

Getting most out of nonlinear decoherence data using machine learning techniques.

Part I: Perturbative approach on 1D model.

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Outline

- 1 Introduction
- 2 How many parameters are relevant to a given set of data
- 3 Perturbative approach for accurate regression
- 4 Conclusion

Introduction

- Goals:

- Get as many information (e.g. physical parameters) as possible from nonlinear decoherence data.
 - (e.g.) Given a set of data, nonlinear PCA / ICA may able to provide relevant number of parameters that can well represent (generate) the data.
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 - (e.g.) Given a set of data, this can be framed by a nonlinear regression problem.

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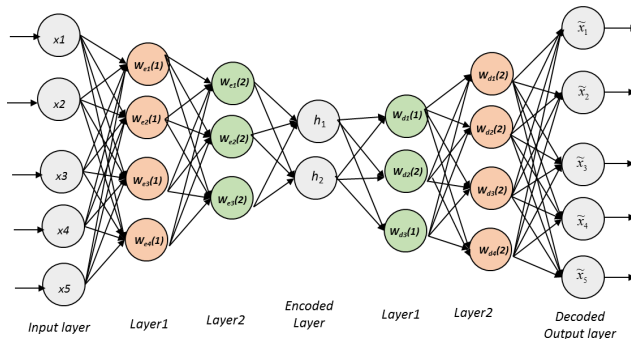
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AutoEncoder



source : <https://medium.com/@venkatakrishna.jonnalagadda/sparse-stacked-and-variational-autoencoder-e5bfe73b64>

Varying the number of latent variable (i.e. number of nodes in encoded layer) while tracking the reconstruction loss (the quality of reconstruction), we can observe how many parameters are relevant for data representation. Therefore, it can be interpreted as a nonlinear PCA.

Variational AutoEncoder

- Differences to the AutoEncoder are...

- Encoded layer act as parameters of a probability that takes an assumed form (e.g. normal distribution)
 - A sample of latent variable is randomly taken (from the probability parameterized by the encoded layer) and sent to decoder.
- One can make the latent variables to be independent of each other by choosing the form of the parametrized probability such that covariance matrix is diagonal. (e.g. multi-dimensional normal distribution with diagonal covariance)
 - In this case, it can be interpreted as a nonlinear ICA.
- Pro & Con
 - Better representation learning but worse reconstruction accuracy (due to sampling procedure, and assumption on probability distribution on latent variable)

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Data model and generation

- Data model: 1D decoherence centroid with first order nonlinear detuning parameter $\partial_l \omega$

$$C(t) \equiv \Re \left\langle \mathcal{A} z_0^{-i(\omega + \partial_l \omega l)t} \right\rangle$$

where $z_t \equiv x_t - ip_t$, $l = zz^*/2$ and \mathcal{A} is the linear transformation operator from normal to physical phase space that takes the following matrix representation.

$$\mathcal{A} \doteq \begin{pmatrix} \sqrt{\beta} & 0 \\ -\alpha/\sqrt{\beta} & 1/\sqrt{\beta} \end{pmatrix}$$

- Data generation
 - Prepare 1D Gaussian beam in normal coordinate so as to model equilibrium beam
 - Sample optics parameters ε , β , α , initial offsets x_0 , p_0 , frequency ω and nonlinear detuning parameter $\partial_l \omega$ from uniform random within rescanable bound. (Remember we have total 7 parameters)
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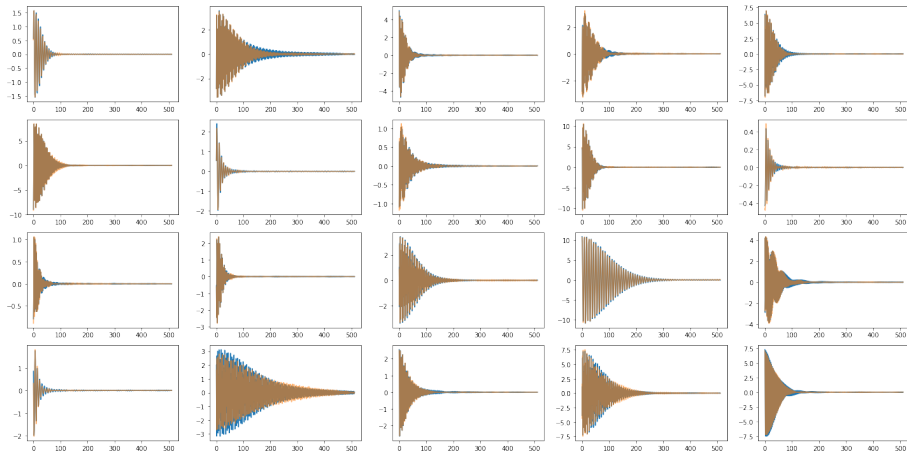
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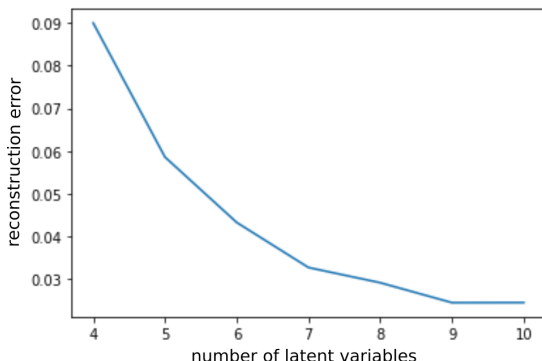
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Reconstruction examples



Reconstructioun loss over number of latent variables

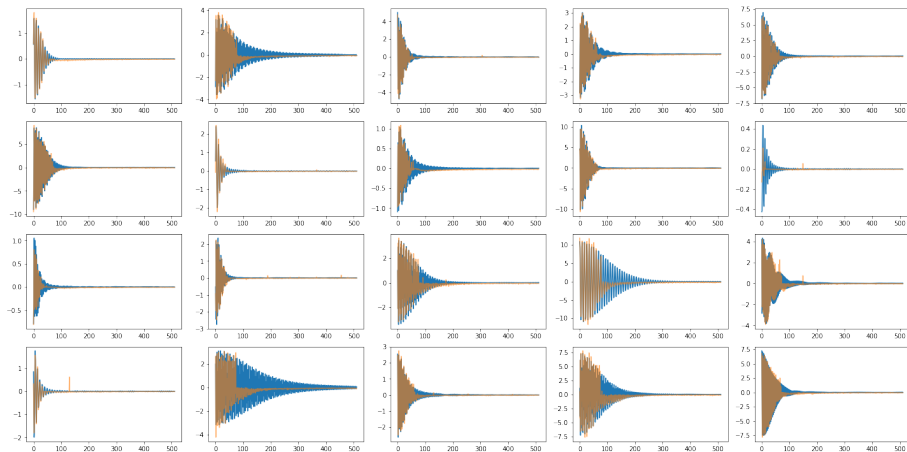


The error keep decreasing as the number of latent variables increase. It is hard to say that we need only 7 parameters to represent the data.

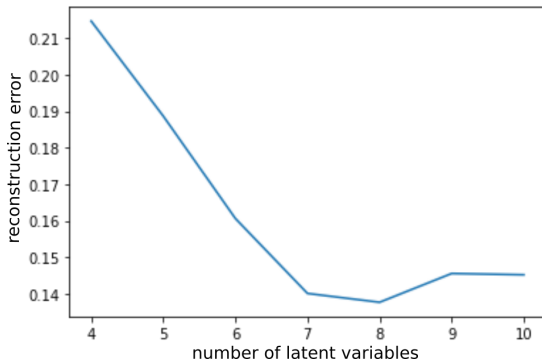
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VAE shows that there are 7 indepedent parameters that represent the data.

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Leading order theory

- Let x, p are normal coordinate normalized by emittance s.t.

$$\int x^2 \rho dx dp = \int p^2 \rho dx dp = 1$$

where $\rho(x, p)$ is the beam density.

- Let the initial kick is $\langle x - ip \rangle_{t=0} = x_0$,
- Then, **in the limit** of $x_0 \gg 1$, the centroid in frequency domain read

$$F(k) \equiv \sum_{t=0}^T \frac{\langle x - ip \rangle_t}{x_0} e^{-i\omega_0 t} e^{-ikt}$$

$$\Re[F(k)] \simeq \frac{1}{2} + \frac{\pi}{|x_0 \omega_l|} \lambda \left(\frac{k}{x_0 \omega_l} \right)$$

where

$$\lambda(x) = \int \rho(x, p) dp$$

$$\omega_0 \equiv \omega(x_0, 0) \quad \omega_l \equiv \frac{\partial \omega}{\partial l}(x_0, 0)$$

- Therefore, we can measure ω_0 , $|\omega_l|$, and λ in the limit of $x_0 \gg 1$.

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 - Input: Particle ID, $C(t)$, $\partial_l \omega$, ω
 - Output: $\partial_l \omega$, ω

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Test1: small x_0

- Given: $x_0 = 1$ and

$$\frac{\omega}{2\pi} = 0.739$$

$$\frac{\partial_I \omega}{2\pi} = \pm 9.24 \times 10^{-3}$$

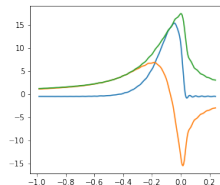
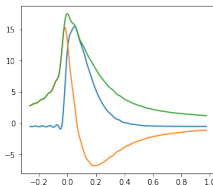
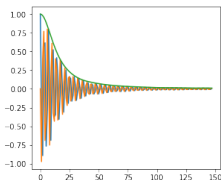
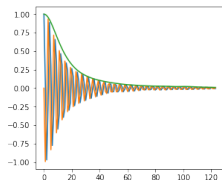
- Estimated:

$$\frac{\omega}{2\pi} = 0.738452$$

$$\frac{\partial_I \omega}{2\pi} = +11.3871 \times 10^{-3}$$

$$\frac{\omega}{2\pi} = 0.740129$$

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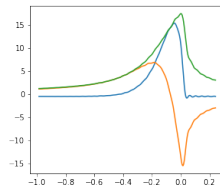
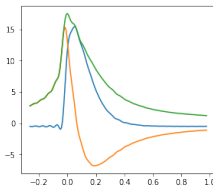
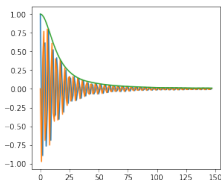
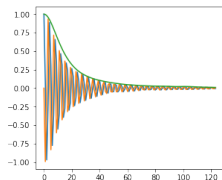
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Test 2: large x_0

- Given: $x_0 = 20$ and

$$\frac{\omega}{2\pi} = 0.235$$

$$\frac{\partial_I \omega}{2\pi} = \pm 4.44 \times 10^{-4}$$

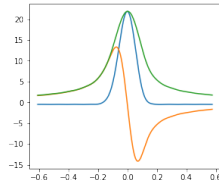
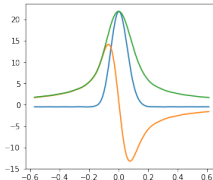
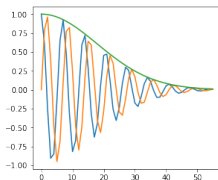
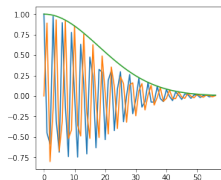
- Estimated:

$$\frac{\omega}{2\pi} = 0.232647$$

$$\frac{\partial_I \omega}{2\pi} = +4.55265 \times 10^{-4}$$

$$\frac{\omega}{2\pi} = 0.237312$$

$$\frac{\partial_I \omega}{2\pi} = -4.55461 \times 10^{-4}$$



Test 2: large x_0

- Given: $x_0 = 20$ and

$$\frac{\omega}{2\pi} = 0.235$$

$$\frac{\partial_I \omega}{2\pi} = \pm 4.44 \times 10^{-4}$$

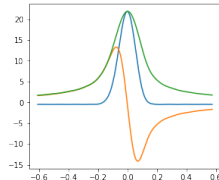
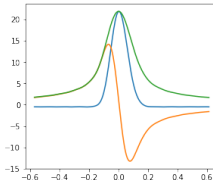
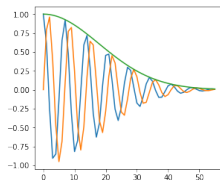
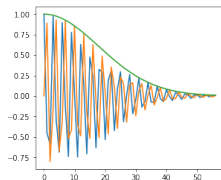
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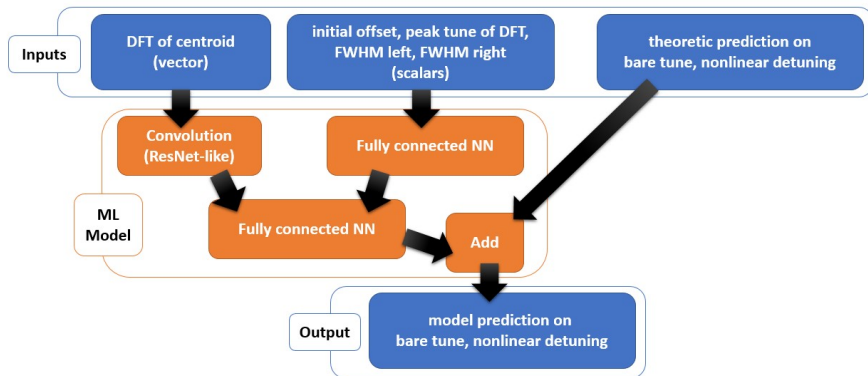
$$\frac{\partial_I \omega}{2\pi} = -4.55461 \times 10^{-4}$$



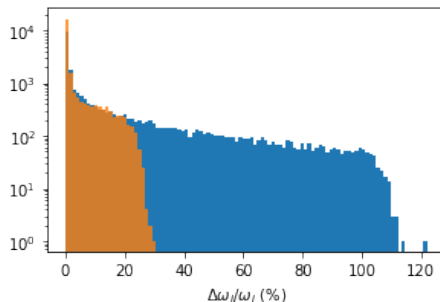
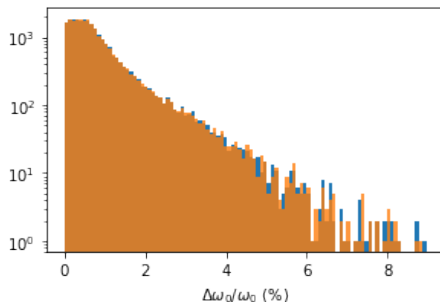
Outline

- 1 Introduction
- 2 How many parameters are relevant to a given set of data
 - Review: AutoEncoder and Variational AutoEncoder
 - Data model and generation
 - AutoEncoder Result
 - Variational AutoEncoder Result
- 3 Perturbative approach for accurate regression
 - Leading order theory
 - Data Model and Generation
 - ML model as a perturbative correction to leading order theory
- 4 Conclusion

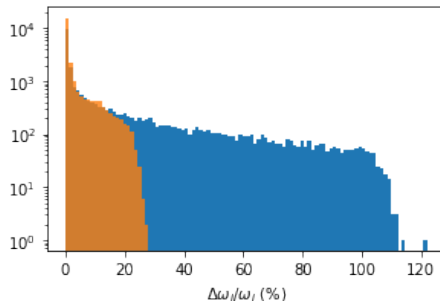
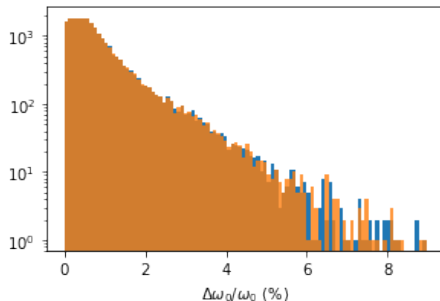
ML Model as a perturbative correction to leading order theory



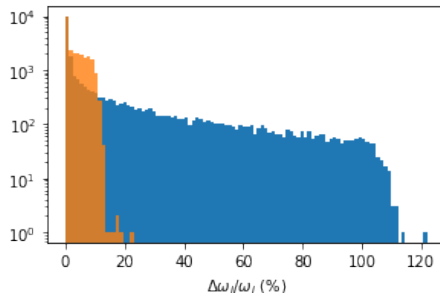
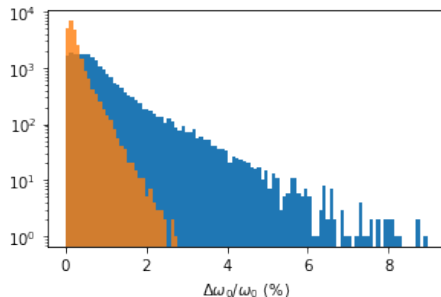
Result: scalar inputs only



Result: scalar inputs only, $1/x_0$



Result: Full and complex model, $1/x_0$



Outline

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- 3 Perturbative approach for accurate regression
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Conclusion

- VAE can be used to get the number of representation parameters of a given data.
 - Given a nonlinear decoherence centroid data, we may able to determine how many informations can be extracted using VAE.
- ML model as a perturbative correction to leading order nonlinear decoherence theory could increase (virtual) measurement accuracy.

Thank you for your attention!

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