Dept. of EE, IIT Tirupati
EE3001 : Electromagnetic Fields (Aug - Nov 2018)
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Tutorial Quiz - 2

Note: The tutorial quizzes are part of your assignment. This is an effort to test your learning outcome while solving assignment problems independently.

1. Given that $\mathbf{H} = 0.5\, e^{-0.1x}\, \sin(10^6 t - 2x)\, \mathbf{a}_z$ A/m, which of these statements are incorrect (with reasons/calculations):
   (a) $\alpha = 0.1$ Np/m
   ✓(b) $\beta =$ -2 rad/m
   (c) $\omega = 10^6$ rad/s
   (d) The wave travels along $\mathbf{a}_x$
   ✓(e) The period of the wave is 1 μs

2. The magnetic field of a collimated laser light falling on a plane surface is given by

$$\mathbf{H} \;=\; 30\; cos(4\pi \;\times\; 10^{14}t \;-\; 6x)\; \mathbf{a}_y \; mA/m$$

   Find:
   (a) the frequency of the incoming laser light.
   (b) the electric field **E**.
   (c) intrinsic impedance.
   (d) the intensity of the laser light falling on the plane surface .

3. At the upper surface of the earth's atmosphere, the time-averaged magnitude of the Poynting vector referred to as solar constant is S = 1.35×10^3 W/m^2.
   (a) Assuming that the Sun's electromagnetic radiation is a plane sinusoidal wave, what are the magnitudes of the electric and magnetic fields ?
   (b) What is the total time-averaged power radiated by the Sun ? The mean Sun to Earth distance is R = 1.5 $\times\, 10^{11}$ m.

4. Given that the skin depth for graphite at 100 MHz is 0.16 mm, determine
   (a) the conductivity of graphite (consider graphite as a non-magnetic material).
   (b) the distance that a 1 GHz wave travels in graphite such that its field intensity is reduced by 30 dB.

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$\vec{H} = 0.5\, e^{-0.1x} \sin(10^6 t - 2x)\, a_z$ A/m.

$\Rightarrow \omega = 10^6$ rad/s

$\beta = 2$ rad/m.

$\alpha = 0.1$ Np/m.

direction of propagation = a_x.

Period $= \dfrac{1}{f} = \dfrac{2\pi}{10^6} = 2\pi \times 10^{-6}$ s

$= 2\pi\,\mu s$.

So, b and e are incorrect.

2) a) $\omega t = 4\pi \times 10^{14} t$

$\vec{H} = 30\cos(4\pi \times 10^{14} t - 6x) a_y$ mA/m

$\omega = 4\pi \times 10^{14}$

$2\pi f = 4\pi \times 10^{14}$

$f = \dfrac{4\pi}{2\pi} \times 10^{14}$

$= 2 \times 10^{14}$ Hz.

$\boxed{f = 200 \text{ THz}}$

b) $\epsilon \dfrac{\partial \vec{E}}{\partial t} = \nabla \times \vec{H}$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 0 & 30\cos(4\pi \times 10^{14} t - 6x) & 0 \end{vmatrix}$$

$$= + \hat{a}_z \left[180 \sin(\omega t - 6x) \right]$$

$$\vec{E} = \frac{+180}{\epsilon} \hat{a}_z \int \sin(\omega t - 6x)\, dt$$

$$= \frac{-180}{\epsilon \omega} \cos(\omega t - 6x)\, \hat{a}_z$$

c) $\eta_{air} = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$\mu = \mu_0$

$\epsilon = \epsilon_0$.

$\sigma = 0$.

$= \sqrt{\dfrac{\mu_0}{\epsilon_0}}$

$= 120\pi \ \Omega$.

~~d) Intensity~~. But $v = \dfrac{\omega}{\beta} = \dfrac{4\pi \times 10^{14}}{6} = \dfrac{c}{n}$.

$\Rightarrow n = \dfrac{c}{0.67\pi \times 10^{14}}$

$\eta = \sqrt{\dfrac{\cancel{\mu}}{\epsilon}} = \sqrt{\dfrac{\mu_0}{\epsilon_0 \epsilon_r}} = \dfrac{\eta_0}{\sqrt{\epsilon_r}} = \dfrac{\eta_0}{n}$

d) Intensity of the Laser light falling on the plane surface : $\dfrac{1}{2} E_0 H_0$.

$= \dfrac{1}{2} \times 30 \times \dfrac{180}{\omega\epsilon}$ $[W/m^2]$

Substitute ϵ from value of 'n'.

3) The time-averaged Poynting vector is related to the amplitude of the electric field by

$$\langle S \rangle = \frac{c}{2} \epsilon_0 E_0^2$$

Thus, the amplitude of the electric field is

$$E_0 = \sqrt{\frac{2\langle S \rangle}{c\epsilon_0}} = \sqrt{\frac{2 \times (1.35 \times 10^3 \text{ W/m}^2)}{(3 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N-m}^2)}}$$

$$= 1.01 \times 10^3 \text{ V/m}$$

The corresponding amplitude of the magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}}$$

$$= 3.4 \times 10^{-6} \text{ T}$$

Note that the associated magnetic field is less than $1/10^{th}$ of the Earth's magnetic field.

b) The total time averaged power radiated by the Sun at the distance R is

$$\langle P \rangle = \langle S \rangle A = \langle S \rangle 4\pi R^2$$

$$= (1.35 \times 10^3 \text{ W/m}^2)\, 4\pi \times (1.5 \times 10^{11} \text{ m})^2$$

$$= 3.8 \times 10^{26} \text{ W}.$$

4.) (a) $f = 100\,MHz$; $\sigma = ?$

$\delta = 0.16\,mm$;

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0.16\,mm.$$

$$\sigma = \frac{1}{\pi f \mu \delta^2}$$

$$= \frac{1}{\pi \times 100 \times 10^6 \times 4\pi \times 10^{-7} \times (0.16)^2 \times (10^{-3})^2}$$

$$\boxed{\sigma = 9.9 \times 10^4\ S/m}$$

(b) Now $f = 1\,GHz = 10^9\,Hz$.

@ $10^9\,Hz$; $\alpha = \sqrt{\pi f \mu \sigma}$

$= 1.98 \times 10^4\ Np/m.$

Now,

$$20 \log_{10} e^{-\alpha x} = -30\,dB.$$

$$x = \frac{1.5}{\alpha \log_{10} e}$$

$$= 1.75 \times 10^{-4}\ m$$

$$\boxed{x = 0.175\ mm.}$$