On representations of Gaussian integers by primes and powers of $1+i$

Nikita Kalinin

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Experimental observation. Let $z = a + bi \in \mathbb{Z}[i]$ with $a + b$ odd. Then z appears to admit a representation of the form

$$z \;=\; \pi \;+\; \varepsilon\,(1+i)^k,$$

where π is a Gaussian prime, $\varepsilon \in \{\pm 1, \pm i\}$ is a unit, and $k \geq 0$ is an integer.

No counterexamples were found within the tested range.

Possible strategy. A conceivable approach toward a proof would be to construct (as in [1]), for each unit $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 = 1, -1, i, -i$, a covering system of moduli

$$\{m_{ij}\}_{j=1}^{4}$$

together with pairwise disjoint sets of distinct Gaussian primes

$$\{p_{ij}\}_{j=1}^{4},$$

such that

$$(1+i)^{m_{ij}} \equiv 1 \pmod{p_{ij}}$$

for all relevant indices. One would then attempt to combine these congruences to force $\pi = z - \varepsilon_j(1+i)^k$ to be divisible by a Gaussian prime from the corresponding set $\{p_{ij}\}$, for each $j = 1, 2, 3, 4$, depending on the residue class of k.

No such covering systems satisfying the required disjointness and congruence conditions were yet found.

References

[1] P. Erdos. On integers of the form $2^k + p$ and some related problems. *Summa Brasil. Math*, 2:113–123, 1950.