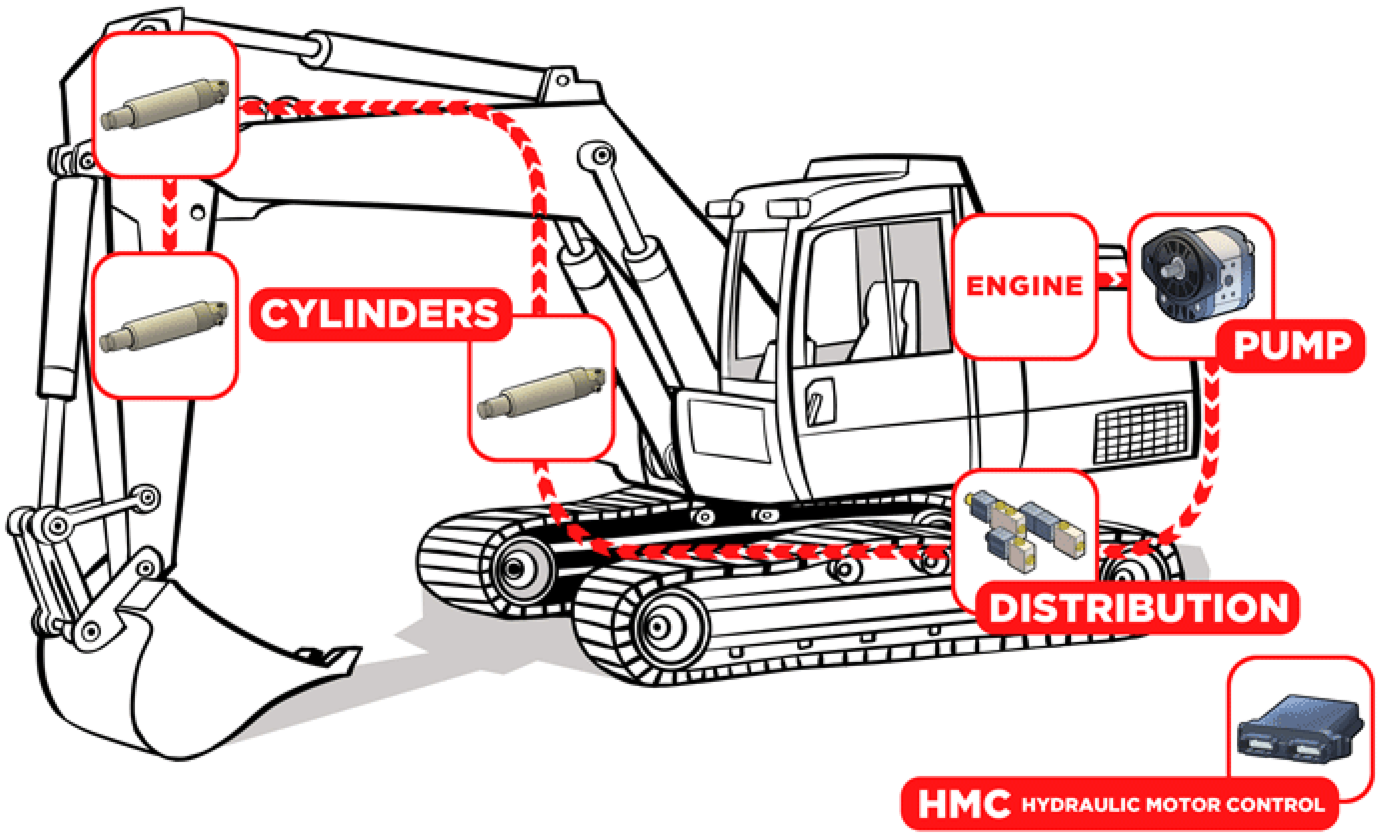
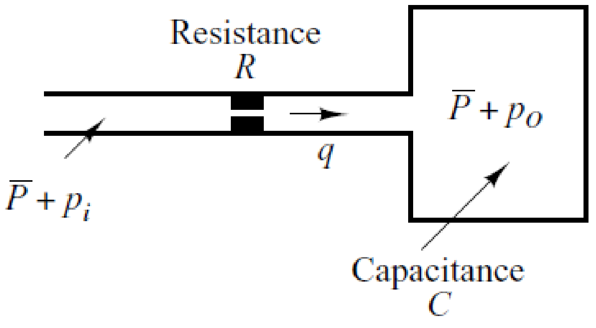
Modelling Pneumatic and Hydraulic Control Systems

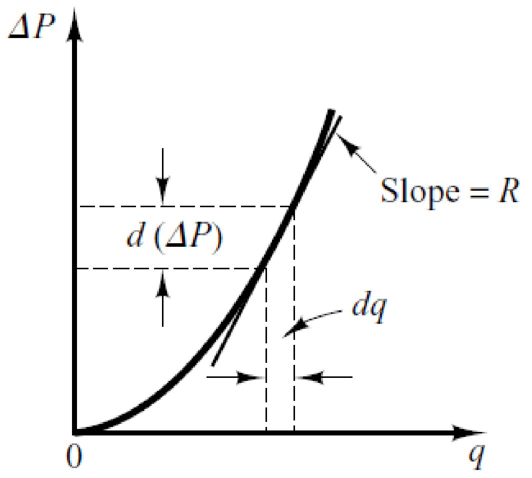


Pneumatic Systems



(a)

- Consider the pressure system shown in Figure (a), The gas flow q through the restriction R is a function of the gas pressure difference $\Delta P = p_i - p_0$, Such a pressure system may be characterized in terms of a resistance R and a capacitance C.
- The gas flow resistance R and Capacitance C may be defined as follows:



(b)

$$R = \frac{\text{change in gas pressure difference, lb}_f/\text{ft}^2}{\text{change in gas flow rate, lb/sec}}$$

$$R = \frac{d(\Delta P)}{dq} \qquad (4\text{–}8)$$

$$C = \frac{\text{change in gas stored, lb}}{\text{change in gas pressure, lb}_f/\text{ft}^2}$$

General Expression

$$C = \frac{dm}{dp} = V\frac{d\rho}{dp} \qquad (4\text{–}9)$$

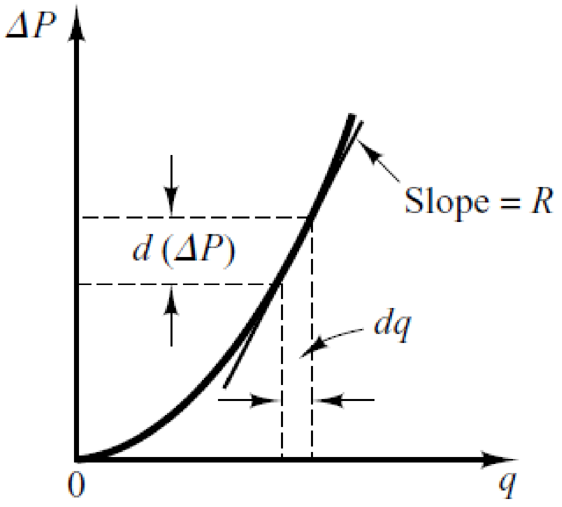
where C = capacitance, lb-ft^2/lb$_f$
m = mass of gas in vessel, lb
p = gas pressure, lb$_f$/ft^2
V = volume of vessel, ft^3
ρ = density, lb/ft^3

Electrical Circuit Analogy

Pressure -----> Voltage
Flow -----> Current
Mass -----> Charge

Pneumatic Systems

- Consider the system shown in Figure (a). If we assume only small deviations in the variables from their respective steady-state values, then this system may be considered linear.



$\bar{P}$ = gas pressure in the vessel at steady state (before changes in pressure have occurred), lb_f/ft^2

p_i = small change in inflow gas pressure, lb_f/ft^2

p_o = small change in gas pressure in the vessel, lb_f/ft^2

V = volume of the vessel, ft^3

m = mass of gas in the vessel, lb

q = gas flow rate, lb/sec

ρ = density of gas, lb/ft^3

For small values of p_i and p_o, the resistance R given by Equation (4–8) becomes constant and may be written as

$$R = \frac{p_i - p_o}{q}$$

The capacitance C is given by Equation (4–9), or

$$C = \frac{dm}{dp}$$

Electrical Circuit Analogy

Pressure -----> Voltage
Flow -----> Current
Mass -----> Charge

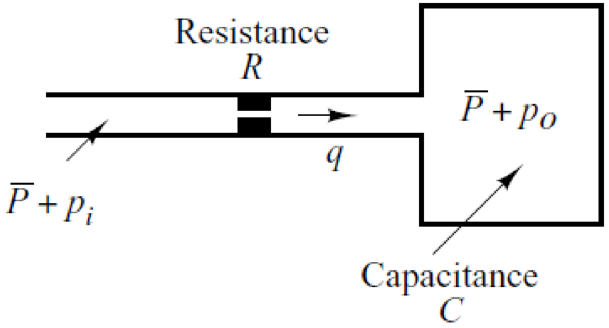
Pneumatic Systems

Since the pressure change dp_o times the capacitance C is equal to the gas added to the vessel during dt seconds, we obtain

$$C\,dp_o = q\,dt$$

or

$$C\frac{dp_o}{dt} = \frac{p_i - p_o}{R}$$



which can be written as

$$RC\frac{dp_o}{dt} + p_o = p_i$$

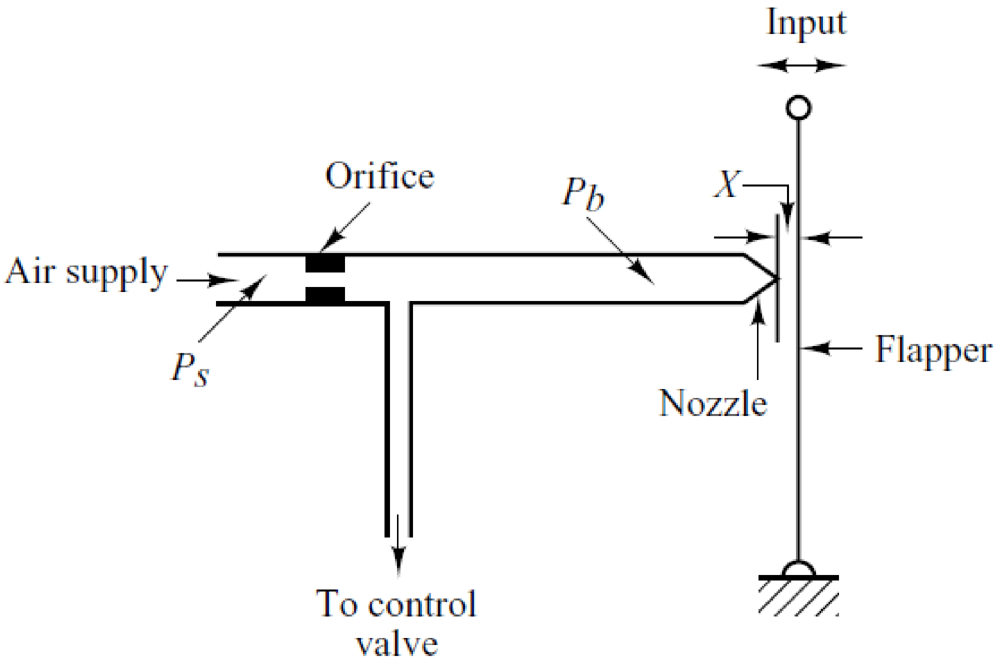
If p_i and p_o are considered the input and output, respectively, then the transfer function of the system is

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1}$$

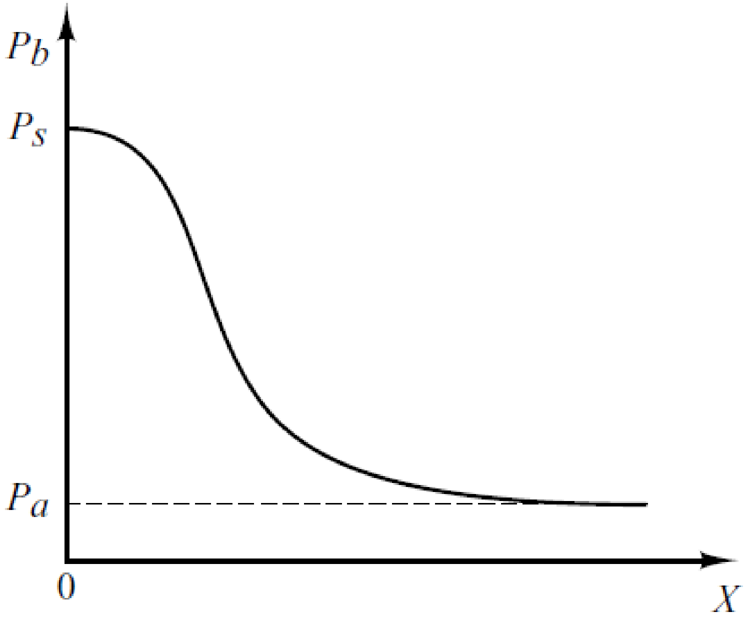
where RC has the dimension of time and is the time constant of the system.

Pneumatic Amplifiers: Nozzle Flapper

- In a Pneumatic control system, the nozzle-flapper amplifier acts as the first stage amplifier and the Pneumatic relay as the second stage amplifier.
- A nozzle-flapper amplifier converts small changes of the flapper into large changes in back pressure in the nozzle. Therefore, a large power output can be controlled by a very little power that is needed to position the nozzle.



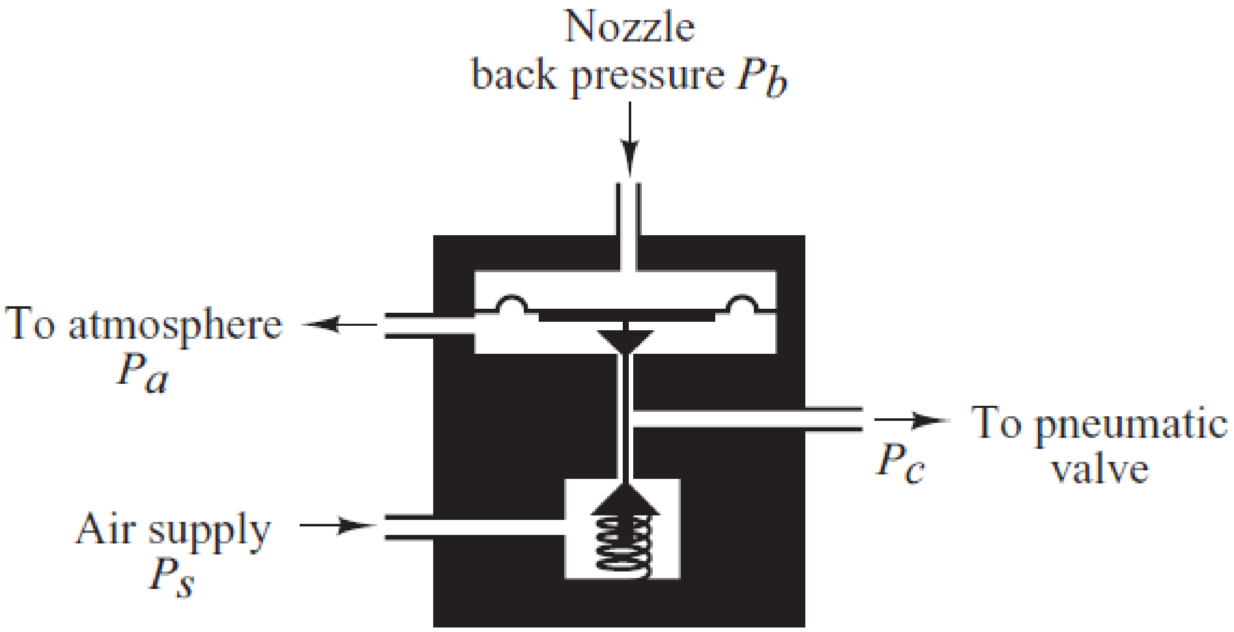
(a)



(b)

First Stage of Amplification

Pneumatic Amplifiers: Pneumatic Relay

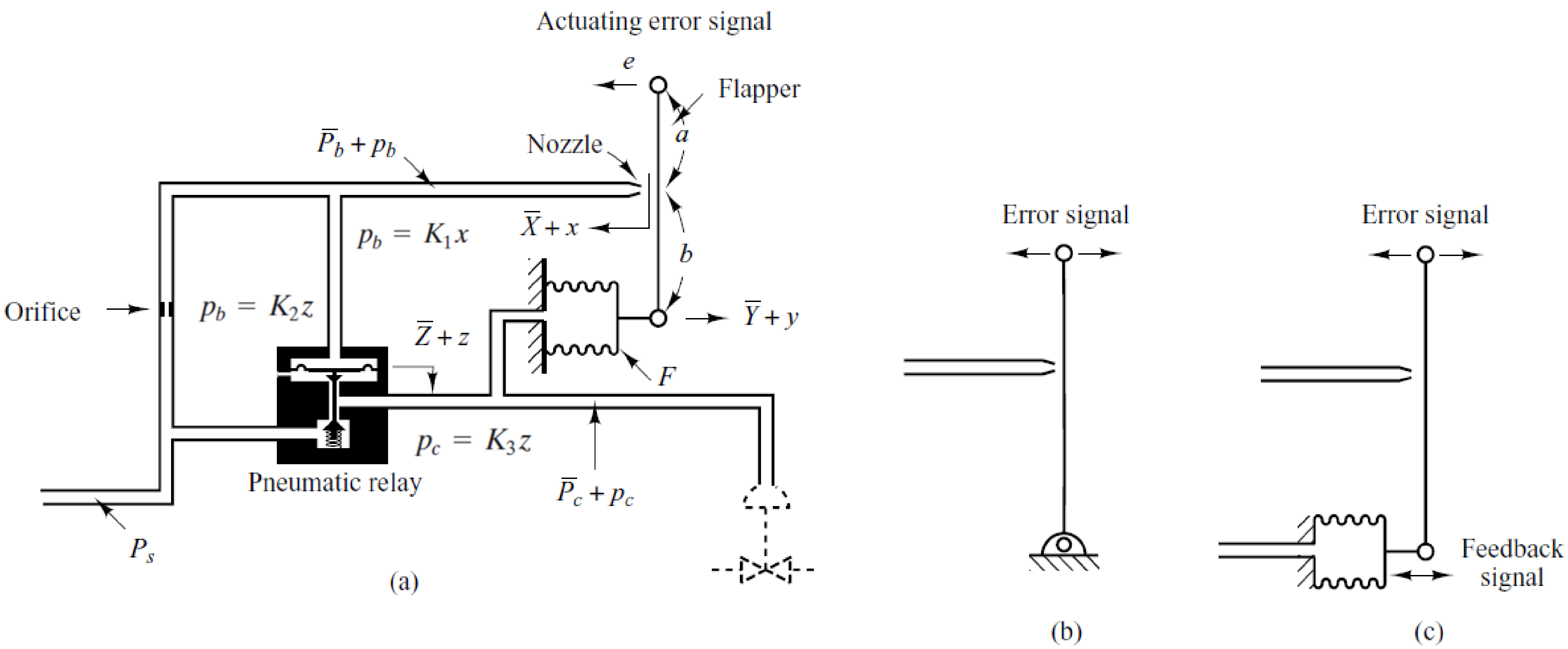


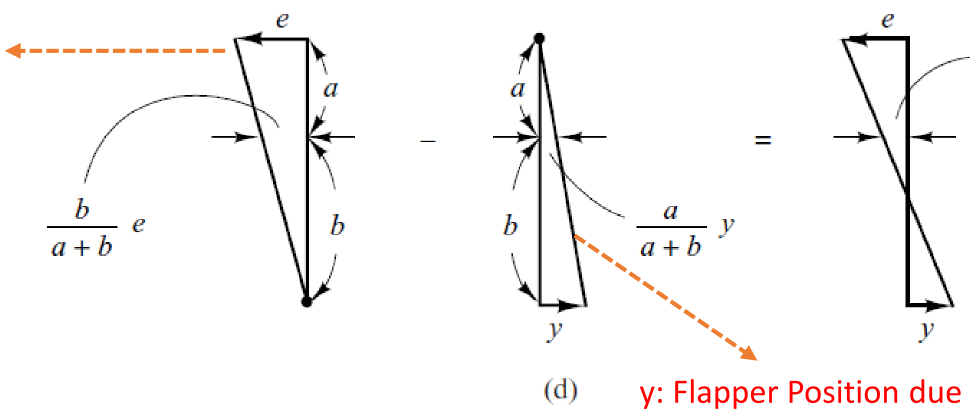
(a)

Second Stage of Amplification:
Used after the Nozzle-flapper output

- As the nozzle back pressure P_b increases, the diaphragm valve moves downwards. Then the opening to atmosphere decreases and the opening to the pneumatic valve increases and vice versa.
- The control pressure P_c can thus be made to vary between 0 to full supply pressure P_s.

Pneumatic Proportional Controllers





e: Flapper Position due to error Signal alone

x: Effective Flapper Position

y: Flapper Position due to Feedback signal alone

“In pneumatic controllers, some type of pneumatic feedback is employed. Feedback of the pneumatic output reduces the amount of actual movement of the flapper. Instead of mounting the flapper on a fixed point, as shown in Figure (b), it is often pivoted on the feedback bellows, as shown in Figure (c).The amount of feedback can be regulated by introducing a variable linkage between the feedback bellows and the flapper connecting point. The flapper then becomes a floating link. It can be moved by both the error signal and the feedback signal”

Steady State or Equilibrium values are:

$\bar{X}$

$\bar{Y}$

$\bar{Z}$

$\bar{P}_c$

$\bar{P}_b$

Pneumatic Proportional Controllers

Assuming that the relationship between the variation in the nozzle back pressure and the variation in the nozzle–flapper distance is linear, we have

$$p_b = K_1 x \tag{4–13}$$

where K_1 is a positive constant. For the diaphragm valve,

$$p_b = K_2 z \tag{4–14}$$

where K_2 is a positive constant. The position of the diaphragm valve determines the control pressure. If the diaphragm valve is such that the relationship between p_c and z is linear, then

$$p_c = K_3 z \tag{4–15}$$

where K_3 is a positive constant. From Equations (4–13), (4–14), and (4–15), we obtain

$$p_c = \frac{K_3}{K_2} p_b = \frac{K_1 K_3}{K_2} x = K x \tag{4–16}$$

where $K = K_1 K_3 / K_2$ is a positive constant. For the flapper, since there are two small movements (e and y) in opposite directions, we can consider such movements separately and add up the results of two movements into one displacement x. See Figure(d).

Thus, for the flapper movement, we have

$$x = \frac{b}{a+b}e - \frac{a}{a+b}y \tag{4–17}$$

The bellows acts like a spring, and the following equation holds true:

$$Ap_c = k_s y \tag{4–18}$$

Pneumatic Proportional Controllers

where A is the effective area of the bellows and k_s is the equivalent spring constant—that is, the stiffness due to the action of the corrugated side of the bellows.

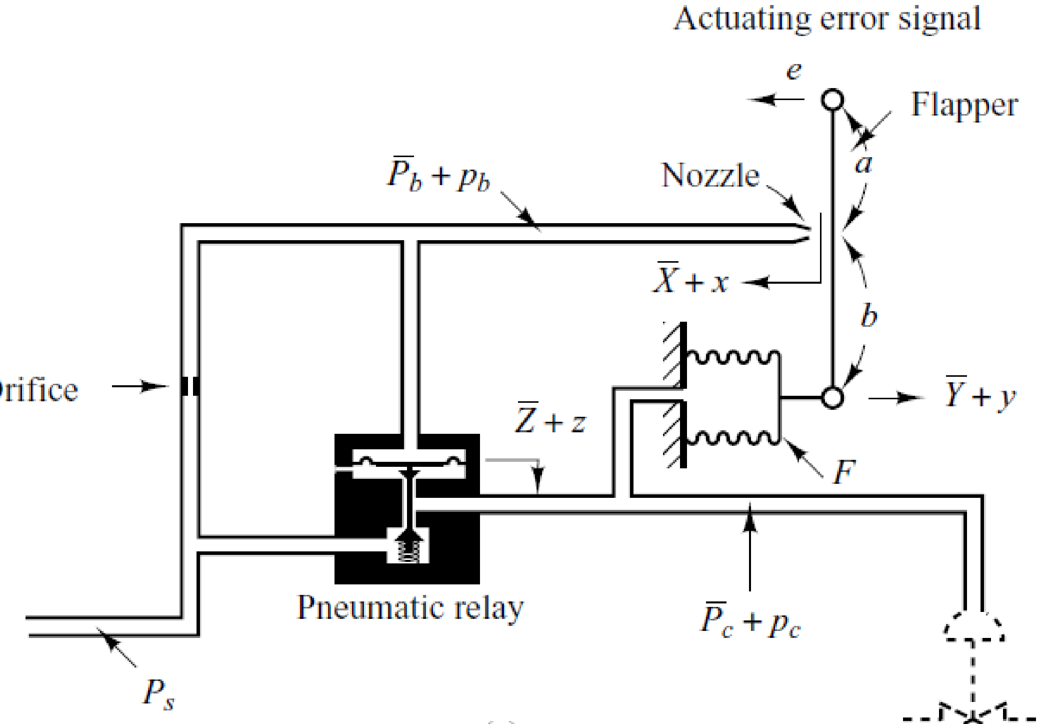
Assuming that all variations in the variables are within a linear range, we can obtain a block diagram for this system from Equations (4–16), (4–17), and (4–18) as shown in Figure 4–8(e). From Figure 4–8(e), it can be clearly seen that the pneumatic controller shown in Figure 4–8(a) itself is a feedback system. The transfer function between p_c and e is given by

Derive from block diagram Next page

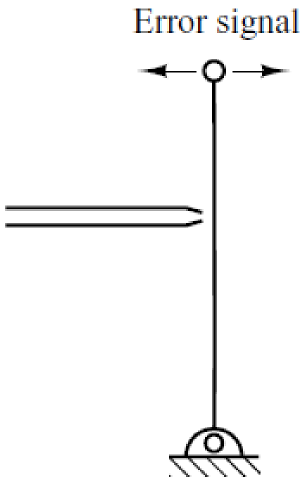
$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b}K}{1 + K\frac{a}{a+b}\frac{A}{k_s}} = K_p \tag{4–19}$$

A simplified block diagram is shown in Figure (f). Since p_c and e are proportional, the pneumatic controller shown in Figure (a) is a *pneumatic proportional controller*.

Pneumatic Proportional Controllers



(a)



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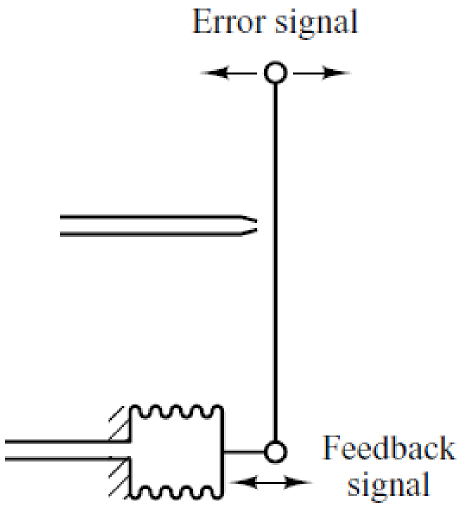
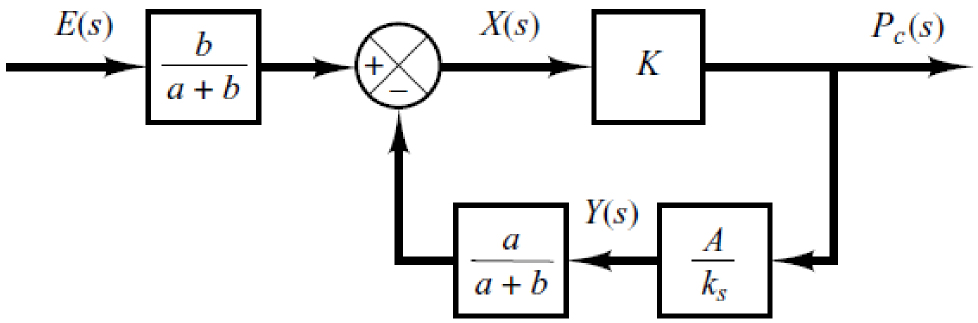


(c)



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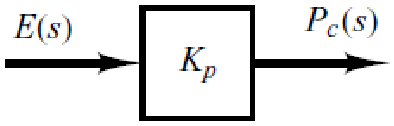
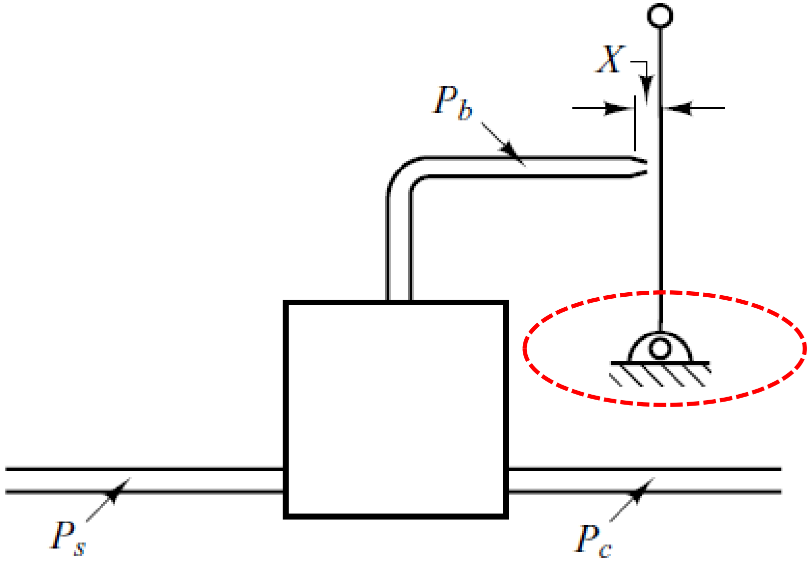
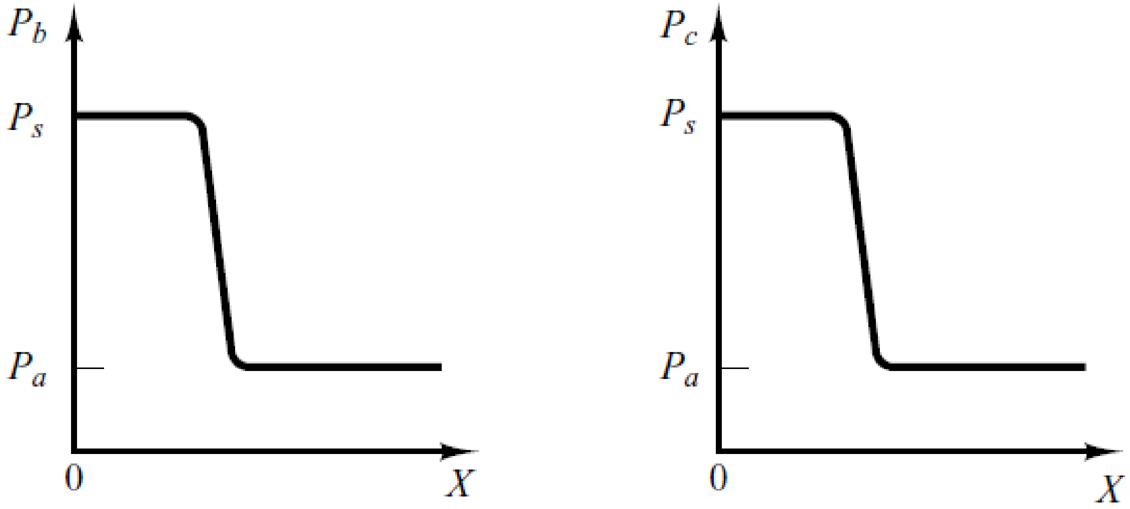


(f)

Pneumatic On-off Controller



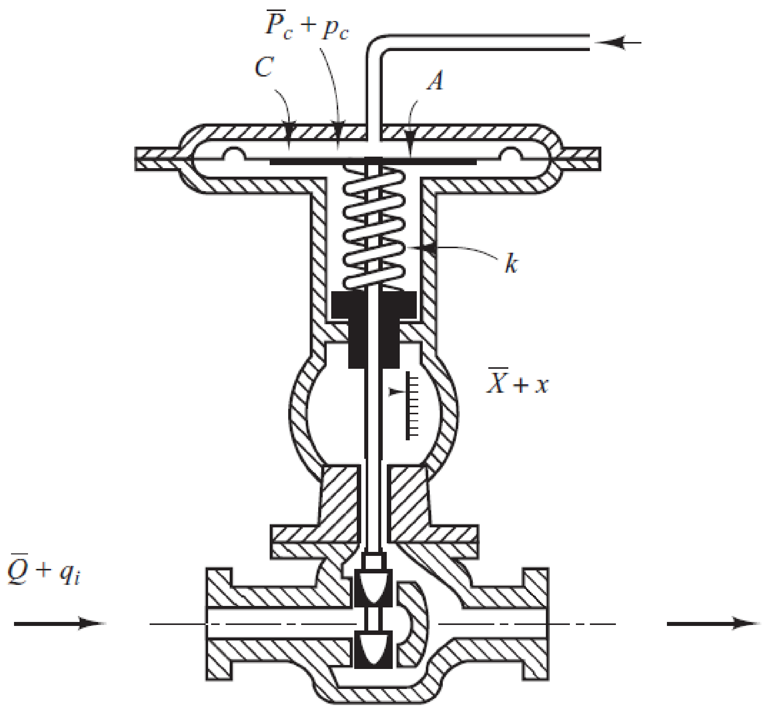
(a)



(b)

Pneumatic controllers that do not have feedback mechanisms [which means that one end of the flapper is fixed, as shown in Figure (a) have high sensitivity and are called *pneumatic two-position controllers* or *pneumatic on–off controllers*.

Pneumatic (Proportional) Actuating Valve



Consider the schematic diagram of a pneumatic actuating valve shown in Figure. Assume that the area of the diaphragm is A. Assume also that when the actuating error is zero, the control pressure is equal to $\bar{P}_c$ and the valve displacement is equal to $\bar{X}$.

Let us define the small variation in the control pressure and the corresponding valve displacement to be p_c and x, respectively. Since a small change in the pneumatic pressure force applied to the diaphragm repositions the load, consisting of the spring, viscous friction, and mass, the force-balance equation becomes

$$Ap_c = m\ddot{x} + b\dot{x} + kx$$

where m = mass of the valve and valve stem

b = viscous-friction coefficient

k = spring constant

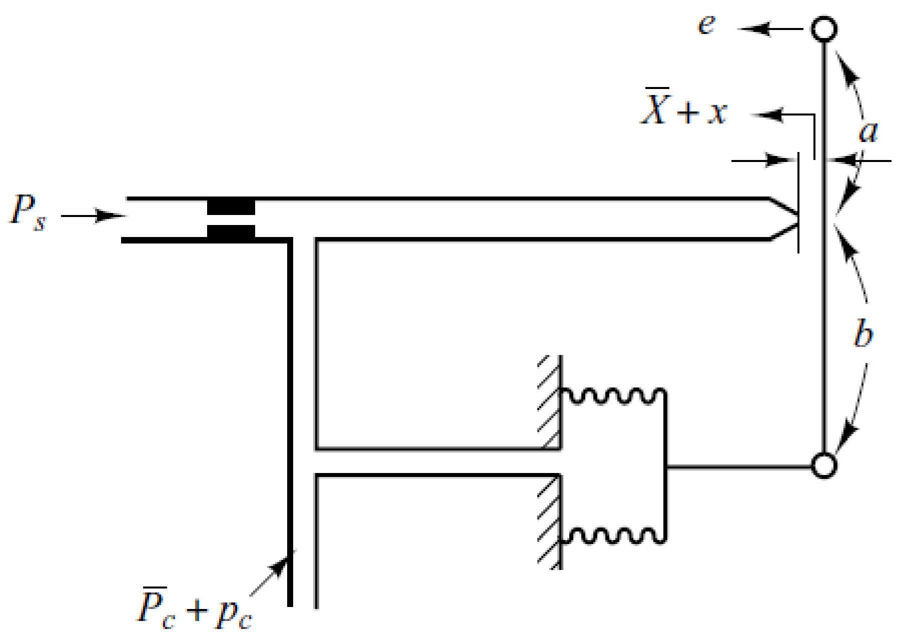
If the force due to the mass and viscous friction are negligibly small, then this last equation can be simplified to

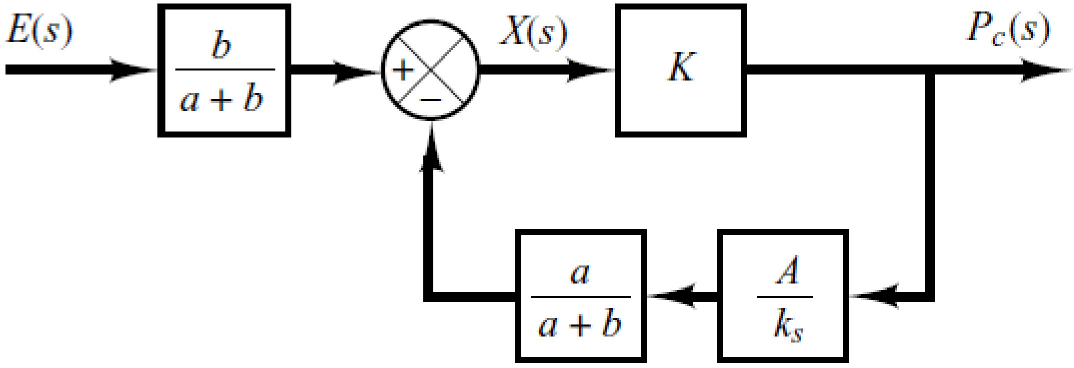
$$Ap_c = kx$$

The transfer function between x and p_c thus becomes

$$\frac{X(s)}{P_c(s)} = \frac{A}{k} = K_c$$

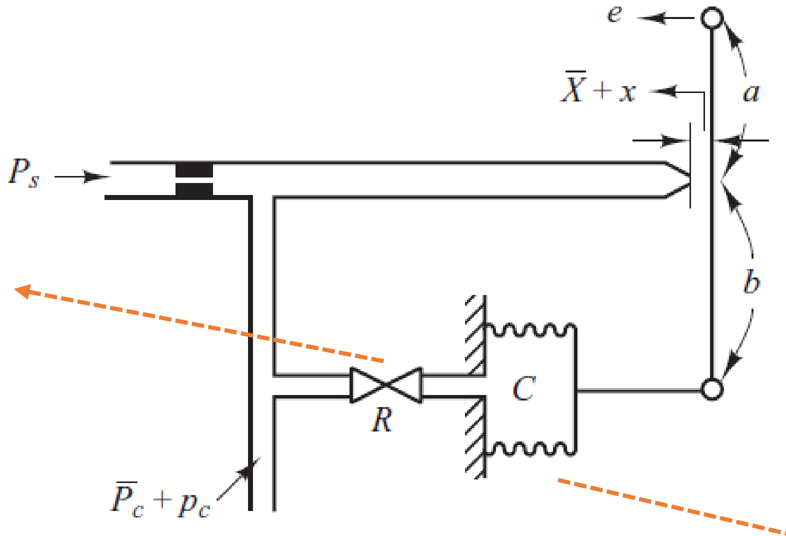
Pneumatic PD Action: Proportional Component



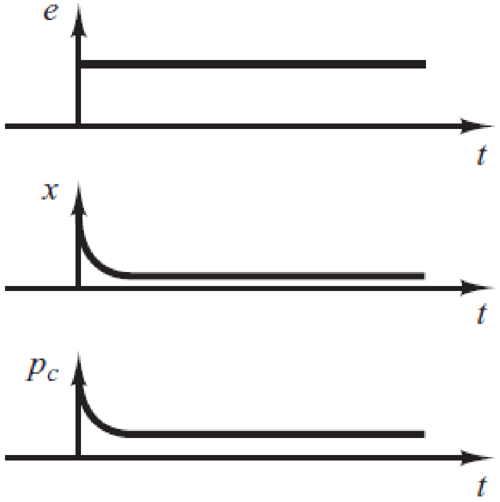


Pneumatic Proportional + Derivative (PD) Action

This restriction will not allow feedback instantaneously, hence the initial control pressure will be Large due to step change in the error signal



(a)



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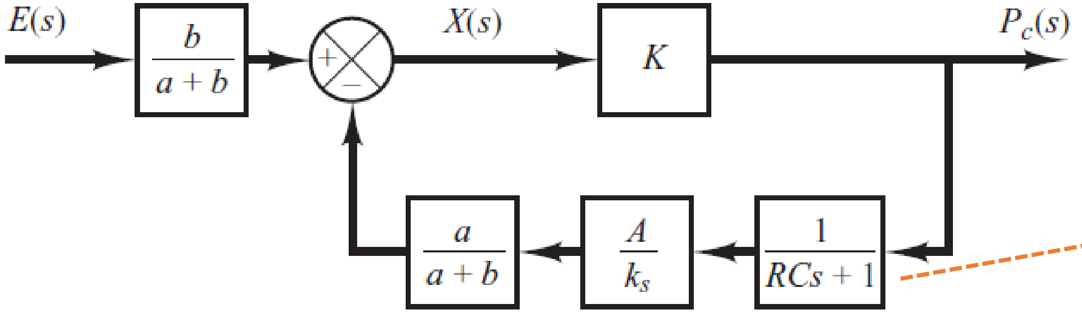

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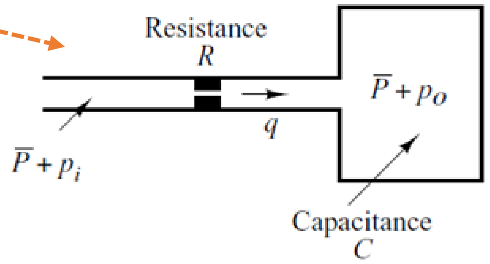
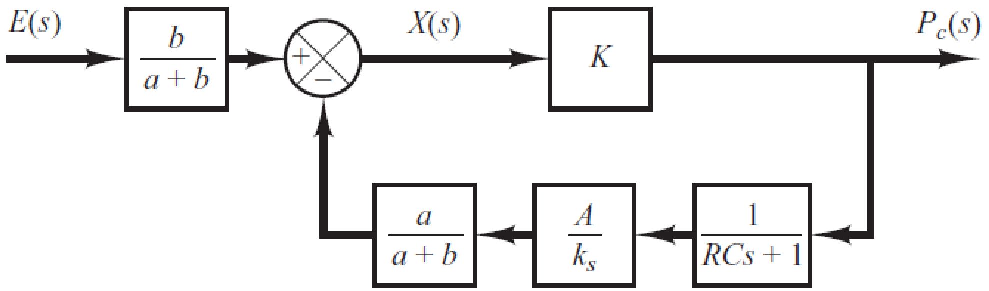


$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1}$$

Pneumatic Proportional + Derivative (PD) Action



$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b}K}{1 + \frac{Ka}{a+b}\frac{A}{k_s}\frac{1}{RCs+1}}$$

In such a controller the loop gain $|KaA/[(a+b)k_s(RCs+1)]|$ is made much greater than unity. Thus the transfer function $P_c(s)/E(s)$ can be simplified to give

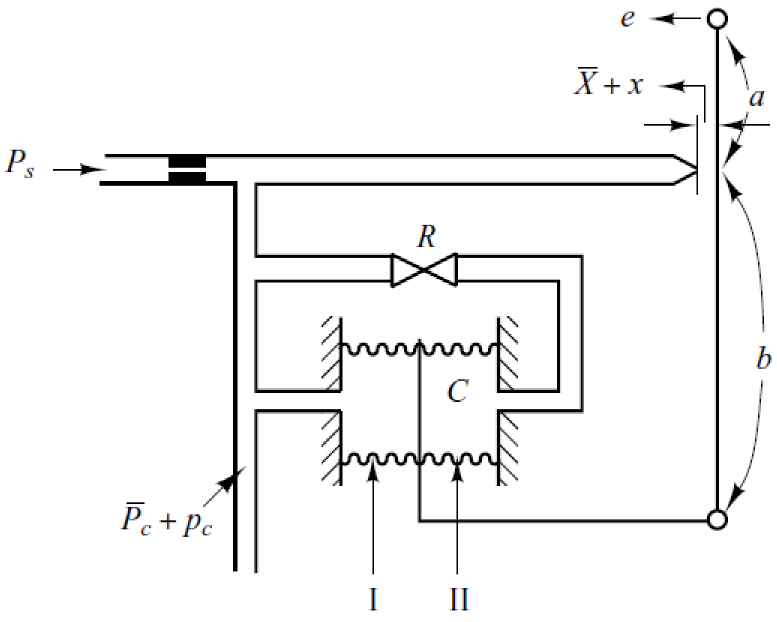
$$\frac{P_c(s)}{E(s)} = K_p(1 + T_d s)$$

where

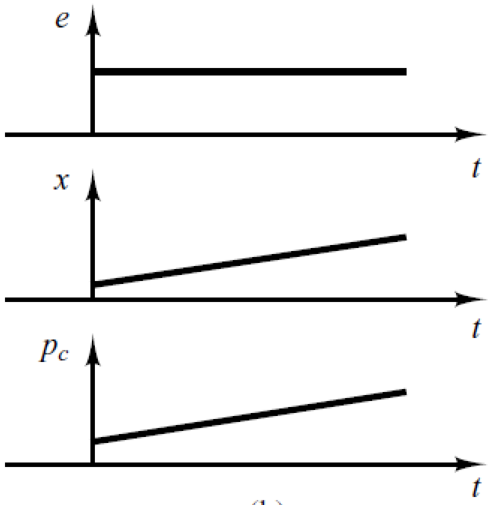
$$K_p = \frac{bk_s}{aA}, \qquad T_d = RC$$

Thus, delayed negative feedback, or the transfer function $1/(RCs+1)$ in the feedback path, modifies the proportional controller to a proportional-plus-derivative controller.

Pneumatic Proportional + Integral (PI) Action



(a)



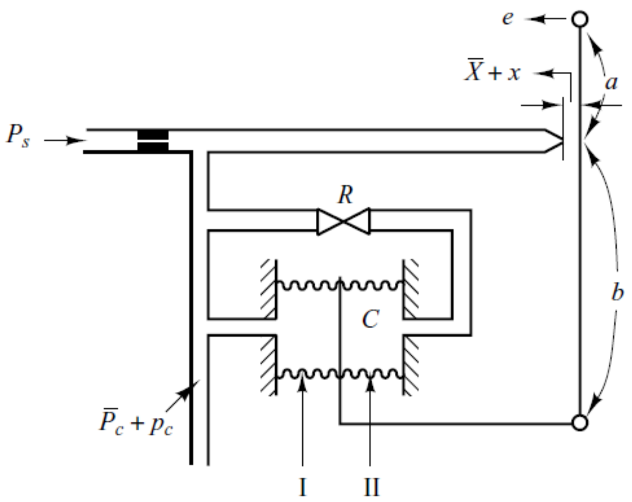
(b)

- Considering small changes in the variables, addition of delayed positive feedback will modify a proportional controller to a proportional-plus-integral controller, or a PI controller.

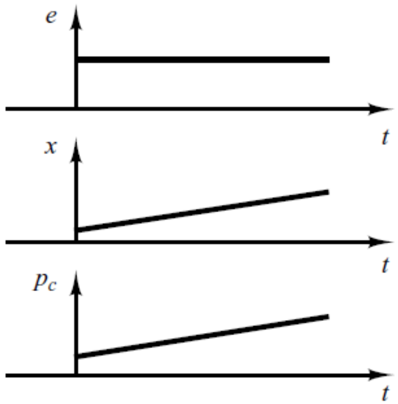
- Consider the pneumatic controller shown in Figure (a). The operation of this controller is as follows: The bellows denoted by I is connected to the control pressure source without any restriction. The bellows denoted by II is connected to the control pressure source through a restriction. Let us assume a small step change in the actuating error. This will cause the back pressure in the nozzle to change instantaneously. Thus a change in the control pressure p_c also occurs instantaneously.

- Due to the restriction of the valve in the path to bellows II, there will be a pressure drop across the valve. As time goes on, air will flow across the valve in such a way that the change in pressure in bellows II attains the value p_c. Thus bellows II will expand or contract as time elapses in such a way as to move the flapper an additional amount in the direction of the original displacement e. This will cause the back pressure p_c in the nozzle to change continuously, as shown in Figure (b).

Pneumatic Proportional + Integral Action



(a)



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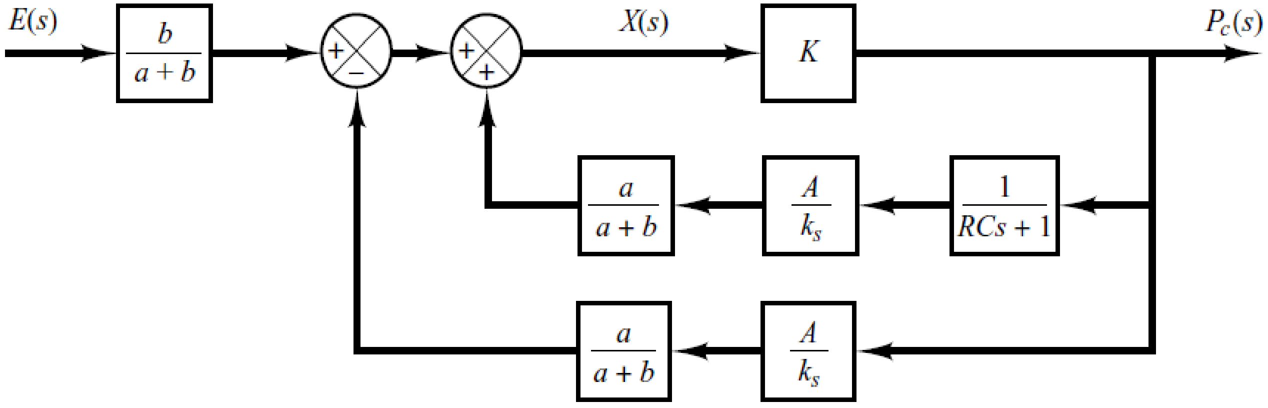

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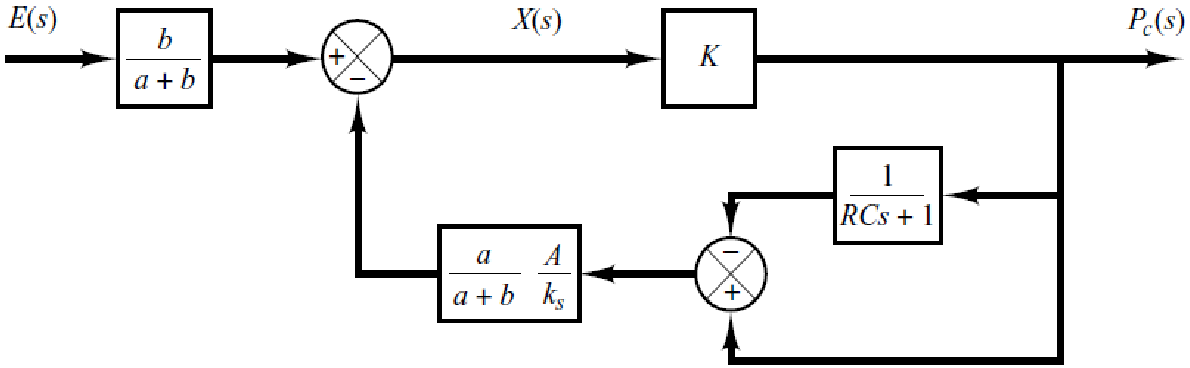
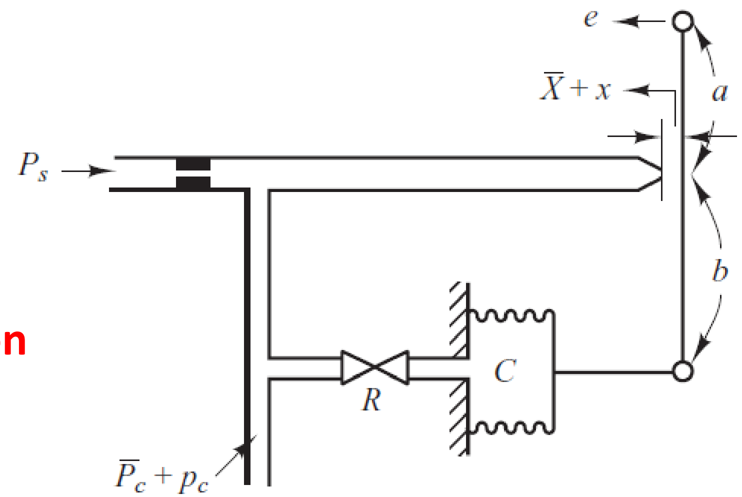


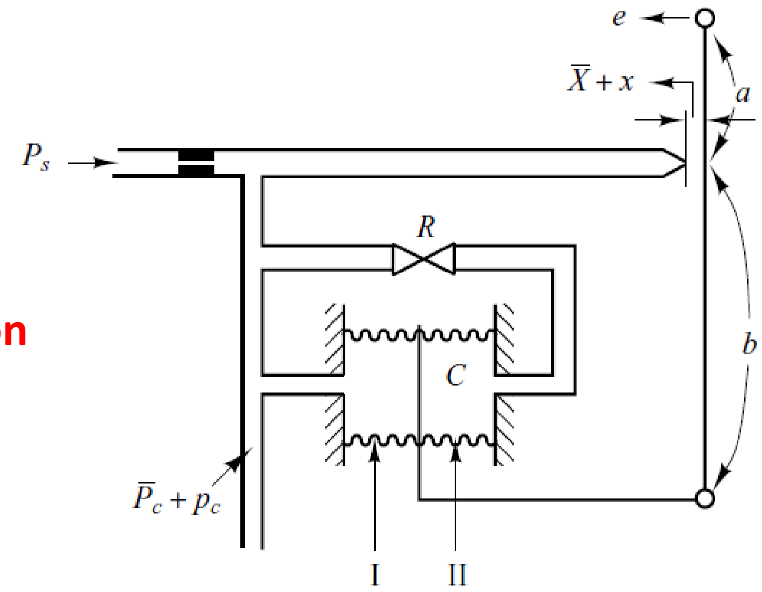
(d)

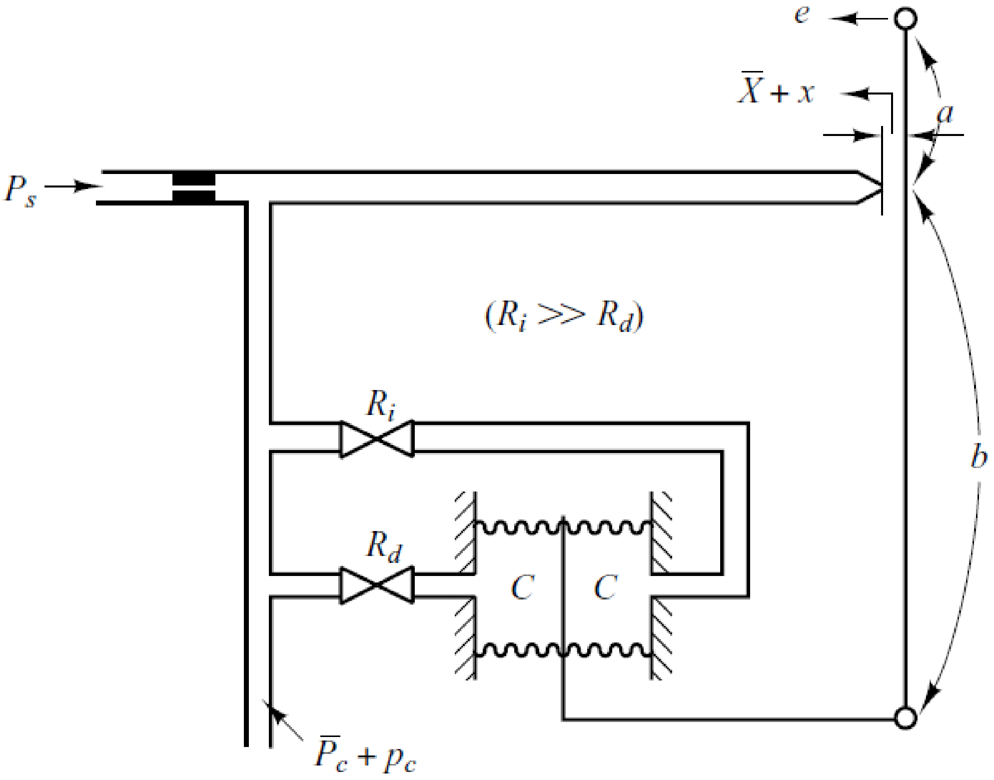
Pneumatic Proportional + Integral + Derivative Action

PD Action



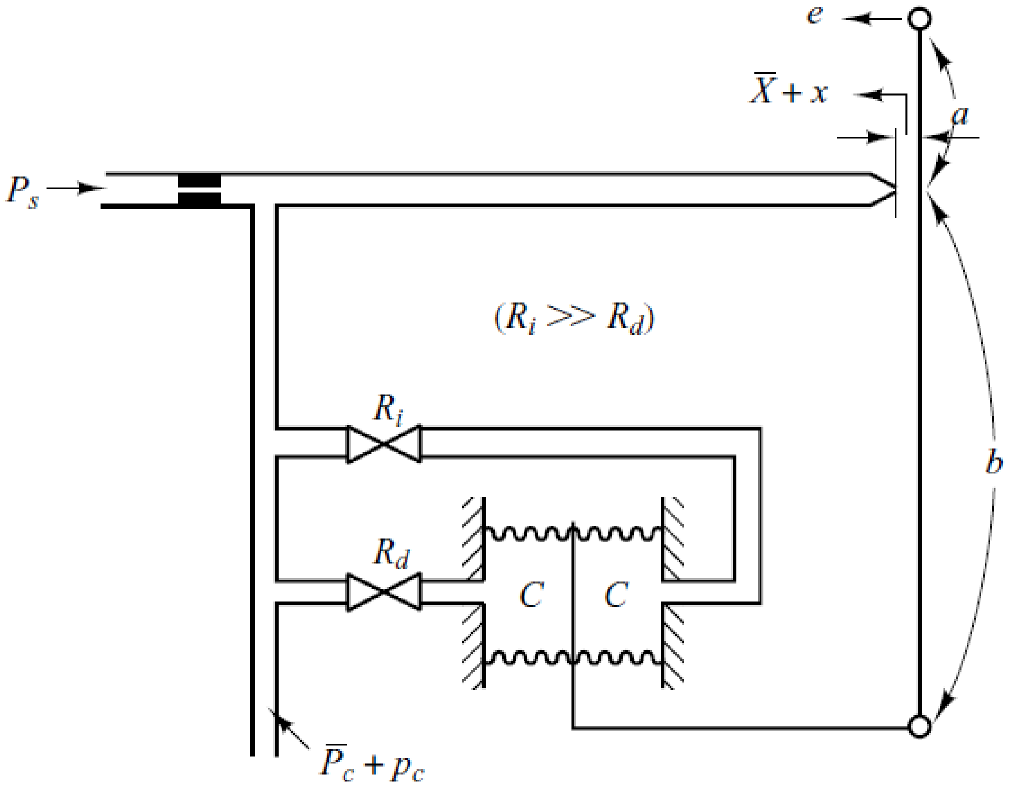
PI Action



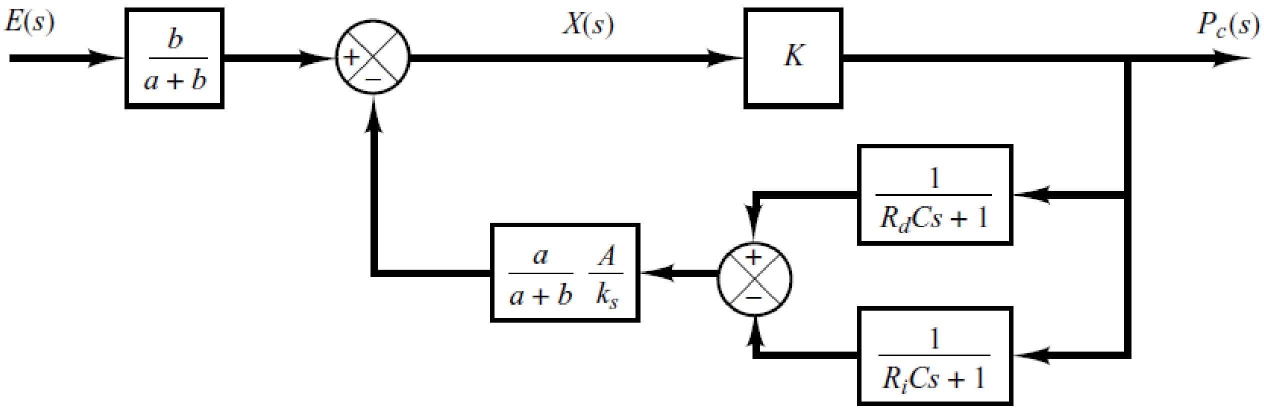


PID Action

Pneumatic Proportional + Integral + Derivative Action



(a)



(b)

Pneumatic PID Action

The transfer function of this controller is

$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{bK}{a+b}}{1 + \dfrac{Ka}{a+b}\dfrac{A}{k_s}\dfrac{(R_iC - R_dC)s}{(R_dCs+1)(R_iCs+1)}}$$

By defining

$$T_i = R_iC, \qquad T_d = R_dC$$

and noting that under normal operation $|KaA(T_i - T_d)s/[(a+b)k_s(T_ds+1)(T_is+1)]| \gg 1$ and $T_i \gg T_d$, we obtain

$$\begin{aligned}\frac{P_c(s)}{E(s)} &\doteqdot \frac{bk_s}{aA}\frac{(T_ds+1)(T_is+1)}{(T_i - T_d)s} \\ &\doteqdot \frac{bk_s}{aA}\frac{T_dT_is^2 + T_is + 1}{T_is} \\ &= K_p\left(1 + \frac{1}{T_is} + T_ds\right) \qquad (4\text{–}24)\end{aligned}$$

where

$$K_p = \frac{bk_s}{aA}$$

THANK YOU