Liquid and Thermal Systems

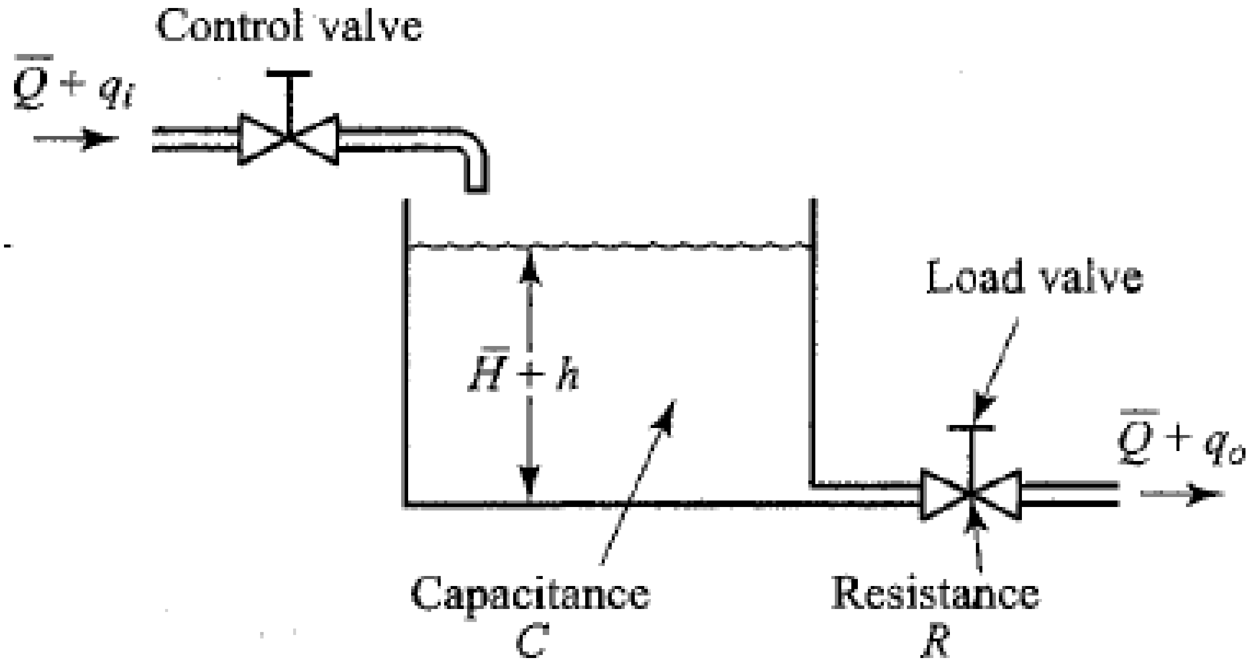
Linear and non linear Flow

- In analyzing systems involving fluid flow, it is necessary to divide flow regimes into laminar flow and turbulent flow, according to the magnitude of the Reynolds number. If the Reynolds number is greater than about 3000 to 4000, then the flow is turbulent.

- The flow is laminar if the Reynolds number is less than about 2000. In the laminar case, fluid flow occurs in streamlines with no turbulence. Systems involving laminar flow may be represented by linear differential equations.

- Industrial processes often involve flow of liquids through connecting pipes and tanks. The flow in such processes is often turbulent and not laminar. Systems involving turbulent flow often have to be represented by nonlinear differential equations. If the region of operation is limited, however, such nonlinear differential equations can be linearized.

Resistance and Capacitance of Liquid-Level Systems



$$R = \frac{\text{change in level difference, m}}{\text{change in flow rate, m}^3/\text{sec}}$$

Resistance and Capacitance of Liquid-Level Systems

In this system the liquid spouts through the load valve in the side of the tank. If the flow through this restriction is laminar, the relationship between the steady-state flow rate and steady-state head at the level of the restriction is given by

$$Q = KH$$

where Q = steady-state liquid flow rate, m^3/sec
K = coefficient, m^2/sec
H = steady-state head, m

For laminar flow, the resistance R_l is obtained as

$$R_l = \frac{dH}{dQ} = \frac{H}{Q}$$

If the flow through this restriction is Turbulent $Q = K\sqrt{H}$

where Q = steady-state liquid flow rate, m^3/sec

K = coefficient, $m^{2.5}/sec$

H = steady-state head, m

The resistance R_t for turbulent flow is obtained from

$$R_t = \frac{dH}{dQ}$$

$$dQ = \frac{K}{2\sqrt{H}}\, dH$$

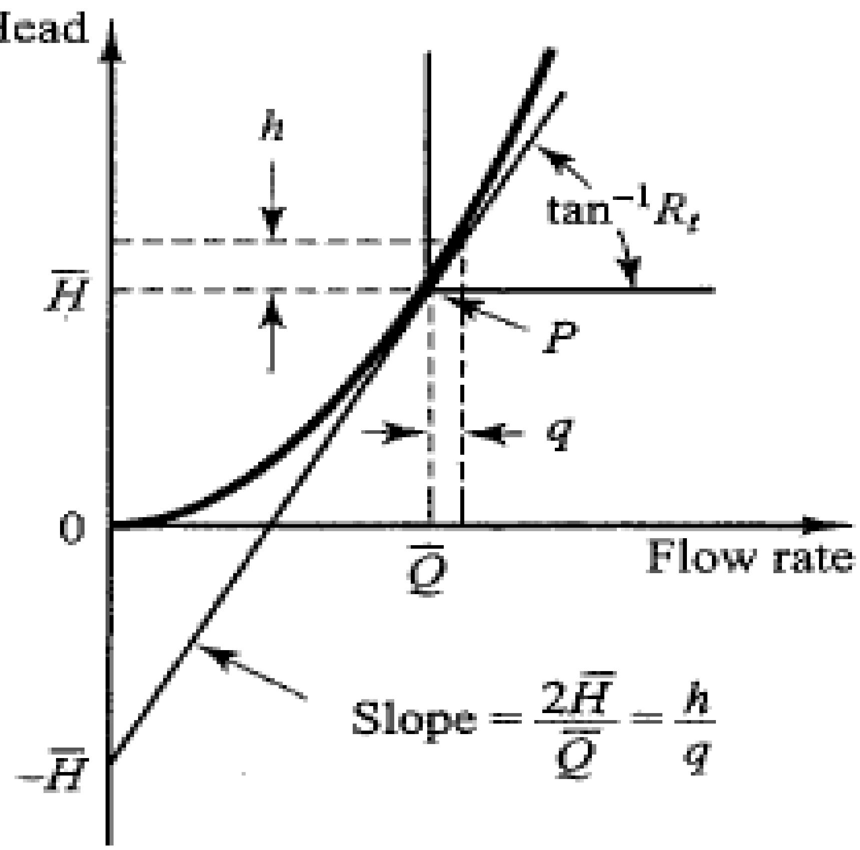
we have

$$\frac{dH}{dQ} = \frac{2\sqrt{H}}{K} = \frac{2\sqrt{H}\sqrt{H}}{Q} = \frac{2H}{Q}$$

Thus,

$$R_t = \frac{2H}{Q}$$

The value of the turbulent-flow resistance R, depends on the flow rate and the head. The value of R, however, may be considered constant if the changes in head and flow rate are small.



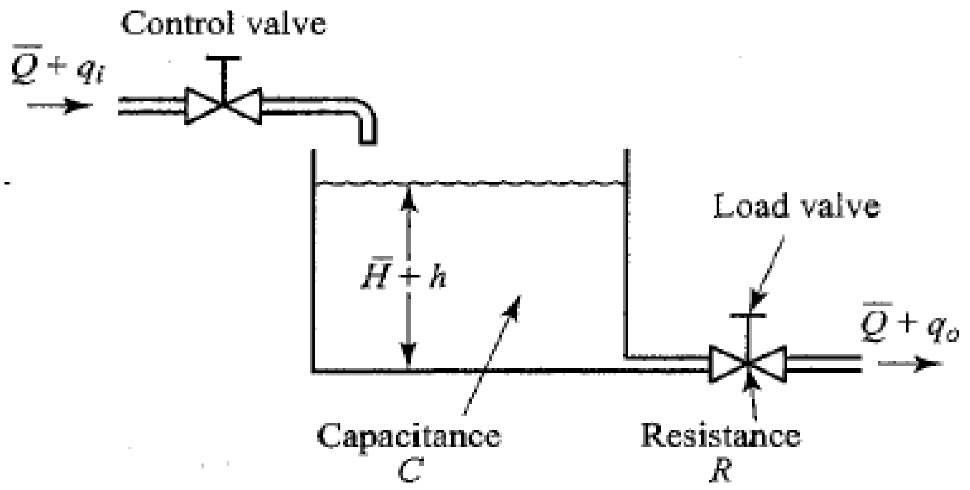
Head vs. Flow Plot is shown in Figure above . In the figure, point P is the steady-state operating point. The tangent line to the curve at point P intersects the ordinate at point (0, -H). Thus, the slope of this tangent line is 2H/Q. Since the resistance R, at the operating point P is given by 2H/Q, the resistance R, is the slope of the curve at the operating point.

Consider the operating condition in the neighborhood of point P. Define a small deviation of the head from the steady-state value as h and the corresponding small change of the flow rate as q. Then the slope of the curve at point P can be given by

$$\text{Slope of curve at point } P = \frac{h}{q} = \frac{2\bar{H}}{\bar{Q}} = R_t$$

The linear approximation is based on the fact that the actual curve does not differ much from its tangent line if the operating condition does not vary too much.

Capacitance of the Tank



The capacitance C of a tank is defined to be the change in quantity of stored liquid necessary to cause a unit change in the potential (head). (The potential is the quantity that indicates the energy level of the system.)

$$C = \frac{\text{change in liquid stored, m}^3}{\text{change in head, m}}$$

$\bar{Q}$ = steady-state flow rate (before any change has occurred), m^3/sec

q_i = small deviation of inflow rate from its steady-state value, m^3/sec

q_o = small deviation of outflow rate from its steady-state value, m^3/sec

$\bar{H}$ = steady-state head (before any change has occurred), m

h = small deviation of head from its steady-state value, m

$$C\,dh = (q_i - q_o)\,dt$$

From the definition of resistance, the relationship between q_o and h is given by

$$q_o = \frac{h}{R}$$

The differential equation for this system for a constant value of R becomes

$$RC\frac{dh}{dt} + h = Rq_i$$

$$(RCs + 1)H(s) = RQ_i(s)$$

where

$$H(s) = \mathscr{L}[h] \qquad \text{and} \qquad Q_i(s) = \mathscr{L}[q_i]$$

If q_i is considered the input and h the output, the transfer function of the system is

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs + 1}$$

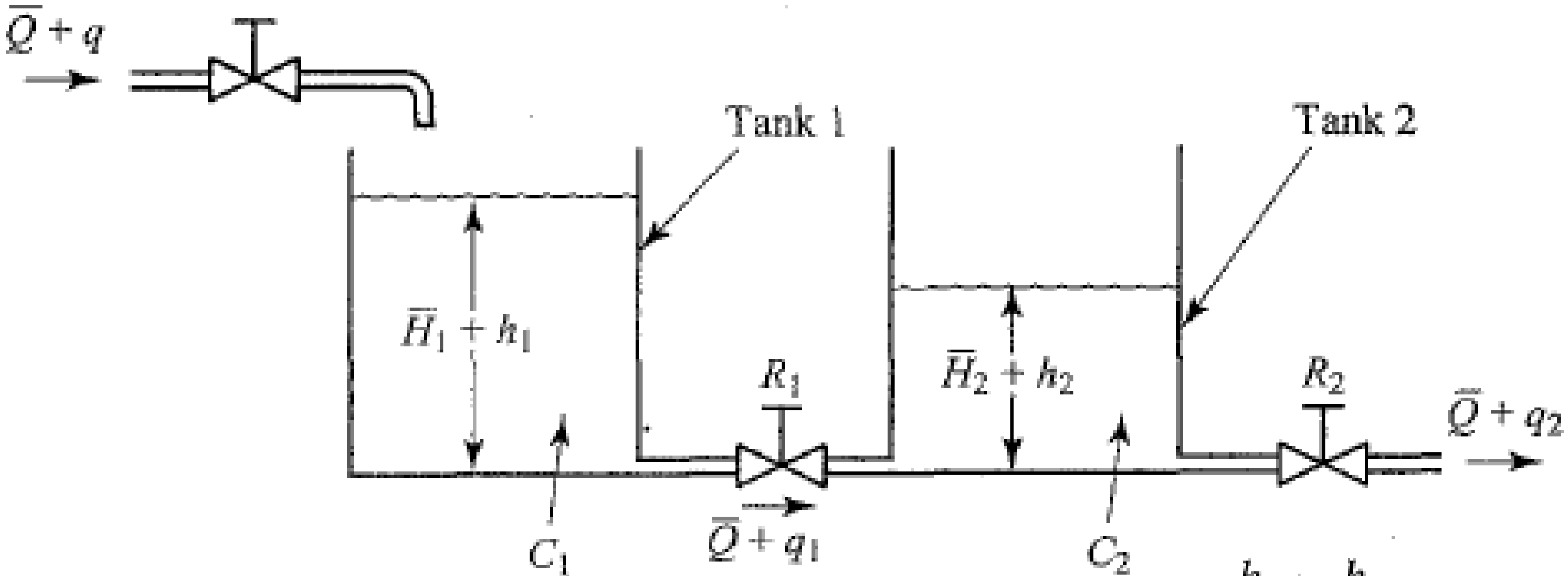
If, however, q_o is taken as the output, the input being the same, then the transfer function is

$$\frac{Q_o(s)}{Q_i(s)} = \frac{1}{RCs + 1}$$

where we have used the relationship

$$Q_o(s) = \frac{1}{R}H(s)$$

Liquid Level Systems with Interaction



$\bar{Q}$: Steady-state flow rate
$\bar{H}_1$: Steady-state liquid level of tank 1
$\bar{H}_2$: Steady-state liquid level of tank 2

$$\frac{h_1 - h_2}{R_1} = q_1$$

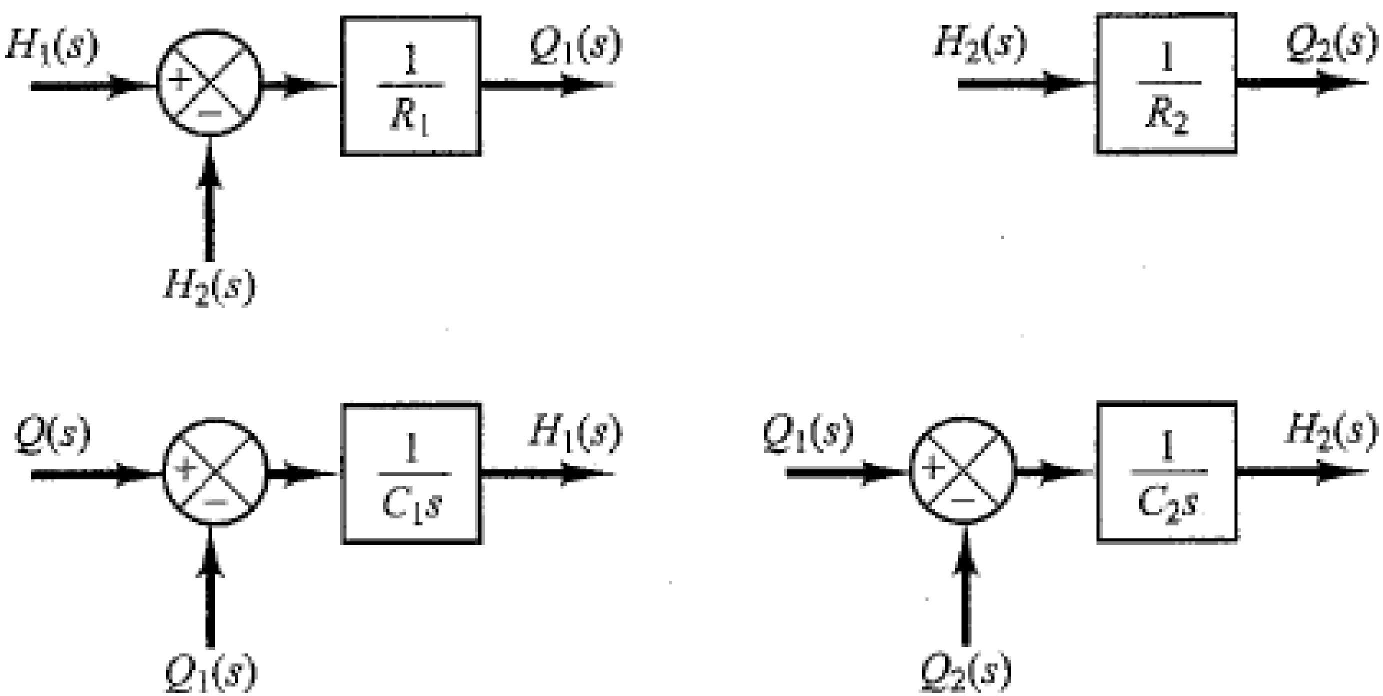
$$C_1 \frac{dh_1}{dt} = q - q_1$$

$$\frac{h_2}{R_2} = q_2$$

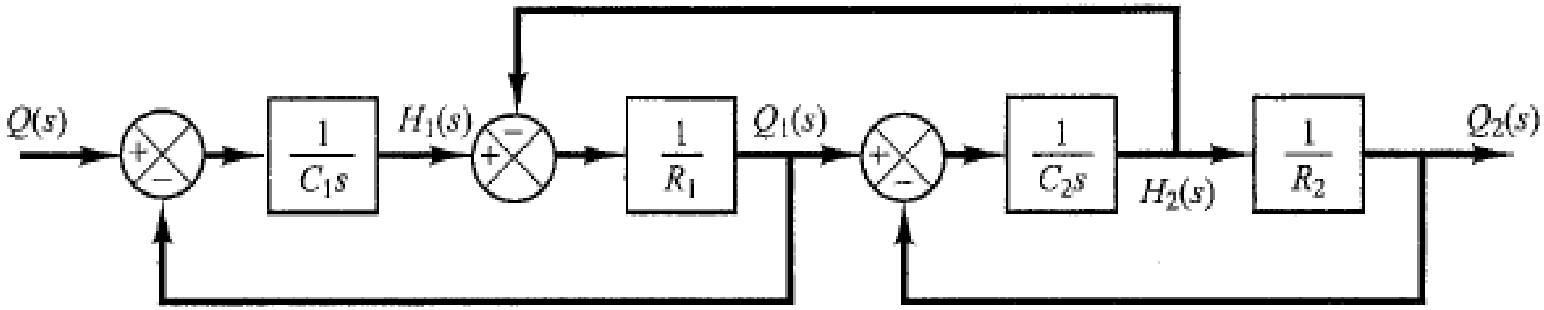
$$C_2 \frac{dh_2}{dt} = q_1 - q_2$$

If q is considered the input and q_2 the output, the transfer function of the system

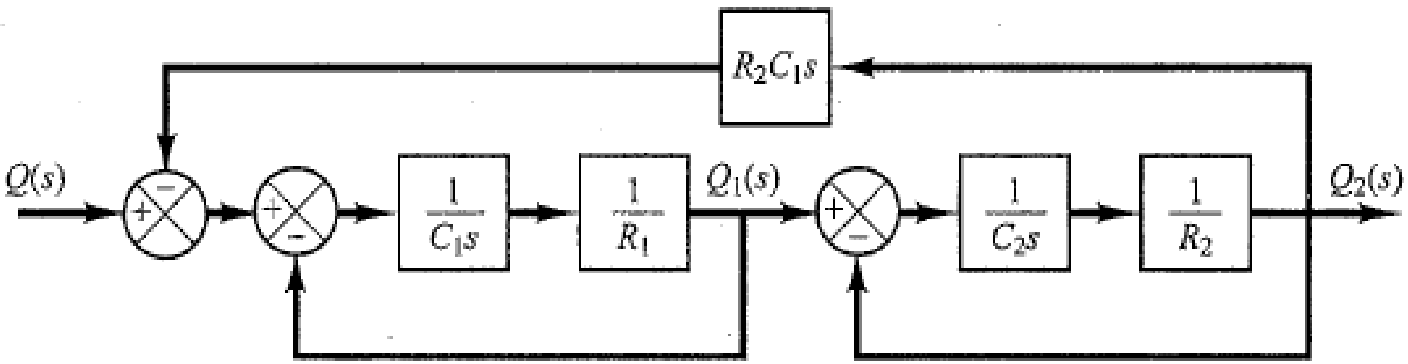
$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1}$$



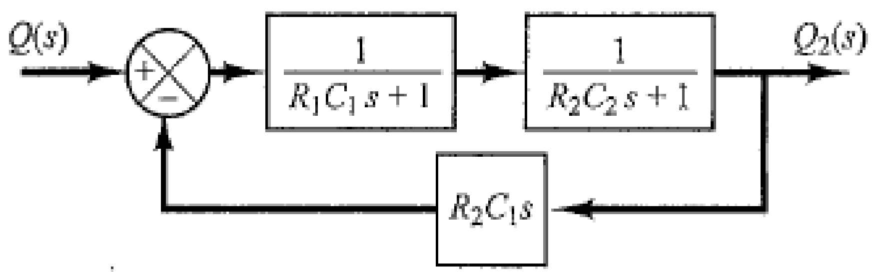
(a)


Q(s)
H1(s)
Q1(s)
Q2(s)
H2(s)
1/C1s
1/R1
1/C2s
1/R2

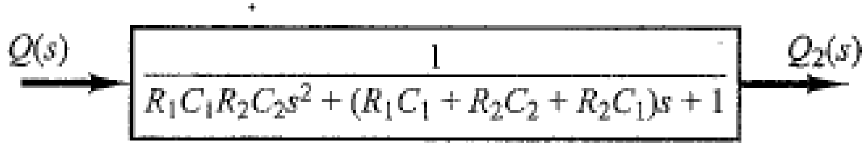
(b)

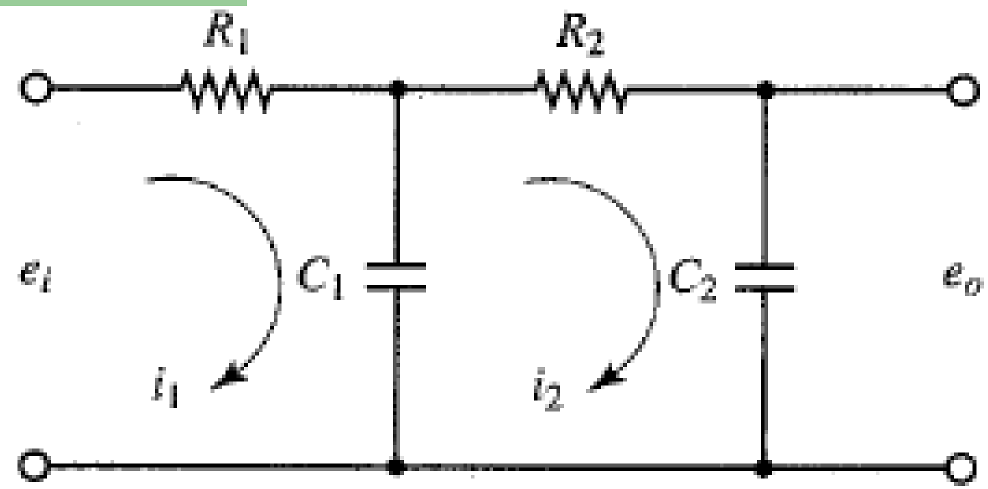

Q(s)
Q1(s)
Q2(s)
R2C1s
1/C1s
1/R1
1/C2s
1/R2

(c)


Q(s)
Q2(s)
1/(R1C1 s + 1)
1/(R2C2 s + 1)
R2C1s

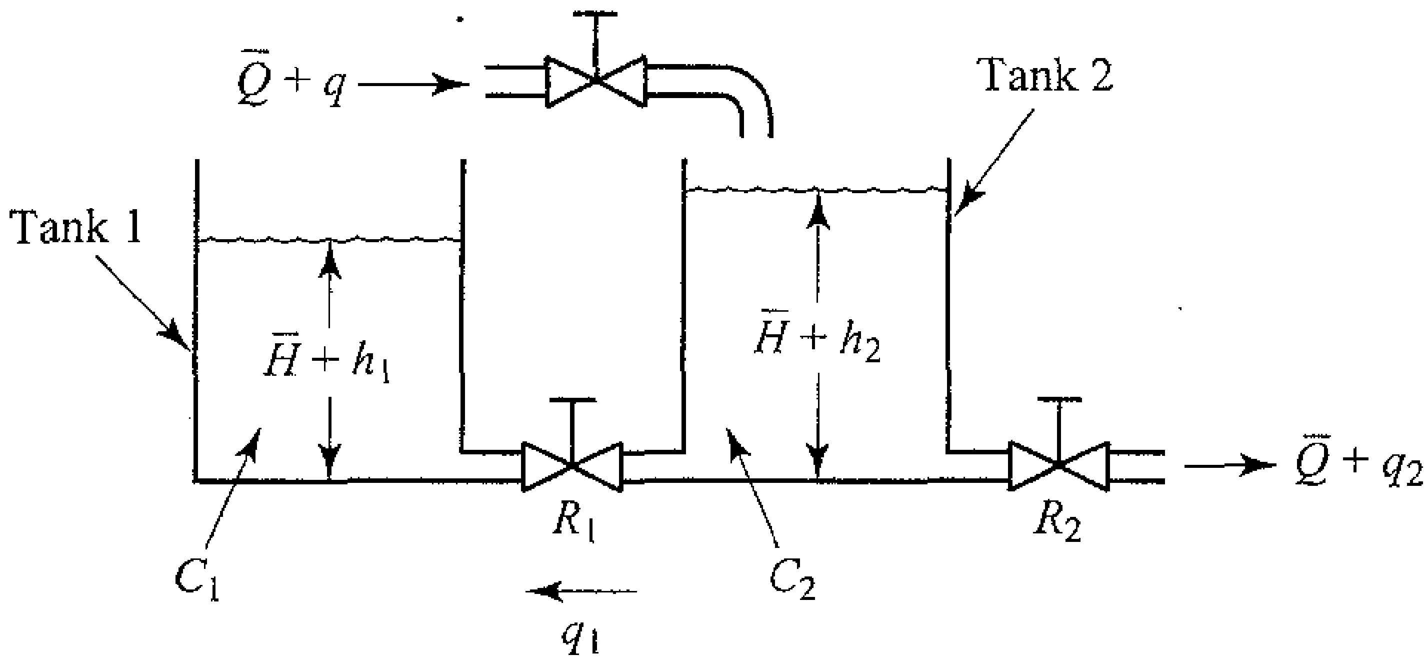
(d)


Q(s)
Q2(s)
1/(R1C1R2C2s^2 + (R1C1 + R2C2 + R2C1)s + 1)



R_1
R_2
e_i
C_1
C_2
e_o
i_1
i_2

Example……….



$\bar{Q} + q$
Tank 2
Tank 1
$\bar{H} + h_1$
$\bar{H} + h_2$
$\bar{Q} + q_2$
R_1
R_2
C_1
C_2
q_1

Thermal Systems

Thermal systems are those that involve the transfer of heat from one substance to another. Thermal systems may be analyzed in terms of resistance and capacitance, i.e. thermal capacitance and thermal resistance.

There are three different ways heat can flow from one substance to another: conduction, convection, and radiation. Here we consider only conduction and convection. (Radiation heat transfer is appreciable only if the temperature of the emitter is very high compared to that of the receiver. Most thermal processes in process control systems do not involve radiation heat transfer.)

For conduction or convection heat transfer,

$$q = K\,\Delta\theta$$

where q = heat flow rate, kcal/sec

$\Delta\theta$ = temperature difference, °C

K = coefficient, kcal/sec °C

The coefficient K is given by

$$K = \frac{kA}{\Delta X}, \qquad \text{for conduction}$$

$$= HA, \qquad \text{for convection}$$

where k = thermal conductivity, kcal/m sec °C

A = area normal to heat flow, m^2

ΔX = thickness of conductor, m

H = convection coefficient, kcal/m^2 sec °C

Thermal Resistance and Thermal Capacitance

The thermal resistance R for heat transfer between two substances may be defined as follows:

$$R = \frac{\text{change in temperature difference, °C}}{\text{change in heat flow rate, kcal/sec}}$$

The thermal resistance for conduction or convection heat transfer is given by

$$R = \frac{d(\Delta\theta)}{dq} = \frac{1}{K}$$

Since the thermal conductivity and convection coefficients are almost constant, the thermal resistance for either conduction or convection is constant.

The thermal capacitance C is defined by

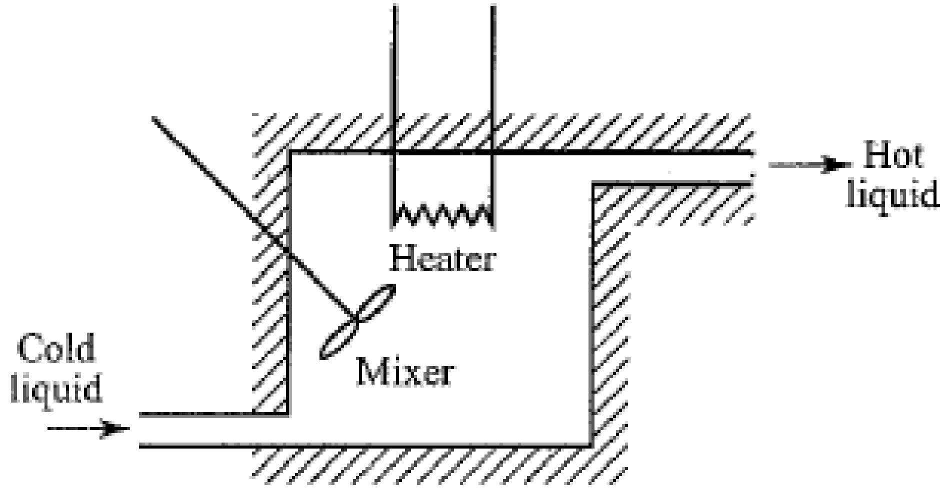
$$C = \frac{\text{change in heat stored, kcal}}{\text{change in temperature, °C}}$$

or

$$C = mc$$

where m = mass of substance considered, kg

c = specific heat of substance, kcal/kg °C



(a)

$\bar{\Theta}_i$ = steady-state temperature of inflowing liquid, °C

$\bar{\Theta}_o$ = steady-state temperature of outflowing liquid, °C

G = steady-state liquid flow rate, kg/sec

M = mass of liquid in tank, kg

c = specific heat of liquid, kcal/kg °C

R = thermal resistance, °C sec/kcal

C = thermal capacitance, kcal/°C

$\bar{H}$ = steady-state heat input rate, kcal/sec

Assume that the temperature of the inflowing liquid is kept constant and that the heat input rate to the system (heat supplied by the heater) is suddenly changed from $\bar{H}$ to $\bar{H} + h_i$, where h_i represents a small change in the heat input rate. The heat outflow rate will then change gradually from $\bar{H}$ to $\bar{H} + h_o$. The temperature of the outflowing liquid will also be changed from $\bar{\Theta}_o$ to $\bar{\Theta}_o + \theta$ For this case, h_o, C, and R are obtained, respectively, as

$$h_o = Gc\theta$$

$$C = Mc$$

$$R = \frac{\theta}{h_o} = \frac{1}{Gc}$$

The heat balance equation for this system is

$$C d\theta = (h_i - h_o)\, dt$$

or

$$C \frac{d\theta}{dt} = h_i - h_o \qquad RC \frac{d\theta}{dt} + \theta = Rh_i$$

Note that the time constant of the system is equal to RC or M/G seconds. The transfer function relating θ and h_i is given by

$$\frac{\Theta(s)}{H_i(s)} = \frac{R}{RCs + 1}$$

THANK YOU