Dept. of EE, IIT Tirupati
EE3001 : Electromagnetic Fields (Aug - Nov 2018)
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Assignment - 1

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1. Identify which of the following expressions are not Maxwell's equations for time varying fields:

   (a) $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho_v}{\partial t} = 0$

   (b) $\vec{\nabla} \cdot \vec{D} = \rho_v$

   (c) $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \vec{B}}{\partial t}$

   (d) $\oint \vec{H}.\mathbf{dl} = \int (\sigma E + \epsilon \frac{\partial \vec{E}}{\partial t}).\mathbf{dS}$

2. Which of the following statements is not true of a phasor?

   (a) It may be a scalar or a vector.
   (b) It is a time-dependent quantity.
   (c) A phasor $\bar{V}_s$ may be represented as $V_0\angle\theta$ or $V_0 e^{j\theta}$ where $V_0 = |\bar{V}|$
   (d) It is a complex quantity.

3. In a certain region with $\sigma = 0$, $\mu = \mu_0$ and $\epsilon = 6.25\ \epsilon_0$, the magnetic field of an EM wave is

$$\vec{H} = 0.6\ cos(\beta x)\ cos(2\pi 10^9 t)\ \hat{a}_z\ A/m$$

   (a) Find β and the corresponding $\vec{E}$ using Maxwell's equations.
   (b) Is it a standing wave or travelling wave?

4. Show that in a source-free region ($\vec{J} = 0,\ \rho_v = 0$), Maxwell's equations can be reduced to two. Identify the two all-embracing differential equations. What would be the possible solutions?

5. An electromagnetic wave

$$\mathbf{E} = \frac{\sqrt{3}}{2} E_0 [\frac{1}{\sqrt{3}} \hat{x}\ cos(3\pi \times 10^9 t - 20\pi z) + \hat{y}\ cos(3\pi \times 10^9 t - 20\pi z)] \quad (1)$$

   (a) what is the frequency of the wave ?
   (b) what is it's wavelength ?
   (c) what is the refractive index of the medium ?
   (d) what is the polarization of the wave ?

6. A medium is characterized by $\sigma = 0$, $\mu = 2\mu_0$ and $\epsilon = 5\epsilon_0$. If $\mathbf{H} = 2\ cos(\omega t - 3y)\mathbf{a}_z$ A/m, calculate ω and $\mathbf{E}$.

7. A conductor with cross-sectional area of 10 cm^2 carries a conduction current $0.2 sin 10^9 t\ mA$. Given that $\sigma = 2.5 \times 10^6\ S/m$ and $\epsilon_r = 6$.

   (a) Calculate the magnitude of the displacement current density.
   (b) What would be the phase difference between conduction current and displacement current?
   (c) Write down the expression for instantaneous total current in the conductor.

8. A parallel plate capacitor with plate area of 6 cm^2 and plate separation of 4 mm has a voltage 100 $sin 10^3 t\ V$ applied to its plates. Calculate the displacement current assuming $\epsilon = 4\epsilon_0$

9. The magnetic field associated with an EM wave in a charge-free region for which $\sigma = 0$, $\epsilon = \epsilon_0\epsilon_r$, and $\mu = \mu_0$ is given by $\vec{H} = 5\ cos(10^{11} t - 4y)\ \hat{a}_z\ A/m$. Find: (a) $\vec{J}_d$ and $\vec{D}$, (b) ϵ_r

10. The ratio of conduction current density to displacement current density is very important at high frequencies - Why? Calculate the ratio at 1 GHz for:

(a) distilled water ($\mu = \mu_0$, $\epsilon_r = 81$, $\sigma = 2$ X 10^{-3} S/m)

(b) sea water ($\mu = \mu_0$, $\epsilon_r = 81$, $\sigma = 25$ S/m)

(c) limestone ($\mu = \mu_0$, $\epsilon_r = 5$, $\sigma = 2$ X 10^{-4} S/m)

11. A slab of perfect dielectric material ($\epsilon_r = 2$) is placed inside a microwave oven.The oven produces an electric field (as well as a magnetic field) Assume that the electric field intensity is uniform in the slab and sinusoidal in form, and that it is perpendicular to the surface of the slab.The oven operates at a frequency of 2.45 GHz and produces an electric field intensity with amplitude 500 V/m inside the dielectric.

    (a) Calculate the displacement current density in the dielectric.

    (b) Is there a displacement current in air? If so, what is the displacement current density in air?

12. In a region of free space, an EM field has an electric field given by

$$\mathbf{E} = E_0 sin(kx) sin(\omega t)\hat{y}$$

    (a) Find the magnetic field.

    (b) Find the phase difference between two fields.

    (c) Is it a travelling wave? Give reasons for your answer.

    ~~(d)~~ Find the energy density of the EM wave in that region.

    ~~(e)~~ If the above wave is confined in the region 0≤x≤L, 0≤y≤w and 0≤z≤d such that sin(kL) = 0, show that the energy density will remain constant over time.

13. What values of A and β are required if the two fields given below satisfy Maxwell's equations in a linear, isotropic, homogeneous medium with $\epsilon_r = \mu_r = 4$ and $\sigma = 0$?

$$\vec{E} = 120\pi\ cos(10^6\pi t - \beta x)\hat{a_y}\ V/m \qquad \vec{H} = A\pi\ cos(10^6\pi t - \beta x)\hat{a_z}\ A/m$$

Assume there are no current or charge densities in space.

14. A plane wave in free space with $\vec{E} = 3.6\ cos(\omega t - 5x)\hat{a_y}\ V/m$ is incident normally on an interface at $x = 0$. If a loss-less medium with $\sigma = 0$, $\epsilon_r = 12.5$ exists for $x \geq 0$ and the reflected wave has $\vec{H_r} = 12.5\ cos(\omega t + 3x)\hat{a_z}\ A/m$. Find μ_2.

~~15.~~ Consider the standard equation for the plane wave. Now write the magnitude of the wave vector k as $k = k_1 + ik_2$ where k_1 and k_2 are real quantities. Substitute this in the wave equation expression and interpret your result in terms of the nature of wave propagation.

16. Consider two concentric spherical shells made of metal.The inner one is of radius 'a' and carries a charge Q(t). The outer one is of radius 'b' and carries an opposite charge of -Q(t). The space between them is filled with ohmic material of conductivity σ. Find the magnetic field inside the space between the shells.

17. A uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz and its magnitude is reduced by 60 percentage for every 2m traveled. Find the skin depth and the speed of the wave.

18. In a good conductor, show that the skin depth is always shorter than the wavelength. An aluminum pipe of 40 m length having inner and outer radii of 9 mm and 12 mm respectively carries a total current of 6 $sin(10^6\pi t)$. Given that $\sigma = 3.5 \times 10^7\ S/m$, $\mu_r = 1, \epsilon_r = 1$. Find the skin depth and the effective resistance of the pipe.

~~19.~~ At some location inside a lossy dielectric material the measured peak electric field of a wave is 10 V/m. The material has relative permittivity of 16 and conductivity of 200 S/m. Find the average power density of the wave at that location. Also find the power density at a distance of 1 cm in the direction of the wave propagation. The frequency of the wave is 300 MHz.

20. Region 1 in a loss-less medium for which y≥0, $\mu = \mu_0$, $\epsilon = 4\epsilon_0$ whereas region 2 is free space, y≤0. If a plane wave $\mathbf{E} = 5$ cos $(10^8$t + βy) $\mathbf{a}_z$ exists in region 1, find:

    (a) the total electric field component of the wave in region 2.

    (b) the time-average Poynting vector in region 1.

    (c) the time-average Poynting vector in region 2.