1. Using truth tables, prove that given two arbitrary Boolean functions F_1 and F_2:

a. $E = F_1 + F_2$ has the minterms that are in either F_1 or F_2 when it is in **Sum of Products** canonical form.

F_1	F_2	$E = F_1 + F_2$
0	0	0
0	1	1
1	0	1
1	1	1

As shown in the truth table, when $E = F_1 + F_2$, the sets of minterms in each function are united together.

b. $G = F_1F_2$ contains only the minterms that are common to both F_1 and F_2.

F_1	F_2	$G = F_1F_2$
0	0	0
0	1	0
1	0	0
1	1	1

As shown in the truth table, when $G = F_1F_2$, the sets of minterms in each function are intersected.

2. Obtain the truth table of the following functions, and express each function in sum of products (SoP) and product of sums (PoS) in canonical forms

a. $(c' + d)(b + c')$

b	c	d	c'	$(c' + d)$	$(b + c')$	$(c' + d)(b + c')$	#
0	0	0	1	1	1	1	0
0	0	1	1	1	1	1	1
0	1	0	0	0	0	0	2
0	1	1	0	1	0	0	3
1	0	0	1	1	1	1	4
1	0	1	1	1	1	1	5
1	1	0	0	0	1	0	6
1	1	1	0	1	1	1	7

Sum of Products (SoP):

$$\sum m_{0,1,4,5,7} = (b'c'd') + (b'c'd) + (bc'd') + (bc'd) + (bcd)$$

Product of Sums (PoS):

$$\prod M_{2,3,6} = (b + c' + d)(b + c' + d')(b' + c' + d)$$

b. $bd' + acd' + ab'c + a'c'$

a	b	c	d	a'	b'	c'	d'	bd'	acd'	$ab'c$	$a'c'$	$bd' + acd' + ab'c + a'c'$	#
0	0	0	0	1	1	1	1	0	0	0	1	1	0
0	0	0	1	1	1	1	0	0	0	0	1	1	1
0	0	1	0	1	1	0	1	0	0	0	0	0	2
0	0	1	1	1	1	0	0	0	0	0	0	0	3
0	1	0	0	1	0	1	1	1	0	0	1	1	4
0	1	0	1	1	0	1	0	0	0	0	1	1	5
0	1	1	0	1	0	0	1	1	0	0	0	1	6
0	1	1	1	1	0	0	0	0	0	0	0	0	7
1	0	0	0	0	1	1	1	0	0	0	0	0	8
1	0	0	1	0	1	1	0	0	0	0	0	0	9
1	0	1	0	0	1	0	1	0	1	1	0	1	10
1	0	1	1	0	1	0	0	0	0	1	0	1	11
1	1	0	0	0	0	1	1	1	0	0	0	1	12
1	1	0	1	0	0	1	0	0	0	0	0	0	13
1	1	1	0	0	0	0	1	1	1	0	0	1	14
1	1	1	1	0	0	0	0	0	0	0	0	0	15

Sum of Products (SoP):

$$\begin{aligned}\sum m_{0,1,4,5,6,10,11,12,14} &= (a'b'c'd') + (a'b'c'd) + (a'bc'd') + (a'bc'd)\\ &(a'bcd') + (ab'cd') + (ab'cd) + (abc'd') + (abcd')\end{aligned}$$

Product of Sums (PoS):

$$\begin{aligned}\prod M_{2,3,7,8,9,13,15} &= (a+b+c'+d)(a+b+c'+d')(a+b'+c'+d')(a'+b+c+d)\\ &(a'+b+c+d')(a'+b'+c+d')(a'+b'+c'+d')\end{aligned}$$

3. Express the complement of the following functions in **Sum of minterms** form:

a. $F(A,B,C,D) = \sum m_{2,4,6,10,12,14} \rightarrow F'(A,B,C,D) = \sum m_{?}$

Consider the truth tables of an arbitrary function F and it's complement: where there are 0s in the result, the other has 1s. Therefore, all minterms should be present between the two funcitons. Therefore, a quick and simple way to compute the minterms of F' is to let the Universe of Discourse be all of the minterms possible for any arbitrary function with the matching number of variables, then take the compliment of the set of minterms in F with respect to this new Universe.

$\underline{U = \{0, 1, \ldots, (2^4 - 1 = 15)\}}$

$\overline{\{2, 4, 6, 10, 12, 14\}} = \{1, 3, 5, 9, 11, 13, 15\}$

$\therefore F'(A,B,C,D) = \sum m_{1,3,5,9,11,13,15}$

b. $F(x, y, z) = \prod M_{3,5,7} \rightarrow F'(x, y, z) = \sum m_{?}$

Again, considering the truth tables of F and it's complement, it follows clearly that if F has some minterm m_n then F's complement should have the MAXTERM M_n, since wherever F has a True result, F' will have a False result.

$F(x, y, z) = \prod M_{3,5,7}$
$F'(x, y, z) = \sum m_{3,5,7}$

4. Convert each of the following to the opposite cannonical form, then simplify if possible:

a. $F(x, y, z) = \sum m_{1,3,5}$

Cannonical Conversion (described above):
$U = \{0, 1, \ldots 6, 7\}$
$\overline{\{1, 3, 5\}} = \{0, 2, 4, 6, 7\}$

$\therefore F = \prod M_{0,2,4,6,7}$.

Simplification:

$$
\begin{aligned}
F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y' + z)(x' + y' + z') \\
&= (x + z)(x' + y + z)(x' + y' + z)(x' + y' + z') \\
&= (x + z)(x' + z)(x' + y' + z') \\
&= (z)(x' + y' + z') \\
&= zx' + zy'
\end{aligned}
$$

b. $F(A, B, C, D) = \prod M_{3,5,8,11}$

Cannonical Conversion (described above):
$U = \{0, 1, \ldots 14, 15\}$
$\overline{\{3, 5, 8, 11\}} = \{0, 1, 2, 4, 6, 7, 9, 10, 12, 13, 14, 15\}$

$\therefore F = \sum m_{0,1,2,4,6,7,9,10,12,13,14,15}$.

Simplification

$$
\begin{aligned}
F &= \sum m_{0,1,2,4,6,7,9,10,12,13,14,15} \\
F' &= \prod M_{0,1,2,4,6,7,9,10,12,13,14,15} \\
&= (A + B + C + D)(A + B + C + D')(A + B + C' + D)(A + B' + C + D) \\
&\quad (A + B' + C' + D)(A + B' + C' + D')(A' + B + C + D') \\
&\quad (A' + B + C' + D)(A' + B' + C + D)(A' + B' + C + D') \\
&\quad (A' + B' + C' + D)(A' + B' + C' + D') \\
&= (A + B + C)(A + B + C' + D)(A + B' + C + D)(A + B' + C' + D) \\
&\quad (A + B' + C' + D')(A' + B + C + D')(A' + B + C' + D) \\
&\quad (A' + B' + C + D)(A' + B' + C + D')(A' + B' + C' + D) \\
&\quad (A' + B' + C' + D')
\end{aligned}
$$

$$
\begin{aligned}
&= (A + B + CD)(A + B' + C + D)(A + B' + C' + D)(A + B' + C' + D') \\
&\quad (A' + B + C + D')(A' + B + C' + D)(A' + B' + C + D) \\
&\quad (A' + B' + C + D')(A' + B' + C' + D)(A' + B' + C' + D') \\
&= (A + BC + BD + CD)(A + B' + C' + D)(A + B' + C' + D') \\
&\quad (A' + B + C + D')(A' + B + C' + D)(A' + B' + C + D) \\
&\quad (A' + B' + C + D')(A' + B' + C') \\
&= (A + BC + BD + CD)(A + B' + C')(A' + B + C + D')(A' + B + C' + D) \\
&\quad (A' + B' + C + D)(A' + B' + C + D')(A' + B' + C') \\
&= (A + BC + BD + CD)(A + B' + C')(A' + B + C + D')(A' + B + C' + D) \\
&\quad (A' + B' + C + D)(A' + B' + C'D') \\
&= (A + BC + BD + CD)(A + B' + C')(A' + B + C + D')(A' + B + C' + D) \\
&\quad (A' + B') \\
&= (A + BC + BD + CD)(A + B' + C')(A' + B + C + D')(A' + B'C' + B'D) \\
&= (A + BC + BD + CD)(A + B' + C')(A' + B'CD + B'C'D') \\
&= AB'C'D' + A'BC'D + B'CD \\
F &= (AB'C'D' + A'BC'D + B'CD)' \\
&= (AB'C'D')'(A'BC'D)'(B'CD)' \\
&= (A' + B + C + D)(A + B' + C + D')(B + C' + D') \\
&= (A'B' + A'D' + BA + BD' + C + DA + DB')(B + C' + D') \\
&= AB + BC + BD' + A'B'C' + AC'D + B'C'D + A'D' + D'C
\end{aligned}
$$