in}(L) = \eta_0$$

$$\Gamma = \frac{\eta_{in}(L) - \eta_0}{\eta_{in}(L) + \eta_0}.$$

$\boxed{\Gamma = 0}$. $\Rightarrow$ No reflection.

The application is : Anti- Reflection coating.