

常见的等价无穷小

$$\sin x, \tan x, \arcsin x, \arctan x \sim x \sim e^x - 1, \ln^{(x+1)}$$

$$1 - \cos x, x - \ln^{(1+x)} \sim \frac{1}{2} x^2$$

$$(1 + \alpha(x))^{\beta(x)} - 1 \sim \alpha(x)\beta(x) \quad (\alpha(x) \rightarrow 0, \alpha(x)\beta(x) \rightarrow 0)$$

$$\sqrt{1+x} - \sqrt{1-x} \sim x$$

$$x - \sin x, \arcsin x - x \sim \frac{1}{6} x^3$$

$$\tan x - x, x - \arctan x \sim \frac{1}{3} x^3$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

无穷大量的比较

$$x \rightarrow +\infty \quad \ln^a(x) \ll x^\beta \ll a^x \quad a > 0 \quad \beta > 0 \quad a > 1$$

$$n \rightarrow \infty \quad \ln^a n \ll n^\beta \ll a^n \ll n! \ll n^n \quad a > 0 \quad \beta > 0 \quad a > 1$$

常见的泰勒公式

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + o(x^n)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + o(x^{2n-1})$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n})$$

$$\ln^{(1+x)} = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \dots + (-1)^n \frac{x^n}{n!} + o(x^n)$$

$$(1+x)^a = 1 + ax + \frac{a(a-1)x^2}{2!} + \dots + \frac{a(a-1)\dots(a-n+1)x^n}{n!} + o(x^n)$$