Assignment - 1

1. (c) is not Maxwell's eqn. for time varying field.

2. (b)

3. $\sigma = 0, \quad \mu = \mu_0, \quad \epsilon = 6.25\,\epsilon_0$

$$\vec{H} = 0.6\cos(\beta x)\cos(2\pi\, 10^9 t)\,\hat{a}_z \ \text{A/m}$$

(a) $$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]}$$

$$\sigma = 0 \Rightarrow \beta = \omega\sqrt{\mu\epsilon}$$

$$= 2\pi\times 10^9\sqrt{4\pi\times10^{-7}\times 6.25\times\frac{1}{36\pi}\times10^{-9}}$$

$$\boxed{\beta = 52.33 \text{ rad/m}}$$

$$\nabla\times\vec{H} = \epsilon\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\times\vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & 0.6\cos(\beta x)\cos(2\pi\times10^9 t)\hat{a}_z \end{vmatrix}$$

$$\nabla \times \vec{H} = -0.6\,\beta \sin(\beta x)\cos(\omega t)\,\hat{a}_y$$

$$\therefore \frac{\partial \vec{E}}{\partial t} = -\frac{0.6\,\beta}{\epsilon} \sin(\beta x)\cos(\omega t)\,\hat{a}_y$$

$$\vec{E} = -\frac{0.6\,\beta}{\epsilon\,\omega} \sin(\beta x)\sin(\omega t)\,\hat{a}_y$$

$$\boxed{\vec{E} = -90.42 \sin(\beta x)\sin(\omega t)\,\hat{a}_y \ \ V/m}$$

(b) It's a standing wave.

4. Source-free region $(\vec{J} = 0,\ \rho_v = 0)$

Maxwell's Eqn.

$$\nabla \cdot \vec{B} = 0 \quad \text{——} \ (1)$$

$$\nabla \cdot \vec{D} = \rho \quad \text{——} \ (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{——} \ (3)$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{——} \ (4)$$

For $\vec{J} = 0,\ \rho_v = 0 \Rightarrow$ Eq (2) $\Rightarrow \nabla \cdot \vec{D} = 0$ — (2a)

Eq (4) $\Rightarrow \nabla \times \vec{H} = \dfrac{\partial \vec{D}}{\partial t}$ — (4a)

$\nabla \cdot \nabla \times \vec{A} = 0$ for any vector field $\vec{A}$.

$$\nabla \cdot \nabla \times \vec{E} = -\frac{\partial \nabla \cdot \vec{B}}{\partial t} = 0 \quad \text{and}$$

$$\nabla \cdot \nabla \times \vec{H} = -\frac{\partial \nabla \cdot \vec{D}}{\partial t} = 0.$$

From Eq (1) $\rightarrow \nabla \cdot \vec{B} = 0$

& Eq (2a) $\rightarrow \nabla \cdot \vec{D} = 0$.

As we see that Eq (1) and Eq (2a) are incorporated in Eq (3) and Eq (4), thus Maxwell's Eqns. can be reduced to Eq (3) and Eq (4a).

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{and}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

5. $$\vec{E} = \frac{\sqrt{3}}{2} E_0 \left[\frac{1}{\sqrt{3}} \hat{x} \cos(3\pi \times 10^9 t - 20\pi z) + \hat{y} \cos(3\pi \times 10^9 t - 20\pi z) \right].$$

(a) $\omega = 3\pi \times 10^9$ rad/s.

$f = \frac{\omega}{2\pi} = 1.5 \times 10^9$ Hz.

$\boxed{\text{frequency} = 1.5 \text{ GHz}}$

(b) $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^9} = 0.2$ m.

(c) $n = \frac{c}{v_p}$. $v_p = \frac{\omega}{\beta} = \frac{3\pi \times 10^9}{20\pi} = 1.5 \times 10^8$ m/s.

$\therefore n = \frac{3 \times 10^8}{1.5 \times 10^8}$

$\boxed{n = 2.}$

(d) Linear polarization; since there is no phase difference between the two components of the E field.

direction: $\theta = \tan^{-1}\left(\frac{1}{1/\sqrt{3}}\right) = \tan^{-1}(\sqrt{3}).$

$= 60°$ with x-axis

6. $\underline{\underline{\sigma = 0}}$; $\mu = 2\mu_0$, $\epsilon = 5\epsilon_0$.

$\hookrightarrow \beta = \omega\sqrt{\mu\epsilon}$

$$\vec{H} = 2\cos(\omega t - 3y)\,\hat{a}_z \quad A/m$$

$\beta = 3$ rad/m .

$$\omega = \frac{\beta}{\sqrt{\mu\epsilon}} = \frac{3}{\sqrt{2 \times \cancel{4\pi} \times 10^{-7} \times 5 \times \frac{1}{\cancel{36\pi}_{9}} \times 10^{-9}}}$$

$$= 2.846 \times 10^{8} \text{ rad/m.}$$

$$\boxed{\omega = 2.85 \times 10^{8} \text{ rad/m}}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(\omega t - 3y)\hat{a}_z \end{vmatrix}$$

$$= 6\sin(\omega t - 3y)\,\hat{a}_x$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{6}{\epsilon}\sin(\omega t - 3y)\,\hat{a}_x$$

$$\vec{E} = \frac{-6}{\epsilon\omega}\cos(\omega t - 3y)\,\hat{a}_x .$$

$$\boxed{\vec{E} = -475.95\cos(\omega t - 3y)\,\hat{a}_x}$$

7. Cross-sectional Area(A) = 10 cm^2

Conduction current (J_c) = $0.2 \sin 10^9 t$ mA

$\sigma = 2.5 \times 10^6$ s/m

$\epsilon_r = 6$.

(a) $$\frac{|J_c|}{|J_{ds}|} = \frac{\sigma}{\omega\epsilon}$$

$$|J_{ds}| = |J_c| \times \frac{\omega\epsilon}{\sigma}$$

$$= \frac{0.2 \times 10^9 \times 6 \times \frac{1}{36} \times 10^{-9}}{2.5 \times 10^6}$$

$$= 4.25 \times 10^{-9} \text{ A/m}^2$$

(b) Phase difference = $\pi/2$

(c) Total current = $I_d + I_c$.

where $I_c = 0.2 \sin 10^9 t$ mA

$I_d = 4.25 \times 10^{-9} \cos 10^9 t$ mA.

8. Area = 6 cm^2

Plate separation = 4 mm

Voltage = $100 \sin 10^3 t$ V

$\epsilon = 4\epsilon_0$

$\vec{D} = \epsilon \vec{E} = \epsilon \frac{V}{d}$.

$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d} \frac{dV}{dt}$

$\therefore \quad I_d = J_d \cdot S = \frac{\epsilon S}{d} \frac{dV}{dt} = C \frac{dV}{dt}$

which is same as conduction current given by:

$I_c = \frac{dQ}{dt} = S \cdot \frac{d\rho_s}{dt} = S \frac{dD}{dt} = \epsilon S \frac{dE}{dt}$

$= \frac{\epsilon S}{d} \frac{dV}{dt} = C \frac{dV}{dt}$

$I_d = \frac{4 \times 10^{-9}}{36\pi} \times \frac{6 \times 10^{-4}}{4 \times 10^{-3}} \times 10^3 \times 100 \cos 10^3 t$

$$\boxed{I_d = 530.51 \cos 10^3 t \ \text{nA}.}$$

9. $\sigma = 0$; $\epsilon = \epsilon_0 \epsilon_r$, $\mu = \mu_0$

$$\vec{H} = 5\cos(10^{11}t - 4y)\,\hat{a}_z \text{ A/m}$$

$$\nabla \times \vec{H} = J_c + \frac{\partial D}{\partial t} \qquad \text{as } \sigma = 0 \Rightarrow J_c = 0.$$

(a)

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = J_d$$

$$J_d = \nabla \times \vec{H} = \frac{\partial}{\partial y}\left[5\cos(10^{11}t - 4y)\right]\hat{a}_x$$

$$\boxed{J_d = 20\sin(10^{11}t - 4y)\,\hat{a}_x \text{ A/m}^2}$$

$$\frac{\partial \vec{D}}{\partial t} = J_d = 20\sin(10^{11}t - 4y)\,\hat{a}_x \text{ A/m}^2$$

$$\therefore \vec{D} = \int 20\sin(10^{11}t - 4y)\,\hat{a}_x\,dt$$

$$\boxed{\vec{D} = -20\times10^{-11}\cos(10^{11}t - 4y)\,\hat{a}_x \text{ C/m}^2}$$

(b)

$$\beta = \omega\sqrt{\mu\epsilon} = 4. \qquad \Big|\ \omega = 10^{11} \text{ rad/s.}$$

$$\Rightarrow \sqrt{\epsilon_r} \times \sqrt{\mu_0\epsilon_0} \times 10^{11} = 4$$

$$\Rightarrow \boxed{\epsilon_r = 1.44\times10^{-4} \text{ F/m}}$$

10. $J_c = \sigma E$

$J_d = \omega \epsilon E$

$$\frac{J_c}{J_d} = \frac{\sigma E}{\omega \epsilon E} = \frac{\sigma}{\omega \epsilon}$$

(a) distilled water : 4.44×10^{-4}

(b) sea water : 5.5533

(c) lime stone : 7.197×10^{-4}

11. Electric field intensity in the dielectric

(a)
$$E = E_0 \, Re\{e^{j\omega t}\}$$
$$= E_0 \cos \omega t .$$

Electric field density in the dielectric

$$D = \epsilon E = \epsilon_0 \epsilon_r E$$
$$= \epsilon_0 \epsilon_r E_0 \cos \omega t .$$

The displacement current density in the dielectrics is

$$J_d = \frac{\partial D}{\partial t} = -\epsilon_0 \epsilon_r \omega E_0 \sin \omega t .$$

$$\boxed{J_d = -136.3 \sin (4.9 \times 10^9 \pi) t \ \ A/m^2}$$

(b) In air, the electric flux density 'D' is same as in the dielectrics ($D_{1n} = D_{2n}$)

Although the E. field intensity is twice as high in air.

Since D is the same, the displacement current density in air is:

$$\boxed{J_D = -136.3 \sin(4.9\times10^9 \pi)t \quad A/m^2}$$

12. $\vec{E} = E_0 \sin(kx)\sin(\omega t)\hat{y}$:

$$\nabla\times\vec{E} = -\mu_0 \frac{\partial H}{\partial t}.$$

(a) $$\nabla\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 \sin(kx)\sin(\omega t)\hat{y} & 0 \end{vmatrix}$$

$$= \hat{z} \quad E_0 k \cos(kx)\sin(\omega t).$$

$$\vec{H} = \frac{E_0 k}{\omega \mu} \cos(\omega t)\cos(kx)\ \hat{z}.$$

(b) at a ~~any~~ particular point of time; the phase difference b/w two fields $= \pi/2$.

(c) Not a Travelling wave.

@ any particular point of time, the E. field is zero throughout the space.

(d). $\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2 \leftarrow$ Energy density

(e) Note: Not energy density; but energy will remain const. over time.

$$\text{Energy} = \int_V \text{Energy density}.$$

$$\cancel{A} = \frac{1}{4}\epsilon E_0^2 (wd).$$

So, over time $\rightarrow \frac{d}{dt}(\text{Energy}) = 0.$

$\Rightarrow$ Const. over time.

13. $\vec{E} = 120\pi \cos(10^6\pi t - \beta x)\,\hat{a}_y$ V/m

$\vec{H} = A\pi \cos(10^6\pi t - \beta x)\,\hat{a}_z$ A/m

$$\eta = \frac{120\pi}{A\pi} = \frac{120}{A} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \cancel{\epsilon_r}}}.$$

given $\mu_r = \epsilon_r = 4$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= 120\pi.$$

$$\therefore \quad \frac{120}{A} = 120\pi$$

$$\Rightarrow \boxed{A = \frac{1}{\pi}.}$$

$$v = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{4}.$$

and $v = \dfrac{\omega}{\beta} \Rightarrow \beta = \dfrac{\omega}{v}$

$$= \frac{\omega}{c/4} = \frac{4\omega}{c}$$

$$\boxed{\beta = \frac{4 \times 10^6 \pi}{3 \times 10^8} \text{ rad/m}.}$$

14. $\rightarrow$ Refer Assignment - 2 Q - 11.

15.) Standard eqn. of plane wave:

$$E = E_0 \, e^{j(\omega t - kz)}$$

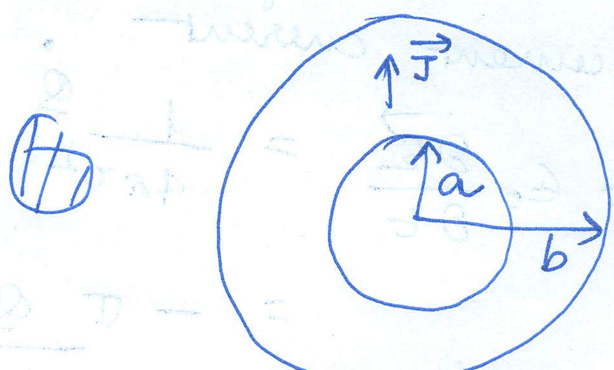
Now, $k = k_1 + i k_2$

$$\Rightarrow E = E_0 \, e^{j(\omega t - (k_1 + i k_2) z)}$$

$$= E_0 \, e^{kz} \, e^{j(\omega t - k_1 z)}.$$

This implies the amplitude of the E. field will decrease with z.

16.)



Space b/w them is filled with Ohmic material. $\Rightarrow$ Radial current flows.

$$\vec{J} = \sigma \vec{E}$$

$$= \sigma \frac{1}{4\pi\epsilon_0 r^2} Q \, \hat{r} \, ;$$

$$I = \frac{-dQ}{dt} = \int \vec{J} \cdot d\vec{a}$$

$$= \frac{\sigma Q}{\epsilon_0}.$$

This is spherically symmetrical.

$\Rightarrow$ Magnetic field has to be zero.

$\rightarrow$ It is not a static case:

$\Rightarrow Q, \vec{E}, \vec{J} \rightarrow$ all are functions of time.

So, Ampere and Biot - Savart's law do not apply.

$\therefore$ displacement current

$$J_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{1}{4\pi r^2} \dot{Q} \hat{r}$$

$$= -\sigma \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

this exactly cancels the conduction current

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}.$$

$\Rightarrow \nabla \times \vec{B} = 0 \Rightarrow$ Magnetic field is zero.

17. Assignment 2 → Q-1.

18. (a) For a good conductor,

$$\alpha = \beta = \frac{1}{\delta}$$

and, $\beta = \frac{2\pi}{\lambda}$

$$\Rightarrow \lambda = \frac{2\pi}{\beta} = 2\pi\delta$$

$$\therefore \delta = \frac{1}{2\pi}\lambda = 0.1592\lambda.$$

$\boxed{\delta = 0.1592\lambda}$ ⇒ Skin depth is always shorter than wavelength

(b) length of Al pipe = 40 m

Inner radius = 9 mm

outer radius = 12 mm

Total current = $6 \sin 10^6 \pi t$

So, $\omega = 10^6 \pi$ rad/s.

$2\pi f = 10^6 \pi$

$\Rightarrow f = 0.5 \times 10^6$ Hz.

Given, $\sigma = 3.5 \times 10^7$ S/m.

$\mu_r = \epsilon_r = 1.$

$$\therefore \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 1.2037 \times 10^{-4} \text{ m}$$

19. E. field = 10 V/m.

$\epsilon_r = 16$

$\sigma = 200$ S/m.

$f = 300$ MHz

$\therefore \omega = 2\pi \times 300 \times 10^6 = 1.8849 \times 10^9$ rad/s

$\therefore$ Intrinsic impedance

$$\eta = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\epsilon_0\epsilon_r}}$$

$= 0.9419$.

Average power density of the wave:

$$P_{av} = \frac{|E_0|^2}{2} Re\left\{\frac{1}{\eta^*}\right\}.$$

$= 53.08$ W/m².

As, conductivity is non-zero, there is attenuation in the medium.

$\therefore$ Propagation Const $\rightarrow \gamma$

$\gamma = \alpha + j\beta$.

$= \sqrt{j\omega\mu_0(\sigma + j\omega\epsilon_0\epsilon_r)}$

$\alpha = \omega^2 \epsilon_0 \epsilon_r \mu_0$. [Np/m]

$\neq$

Attenuation const $\rightarrow \alpha$.

$\therefore$ Power density at 1cm distance is

$$P_{av} \, e^{-2 \times 0.01 \times \alpha} \quad [mW/m^2].$$

20. R-2 | R-1.

$\sigma = 0$, μ_0, ϵ_0 (R-2)

$\sigma = 0$, $\mu = \mu_0$, $\epsilon = 4\epsilon_0$ (R-1)

$y = 0$.

$$\vec{E} = 5 \cos(10^8 t + \beta y)\, \hat{a}_z \rightarrow \text{Region-1}.$$

$$R\text{-}1 \rightarrow \eta_1 = \frac{\eta_0}{\sqrt{4}} = \frac{\eta_0}{2}$$

$$R\text{-}2 \rightarrow \eta_2 = \eta_0.$$

$\therefore \tau = \frac{2\eta_2}{\eta_2 + \eta_1}$

Transmission Coeff

$$= \frac{2\eta_0}{\eta_0 + \frac{\eta_0}{2}} = \frac{4}{3}\frac{\eta_0}{\eta_0}$$

$$\boxed{\tau = 1.33}$$

$\beta_t \rightarrow \omega\sqrt{\mu\epsilon}$

$= \beta \sqrt{4}$

$= 2\beta$

$\therefore \quad E_t = \tau E_i \quad$ with (β_t).

$= (1.33 \times 5) \cos(10^8 t + 2\beta y) a_z$.

(b) Time-averaged Poynting vector in Region-1 $= \frac{1}{2}\left(E_0^2/\eta_1\right)$.

$= \frac{1}{\cancel{2}} \times \frac{25}{\eta_0/\cancel{2}}$

$= \frac{25}{\eta_0}$. W/m^2

(c) Region-2 $= \frac{1}{2}\left(E_t^2/\eta_2\right)$

$= \frac{1}{2\eta_0}(1.33 \times 5)^2$

$= \frac{22.11}{\eta_0}$ W/m^2