Supplemental Material for "Sticky Discount Rates"*

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February 2024

Supplement A Comparing Different Investment Rules

The aim of this section is to explain why investment rules based on the stochastic discount factor and on a discount rate lead to similar investment decisions, as long as the discount rate is chosen in a certain way. Moreover, the section clarifies why textbooks recommend that firms should set their discount rate equal to the cost of capital. The discussion here is based on Gormsen and Huber (2023).

Setup In models with uncertainty, firms can generally maximize market value by using the stochastic discount factor to discount future cash flows. Textbooks aimed at managers nonetheless tend to present simpler rules based on a discount rate. We illustrate that the two methods lead to similar investment outcomes using the example of a simple project with uncertain returns. This project generates expected revenue $\mathbb{E}_t[\text{Revenue}_{t+j}]$ in period $t+j$ and costs Cost_t in period t.

Using the Stochastic Discount Factor The first decision rule states that the firm accepts the project if the net present value, discounted using the stochastic discount factor M_{t+j}, is positive:

$$\mathbb{E}_t\left[M_{t+j}\text{Revenue}_{t+j}\right] - \text{Cost}_t > 0. \tag{S1}$$

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Using the definition of covariance, we can rewrite equation S2 as:

$$\mathbb{E}_t\left[\text{Return}_{t,t+j}\right] > R^f_{t,t+j} - \text{Cov}_t\left[\text{M}_{t+j}, \text{Return}_{t,t+j}\right] R^f_{t,t+j}, \tag{S2}$$

where $R^f_{t,t+j} = \mathbb{E}_t\left[\text{M}_{t+j}\right]^{-1}$ is the risk-free interest rate between t and $t+j$ and $\text{Return}_{t,t+j} = \frac{\text{Revenue}_{t+j}}{\text{Cost}_t}$ is the return to the project.

Using a Discount Rate The second rule states that the firm accepts the project if the net present value of the project, discounted using a discount rate δ_t, is positive:

$$\sum_{s=0}^{\infty}(1+\delta_t)^{-s}\,\mathbb{E}_t[\text{Revenue}_{t+s} - \text{Cost}_{t+s}] = (1+\delta_t)^{-j}\,\mathbb{E}_t[\text{Revenue}_{t+j}] - \text{Cost}_t > 0. \tag{S3}$$

This rule can also be rewritten as saying that the firm should invest if the return to the project exceeds a "hurdle" rate, such that:

$$\mathbb{E}_t\left[\text{Return}_{t,t+j}\right] > (1+\delta_t)^j. \tag{S4}$$

The two rules in equations S2 and S4 are equivalent, as long as the firm sets the discount rate such that:

$$(1+\delta_t)^j = R^f_{t,t+j} - \text{Cov}_t\left[\text{M}_{t+j}, \text{Return}_{t,t+j}\right] R^f_{t,t+j}. \tag{S5}$$

Hence, for a given project, the rules based on the stochastic discount factor and the discount rate lead to the same investment outcome if the chosen discount rate satisfies equation S5.

Choosing the Discount Rate and the Cost of Capital To determine the discount rate given by equation S5, the firm can use financial prices. Assume that the firm issues just one financial asset (e.g., only equity). By definition, the expected return to the financial asset of firm i over one period is equal to 1 plus the firm's "financial cost of capital," given by r^{fin}_{it}. The basic asset pricing equation implies that the expected return to the financial asset over the lifetime of the project is:

$$(1 + r_{it}^{\text{fin}})^j = \mathbb{E}_t \left[R_{t,t+j}^i \right] = R_{t,t+j}^f - \text{Cov}_t \left[\text{M}_{t+j}, R_{t,t+j}^i \right] R_{t,t+j}^f. \tag{S6}$$

If the covariance between the stochastic discount factor and the project return is identical to the covariance between the stochastic discount factor and the financial asset return (i.e., $\text{Cov}_t \left[\text{M}_{t+j}, R_{t,t+j}^i \right] = \text{Cov}_t \left[\text{M}_{t+j}, \text{Return}_{t,t+j} \right]$), then the rules in equations S2 and S4 are equivalent for a firm that sets the discount rate equal to its financial cost of capital. Intuitively, if the project under consideration exhibits the same risk profile as the firm's existing investments, then the financial cost of capital tells the firm how financial markets price the risk of the project.

Generalizations The above results generalize to firms with multiple liabilities (e.g., debt and equity). In such cases, r_{it}^{fin} is the weighted average cost of capital, where the expected return is separately estimated for each asset type and weights are calculated using the value of outstanding assets of that type relative to firm total assets, accounting for differential tax treatments of different assets.

The results can also be extended to investments with more complex cash flows. For instance, consider an investment consisting of multiple sub-projects, indexed by s, where each project requires a cost in period t and pays uncertain revenue in one period $t + j$. In that case, the firm could still apply a decision rule as in equations S2 and S4, by summing over the individual sub-projects s.

If $\text{Cov}_t \left[\text{M}_{t+j}, R_{t,t+j}^i \right] \neq \text{Cov}_t \left[\text{M}_{t+j}, \text{Return}_{t,t+j} \right]$, firms cannot infer the riskiness of an individual project using expected returns on the firm's existing financial assets. Instead, firms should then adjust the discount factor by a project-specific risk premium.

Supplement B Details on the New Keynesian Analysis

Supplement B.1 Non-Linear Characterization of the Firm Problem

Due to the constant returns to scale assumptions, each individual firm's problem is independent of its size, k_t. Let $\iota_t = I_t/k_t$ be the investment rate of the firm. It is easy to verify that the firm's value functions is linear in the capital stock:

$$V_t^I(k_t, \delta) = v_t^I(\delta) P_t k_t, \tag{S7}$$

where $v_t^d(\delta)$ denotes the real marginal value of unit capital. It solves the following Bellman equation:

$$v_t^I(\delta) = \max_{\iota_t} \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + \delta} \left\{ (1 - \xi) + \iota_t \right\} v_{t+1}^I(\delta), \tag{S8}$$

where $1 + \pi_{t+1} \equiv P_{t+1}/P_t$ is the gross inflation rate, ω_t are real profits from unit capital,

$$\omega_t \equiv \max_l \frac{1}{P_t} (p_t F_t(1, l) - W_t l), \tag{S9}$$

and $\varphi(\iota) \equiv \Phi(\iota, 1)$. The first-order optimality condition for investment is

$$1 + \varphi'(\iota_t) = \mathbb{E}_t \frac{1 + \pi_{t+1}}{1 + \delta} v_{t+1}^I(\delta) \tag{S10}$$

Likewise, the firm's financial market value is also linear in capital:

$$V_t^a(k) = v_t^a P_t k, \quad V_t^n(k, \delta) = v_t^n(\delta) P_t k, \tag{S11}$$

where v_t^a and v_t^n solve the following recursion

$$v_t^a = \max_{\delta^*} \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1+\pi_{t+1}}{1+i_t} \{(1-\xi) + \iota_t\} [\theta v_{t+1}^n(\delta^*) + (1-\theta) v_{t+1}^a] \tag{S12}$$

$$\text{s.t.} \quad \iota_t = \bar{\iota}_t(\delta^*) \tag{S13}$$

and

$$v_t^n(\delta^*) = \omega_t - \iota_t - \varphi(\iota_t) + \mathbb{E}_t \frac{1+\pi_{t+1}}{1+i_t} \{(1-\xi) + \iota_t\} [\theta v_{t+1}^n(\delta^*) + (1-\theta) v_{t+1}^a] \tag{S14}$$

$$\text{s.t.} \quad \iota_t = \bar{\iota}_t(\delta^*) \tag{S15}$$

The first-order optimality condition for the choice of the discount rate is

$$\begin{aligned} &\mathbb{E}_t \left[\frac{1+\pi_{t+1}}{1+i_t} [\theta v_{t+1}^n(\delta^*) + (1-\theta) v_{t+1}^a] - (1+\varphi'(\iota_t)) \right] \frac{d\bar{\iota}_t}{d\delta_t^*} \\ &\qquad + \mathbb{E}_t \frac{1+\pi_{t+1}}{1+i_t} \{(1-\xi) + \iota_t\} \, \theta \frac{d v_{t+1}^n(\delta^*)}{d\delta^*} = 0. \end{aligned} \tag{S16}$$

Supplement B.2 Proof of Proposition 1

A first-order approximation of (S16) around the deterministic steady state gives

$$\begin{aligned} \mathbb{E}_t \Bigg[\frac{d\bar{\iota}}{d\delta^*} \Bigg(v \frac{1}{1+r} \Big[d\pi_{t+1} - d\ln(1+i_t) + d\ln v_{t+1}^a + \theta \underbrace{v^{n\prime}(\delta^*)}_{=0} \frac{1}{1+r} d\ln(1+\delta_t^*) \Big] - \varphi''(\iota) d\iota_t \Bigg) \\ + \underbrace{\left[\frac{1}{1+r} v - (1+\varphi'(\iota)) \right]}_{=0} d\left(\frac{d\bar{\iota}_t}{d\delta_t^*} \right) \\ + \underbrace{v^{n\prime}(\delta^*)}_{=0} d\left(\frac{1+\pi_{t+1}}{1+i_t} \{(1-\delta^*) + \iota_t\} \theta \right) \\ + \frac{1}{1+r} \theta d \left(\frac{d v_{t+1}^n(\delta^*)}{d\delta^*} \right) \Bigg] = 0, \end{aligned} \tag{S17}$$

which, in turn, simplifies as follows, since many terms disappear owing to envelope conditions:

$$\mathbb{E}_t\left[\frac{d\bar{\iota}}{d\delta^*}\left(v\frac{1}{1+r}\left[d\pi_{t+1} - d\ln(1+i_t) + d\ln v^a_{t+1}\right] - \varphi''(\iota)d\iota_t\right)\right.$$
$$\left. + \frac{1}{1+r}\theta d\left(\frac{dv^n_{t+1}(\delta^*)}{d\delta^*}\right)\right] = 0. \tag{S18}$$

Linearizing (S10) gives

$$\varphi''(\iota)d\iota = \frac{1}{1+r}v\left(d\pi_{t+1} - d\ln(1+\delta^*_t) + d\ln v^d_{t+1}(\delta^*)\right). \tag{S19}$$

Combining (S18) and (S19),

$$\mathbb{E}_t\left[\frac{d\bar{\iota}}{d\delta^*}v\frac{1}{r}\left[d\ln(1+\delta^*_t) - d\ln(1+coc_t)\right] + \frac{1}{1+r}\theta d\left(\frac{dv^n_{t+1}(\delta^*)}{d\delta^*}\right)\right] = 0, \tag{S20}$$

where

$$d\ln(1+coc_t) = \frac{r}{1+r}\mathbb{E}_t\sum_{s=t}^{\infty}\frac{1}{(1+r)^{s-t}}d\ln(1+i_s) \tag{S21}$$

denote the long-run cost of capital, which is a weighted average of future short-term interest rates with weights that depend on the steady-state interest rate.

Following the same steps as above, we can show that

$$d\left(\frac{dv^n_t(\delta^*_t)}{d\delta^*}\right) = \mathbb{E}_t\left[\frac{d\bar{\iota}}{d\delta^*}v\frac{1}{r}\left[d\ln(1+\delta^*_t) - d\ln(1+coc_t)\right] + \frac{1}{1+r}\theta d\left(\frac{dv^n_{t+1}(\delta^*_t)}{d\delta^*}\right)\right]. \tag{S22}$$

Using (S22), we can iterate (S20) forward to obtain the expression for the optimal choice of discount rate:

$$d\ln(1+\delta^*_t) = \frac{1+r-\theta}{1+r}\sum_{s=t}^{\infty}\left(\frac{\theta}{1+r}\right)^{s-t}d\ln(1+\widehat{coc}_s), \tag{S23}$$

which we can write recursively as

$$d \ln(1+\delta_t^*) = \frac{1+r-\theta}{1+r} d \ln(1+i_t^l) + \frac{\theta}{1+r} d \ln(1+\delta_{t+1}^*). \tag{S24}$$

The average discount rate in the economy, which we denote as δ_t, evolves according to

$$d \ln(1+\delta_t) = \theta d \ln(1+\delta_{t-1}) + (1-\theta) d \ln(1+\delta_t). \tag{S25}$$

We define the adjusted marginal value of capital (marignal Q) as

$$q_t^{adj} = \mathbb{E}_t \frac{1+\pi_{t+1}}{1+\delta_t} v_{t+1}^I(\delta) \tag{S26}$$

and its first-order approximation as

$$d \ln q_t^{adj}(\delta) = \mathbb{E}_t \left[d \ln(1+\pi_{t+1}) - d \ln(1+\delta_t) + d\omega_{t+1} + d \ln q_{t+1}^{adj}(\delta) \right]. \tag{S27}$$

Using hat notation, we can rewrite this equation as

$$\hat{q}_t^{adj} = \hat{q}_t - (\hat{\delta}_t - \widehat{coc}_t), \tag{S28}$$

where $\hat{q}_t$ is the frictionless Tobin's Q:

$$\hat{q}_t = \mathbb{E}_t \left[\hat{\pi}_{t+1} - \hat{i}_t + \frac{1}{1+r} \left((r+\xi)\hat{\omega}_{t+1} + \hat{q}_{t+1} \right) \right]. \tag{S29}$$

The firm's optimal labor demand solves

$$\Omega_t = \max_{l_t} p_t A_t k_t^{\alpha} (l_t)^{1-\alpha} - W_t l_t, \tag{S30}$$

which results in

$$l_t = \left((1-\alpha) \frac{p_t A_t}{W_t} \right)^{\frac{1}{\alpha}}. \tag{S31}$$

Substituting back into the objective function, we have

$$\Omega_t = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha A_t^{1/\alpha} \left(\frac{p_t}{W_t}\right)^{\frac{1}{\alpha}} W_t k_t. \tag{S32}$$

In turn, real profits per unit capital, $\omega_t \equiv \Omega_t/(P_t k_t)$, are

$$\omega_t = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \alpha A_t^{1/\alpha} \left(\frac{p_t}{W_t}\right)^{\frac{1}{\alpha}} \frac{W_t}{P_t}. \tag{S33}$$

Log-linearizing, we write that

$$\hat{\omega}_t = \frac{1}{\alpha}\hat{A}_t + \frac{1}{\alpha}\hat{p}_t - \frac{1-\alpha}{\alpha}\hat{W}_t - \hat{P}_t. \tag{S34}$$

Aggregate investment dynamics follow

$$\hat{\iota}_t = \frac{1}{\varphi''(\iota)\xi}\hat{q}_t. \tag{S35}$$

This completes the characterization of Proposition 1.

Supplement B.3 Proof of Propositions 2 and 3

We first describe the case with commitment. The optimal monetary policy problem is

$$\min_{\{\hat{\iota}_t, \hat{C}_t, \hat{L}_t, \hat{K}_{t+1}, \hat{I}_t, \hat{\pi}_t^w, \hat{\pi}_t, \hat{\delta}_t, \chi_t^p, \chi_t^h\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \mathbb{L}_t, \tag{S36}$$

where

$$\mathbb{L}_t = \omega_{KL}(\hat{L}_t - \hat{K}_t)^2 + \omega_{IK}(\hat{I}_t - \hat{K}_t)^2 + \omega_C \hat{C}_t^2 + \omega_L \hat{L}_t^2 + \omega_\pi^w (\hat{\pi}_t^w)^2 + \omega_\delta(\hat{\delta}_t - \hat{\delta}_{t-1})^2$$
$$+\omega_\chi \sum_{i\in\{p,h\}} \mu^i \frac{1}{\chi^i}(\chi_t^i - \chi^i)^2 - 2\sum_{s=0}^{t} \hat{\beta}_s (C\hat{C}_t - (1-\alpha)Y\hat{L}_t) - 2Y\hat{A}_t(\alpha\hat{K}_t + (1-\alpha)\hat{L}_t), \tag{S37}$$

subject to (with the relevant Lagrangian multiplier given on the left-hand side)

$$\lambda_{rc}: \quad C\hat{C}_t + I\hat{I}_t + Y\hat{G}_t = Y\hat{A}_t + \alpha Y\hat{K}_t + (1-\alpha)Y\hat{L}_t \tag{S38}$$

$$\lambda_K: \quad \hat{K}_{t+1} = (1-\xi)\hat{K}_t + \xi\hat{I}_t \tag{S39}$$

$$\lambda_I: \quad \hat{I}_t - \hat{K}_t = \frac{1}{\xi\phi}\left(\hat{q}_t - \frac{1+r}{r}(\hat{\delta}_t - i_t^l)\right) \tag{S40}$$

$$\lambda_\delta: \quad \hat{\delta}_t = \theta\hat{\delta}_{t-1} + (1-\theta)\hat{\delta}_t^* \tag{S41}$$

$$\lambda_*: \quad \hat{\delta}_t^* = \frac{1+r-\theta}{1+r}\hat{i}_t^l + \frac{\theta}{1+r}\hat{\delta}_{t+1}^* \tag{S42}$$

$$\lambda_q: \quad \hat{q}_t = -i_t + \hat{\pi}_{t+1} + \frac{r+\xi}{1+r}\left[\hat{A}_{t+1} - (1-\alpha)(\hat{K}_{t+1} - \hat{L}_{t+1})\right] + \frac{1}{1+r}\hat{q}_{t+1} \tag{S43}$$

$$\lambda_l: \quad \hat{i}_t^l = \frac{r}{1+r}\hat{i}_t + \frac{1}{1+r}\hat{i}_{t+1}^l \tag{S44}$$

$$\lambda_C: \quad \hat{C}_t = \mathbb{E}_t\left[\hat{C}_{t+1} - \xi_r(1/\sigma)\left[\hat{\beta}_{t+1} + \hat{r}_{t+1}\right] + \xi_I\left[\hat{I}_t - \hat{I}_{t+1}\right]\right] \tag{S45}$$

$$\lambda_w: \quad \hat{\pi}_t^w = \psi_w\left[\sigma(\sum_i \vartheta^i \frac{1}{\chi^i} d\chi_t^i) + \sigma\hat{C}_t + \nu\hat{L}_t - \alpha(\hat{K}_t - \hat{L}_t)\right] + \beta\hat{\pi}_{t+1}^w \tag{S46}$$

$$\lambda_p: \quad \hat{\pi}_t = \hat{\pi}_t^w - \hat{A}_t - \alpha(\hat{K}_t - \hat{L}_t) + \hat{A}_{t-1} + \alpha(\hat{K}_{t-1} - \hat{L}_{t-1}) \tag{S47}$$

$$\lambda_{\chi^h}: \quad \chi_t^h - \chi^h = \underbrace{(1-\alpha-\chi^h)}_{\equiv\iota_C}\hat{C}_t + (1-\alpha)\frac{1}{C}G_t - \frac{1}{C}T_t + \underbrace{(1-\alpha)\frac{I}{C}}_{\equiv\iota_I}\hat{I}_t \tag{S48}$$

$$\lambda_{\chi^p}: \quad 0 = \mu(\chi_t^h - \chi^h) + (1-\mu)(\chi_t^p - \chi^p). \tag{S49}$$

The first-order condition with respect to $\hat{\delta}_t$ is

$$\omega_\delta(\hat{\delta}_t - \hat{\delta}_{t-1}) - \beta\omega_\delta(\hat{\delta}_{t+1} - \hat{\delta}_t) + \lambda_{I,t}\frac{1+r}{r}\frac{1}{\xi\phi} + \lambda_{\delta,t} - \beta\theta\lambda_{\delta,t+1} = 0. \tag{S50}$$

Solving for $\lambda_{\delta,t}$ by iterating forward,

$$\lambda_{\delta,t} = \sum_{s=t}^{\infty}(\beta\theta)^{s-t}\left\{\omega_\delta(\hat{\delta}_s - \hat{\delta}_{s-1}) - \beta\omega_\delta(\hat{\delta}_{s+1} - \hat{\delta}_s) + \lambda_{I,s}\frac{1+r}{r}\frac{1}{\xi\phi}\right\}. \tag{S51}$$

The first-order condition for $\hat{\delta}_t^*$ is

$$-(1-\theta)\lambda_{\delta,t} + \lambda_{*,t} - \theta\lambda_{*,t-1} = 0. \tag{S52}$$

Iterating backwards,

$$\lambda_{*,t} = \sum_{s=0}^{t} \theta^{t-s}(1-\theta)\lambda_{\delta,s}. \tag{S53}$$

The first-order condition for $\widehat{coc}_t$ is

$$\lambda_{l,t} - \lambda_{l,t-1} - \frac{1+r-\theta}{1+r}\lambda_{*,t} - \frac{1}{\xi\phi}\frac{1+r}{r}\lambda_{I,t} = 0. \tag{S54}$$

Combining (S51), (S53), and (S54),

$$\begin{aligned}\lambda_{l,t} &= \lambda_{l,t-1} - \frac{1}{\xi\phi}\frac{1+r}{r}\lambda_{I,t} \\ &\quad + \sum_{s=0}^{t}\theta^{t-s}(1-\theta)\sum_{u=s}^{\infty}(\beta\theta)^{u-s}\left\{\omega_\delta(\hat{\delta}_u - \hat{\delta}_{u-1}) - \beta\omega_\delta(\hat{\delta}_{u+1} - \hat{\delta}_u) + \lambda_{I,u}\frac{1+r}{r}\frac{1}{\xi\phi}\right\}.\end{aligned} \tag{S55}$$

Note that when $\theta = 0$ (flexible discount rates), the above expression collapses to

$$\lambda_{l,t} = \lambda_{l,t-1}, \tag{S56}$$

which would imply $\lambda_{l,t} = 0$ for all t given $\lambda_{l,-1} = 0$. Away from such a case, (S55) shows that the process for $\lambda_{l,t}$ is nonstationary. This means that, in general,

$$\lambda_{l,\infty} \neq 0. \tag{S57}$$

The first-order conditions with respect to π_t and i_t are

$$-\beta^{-1}\lambda_{q,t-1} - \beta^{-1}\lambda_{C,t-1}(1/\sigma)\zeta_r + \lambda_{p,t} = 0 \tag{S58}$$

$$\lambda_{q,t} - \lambda_{l,t}\frac{r}{1+r} + \lambda_{C,t}(1/\sigma)\zeta_r = 0. \tag{S59}$$

Combining the two,

$$\lambda_{p,t} = r\lambda_{l,t}. \tag{S60}$$

The first-order condition for π_t^w is

$$\omega_\pi^w \hat{\pi}_t^w + \lambda_{w,t} - \lambda_{w,t-1} - \lambda_{p,t} = 0, \tag{S61}$$

which in turn equals

$$\omega_\pi^w \hat{\pi}_t^w + \lambda_{w,t} - \lambda_{w,t-1} - r\lambda_{l,t} = 0, \tag{S62}$$

using (S60).

In the limit as $t \to \infty$,

$$\lim_{t\to\infty} \pi_t^w = \frac{r}{\omega_\pi^w}\lambda_{l,\infty} \tag{S63}$$

Equations (S63) and (S57), as well as (S47), show $\lim_{t\to\infty} \pi_t \neq 0$ under commitment.

We now turn to the case with discretion. At any time t, the central bank minimizes $\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{1}{2} \mathbb{L}_s$, taking private sector expectations as given. The set of first-order conditions for $t \to \infty$ are

$$C_t: \quad \omega_C \hat{C}_t - \lambda_{rc,t} C + \lambda_{C,t} - \lambda_{w,t}\psi_w \sigma - \lambda_{\chi^h,t}\iota_C = 0 \tag{S64}$$

$$L_t: \quad \omega_L \hat{L}_t \tag{S65}$$

$$- \lambda_{rc,t}(1-\alpha)Y - \lambda_{w,t}\psi_w(\nu+\alpha) \tag{S66}$$

$$I_t: \quad -\lambda_{K,t}\xi + \lambda_{rc,t} I + \lambda_{I,t} - \lambda_{C,t}\zeta_I - \lambda_{\chi^h,t}\iota_I = 0 \tag{S67}$$

$$K_{t+1}: \quad -\beta\lambda_{rc,t+1}\alpha Y \tag{S68}$$

$$+ \lambda_{K,t} - \beta\lambda_{K,t+1}(1-\xi) - \beta\lambda_{I,t+1} + \lambda_{q,t}\frac{r+\xi}{1+r}(1-\alpha) \tag{S69}$$

$$+ \beta\lambda_{w,t+1}\psi_w\alpha = 0 \tag{S70}$$

$$\pi: \quad \lambda_{p,t} = 0 \tag{S71}$$

$$\pi^w: \quad \omega_\pi^w \hat{\pi}_t^w + \lambda_{w,t} = 0 \tag{S72}$$

$$q: \quad \lambda_{q,t} - \frac{1}{\xi\phi}\lambda_{I,t} = 0 \tag{S73}$$

$$i_t: \quad \lambda_{q,t} - \lambda_{l,t}\frac{r}{1+r} + \lambda_{C,t}(1/\sigma)\zeta_r = 0 \tag{S74}$$

$$\delta: \quad \lambda_{I,t}\frac{1+r}{r}\frac{1}{\xi\phi} + \lambda_{\delta,t} - \beta\theta\lambda_{\delta,t+1} = 0 \tag{S75}$$

$$\delta^*: \quad -(1-\theta)\lambda_{\delta,t} + \lambda_{*,t} = 0 \tag{S76}$$

$$i^l: \quad \lambda_{l,t} - \frac{1+r-\theta}{1+r}\lambda_{*,t} - \frac{1}{\xi\phi}\frac{1+r}{r}\lambda_{I,t} = 0 \tag{S77}$$

$$\chi^h: \quad \omega_C\mu\frac{1}{\chi^h}(\chi_t^h - \chi^h) + \lambda_{\chi^h} - \mu\lambda_{\chi^p} = 0 \tag{S78}$$

$$\chi^p: \quad \omega_C(1-\mu)\frac{1}{\chi^p}(\chi_t^p - \chi^p) - (1-\mu)\lambda_{\chi^p} = 0, \tag{S79}$$

where we use the fact that (S38)-(S49) imply that as $t \to \infty$,

$$\hat{I}_t - \hat{K}_t = 0 \tag{S80}$$

$$\hat{L}_t - \hat{K}_t = 0. \tag{S81}$$

These optimality conditions do not involve any state variables. Therefore, the efficient steady-state allocations with all Lagrangian multipliers being zero constitute the solutions at $t \to \infty$. This implies that

$$\lim_{t \to \infty} \pi_t^w = -\lambda_{w,t} / \omega_\pi^w = 0. \tag{S82}$$

Proposition 3 follows from the fact that the proof above holds setting $\omega_\delta = \omega_\chi = 0$.

Supplement C Heterogeneous-Agent Model with Sticky Discount Rates

We consider a richer heterogeneous-agent New Keynesian model with investment along the lines of Kaplan et al. (2018) and Auclert et al. (2020). Instead of assuming that a fraction μ of households is hand-to-mouth, we allow these households to access financial markets subject to uninsurable idiosyncratic income risk. We call these households workers. We index workers with superscript h and permanent income households with superscript p.

Workers experience idiosyncratic productivity shocks e, which follow a Markov process. Workers save in risk-free assets that give a deterministic return of $1 + r_t$. Workers face a borrowing constraint of the form $b_t \geq \underline{b}$ for liquid asset holdings. As in the baseline, unions make labor supply decisions that are the same for all households, L_t. We assume that lump-sum transfers are imposed proportionally to household idiosyncratic productivity. The worker's problem in recursive form is

$$U_t(b, e) = \max_{c, b' \geq \underline{b}} u(c_t) + \beta^h \mathbb{E}_t [U_{t+1}(b', e')]$$
$$\text{s.t.} \quad c + b' = (1 + r_t) b + e \left[(W_t / P_t) L_t (1 - \tau^l) - T_t \right].$$

Let $c_t^h(b, e, a)$ denote the policy function. Aggregate consumption of workers is $C_t^h \equiv \int c_t^h(b, e) dH_t(b, e)$, where H_t denotes the joint distribution of assets and idiosyncratic worker productivity in period t.

Permanent-income households solve the identical problem as in the main text and face no idiosyncratic risk. Aggregate consumption is

$$C_t = \mu C_t^h + (1-\mu)C_t^p. \tag{S83}$$

Our baseline two-agent structure is nested as a special case where $\beta^h \to 0$.

There is a risk-neutral financial intermediary that issues liquid deposits to households and invests them in a diversified portfolio of firm shares and government bonds. The no-arbitrage conditions imply

$$\mathbb{E}_t\left[\frac{V_{t+1}+D_{t+1}}{V_t}\right] = \mathbb{E}_t[1+r_{t+1}^p] = \mathbb{E}_t[1+r_{t+1}], \tag{S84}$$

where V_t is the value of firm shares and D_t is the dividend paid by firms.

Following Auclert et al. (2023), we assume wages are sticky and prices are flexible. The wage Phillips curve is becomes

$$\hat{\pi}_t^w = \psi_w\left[\sigma\hat{C}_t + \sigma\int_0^1 \vartheta^i(\chi_t^i - \chi^i)/\chi^i di + \nu\hat{L}_t - \hat{W}_t + \hat{P}_t\right] + \frac{1}{1+r}\mathbb{E}_t\hat{\pi}_{t+1}^w, \tag{S85}$$

where $\vartheta^i = \frac{\tilde{\vartheta}^i u'(C^i)}{\int_0^1 \tilde{\vartheta}^i u'(C^i)di}$ is the weight placed on the utility of household $i \in [0,1]$, and $\psi_w \equiv (1-\gamma_w)(1-\beta\gamma_w)/\gamma_w$. The rest of the model remains unchanged.

We calibrate the economy as follows. We set the share of workers to 80%, $\mu = 0.8$, reflecting that the majority of household assets are concentrated at the top of the wealth distribution. The discount factor of permanent-income households matches the steady-state annualized cost of capital of 7%. The income process is an AR(1) process with autocorrelation 0.966 and a standard deviation of 0.13, following McKay et al. (2016). We discretize the income process using the methodology in Rouwenhorst (1995) with 7 grid points. The borrowing limit of workers is zero, $\underline{b} = 0$. We choose the discount factor of workers, β^h, to target an average annual marginal propensity to consume (MPC) out of a one-time transfer of 0.5, in line with the estimates of Fagereng et al. (2021). Finally, we set $\vartheta^i = 1$ so that the New Keynesian wage Phillips curve only depends on aggregate values. The wage stickiness parameter is $\gamma_w = 0.85$ and prices are flexible, $\gamma_p = 0$. The remaining calibration is the same as in the baseline model.

Figure S1, S2, and S3 present impulse response functions to the government spending, patience, and inflation target shocks. The qualitative conclusions are

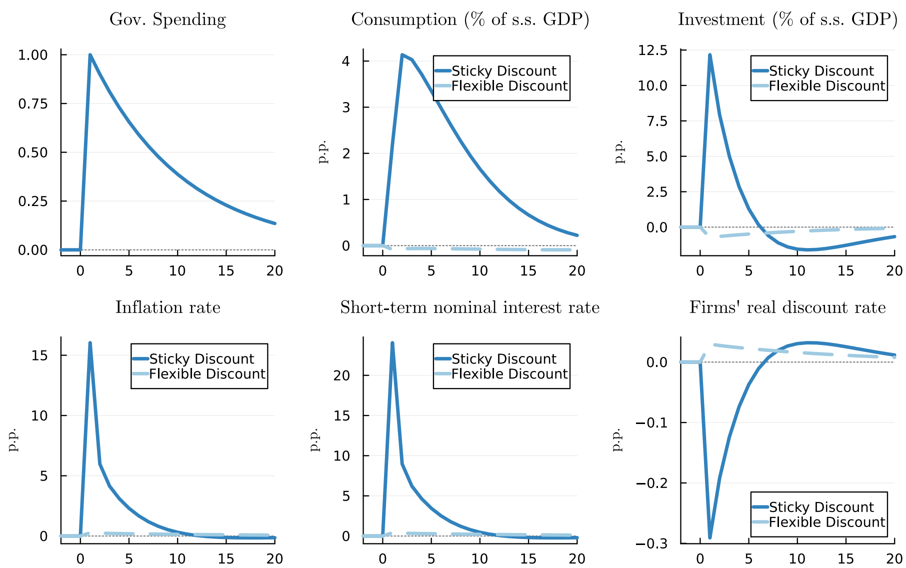

Figure S1: Heterogeneous Agents: Impulse Responses to a Government Spending Shock

The figure plots the impulse response to a government spending shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

similar to the baseline model. Quantitatively, we find a larger amplification effect relative to our baseline economy even though the impact MPC is similar in both models, consistent with Auclert et al. (2023).[S1]

[S1] In our baseline model, the MPC of hand-to-mouth households is 1 and the MPC of permanent income households is $1 - \beta = 0.017$. Since 30% of households are hand-to-mouth, the average MPC is $0.3 \times 1 + 0.7 \times 0.017 \approx 0.31$. In our heterogenous-agent model, the average quarterly impact MPC is 0.25.

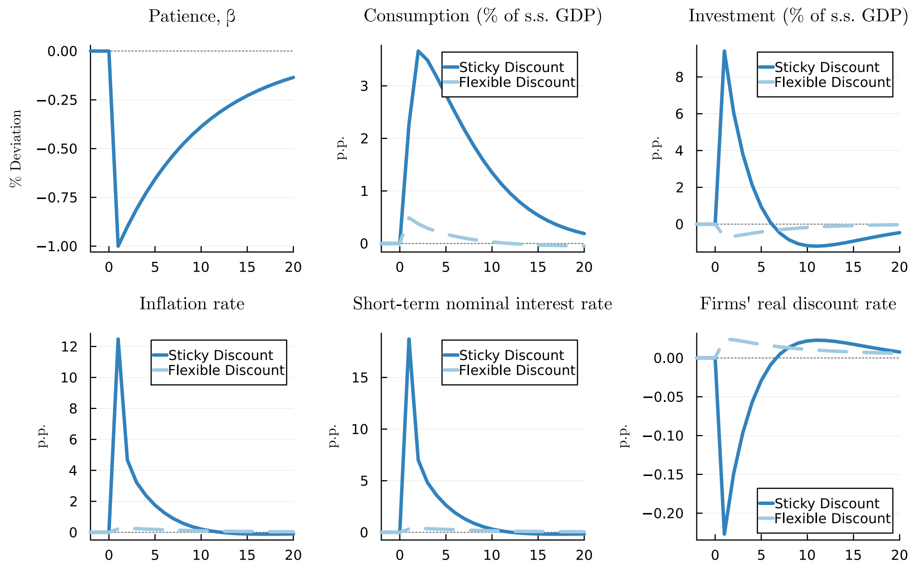

Figure S2: Heterogeneous Agents: Impulse Responses to a Household Patience Shock

The figure plots the impulse response to a household patience shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

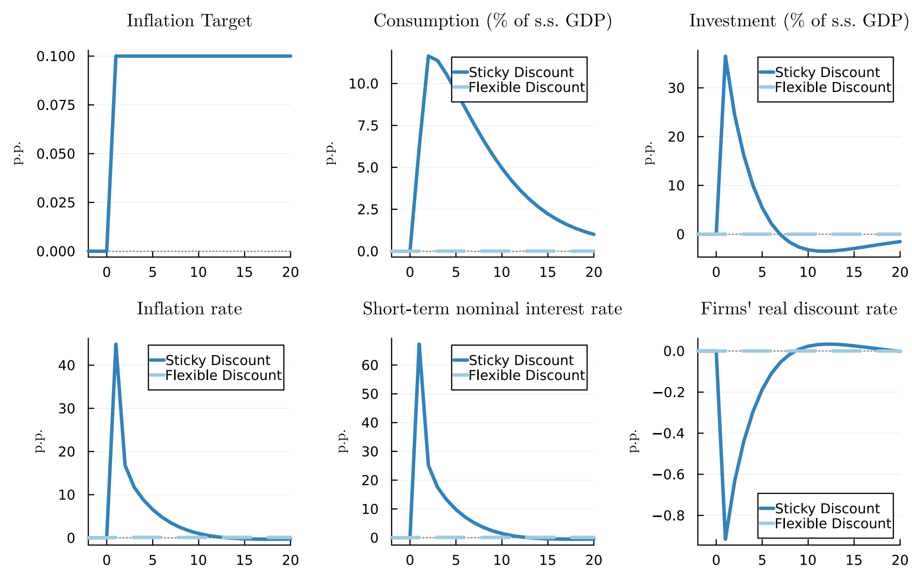

Figure S3: Heterogeneous Agents: Impulse Responses to an Inflation Target Shock

The figure plots the impulse response to an inflation target shock for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

Supplement D Real Business Cycle Model with Sticky Discount Rates

We present a simple representative-agent real business cycle (RBC) model augmented with sticky discount rates by setting $\mu = 0$ and $\gamma_p = \gamma_w = 0$.

Figures S4 and S5 present impulse responses to a government spending shock and a household patience shock under $\mu = 0$ and $\gamma_w = \gamma_p = 0$. In both cases, sticky discount rates reverse the sign of the investment response. Unlike the two-agent New Keynesian model, the RBC model does not generate comovement between consumption and investment following the patience shock.

Figure S6 shows impulse responses to an inflation target shock. With flexible discount rates, the inflation target is entirely neutral for real outcomes. With sticky discount rates, the inflation target shock stimulates investment because of the direct link between expected inflation and real discount rates. This finding highlights that sticky discount rates are an independent source of monetary non-neutrality.

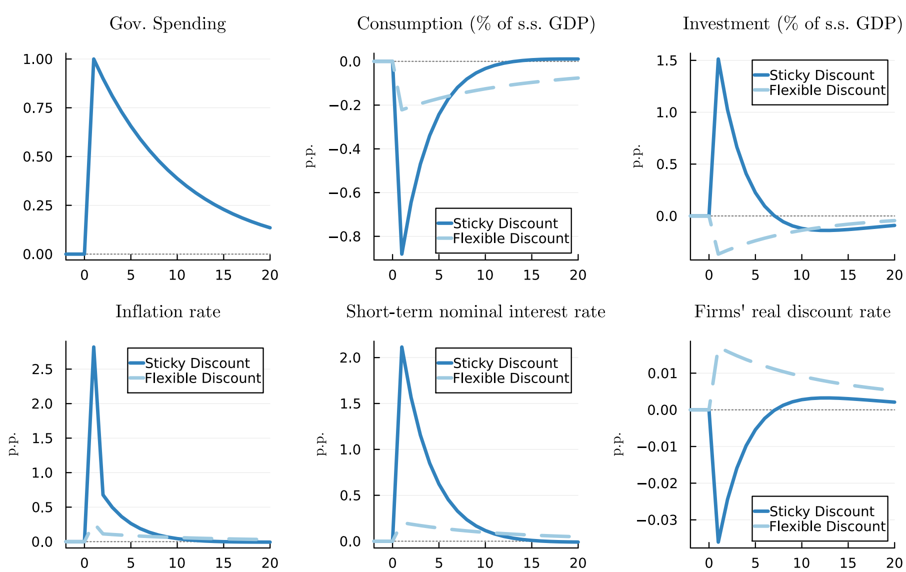

Figure S4: RBC: Impulse Responses to a Government Spending Shock

The figure plots the impulse response to a government spending shock under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

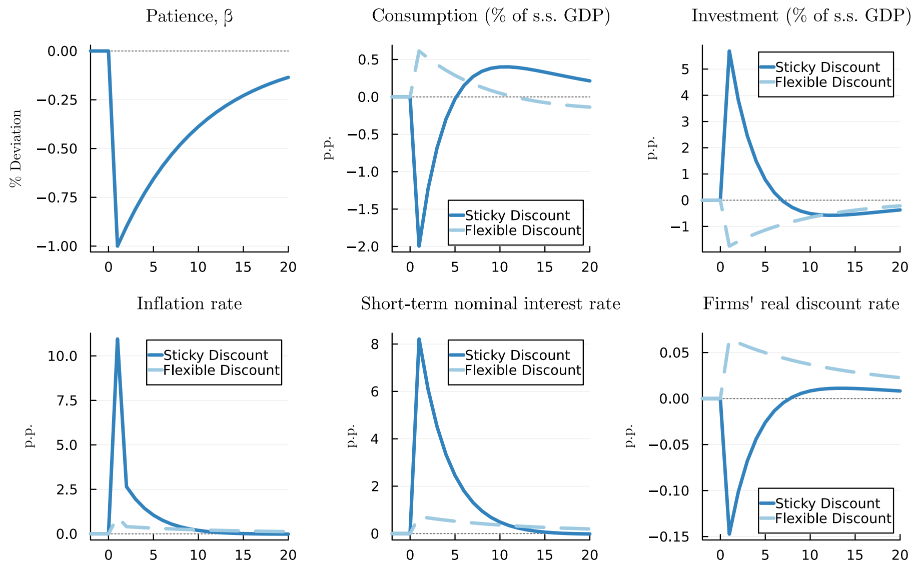

Figure S5: RBC: Impulse Responses to a Household Patience Shock

The figure plots the impulse response to a patience shock under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

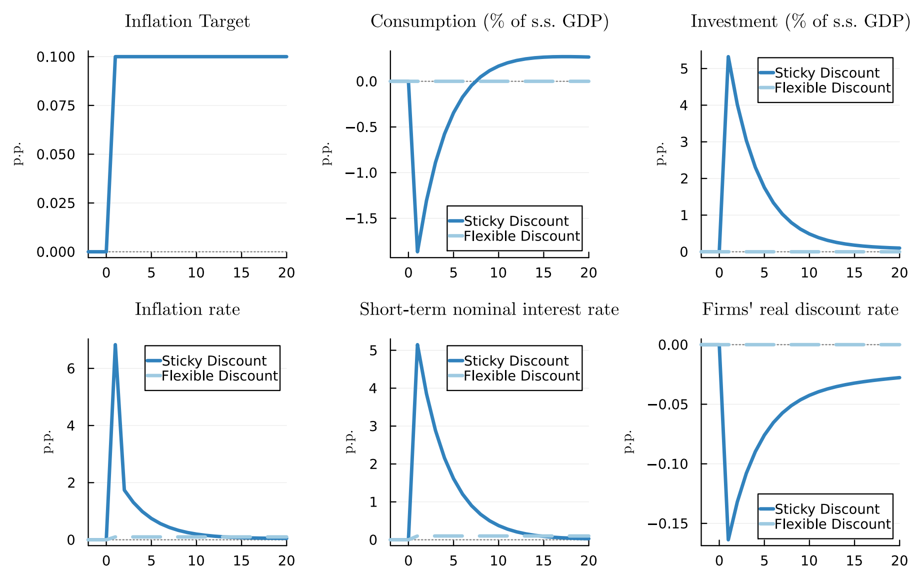

Figure S6: RBC: Impulse Responses to an Inflation Target Shock

The figure plots the impulse response to an increase in long-run inflation target by 0.1 p.p. under flexible prices ($\gamma_p = 0$) and no hand-to-mouth households ($\mu = 0$)for two different values of discount rate stickiness, $\theta \in \{0, 0.95\}$.

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