1. Using truth tables or boolean postulates, prove that **XOR** is:

a. Commutative: $x \oplus y = y \oplus x$

x	y	$x \oplus y$	$y \oplus x$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	0	0

$\therefore$ Since the highlighted columns match, the **XOR** operation is commutative.

b. Associative: $x \oplus (y \oplus z) = (x \oplus y) \oplus z = x \oplus y \oplus z$

x	y	z	y⊕z	x⊕(y⊕z)	x⊕y	(x⊕y)⊕z
1	1	1	0	1	0	1
1	1	0	1	0	0	0
1	0	1	1	0	1	0
1	0	0	0	1	1	1
0	1	1	0	0	1	0
0	1	0	1	1	1	1
0	0	1	1	1	0	1
0	0	0	0	0	0	0

$\therefore$ Since the highlighted columns match, the **XOR** operation is associative.

2. Prove that **XOR** is not distributive over **AND**, i.e., $x \oplus (yz) \neq (x \oplus y)(x \oplus z)$

x	y	z	yz	x⊕(yz)	x⊕y	x⊕z	(x⊕y)(x⊕z)
1	1	1	1	0	0	0	0
1	1	0	0	1	0	1	0
1	0	1	0	1	1	0	0
1	0	0	0	1	1	1	1
0	1	1	1	0	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

$\therefore$ Since the highlighted columns do not match, the **XOR** operation is not distributive over **AND**.